Jumps and the Correlation Risk Premium: Evidence from Equity Options

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Theoretical Motivation

Summary

This paper breaks the correlation risk premium down into two components: a premium related to the **correlation of continuous stock price movements** and a premium for bearing the risk of **co-jumps**. We propose a novel way to identify both premiums based on dispersion trading strategies that go long an index option portfolio and short a basket of option portfolios on the constituents. The **option portfolios** are constructed to only load on either diffusive volatility or jump risk. We document that both risk premiums are economically and statistically significant for the **S&P 100 index**. In particular, selling insurance against co-jumps generates a sizable annualized Sharpe ratio of 0.85.

Index Variance Risk Premium

Investors dislike times of stock market turbulence

- investors fear stock market volatility
  → pay variance risk premium (VRP) to hedge against states of high market volatility
- stock correlations go up when markets are volatile
  → pay correlation risk premium (CRP) to eliminate the risk of high correlations

There is a close theoretical link between the two risk premiums

- index variance is the sum of individual stocks' variances and covariances

$$VRP_{I,t} = \sum_{i=1}^{N} \omega_{i,t}^2 VRP_{i,t} + CovRP_t$$

- literature shows that **index VRP is largely driven by CRP** (e.g. Driessen et al., 2009, Carr and Wu, 2009, Buss et al., 2018)

Research question: What are the drivers of the CRP?

Stock correlations may stem from...

1 continuous movements in the same direction
2 common discontinuous movements on rare occasions (co-jumps)

Continuous and Discontinuous Co-Movements

The index comprises constituents $i = 1, \dots, N$ whose stock prices follow **jump-diffusions**

$$\begin{aligned} \frac{dS_{i,t}}{S_{i,t-}} &= \mu_{S_i} dt + \sqrt{V_{i,t}} dW_t^{S_i} + \frac{\Delta S_{i,t}}{S_{i,t-}} \\ dV_{i,t} &= \mu_{V_i} dt + \sigma_{V_i} \sqrt{V_{i,t}} dW_t^{V_i} \end{aligned}$$

The index is the weighted sum of all constituents

$$S_{I,t} = \sum_{i=1}^{N} \omega_{i,t} S_{i,t} \quad \text{and} \quad V_{I,t} = \sum_{i=1}^{N} \omega_{i,t}^2 V_{i,t} + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \omega_{i,t} \omega_{j,t} \sqrt{V_{i,t}} \sqrt{V_{j,t}} \rho_{ij,t}$$

The index variance risk premium can be decomposed into

$$\begin{aligned} VRP_{I,t} = &\sum_{i=1}^{N} \omega_{i,t}^2 \underbrace{\left(E_t^{\mathbb{P}}[CV_{i,[t,t+\tau]}] - E_t^{\mathbb{Q}}[CV_{i,[t,t+\tau]}]\right)}_{\text{continuous variation risk premium}} + \sum_{i=1}^{N} \omega_{i,t}^2 \underbrace{\left(E_t^{\mathbb{P}}[JV_{i,[t,t+\tau]}] - E_t^{\mathbb{Q}}[JV_{i,[t,t+\tau]}]\right)}_{\text{jump variation risk premium}} \\ &+ \underbrace{\sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \omega_{i,t} \omega_{j,t} \left(E_t^{\mathbb{P}}[CV_{ij,[t,t+\tau]}] - E_t^{\mathbb{Q}}[CV_{ij,[t,t+\tau]}]\right)}_{\text{continuous covariation risk premium}} + \underbrace{\sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \omega_{i,t} \omega_{j,t} \left(E_t^{\mathbb{P}}[JV_{ij,[t,t+\tau]}] - E_t^{\mathbb{Q}}[JV_{ij,[t,t+\tau]}]\right)}_{\text{jump covariation risk premium}} \end{aligned}$$

Investors pay the index VRP for very different reasons

→ **how large are the risk premiums for continuous correlation and co-jumps?**

Empirically identifying these risk premiums is challenging

- option returns contain rich information about several risk premiums
- specifically constructed **option portfolios** provide a simple and legitimate way to solve the identification problem

