A Theory of Collateral Requirements for Central Counterparties*

Jessie Jiaxu Wang
Arizona State University

Agostino Capponi
Columbia University

Hongzhong Zhang
Columbia University

Abstract

This paper develops a framework for designing collateral requirements in a centrally cleared market. Clearing members post collateral—initial margins and default funds—to increase their pledgeable income, thereby committing to risk management. The two types of collateral, however, are not perfect substitutes. Due to its loss-mutualization role, the default fund is more effective than initial margin in aligning members' incentives ex-ante. The optimal mix of collateral allocated as initial margin and default fund balances their relative effectiveness in providing incentives with their relative opportunity costs. Our model predicts increasing use of initial margin when capital requirements become more stringent, and of default funds under distressed market scenarios.

Keywords: Central counterparty (CCP), collateral requirements, initial margin, default fund, macro-prudential regulation

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1 Introduction

A key reform after the financial crisis of 2007–2009 is the mandatory clearing of standardized over-the-counter derivative contracts. After a trade is established between two counterparties, the contractual obligations are replaced by equivalent positions between the two original parties and the central counterparty (CCP). The latter, also known as the clearinghouse, acts as the buyer to the original seller and as the seller to the original buyer. This way, the original counterparties become insulated from each other's default risk—provided that the CCP meets its own obligations. Given the increasing importance of CCPs,[1] their risk management has drawn the attention of policymakers and the public alike (Cunliffe, 2018; Duffie, 2018, 2019). To ensure its resilience, the CCP relies on a so-called default waterfall and collects two types of prepaid collateral from its members: initial margin and default funds. Initial margins are only used to absorb losses triggered by the default of the posting member. Default funds, instead, are shared across members and thus allow for loss mutualization (Pirrong, 2011). Should all collateral resources be depleted, surviving members are responsible for additional losses as part of the CCP recovery mechanism. Despite extensive regulatory debates on the clearing reforms, there is little academic research on the structure of the CCP's default waterfall, and in particular on the design and regulation of margins and default fund requirements.

This paper provides the first framework to analyze the *joint* determination of collateral allocated as initial margins and default funds. While posting collateral increases members' pledgeable income and enhances their risk-management incentives, the two collateral types are imperfect substitutes. Unlike initial margin, default funds are shared across members to mutualize losses ex-post; hence, they are more effective than initial margin in strengthening CCPs' default waterfall, thereby reducing the burden of surviving members at the end of the waterfall. This unique feature makes default fund more effective in aligning members'

[1]The volume of cleared contracts has been growing steadily. The global clearing rates for interest-rate and credit derivatives rose from 55% in 2010 to 75% in 2017 for interest rate derivatives, and over the same period they increased from 10% to 55% for credit default swaps (see Bank for International Settlements, 2019).

incentives ex-ante. The optimal mix of the two collateral types balances the relative effectiveness of default fund and initial margin in providing incentives with their relative opportunity costs. As a result, our findings shed new light on the regulation of collateral requirements and CCP resilience.

Our baseline model considers a finite number of risk-neutral dealers, a continuum of risk-averse protection buyers, and the clearinghouse. To seek insurance against credit risk, protection buyers purchase contracts that resemble credit default swaps (CDS) from dealers. A dealer fully invests his endowment plus the payment collected from the buyer in a risky asset. Because of limited liability, a dealer defaults if the risky asset has a bad state realization; the state realizations are independent across dealers. To partially hedge the asset risk, the dealer can engage in risk management at the cost of a lower return in the good state. However, the risk-management choice is unobservable. This self-interested unobservable action by dealers, combined with their limited liability, generates moral hazard in terms of insufficient risk management, which we refer to as risk-shifting.

The novelty of our study is to model the clearinghouse default waterfall and stress the distinctive role of default funds relative to initial margins in maintaining clearing members' risk-prevention incentives. A default waterfall enables the clearinghouse to guarantee all obligations of its members. To achieve this outcome, the clearinghouse requires its members to post two types of collateral—initial margin and default fund. Depositing assets as collateral cuts down the members' investment scale, thus generating an opportunity cost. Compared with initial margins, the default fund bears a higher unit cost, due to its larger risk weight in the capital requirements. If a member defaults, the loss that exceeds the member's total collateral is mutualized among surviving members using the shared default funds. Because the clearinghouse insulates each member against counterparty risk, the risk-averse CDS buyers are willing to pay a premium for transacting a centrally cleared CDS. This premium motivates dealers to become clearing members and participate in central clearing, despite the cost of posting collateral. The clearinghouse, acting as the social planner, sets the optimal collateral requirements to maximize the value of all market participants, subject to its members' incentive-compatibility constraints.

