Information Design in Simultaneous All-Pay Auction Contests

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Motivation

- Contests are prevalent and essential in real world.
- Contestants expend irreversible resources to win a reward.
- Contestants have private information.
- Organizer maximizes the expected total effort of contestants.
- Organizer influences contestants' beliefs about each other by information disclosure.
- **Real World Examples**.
    - Politics: lobbying, president election.
    - Enterprise: job promotion, oligopoly.
    - Innovation: patent race, crowd sourcing.
    - Academic: grants competition, application.
    - ...

Motivation

Contests	Information	Disclosure
Patent race	Productivity	Announcements
Lobbying	Financial	Announcements
Grants competition	Capabilities	List of applicants Proposal's quality
Job promotion	Capabilities	Work Performance Education Experience

An Illustrative Example

In a simultaneous all-pay auction contest, the prior type distributions for both players are independently and identically distributed

$$v_i = \begin{cases} 1 & \text{w.p.} \quad 0.5 \\ 2 & \text{w.p.} \quad 0.5 \end{cases}$$

Previous literature (Lu, Ma, and Wang, 2018) characterized four type-dependent disclosure policies.

Strategy	Expected total effort
(C,C,C)	5/4
(D,C,C)	7/6
(D,D,D)	9/8
(C,C,D)	3/4

Table: Expected Total Effort

An Illustrative Example

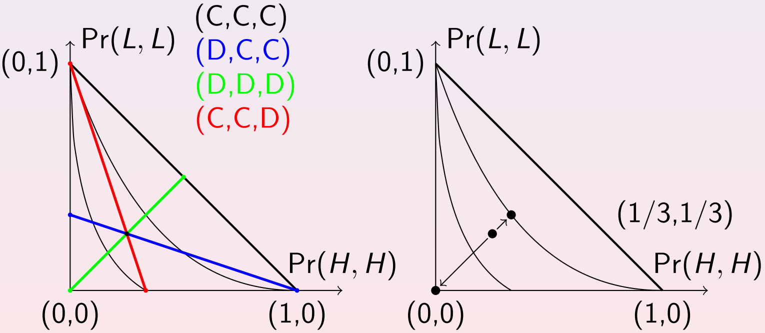


Figure: Strategy in LMW (2018) (left) and This Paper (right)

An Illustrative Example

However, we can design the following two posteriors with correlated distributions,

	High	Low
High	1/3	1/6
Low	1/6	1/3

and

	High	Low
High	0	1/2
Low	1/2	0

with probability 3/4 and 1/4 respectively.
Then the expected total effort will be

$$\frac{3}{4} \times \frac{5}{3} + \frac{1}{4} \times \frac{3}{4} = \frac{23}{16} > \frac{5}{4} = \frac{20}{16} \quad \text{better than } (C, C, C)$$

What We Do

- We study the information design problem of the designer in a simultaneous all-pay auction contest environment.
- We allow the information disclosure policy to take the stochastic approach of Bayesian persuasion (KG2011).
- We incorporate our results using the surplus triangle (BBM2015, BBM2017, RS2017, KZ2019).
- **Key Feature**. Two-sided Information Asymmetry. Higher Dimension Concavification.
- **Methodology**. Elimination of **Strictly** Dominated Posteriors.
- **Results**. Answer the two core questions,
  - When does contest designer benefit from Bayesian persuasion?
  - What is the optimal Bayesian persuasion signal?

Literature

- Bayesian Persuasion and Information Design:
  - Kamenica and Gentzkow (2011)
  - Bergemann and Morris (2016)
  - Mathevet, Perego, and Taneva (2017)
- Contest with Correlated Information:
  - Siegel (2014)
  - Liu and Chen (2016)
  - Rentschler and Turocy (2016)
  - Lu and Parreiras (2017)
  - Chi, Murto and Valimaki (2017)
- Surplus Triangle:
  - Bergemann, Brooks and Morris (2015,2017)
  - Roesler, Szentes (2017)
  - Kartik, Zhong (2019)