Empirical Strategy

Option Returns

The price of any option follows

$$\begin{aligned} dO_{i,t} - r_{f,t} O_{i,t} dt \approx &\frac{\partial O_i}{\partial S_i} \left(dS_{i,t} - E_t^{\mathbb{Q}}[dS_{i,t}]\right) \\ &+ \frac{\partial O_i}{\partial V_i} \underbrace{\left(dV_{i,t} - E_t^{\mathbb{Q}}[dV_{i,t}]\right)}_{\text{volatility risk premium}} + \frac{1}{2} \frac{\partial^2 O_i}{\partial S_i^2} \underbrace{\left((\Delta S_{i,t})^2 - E_t^{\mathbb{Q}}[(\Delta S_{i,t})^2]\right)}_{\text{jump risk premium}} \end{aligned}$$

We construct **delta-gamma-neutral** (VOL) and **delta-vega-neutral** ($JUMP$) option portfolios for the index and all constituents (Cremers et al., 2015)

$$\begin{aligned} dVOL_{I,t} - r_{f,t} VOL_{I,t} dt &\approx \frac{\partial VOL_I}{\partial V_I} \left(dV_{I,t} - E_t^{\mathbb{Q}}[dV_{I,t}]\right) \\ \text{where} \quad dV_{I,t} &= \underbrace{\sum_{i=1}^{N} \frac{\partial V_I}{\partial V_i} dV_{i,t}}_{\text{individual volatilities}} + \underbrace{\frac{\partial V_I}{\partial \rho} d\rho_t}_{\text{continuous correlation}} \\ dJUMP_{I,t} - r_{f,t} JUMP_{I,t} dt &\approx \frac{1}{2} \frac{\partial^2 JUMP_I}{\partial S_I^2} \left((\Delta S_{I,t})^2 - E_t^{\mathbb{Q}}[(\Delta S_{I,t})^2]\right) \\ \text{where} \quad (\Delta S_{I,t})^2 &= \underbrace{\sum_{i=1}^{N} \omega_{i,t}^2 (\Delta S_{i,t})^2}_{\text{individual jumps}} + \underbrace{\sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \omega_{i,t} \omega_{j,t} \Delta S_{i,t} \Delta S_{j,t}}_{\text{co-jumps}} \end{aligned}$$

The portfolios are only exposed to volatility or jump risks

- VOL_I returns depend on individual volatility and continuous correlation risk premiums
- $JUMP_I$ returns depend on individual jump and co-jump risk premiums

To isolate the correlation risk premiums, we implement **dispersion trades**: we go long the index option portfolio and short the basket of option portfolios on the constituents (Driessen et al., 2009)

$$\begin{aligned} dCRP_{VOL,t} - r_{f,t} CRP_{VOL,t} dt &\approx \frac{\partial VOL_I}{\partial V_I} \frac{\partial V_I}{\partial \rho} \left(d\rho_t - E_t^{\mathbb{Q}}[d\rho_t]\right) \\ dCRP_{JUMP,t} - r_{f,t} CRP_{JUMP,t} dt &\approx \frac{1}{2} \frac{\partial^2 JUMP_I}{\partial S_I^2} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \omega_{i,t} \omega_{j,t} \left(\Delta S_{i,t} \Delta S_{j,t} - E_t^{\mathbb{Q}}[\Delta S_{i,t} \Delta S_{j,t}]\right) \end{aligned}$$

⇒ portfolios pay the risk premiums for continuous correlation and co-jumps, respectively

Data

- sample: S&P 100 index + constituents, 01/1996 – 12/2017, daily frequency
- data sources: OptionMetrics, CRSP, Compustat
- filters: similar to Goyal and Saretto (2009)
- additional complication: American-style options
  → strip off early exercise feature in CRR-trees
  → compute European option prices and Black-Scholes greeks

Every day, we construct VOL and $JUMP$ portfolios for the S&P 100 index and all constituents

- set up straddles with different times to maturity between 14 and 365 days
- select the **2 straddles** that are nearest-the-money
- take positions in these 4 options while
  → restricting delta, vega & gamma
  → aiming for balanced allocation of wealth across options

$$\min_{\boldsymbol{\omega}} \left\| \frac{\boldsymbol{\omega} \circ \mathbf{O}}{abs(\boldsymbol{\omega}^\top)\mathbf{O}} \right\|_2^2$$

VOL portfolio s.t.