In the first-best benchmark, all members engage in risk management; costly collateral of neither type is used as long as the marginal financing cost for end-of-waterfall is not too high. By contrast, when members' risk-management choices are unobservable, moral hazard occurs, and members prefer not to hedge their risks. Posting a sufficient amount of collateral increases members' pledgeable income (Holmstrom and Tirole, 1997), allowing them to achieve efficient risk management.

While both types of collateral are instruments to encourage risk management, they are not perfect substitutes. Our analysis shows the advantage of default fund in enhancing CCP's resilience and members' risk-management incentives. We show that an additional unit of collateral posted as default fund is more effective in aligning members' incentives than if posted as initial margin. Intuitively, members' ex-ante risk-management incentives depend on their expected contribution at the end of the waterfall. Such ex-post contribution can be costly to finance because it occurs when multiple members have defaulted and the market is distressed. Importantly, as default fund is shared across members to mutualize losses, it is more effective than initial margin in reducing the ex-post cost of financing end-of-waterfall resources. Consequently, default fund is more productive than initial margin in ex-ante incentive provision.

The optimal incentive-compatible collateral arrangements depend on the relative effectiveness of default fund and initial margin in providing incentives and their relative opportunity costs. Consider the impact of an additional unit of initial margin on a member's value. On the one hand, this generates a "cost-saving effect" by reducing the total opportunity cost of posting collateral (as initial margin imposes a lighter capital requirement). On the other hand, initial margin is less effective in providing incentives; thus, increasing initial margin raises the total amount of collateral needed from a member to prevent risk-shifting, thereby generating a countervailing effect, which we refer to as the "incentive effect." Based on this fundamental trade-off, our model delivers an explicit solution for the optimal mix of collateral. When the capital requirement is more stringent, the optimal collateral policy relies more on initial margin. By contrast, when concerns about financing the end-of-waterfall resources dominate (e.g., during periods of market distress), demanding more default fund contributions from members to mutualize losses is socially desirable. The optimal mix of these two collateral types balances the "cost-saving effect" with the "incentive effect". Altogether, our model provides insights on the relative cost and benefit of the collateral layers that constitute the CCP's default waterfall structure.

Our finding offers a rationale for collecting default funds during periods when financing end-of-waterfall resources demanded by the CCP's recovery mechanism is a nontrivial endeavor for surviving members. Moreover, the default fund level implied by our model differs from the current CPSS-IOSCO (2012) international regulatory guideline known as the "Cover 2" rule.[2] We show that, for clearinghouses consisting of many members, the total default funds collected should cover the shortfalls of a certain fraction of clearing members. This finding is in line with Murphy and Nahai-Williamson (2014) who argue that the "Cover 2" standard is far from prudent.[3]

We further examine the robustness of our results to size heterogeneity in members' outstanding positions, thus capturing the fact that CCPs' exposures tend to concentrate in a few large clearing members (e.g., Office of Financial Research, 2017). We show that our results do not qualitatively change, and the optimal collateral requirements differ by member size. Unlike the banking literature, which has shown that big banks tend to take excessive risk (e.g., O'Hara and Shaw, 1990; Nosal and Ordonez, 2016; Davila and Walther, 2020), our results show that one should be more concerned about the risk management incentives of small members rather than those of big members. Different from big banks that are bailed

[2]The Committee on Payment and Settlement Systems (CPSS) and the International Organization of Securities Commissions (IOSCO) issued principles that require CCPs to maintain a default fund large enough to cover the default of two largest members in extreme yet plausible market scenarios. Clearinghouses in the US must abide by a "Cover 1" system at a minimum, while international and systemic US clearinghouses must comply with the CPSS-IOSCO (2012) regulatory guidelines. Major derivative clearinghouses, such as ICE Clear Credit, CME Clearing, ICE Clear, and LCH.Clearnet, adopt the "Cover 2" rule (Armakola and Laurent, 2017).

[3]Murphy and Nahai-Williamson (2014)[pg. 17] argue that higher levels of financial resources may be needed to ensure the robustness of the clearinghouse: "Perhaps a simple backstop to cover 2 could be considered, such as demanding that the default fund in addition meets the requirement that it is larger than some fixed percentage of the 'cover all' requirement."

out using taxpayers' money, clearing members bail out each other, and their risk management actions impose externalities on each other. Defaults cause a significant shortfall to a small member but appear less significant to a big member.[4] A big member effectively acts as an *internally coordinated group* of small members, and thus finds it easier to internalize the externality and undertakes efficient risk management. This mechanism explains why big members contribute less collateral than small members on a per-unit notional basis.