Literature

- Information Disclosure in Contests:
  - Fu, Jiao and Lu (2011)
  - Fu, Jiao and Lu (2014)
  - Kovenock, Morath, and Munster (2015)
  - Wu and Zheng (2017)
  - Lu, Ma, and Wang (2017)
  - Jiao, Lien, and Zheng (2017)
  - Serena (2017)
  - Chen, Kuang and Zheng (2017a)
  - Konrad and Morath (2018)
  - Cai, Jiao, Lu and Zheng (2019)
- Bayesian Persuasion and Information Design in Contests:
  - Zhang and Zhou (2016)
  - Chen, Kuang and Zheng (2017b)
  - Kuang and Zheng (2018)
  - Kuang (2019)

General Setup

- 2 risk neutral players ($i = 1, 2$) participate in a single-prize All-pay Auction contest, where players 1 and 2 move simultaneously.
- The success function of contestant $i \in \{1, 2\}$ under effort portfolio (x_1, x_2) is given by

$$p_i(x_i, x_{-i}) = \begin{cases} 1, & \text{if} \quad x_i > x_{-i} \\ 0, & \text{if} \quad x_i < x_{-i} \end{cases}$$

- Contestant's payoff has a linear form $\Pi_i = p_i v_i - x_i$.
- Contestants' surplus $\Pi_C = \Pi_1 + \Pi_2$.
- Organizer's surplus $\Pi_O = x_1 + x_2$.

General Setup

- Both contestants' valuations of winning are discrete variables chosen from prior joint distribution.
- v_i is a discrete random variable with two values $v_L < v_H$.
- Define $d = \frac{v_H}{v_L}$ and normalize $v_L = 1$.
- It is commonly known that the joint probability distribution of types is $\Pr(v_i, v_{-i})$, which is symmetric: $\Pr(H, L) = \Pr(L, H)$.
- Efficient frontier $\Pi_O + \Pi_C \leq \mathbb{E}(\max(v_1, v_2)) = d(1-q) + q$.

	High	Low
High	p	$\frac{1-p-q}{2}$
Low	$\frac{1-p-q}{2}$	q

Table: General Form of Joint Distribution

Signal Decomposition

- The information design problem can be decomposed as the following two steps.
- **Step 1**. Optimizing over private information. (Private Signal)
- **Step 2**. Adding an optimal public signal. (Public Signal)
- The private signal is received separately after both contestants observing the public signal.

Public Signal

- The public signal π consists of a realization space S and a family of likelihood distributions $\pi = \{\pi(\cdot|v_1, v_2)\}_{v_1, v_2 \in \{v_H, v_L\}}$ over S.
- The public signal is interpreted as a mapping

$$\pi : \{v_H, v_L\}^2 \to \Delta(S)$$

- For each pair of (v_1, v_2), the signal generates a distribution over the signal space S.
- We focus on anonymous public signal that requiring the symmetric condition $\pi(s|v_H, v_L) = \pi(s|v_L, v_H)$.
- This constraint guarantee symmetric posterior distributions.
- For each realization $s \in S$, the posterior distribution is derived as (p_s, q_s) by Bayes rule.

Private Signal

- Private persuasion comes after this updating procedure.
- The private signal σ consists of two realization spaces X_1, X_2, where X_1, X_2 denotes the private signal set for player i, and a family of likelihood distributions $\sigma = \{\sigma(\cdot, \cdot | s, v_1, v_2)\}_{s \in S, v_1, v_2 \in \{v_H, v_L\}}$ over $X_1 \times X_2$.
- A private signal is interpreted as a mapping

$$\sigma : \{v_H, v_L\}^2 \times S \to \Delta(X_1 \times X_2)$$

- By the revelation principle, it is without loss of generality focus on direct signaling policies.
- The outcome can be implemented if and only if σ constructs a Bayes correlated equilibrium.

Timing of the Game

- The contest designer chooses and pre-commits to a public signal π and a private signal σ.
- Nature moves and draws a valuation profile $\mathbf{V} = (v_1, v_2)$ from prior distribution.
- The contest designer carries out his commitment and a public signal realization $s \in S$ is generated according to $\pi(s|\mathbf{V})$.
- Signal realization s is observable by the public and both players update their common posterior belief as (p_s, q_s).
- The contest designer carries out his commitment and a pair of recommended strategies (x_1, x_2) is generated according to $\sigma(x_1, x_2|\mathbf{V})$. x_1 is privately sent to player 1 and x_2 is privately sent to player 2.
- The contest takes place, and all contestants choose efforts simultaneously.