$$\boldsymbol{\omega}^\top [\boldsymbol{\Delta}, \boldsymbol{\mathcal{V}}, \boldsymbol{\Gamma}] = [0, 200, 0] \qquad \boldsymbol{\omega} \circ [-1, -1, 1, 1]^\top \geq \mathbf{0}$$

$JUMP$ portfolio s.t.

$$\boldsymbol{\omega}^\top [\boldsymbol{\Delta}, \boldsymbol{\mathcal{V}}, \boldsymbol{\Gamma}] = [0, 0, 0.01] \qquad \boldsymbol{\omega} \circ [1, 1, -1, -1]^\top \geq \mathbf{0}$$

where $\boldsymbol{\omega} = [\omega_{call,T_1}, \omega_{put,T_1}, \omega_{call,T_2}, \omega_{put,T_2}]^\top$

We hold the positions for 1 trading day and calculate excess returns at recorded closing prices

→ if necessary, we interpolate using a kernel smoother (maturity, moneyness, put-call identifier)

Results

Correlation Risk Premium

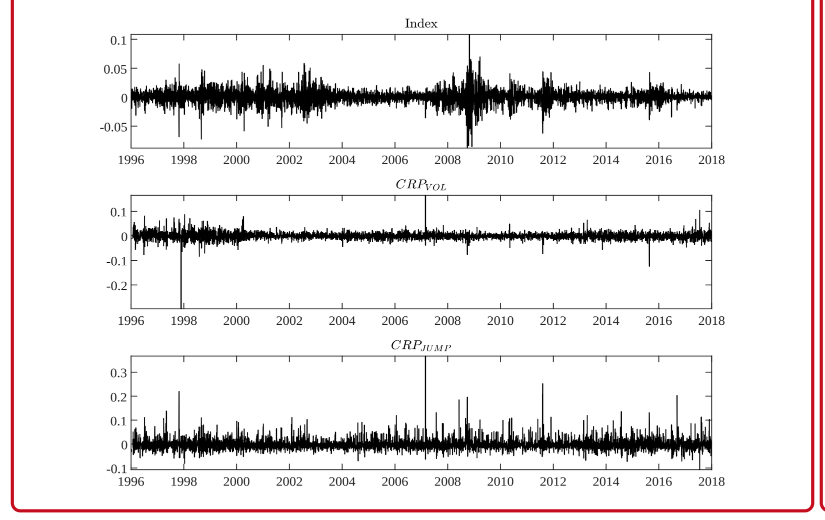
Summary Statistics

Mean	−0.1321	−0.3067
Standard Deviation	0.2185	0.3617
Sharpe Ratio	−0.6046	−0.8480
Median	−0.0009	−0.0042
Skewness	−1.3525	2.7530
Kurtosis	50.7880	27.5230

- investors pay premium of 13.21% p.a. to hedge against high continuous correlation
- investors pay large premium of 30.67% p.a. to hedge against co-jumps

→ returns are scaled by greek → Sharpe ratios are more informative (0.85 vs. 0.60)
→ **risk premium for co-jumps is larger than for continuous correlation**

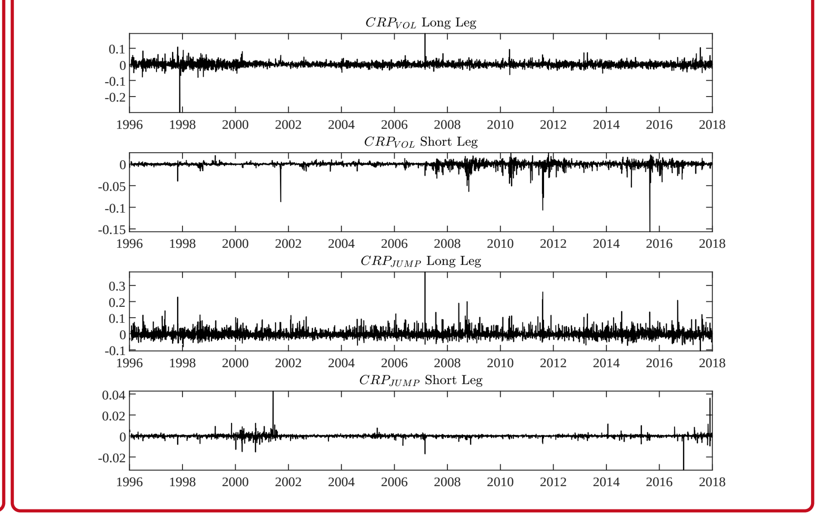
Time Series



- CRP_{VOL}: negative on 53% of days; no sudden changes during crash periods
- CRP_{JUMP}: negative on 61% of days; occasional extreme positive returns coincide with extreme index returns

