Systemic Portfolio Diversification

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Is diversification always desirable?

- The intuition behind why diversification is desirable is based on "convexity"
  - With convex technologies and concave utility functions, risk sharing is always beneficial
  - If technologies are not convex, then risk sharing can lower expected utility
  - Plenty of non-convexities in the real world
    - Fire-sale costs (**this paper**)

Interconnectedness and risk

- In an interconnected system, shocks to one unit of system may (are likely to) have effects on others
  - But in some cases, impacts can be spread throughout the system
  - Net effect is limited (approaches zero with sufficient diversification)
- Advocates of global financial integration talk about the advantages of risk sharing
- But in the context of crises, they worried about contagion:
  - credit contagion through counterparty obligations
  - price mediated contagion through balance sheet commonalities

Transmission of shocks

- Even without *direct* financial market interlinkages, there can be extensive interdependencies through which a shock in one part of the system can be transmitted to others.
  - Liquidity crises are associated with forced sales of assets, leading to price declines
    - Bernanke estimated that Bear Stearns' rescue prevented a potential fire sale of nearly $210 billion of Bear Stearns' assets
- Financial linkages, while they may enhance risk sharing, may increase these adverse effects.

Research Question

- How do institutions *ex ante* structure their balance sheets when they account for the systemic impact of other large institutions?
- Financial institutions may be forced to liquidate assets on a short notice to raise immediacy (margin calls, mutual funds' redemptions, regulatory leverage requirements...)
- Sell-offs affect several institutions simultaneously and exacerbate liquidation costs.
- **Should we be concerned about a different (systemic) kind of diversification?**

The Model

- One period timeline
- Economy with N banks and K assets
- Initial asset prices normalized to 1\$
- Bank i's balance sheet:

  d_i debt,
  e_i equity,
  $w_i := d_i + e_i$ asset value,
  $\lambda_i := d_i / e_i$ leverage ratio,
  $\pi_{i,k}$ weight of asset k in bank i's portfolio

The Model

- Let $Z = (Z_1, \ldots, Z_K)$ be the vector of asset return shocks, where Z_i's are i.i.d. random variables
- Bank i's return is $R_i := \pi_i \cdot Z = \sum_k \pi_{i,k} Z_k$
- **Control variables**: each bank i chooses its asset allocation weights π_i.
- **Objective function**: banks maximize expected portfolio returns:

$$\mathrm{PR}_i(\pi_i, \pi_{-i}) := E[\pi_i^T Z - \mathrm{cost}_i(\pi_i, \pi_{-i}, Z)].$$

Model Parameters

- w: size of the banks
- λ: leverage of the banks
- γ: illiquidity of the assets

The Model

1. **Financial Constraints**: Bank i liquidates assets if its leverage threshold $\lambda_{M,i}$ is breached.
   - Bank i liquidates the minimum amount necessary to restore its leverage at the threshold.

2. **Assumption 1.** Exposures remain fixed: Banks liquidate (or purchase) assets proportionally to their initial allocations.

3. **Assumption 2.** The cost of fire sales, i.e., the execution price, is linear in quantities.
   - A trade of q_k units of asset k is executed at the price $1 + \gamma_k q_k$ per asset share.

4. **Assumption 3.** Ignore the possibility of default.
   - If $R_i \leq -\frac{1}{\lambda_i}$, the bank's equity is negative.

Equilibrium Asset Holdings

Nash equilibrium

A (pure strategy) Nash equilibrium is a strategy $\{\pi_i^*\}_{1\leq i\leq N} \subset X$, where $X := \{x \in [0,1]^K \ : \ \sum_{k=1}^K x_k = 1\}$, such that for every $1 \leq i \leq N$ we have

$$\mathrm{PR}_i(\pi_i^*, \pi_{-i}^*) \geq \mathrm{PR}_i(\pi_i, \pi_{-i}^*) \qquad \text{for all } \pi_i \in X.$$

Because assets' returns are identically distributed, the optimization problem of bank i is equivalent to minimizing $cost_i(\pi_i^*, \pi_{-i}^*)$.