Bayesian Plausible

- Private signal is ineffective under this setting (Kuang, 2019).
- It is without loss of generality to consider public persuasion only.
- Posterior joint distribution receiving signal s, $\mu_s \in \Delta^2$
- τ is a random variable that takes value in the simplex Δ^2. Namely, it assigns a probability measure on the posteriors in the support of Δ^2.
- We call τ *Bayesian-plausible* if the expected posterior probability equals the prior.
- Kamenica and Gentzkow (2011) shows that finding optimal signal π is equivalent to searching over Bayesian-plausible distribution of posteriors.
- The optimal signal always exists and achieves an expected total effort equal to $\mathbf{cav}\Pi_O(\mu_0)$.

Posterior Contest Game

- Symmetric Beliefs.

$$\Pr(H|H) = \frac{\Pr(H,H)}{\Pr(H)} = \frac{2p}{1+p-q} \qquad \Pr(L|H) = \frac{1-p-q}{1+p-q}$$

$$\Pr(L|L) = \frac{\Pr(L,L)}{\Pr(L)} = \frac{2q}{1-p+q} \qquad \Pr(H|L) = \frac{1-p-q}{1-p+q}$$

- Equilibrium strategies have three possibilities,
  1. Strong negative correlation
  2. Weak correlation
  3. Strong positive correlation

Total Effort Function

- Expected Total Effort.
  $\Pi_O(\mu_s) = \Pi_O(p,q) = (1+p-q)\mathbb{E}(x_H) + (1-p+q)\mathbb{E}(x_L)$.
- Case 1, **Strong negative correlation**,

$$\Pi_O(p,q) = 1 + \frac{(d-1)(1+p-q)\left[4q-(1-p+q)^2\right]}{2(1-p+q)(1-p-q)d - 4q(1+p-q)}$$

- Case 2, **Weak correlation**,

$$\Pi_O(p,q) = q + pd + \frac{2q(1+p-q)}{1-p+q}$$

- Case 3, **Strong positive correlation**,

$$\Pi_O(p,q) = 1 + \frac{(d-1)(1+p-q)\left[d(1+p-q)(1-p+q) - 2(1-p-q)\right]}{4pd(1-p+q) - 2(1-p-q)(1+p-q)}$$

2-dimensional Properties

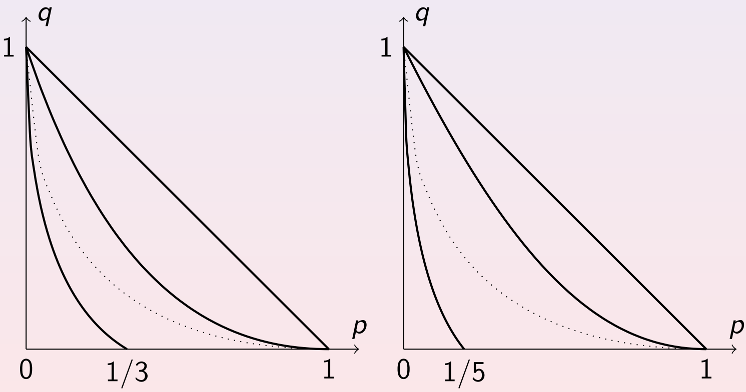
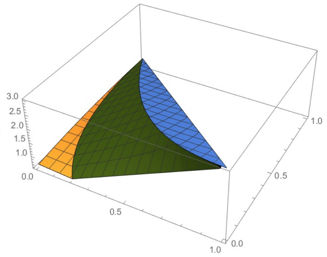


Figure: Left ($d = 2$), Right ($d = 3$)

