GOETHE UNIVERSITÄT FRANKFURT AM MAIN

DEUTSCHE BUNDESBANK EUROSYSTEM

Robust Inference in Time-Varying Structural VAR models: The DC-Cholesky Multivariate Stochastic Volatility Model

Benny Hartwig (Goethe University Frankfurt and Deutsche Bundesbank)

Motivation

- Cholesky multivariate stochastic volatility (CMSV) model commonly used to specify dynamic covariance matrices of a n-dimensional vector y_t:

$$y_t \sim N(0,\Sigma_t), \quad \text{assume } \Sigma_t = A_t^{-1}D_tD_t'A_t^{-1'}$$
$$\Rightarrow y_t = A_t^{-1}D_t\epsilon_t, \quad \epsilon_t \sim N(0,I_n)$$

i.e. Σ_t implicitly modelled by specifying A_t^{-1} and D_t

- **But:** estimates of Σ_t may be sensitive to the ordering variables in y_t, see e.g., Primiceri (2005)
  - $\Rightarrow$ inference may hinge on a chosen variable ordering
  - $\Rightarrow$ majority of applied literature ignores this property
- **Illustration:** Effect of alternative variable orderings on dynamics of Σ_t in Primiceri's (2005) application
  - $\Rightarrow$ volatilities similar, covariances differ in stagflation

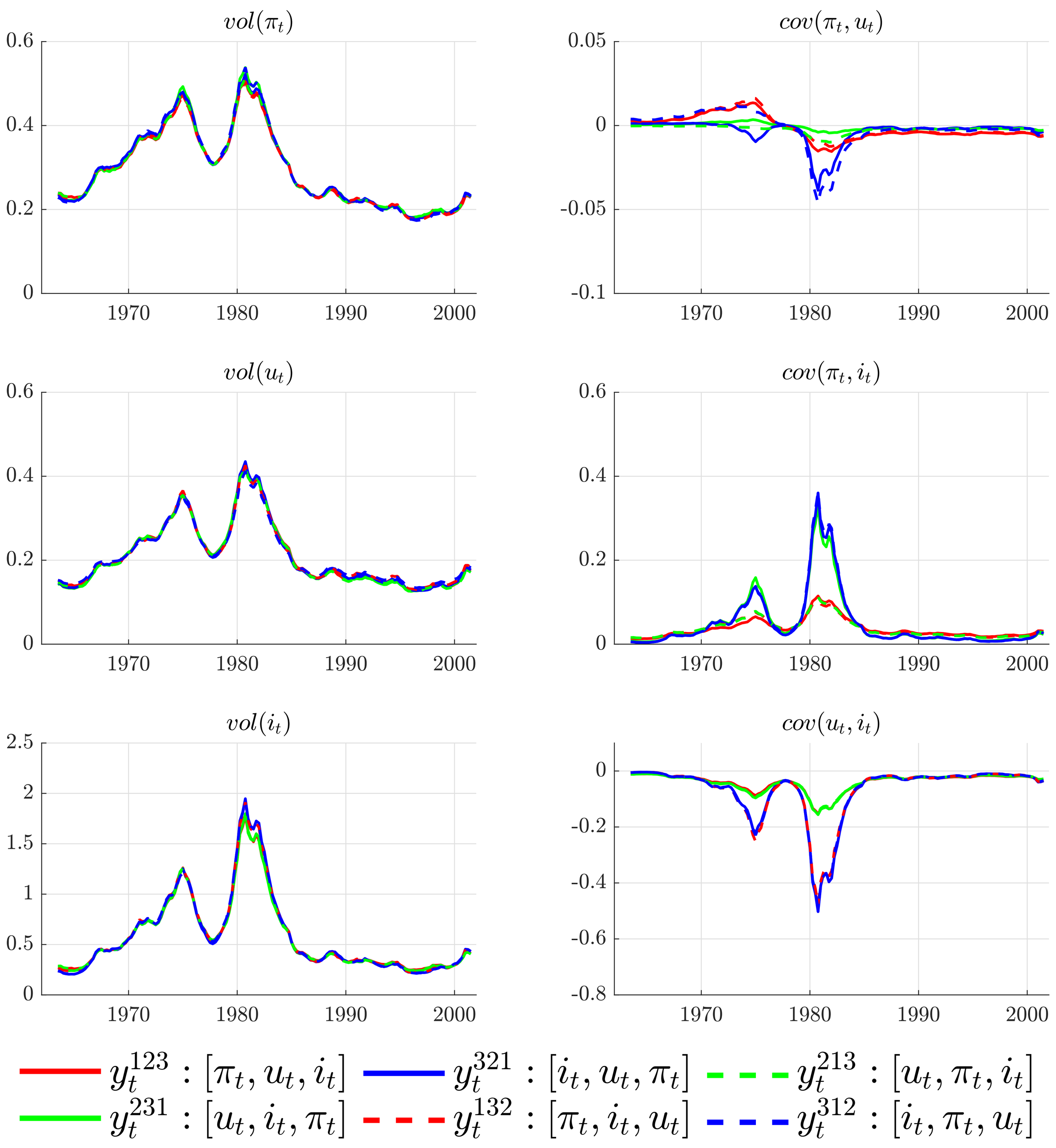

Figure: Estimated reduced-form covariance matrix

Research questions

- Role of variable ordering on the dynamics of Σ_t?
- Variable ordering important for conclusions?
- How to mitigate the ordering sensitivity?