Potential Game

- To start with, assume $N = 2$ and $K = 2$
- Best response strategy of bank 1 is

$$\pi^*_{1,1} = \text{argmin}_{\pi_{1,1}} \Big\{ \lambda^2_{M,1} E\big[w_1^2 (\pi_1 \cdot Z + \ell_1)^2 (\pi^2_{1,1}\gamma_1 + (1-\pi_{1,1})^2\gamma_2)\mathbf{1}_{A_1}\big] +$$

$$\lambda_{M,1}\lambda_{M,2} E\big[w_1 w_2 (\pi_1 \cdot Z + \ell_1)(\pi_2 \cdot Z + \ell_2)(\pi_{1,1}\pi_{1,2}\gamma_1 + (1-\pi_{1,1})(1-\pi_{1,2})\gamma_2)\mathbf{1}_{A_1 \cap A_2}\big] \Big\}$$

$$= \text{argmin}_{\pi_{1,1}} \Big\{ \cdots + \lambda^2_{M,2} E\big[w_2^2 (\pi_2 \cdot Z + \ell_2)^2 (\pi^2_{2,1}\gamma_1 + (1-\pi_{2,1})^2\gamma_2)\mathbf{1}_{A_2}\big] \Big\}.$$

Both banks minimize the same function!

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Both banks minimize the same function!

Single Bank Benchmark

- Assume a single bank system.
- Bank seeks diversification to reduce likelihood of liquidation.
- Bank seeks a larger position in the more liquid asset to reduce realized liquidation costs.

Proposition

Let $N = 1$, $K = 2$, and $\gamma_1 < \gamma_2$. Then

- $\pi^S_{1,1} \in (\frac{1}{2}, \frac{\gamma_2}{\gamma_1+\gamma_2})$, *where* $(\pi^S_{1,1}, 1 - \pi^S_{1,1})$ *minimizes the bank's expected liquidation costs.*
- $\pi^S_{1,1}(\lambda)$ *is increasing in* λ.

Homogeneous Economy

- If there is no heterogeneity in the system (across assets or across banks), then in equilibrium all banks hold the same portfolio.
- In the presence of equally leveraged banks, assets become more "expensive", but the banks' relative preferences do not change.
- The system behaves as a single representative bank.

Proposition

- *If $\gamma_1 = \gamma_2$, then $\pi_{i,1} = 50\%$ for all i.*
- *Let $\bar{\pi}$ be the optimal allocation in asset 1 of a bank with leverage $\bar{\lambda}$, when $N = 1$. If $\lambda_i = \bar{\lambda}$ for all i, then $\pi_{i,1} = \bar{\pi}$ for all i.*

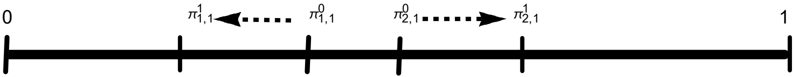
Heterogeneous Economy

Proposition

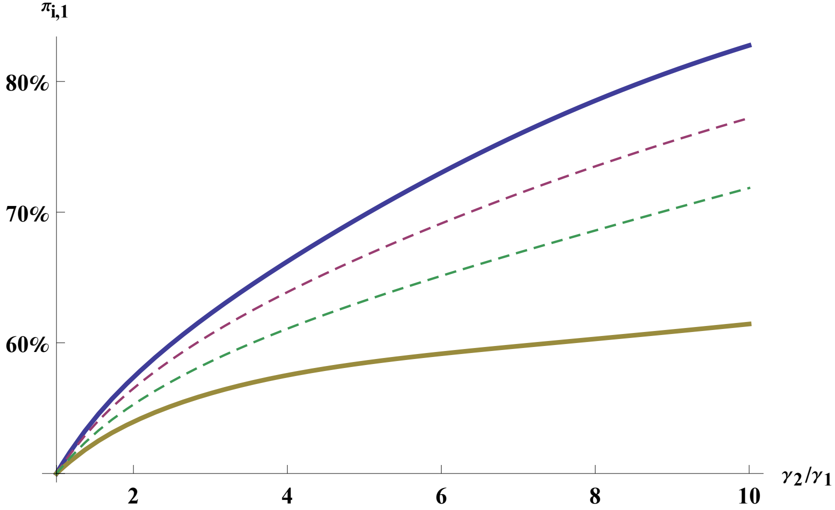
Assume $N = 2$, $\gamma_1 < \gamma_2$ and $\lambda_1 < \lambda_2$.

- $|\pi^*_{1,1} - \pi^*_{2,1}| > |\pi^S_{1,1} - \pi^S_{2,1}|$, *where $\pi^S_{i,1}$ is the bank i's optimal asset 1 allocation in the single agent case.*

- Let f_i be the best response function of bank i, $i = 1, 2$.
- Let $\pi^0_{1,1}$ be the optimal allocation of bank 1, if bank 2 has the same leverage ratio as bank 1.
- Recursively, $\pi^n_{1,1} := f_1(\pi^{n-1}_{2,1})$, $\pi^n_{2,1} := f_2(\pi^{n-1}_{1,1})$
    - banks are more and more **diverse**, until an equilibrium is reached.