2-dimensional Properties

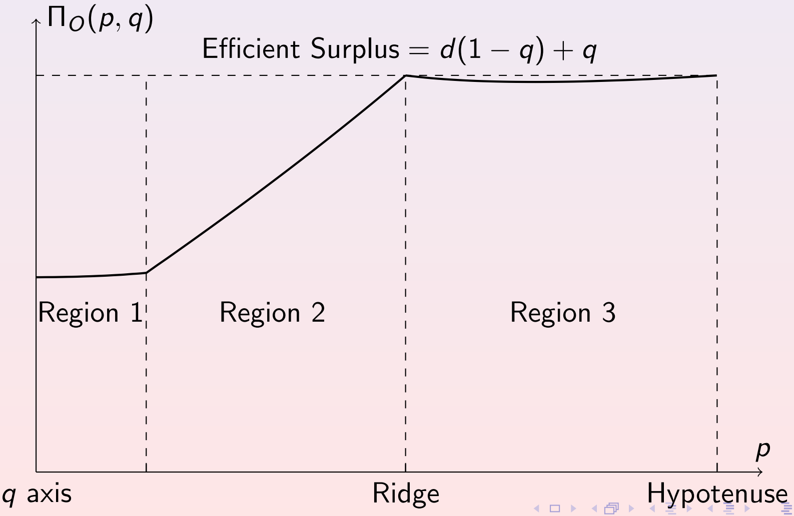
- Boundary Curve between Region 1/2.
  $(2d-1)p^2 + q^2 - 2dpq - 2dp - 2q + 1 = 0$
- Boundary Curve between Region 2/3.
  $dp^2 + (2-d)q^2 - 2pq - 2dp - 2q + d = 0$
- As d grows: Region 2 expands while region 1 and 3 shrink.
- Convexity: Region 3 is convex.

3-dimensional Properties

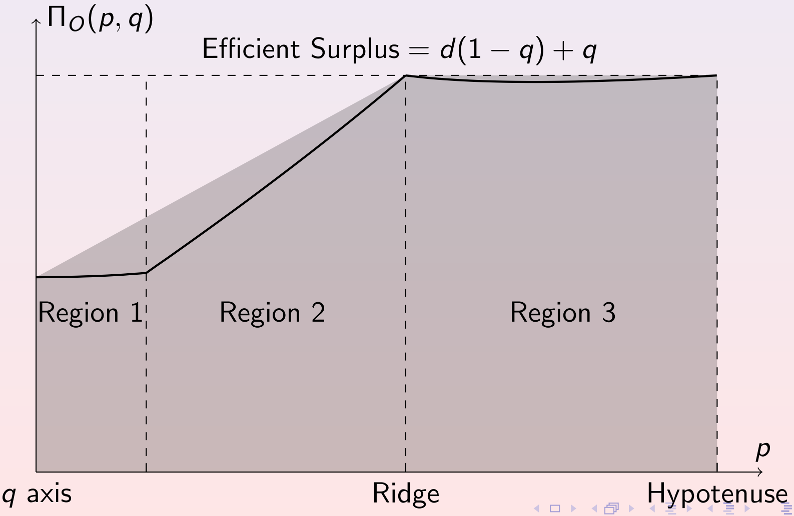


- q-axis. Increasing and concave with respect to q.
- p-axis. Piecewise linear with respect to p. Convex.
- Hypotenuse. Linear with respect to q.
  $\Pi_O(1-q,q) = d + q - dq$
- Ridge (boundary between region 2/3). Linear with respect to q. $\Pi_O(p_r(q),q) = d + q - dq = \Pi_O(1-q,q)$

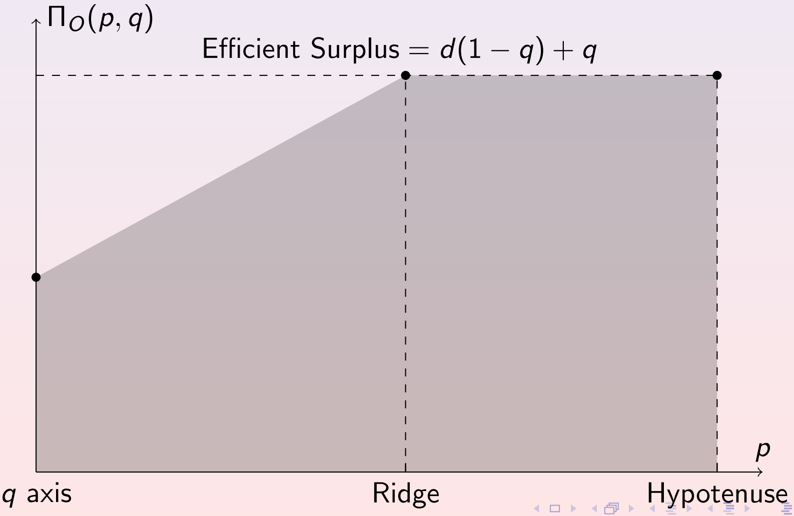
Elimination of Weakly Dominated Posteriors (Fixed q)