Contributions and Results

1. Ordering sensitivity not negligible in CMSV model!
   - $\Rightarrow$ volatility pattern impose alternative restrictions
2. Propose a robust modelling alternative
   - $\Rightarrow$ dynamic correlation Cholesky MSV (DC-CMSV)
3. Monte Carlo simulation to fit Σ_t
   - $\Rightarrow$ Estimated correlations almost ordering insensitive when there is no volatility (CMSV & DC-CMSV)
   - $\Rightarrow$ Estimated covariances of CMSV model more distinct when there are stronger idiosyncratic volatility clusters, while covariances hardly affected by alternative volatility pattern under DC-CMSV
4. Inference may hinge on a ordering for estimating Σ_t!
   - (1) US monetary policy during stagflation
   - $\Rightarrow$ unchanged or more aggressive response?
   - (2) Predictability of US inflation-gap
   - $\Rightarrow$ gradual or abrupt improvement in predictability?

On the Cholesky MSV model

Let y_t be a 2-dimensional vector (tractability)

How does the CMSV structure affect dynamics of Σ_t?

- CMSV model: $\Sigma_t = A_t^{-1}D_tD_tA_t^{-1'}$
  - a_t off-diag. of A_t and g_t log-vol. process of D_t
  - a_t and g_t are Gaussian random walk (RW)

Properties of Σ_t under CMSV

1. the ratio of volatilities $\frac{\sigma_{22,t}}{\sigma_{11,t}}$ is time-varying
2. the correlation ρ_t evolves nonlinearly
$$\rho_t = \rho_{t-1}\frac{\exp(\eta^g_{1,t})}{\exp(\eta^{g**}_{2,t})} + \epsilon^a_t\frac{\sigma_{11,t}}{\sigma_{22,t}}$$
3. the contemporaneous relation a_t evolves linearly
4. the dynamic structure of Σ_t cannot be generated by an analogously setup CMSV model for $\tilde{y}_t$
5. dynamic restrictions increase in the variability of idiosyncratic volatility patterns

Comparison to separated volatilities and correlations?

- DC-MSV model: $\Sigma_t = D_tR_tD_t'$ (Yu and Meyer, 2006)
  - h_t log-vol. process of D_t and ρ_t correlation of R_t
  - $\rho_t(m_t) = \frac{\exp(m_t)-1}{\exp(m_t)+1}$, m_t and h_t are Gaussian RW
  - applicable only to $n \leq 3$ (psd of R_t not guaranteed)

Properties of Σ_t under DC-MSV

1. the ratio of volatilities time-varying or constant
2. the correlation ρ_t evolves approximately linearly
3. the contemp. relation a_t evolves nonlinearly
$$a_t = a_{t-1}\frac{\exp(\eta^h_{2,t})}{\exp(\eta^h_{1,t})} + \eta^\rho_t\frac{\exp(h_{2,t})}{\exp(h_{1,t})}$$

Fit Σ_t with CMSV, when y_t generated by DC-MSV?

- nonlinear transformation of a_t as volatilities switch position ($a_t = \rho_t\frac{\exp(h_{2,t})}{\exp(h_{1,t})}$, $\tilde{a}_t = \rho_t\frac{\exp(h_{1,t})}{\exp(h_{2,t})}$)
- systematically different paths of the covariance (a_t underestimated in one ordering, while mechanically overestimated in reverse ordering)

Special cases:

1. $\rho_t = \rho, \forall t$: effect more severe (no offsetting by η^ρ_t)
2. $h_t = h, \forall t$: a_t is almost ordering insensitive

The DC-Cholesky MSV model

- Let y_t be a n-dimensional vector with $y_t \sim N(0,\Sigma_t)$
- DC-CMSV model: $\Sigma_t = D_tR_tD_t'$
  - $\Rightarrow y_t = D_t\epsilon_t, \epsilon_t \sim N(0,R_t)$
  - $\Rightarrow$ estimate auxilary matrix $Q_t = A_t^{*-1}D_t^*D_t^{*'}A_t^{*'-1}$ using the CMSV model on stand. data $\epsilon_t = D_t^{-1}y_t$
  - $\Rightarrow$ estimate correlations via Engle's (2002) formulas

$$R_t = Q_t^{*-\frac{1}{2}}Q_tQ_t^{*-\frac{1}{2}}, \qquad Q_t^* = \text{diag}[\text{vecd}(Q_t)]$$

where $\text{vecd}(Q_t)$ selects the diagonal of Q_t.