Comparative Statics



Increasing heterogeneity across assets.

Endogenous Probability of Liquidation

- $A_{\text{liq}}(\pi_1, \pi_2)$, the event that at least one bank liquidates assets, given portfolio holdings π_1 and π_2
- $A_{\text{sim}}(\pi_1, \pi_2)$ the event that both banks liquidate assets.

Proposition

Let $N = 2$, $K = 2$, $\gamma_1 < \gamma_2$ and $\lambda_1 > \lambda_2$.

- $P(A_{liq}(\pi_1^*, \pi_2^*)) > P(A_{liq}(\pi_1^S, \pi_2^S))$,
- $P(A_{sim}(\pi_1^*, \pi_2^*)) < P(A_{sim}(\pi_1^S, \pi_2^S))$.

Systemic Diversification: In equilibrium, the system diversifies the likelihood of asset liquidation across banks, so to reduce the probability of a widespread fire-sale event.

Social Planner

- Are banks behaving as a benevolent social planner would like?
- If not, what are the social costs?
- Social planner minimizes objective function

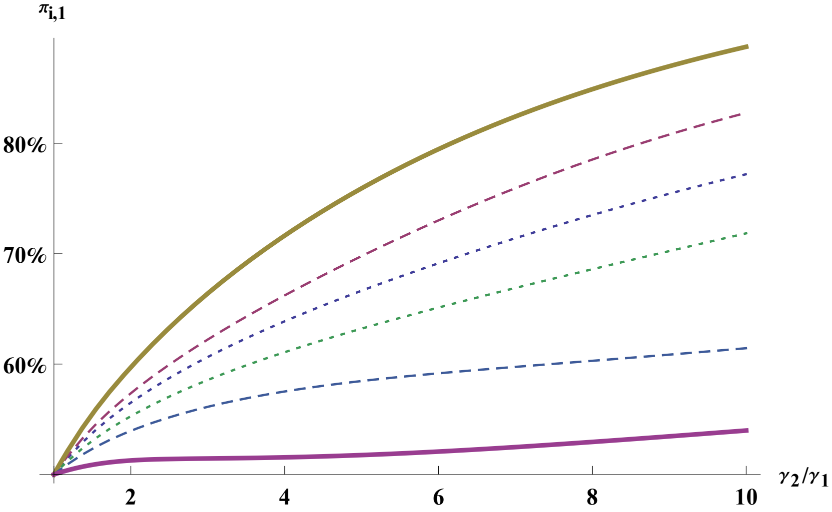
$$TC(\pi_1, \cdots, \pi_N) := \sum_{i=1}^{N} cost_i(\pi_i, \pi_{-1})$$

Proposition

- *If $\ell_i = \bar{\ell}$ for all i, the minimizer π^{SP} of TC is the unique Nash equilibrium.*
- *Assume $N = 2$. If $\lambda_1 \neq \lambda_2$, then π^{SP} is not a Nash equilibrium. In particular, $|\pi^{SP}_{1,1} - \pi^{SP}_{2,1}| > |\pi^*_{1,1} - \pi^*_{2,1}|$.*

- In equilibrium, banks are not **diverse** enough!
- Each bank accounts for the price-impact of other banks on its execution costs, but neglects the externalities it imposes on the other banks.

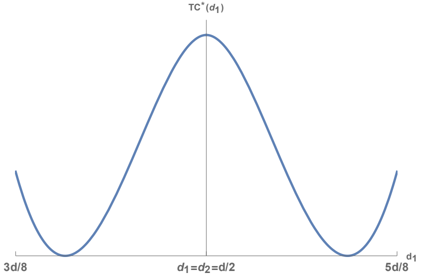
Social Planner



Is Higher Heterogeneity Socially Desirable?

Proposition

Assume the system has two banks and two assets with aggregate asset value w and debt d.
Assume $w_1 = w_2 = \frac{w}{2}$ and $d_2 = d - d_1$. Define $TC^*(d_1)$ as the total expected liquidation costs in equilibrium as function of d_1. Then $d/2$ is a local maximum for $TC^*(d_1)$.