Elimination of Weakly Dominated Posteriors (Fixed q)



Elimination of Weakly Dominated Posteriors (Fixed q)



Remaining Posteriors

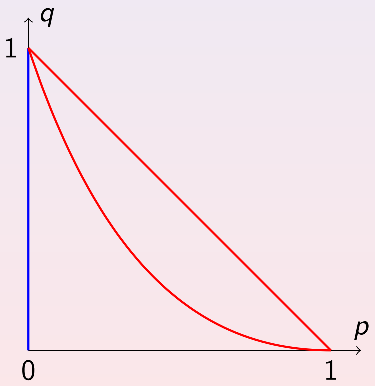


Figure: Left ($d = 2$), Right ($d = 3$)

Region 3: Optimal Solution

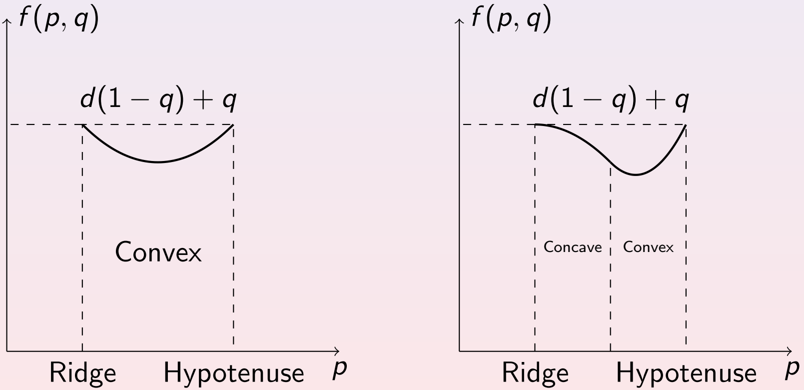
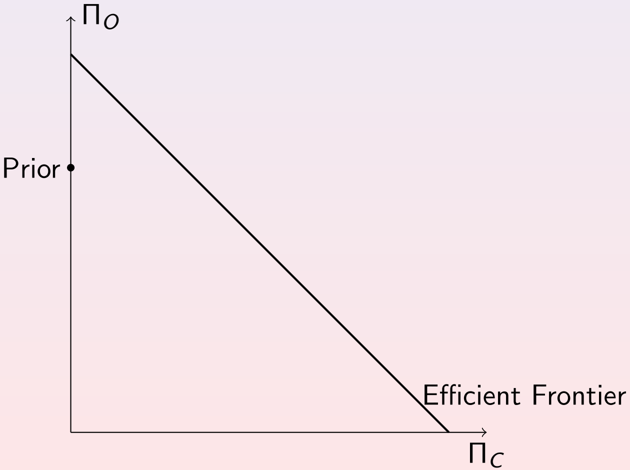
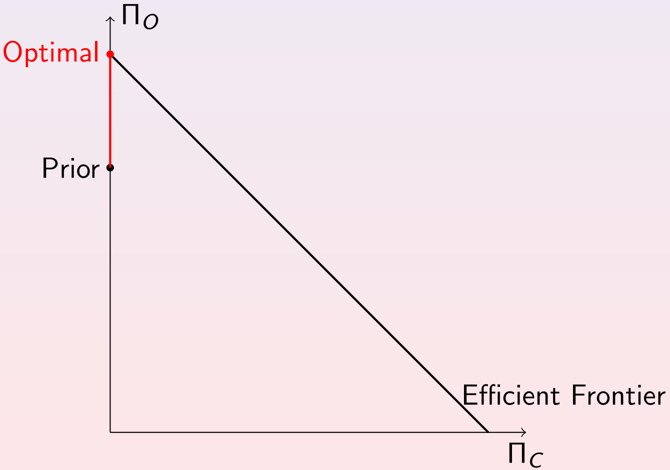