Further assumptions

- State dynamics: RW, stationary or combination
- Independent innovations of volatility and correlation

Monte Carlo Simulation

DGP: Correlations from Engle (2002) w/o SV

(1) Fitting correlations with CMSV model (wo SV)

	ρ_t	$a_t - \tilde{a}_t$	a_t	$\tilde{a}_t$
const	**0.008**	0.084	0.018	0.019
sine	**0.022**	0.086	0.035	0.034
fsine	**0.016**	0.070	0.020	0.020
step	**0.010**	0.076	0.018	0.018
ramp	**0.023**	0.087	0.037	0.037

Table: Mean absolute distance (MAD) without stochastic volatility

- MAD lowest for ρ_t ($R_t = \Sigma_t^{*-1/2}\Sigma_t\Sigma_t^{*-1/2}$)
  - $\Rightarrow$ implied ρ_t almost ordering insensitive

(2) Fitting covariances (with SV)

	High Vol		Low Vol	
	CMSV	DC-CMSV	CMSV	DC-CMSV
const	0.207	**0.037**	0.153	**0.026**
sine	0.179	**0.021**	0.081	**0.023**
fsine	0.210	**0.012**	0.049	**0.015**
step	0.169	**0.021**	0.089	**0.019**
ramp	0.183	**0.023**	0.085	**0.025**

Table: Mean absolute distance (MAD) with stochastic volatility

- CMSV: MAD of $\sigma_{12,t}$ increases for high vol. DGP
  - $\Rightarrow$ DC-CMSV: almost insensitive to alt. DGPs

Empirical Application

(1) Evolution of US monetary policy (Primiceri, 2005)

- unchanged or more aggressive response?
  - $\Rightarrow$ ambiguous with CMSV model
  - $\Rightarrow$ DC-CMSV model suggest that the Fed counteracted π and UR more aggressively!

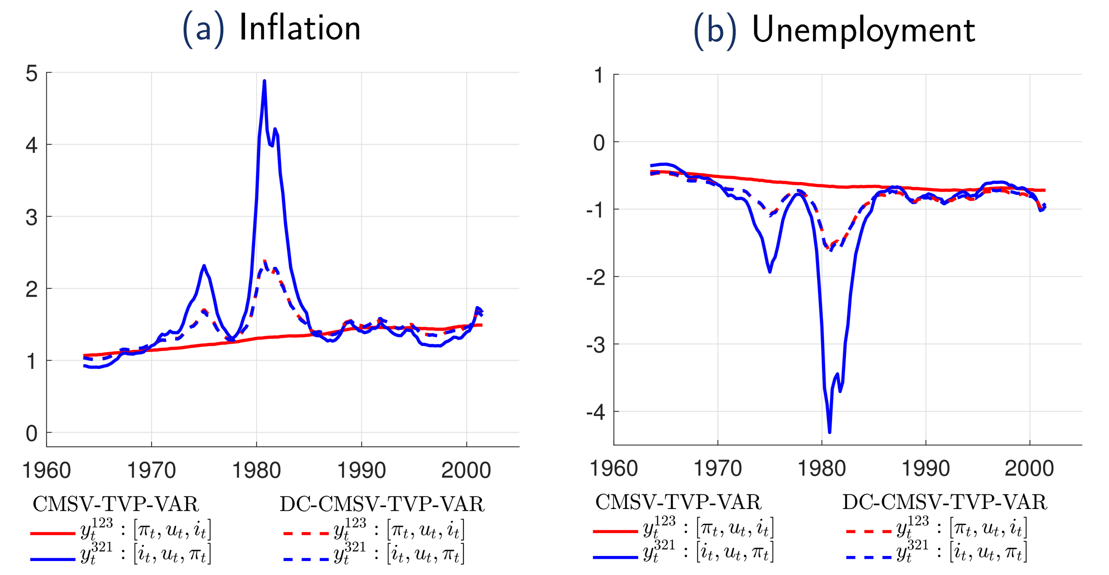

Figure: Estimated long-run US systematic interest rate response

(2) Properties of US inflation-gap persistence (Cogley, Primiceri, and Sargent, 2010)

- decline after great inflation or unchanged?
  - $\Rightarrow$ ambiguous with CPS-TVPSV-VAR model; driven by CMSV heteroskedasticity in TVP
- without CMSV in TVP, unambiguous conclusion!
  - $\Rightarrow$ persistence declined after 1980s

Conclusion

- Variable ordering in CMSV model important!
  - $\Rightarrow$ volatility pattern imposes restrictions
  - $\Rightarrow$ ambiguous conclusions in applications
  - $\Rightarrow$ idiosyncratic volatility pattern not uncommon
- DC-CMSV model as robust alternative
  - $\Rightarrow$ estimates almost ordering invariant
  - $\Rightarrow$ nonlinear contemporaneous relations

benny.hartwig90@gmail.com