→ **co-jumps materialize in crash periods**

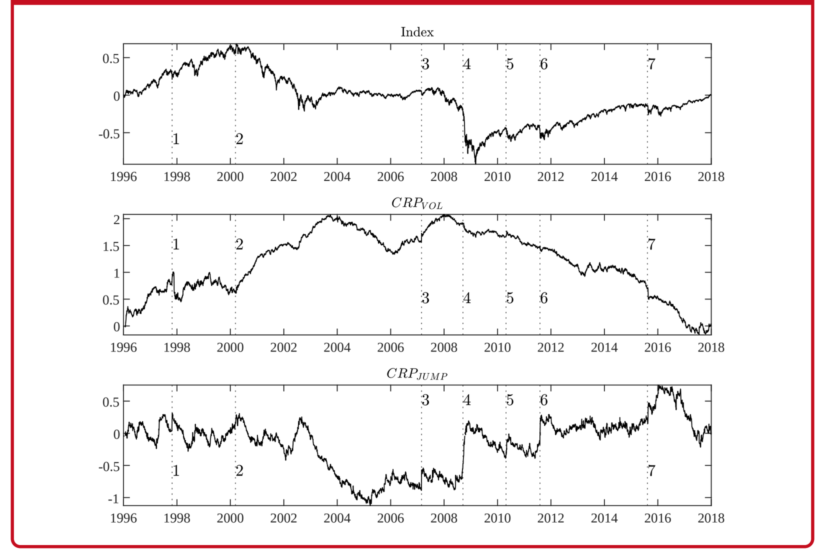
Dispersion Trade Components



- CRP_{VOL}: both legs are of same magnitude; short leg is less volatile
- CRP_{JUMP}: short leg is much smaller and exhibits fewer spikes

→ **index jumps almost exclusively come from co-jumps**

Detrended Cumulative Excess Returns



- CRP_{VOL}: slow-moving; prolonged gradual increase after burst of dot-com bubble

→ **insures against the long-term risk of worsening investment opportunities**

- CRP_{JUMP}: prompt reactions to major events; large positive payoff around default of Lehman Brothers

→ **insures against the short-term risk of simultaneous crashes**

Predictive Power

Predictive Regressions

	1 month	3 months	6 months	12 months	24 months
Intercept	0.0055	0.0276**	0.0524***	0.0588*	0.0131
	(0.0065)	(0.0123)	(0.0207)	(0.0332)	(0.0620)
	[0.0065]	[0.0156]	[0.0336]	[0.0770]	[0.1835]
CRP_{VOL}	0.0037	−0.0392	−0.0183	−0.2282***	−0.2124***
	(0.0423)	(0.0437)	(0.0465)	(0.0433)	(0.0482)
	[0.0427]	[0.0453]	[0.0821]	[0.1203]	[0.1119]
CRP_{JUMP}	−0.0910***	−0.0215	0.0423	0.0205	−0.0405
	(0.0305)	(0.0300)	(0.0324)	(0.0387)	(0.0461)
	[0.0306]	[0.0524]	[0.0355]	[0.0593]	[0.0981]
VRP	−0.0004***	−0.0012***	−0.0016***	−0.0020***	−0.0023***
	(0.0001)	(0.0002)	(0.0004)	(0.0005)	(0.0007)
	[0.0001]	[0.0003]	[0.0004]	[0.0008]	[0.0013]
P/D	−0.0002**	−0.0007***	−0.0009***	−0.0015***	−0.0020***
	(0.0001)	(0.0002)	(0.0002)	(0.0003)	(0.0005)
	[0.0001]	[0.0002]	[0.0003]	[0.0006]	[0.0012]
adj R^2	7.7512	16.7932	12.3407	26.7099	21.2814

- CRP_{VOL}: predicts index returns for horizons of 12 and 24 months
- CRP_{JUMP}: has short-term predictive power over 1 month

→ **the two risk premiums compensate different economic risks**

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