Figure: Convex (left) and Concave-Convex (right)

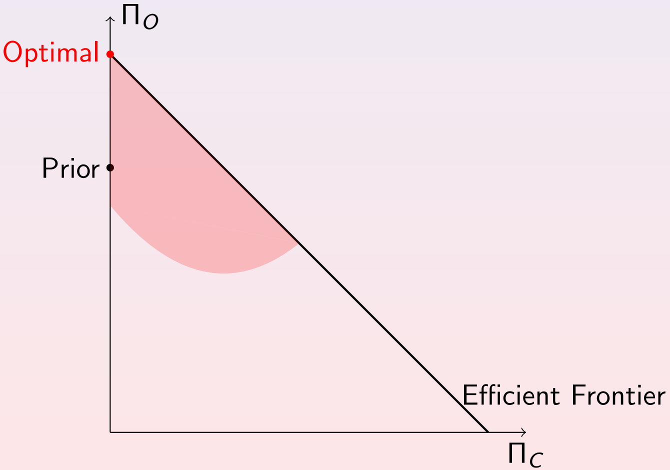
Region 3: Surplus Triangle



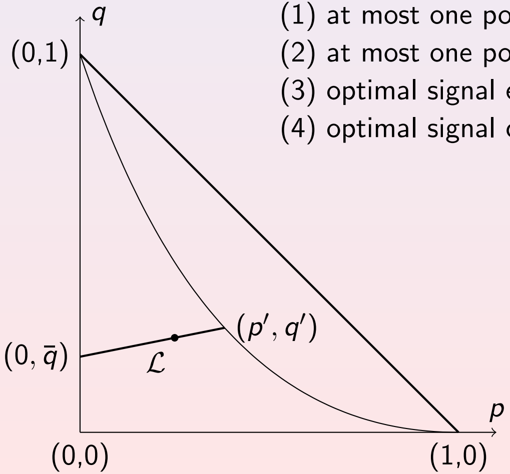
Region 3: Surplus Triangle



Region 3: Surplus Triangle

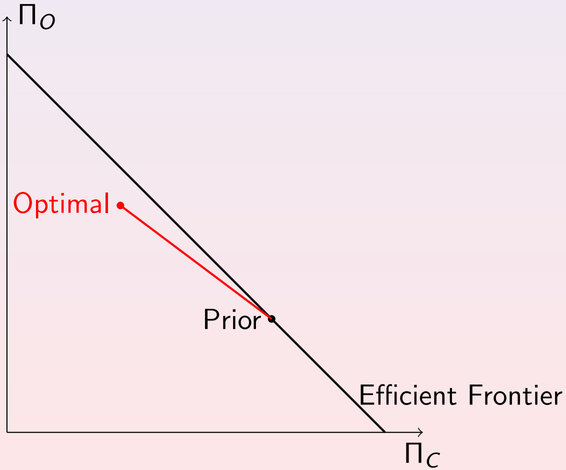


Region 1/2: Optimal Design

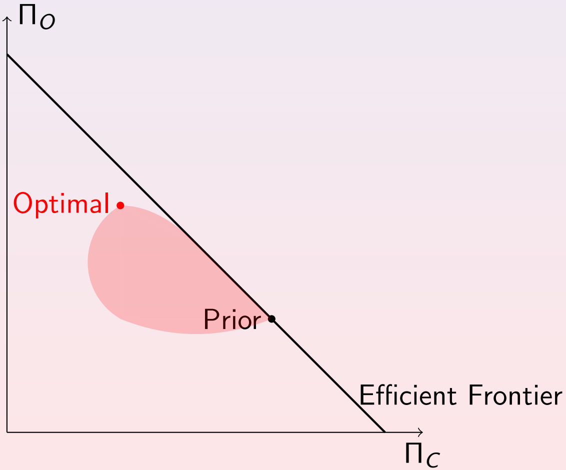


(1) at most one posterior in q axis
(2) at most one posterior on the ridge
(3) optimal signal exists uniquely
(4) optimal signal can be solved numerically