Total expected liquidation costs for different levels of leverage heterogeneity

Tax Systemic Risk

Proposition

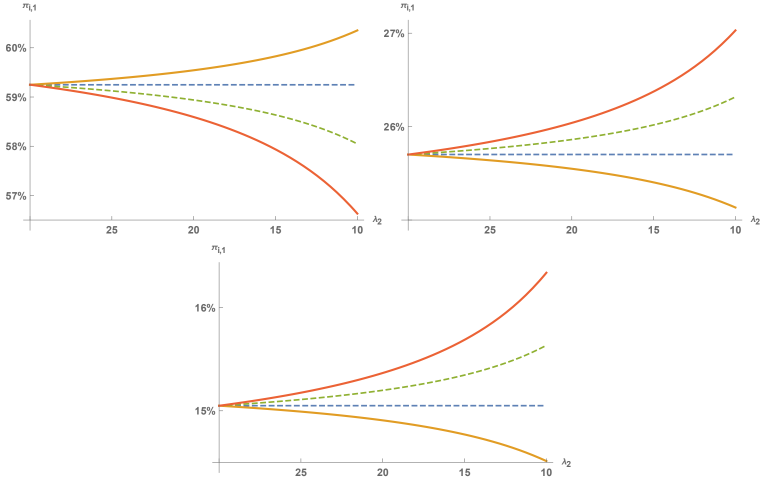
If each bank i pays a tax equal to

$$T_i(\pi) := \sum_{j \neq i} M_{i,j}(\pi),$$

where
$M_{i,j}(\pi_i, \pi_j) := \lambda_{M,i}\lambda_{M,j} w_i w_j E\left[(R_i + \ell_i)^- (R_j + \ell_j)^- \pi_i^T Diag[\gamma]\pi_j\right]$*, then the equilibrium allocation is first best.*

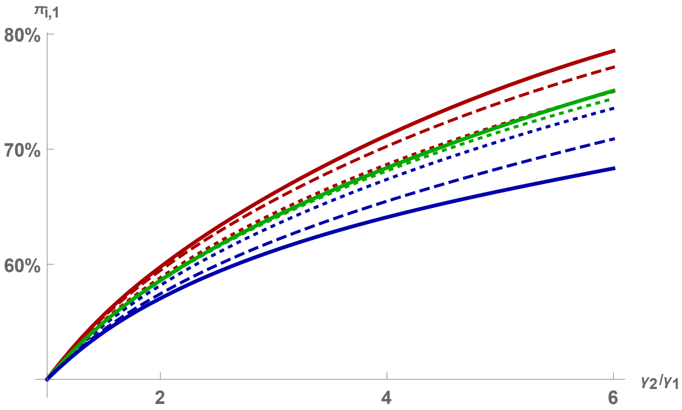
- $M_{i,j}(\pi_i, \pi_j)$ are the externalities that bank i imposes on bank j.
- By internalizing the externalities imposed on the systems, the objectives of the banks become aligned with the social planner's objective.

Multiple Assets



Banks reduce portfolio overlap in *each* asset.

Multiple Banks



Most (resp. least) leveraged bank increases its position in the most (resp. least) liquid asset even further.

Conclusions

- Develop a framework to analyze how fire-sale risk affects banks' ex-ante asset holding decisions.
- Systemic liquidation risk incentivizes banks to reduce portfolio overlap at expenses of diversification benefits
- To achieve the socially optimal allocation, banks should reduce portfolio commonality even further
- Tax on portfolio overlapping may be combined with the initiation of an asset purchase program:
    - The tax would incentivize banks to reduce common exposures, and fund such a relief program to mitigate fire-sale losses during crisis

Equilibrium Asset Holdings

Each bank maximizes an objective function given by its expected portfolio return, i.e.,

$$\mathrm{PR}_i(\pi_i, \pi_{-i}) := E[\pi_i^T Z - \mathrm{cost}_i(\pi_i, \pi_{-i}, Z)].$$

Total liquidation costs of bank i:

$$cost_i(\pi_i, \pi_{-i}) := E\left[\underbrace{\lambda_{M,i} w_i \left(\pi_i \cdot Z + \ell_i\right)^- \pi_i^T}_{\text{assets liquidated by bank } i} \mathrm{Diag}[\gamma] \underbrace{\sum_{j=1}^{N} \pi_j \lambda_{M,j} w_j \left(\pi_j \cdot Z + \ell_j\right)^-}_{\text{total quantities traded}} \right]$$

Model Limitations

- We ignore the possibility of default.
  - If $R_i \leq -\frac{1}{\lambda_i}$, the bank's equity is negative.
- We assume only one round of deleveraging.
  - Due to price impact, banks may engage in several rounds of deleveraging (Capponi and Larsson (2015)).