Region 2: Surplus Triangle



Region 2: Surplus Triangle



Information on Individual Values

- In previous sections, we assume that players know their true valuations before posterior contest game, thus the contest designer cannot manipulate players about their own types.
- However, if players do not know their realized type, should the designer disclose their own type information to them?
- Now we consider three different scenarios on players' information regarding their **own types:**
    1. **No Information. Denoted as $\mathbb{N}$. (Neither player knows his/her own winning value.)**
    2. **Private Information. Denoted as $\mathbb{P}$. (Both players know their own winning values.)**
    3. **Asymmetric Information. Denoted as $\mathbb{A}$. (Exactly one player know his/her own winning value.)**

Information on Individual Values (Cont.)

Theorem

From the contest designer's perspective, for any prior (p_0, q_0)*,*

$$\mathbb{N} \succ \mathbb{A}$$

Theorem

The contest designer's preference over $\mathbb{P}$ *and* $\mathbb{N}/\mathbb{A}$ *depends on the values of parameters. In other words, the following three circumstances are all possible:*

$$\mathbb{P} \succ \mathbb{N} \succ \mathbb{A}$$
$$\mathbb{N} \succ \mathbb{P} \succ \mathbb{A}$$
$$\mathbb{N} \succ \mathbb{A} \succ \mathbb{P}$$

Ridge Phenomenon of Positive Correlation

- Most of our results in the benchmark model depend on the ridge phenomenon of positive correlation.
- For both validity and optimality issues, we require one posterior located on ridge.
- For ridge distribution, resulting equilibrium is both efficient and exploitative.
- We show that given the marginal distribution of the winning value, there exists **unique** joint distribution that satisfies the efficient condition and the exploitative condition.
- The Bayes Nash equilibrium under the following joint distribution is both efficient and exploitative,

$$\Pr(v_k, v_j) = \left(\Pr(v_j) - \sum_{i=1}^{j-1} \Pr(v_i, v_j)\right)\left(\sum_{i=j}^{N} \frac{\Pr(v_i)}{v_i}\right)^{-1} \frac{\Pr(v_k)}{v_k}, k \geq j$$

Ridge Phenomenon of Positive Correlation

We consider the ternary distribution with $(v_1, v_2, v_3) = (2, 3, 6)$.

If $\Pr(v_1) = \Pr(v_2) = \Pr(v_3) = \frac{1}{3}$, distribution: $\begin{bmatrix} \frac{1}{6} & \frac{1}{9} & \frac{1}{18} \\ \frac{1}{9} & \frac{4}{27} & \frac{2}{27} \\ \frac{1}{18} & \frac{2}{27} & \frac{11}{54} \end{bmatrix}$

If $\Pr(v_1) = \frac{1}{2}, \Pr(v_2) = \Pr(v_3) = \frac{1}{4}$, distribution: $\begin{bmatrix} \frac{1}{3} & \frac{1}{9} & \frac{1}{18} \\ \frac{1}{9} & \frac{5}{54} & \frac{5}{108} \\ \frac{1}{18} & \frac{5}{108} & \frac{4}{27} \end{bmatrix}$

If $\Pr(v_2) = \frac{1}{2}, \Pr(v_1) = \Pr(v_3) = \frac{1}{4}$, distribution: $\begin{bmatrix} \frac{3}{32} & \frac{1}{8} & \frac{1}{32} \\ \frac{1}{8} & \frac{3}{10} & \frac{3}{40} \\ \frac{1}{32} & \frac{3}{40} & \frac{23}{160} \end{bmatrix}$

If $\Pr(v_3) = \frac{1}{2}, \Pr(v_1) = \Pr(v_2) = \frac{1}{4}$, distribution: $\begin{bmatrix} \frac{3}{28} & \frac{1}{14} & \frac{1}{14} \\ \frac{1}{14} & \frac{5}{56} & \frac{5}{56} \\ \frac{1}{14} & \frac{5}{56} & \frac{19}{56} \end{bmatrix}$

Thank You!