Our paper adds to the literature on central clearing of derivatives contracts (see, for example, Biais et al., 2012, 2016). Biais et al. (2012) analyze the limitations of central clearing in mutualizing losses. In their model, the risk exposure of protection buyers has an aggregate component, and there is moral hazard among protection buyers. Optimal contracts give protection buyers partial insurance against counterparty risk, and thus incentivize them to search for creditworthy counterparties; however, Biais et al. (2012) do not consider collateral. In Biais et al. (2016), bad news about derivative positions undermines a protection seller's risk-prevention incentive, whereas margin calls can strengthen such an incentive. While they consider the optimal design of margins and central clearing, the two mechanisms are substitutes: when margins are set to improve risk-prevention incentives, clearing is superfluous because counterparty risk is eliminated. Notably, Biais et al. (2016) do not model default fund. By contrast, in our model, posting collateral reduces but not eliminates default risk; hence, collateral and central clearing are complements, making the default waterfall essential to the design of a clearinghouse. A key novelty of our analysis is to explicitly model the different economic roles of default fund and initial margin, and how end-of-waterfall obligations feed back into members' incentives. Due to its unique role in achieving loss mutualization, we show that default fund is more effective than initial margin in aligning members' incentives.

More broadly, our study contributes to the literature on the role of collateral in derivative contracts. Parlatore (2019) provides a microfoundation for the use of financial assets as collateral in the repo market. Bolton and Oehmke (2015) show that granting seniority to

[4]More specifically, the expected contribution to loss-sharing at the end of the waterfall is disproportionately smaller for a bigger surviving member. Accordingly, bigger members find surviving more attractive, thereby facing a less severe incentive problem than a smaller member.

derivatives counterparties in bankruptcy leads to inefficient collateral requirements ex-ante. Oehmke (2014) develops a model of large-scale liquidations of repo collateral that highlights the importance of creditor structure. Biais et al. (2020) further examine the constrained-efficiency of variation margins when margin calls trigger fire sales. We refer to Anderson and Jõeveer (2014) for a discussion of initial margin rules in the context of central clearing, and to Duffie et al. (2015), Capponi et al. (2020), and Cruz Lopez et al. (2017) for empirical analysis of initial margin requirements of CDS clearinghouses. Our paper extends this literature by highlighting that the composition of collateral matters in central clearing.

The loss mutualization among clearing members through default funds shares some commonality with the insurance industry (see Arrow (1974); Raviv (1979); Parlour and Plantin (2008) for related studies). Like the insurance industry, risk-shifting arises when risk is not entirely borne by the agent. Nevertheless, central clearing presents distinguishing features from insurance. Guaranty funds in insurance operate on a post-assessment basis;[5] a higher post-assessment exacerbates ex-ante risk-shifting (Lee et al., 1997). In comparison, default funds are collected as segregated collateral by a clearinghouse; by allowing for ex-post loss mutualization, default fund is more effective in reducing the cost of loss-sharing for a member and thus is more effective in preventing members from shifting risk than initial margin.

Our paper contributes to the debate about the benefits and repercussions of central clearing. While clearing is expected to improve financial stability,[6] a growing literature highlights various forms of inefficiency that potentially arise under central clearing (see, for example, Duffie and Zhu, 2011).[7] Other studies that quantitatively assess the resilience of

[5] After an insurance provider becomes insolvent, the solvent companies are assessed an amount equal to the shortfall at the insolvent firm (Cummins, 1988).

[6] Some theoretical studies analyze the potential advantages of central clearing in increasing transparency (Acharya and Bisin, 2014), economizing on settlement costs (Koeppl et al., 2012), achieving loss mutualization (Zawadowski, 2013), and pooling counterparty risks (Stephens and Thompson, 2014). A few empirical studies also document the value of central counterparty in reducing counterparty risk (Loon and Zhong, 2014; Bernstein et al., 2019), enhancing price stability (Menkveld et al., 2015), completing markets (Vuillemey, 2020), and mitigating fire sales (Vuillemey, 2019).

[7] Duffie and Zhu (2011) show that clearing could increase counterparty risk if the clearing process is fragmented across multiple CCPs. Relatedly, Duffie et al. (2015) use CDS data to show that central clearing does not necessarily reduce collateral demand; Huang (2019) shows that a for-profit clearinghouse may have incentives misaligned with financial stability, in that it contributes less equity and charges a collateral amount lower than the socially optimal level; Pirrong (2014) and Spatt (2017) argue that central clearing reforms

clearinghouses via stress testing call for better design of the default waterfall mechanism (Boissel et al., 2017; Menkveld, 2017; Paddrik et al., 2020). Legal scholars echo existing concerns on the current design of clearinghouses (see, for instance, Yadav (2013) and Roe (2013)). We contribute to this policy debate by offering a normative analysis of collateral requirements. Our model accounts for scenarios when prefunded collateral resources are depleted and highlights how the collateral composition impacts members' liability at the end of the waterfall, which in turn determines their risk-management incentives ex-ante.

The paper proceeds as follows. Section 2 describes the model. Section 3 studies the optimal design of collateral requirements in the baseline model. Section 4 studies the robustness of our results when we account for heterogeneity in exposures among members. We discuss the policy and empirical implications in Section 5. Section 6 concludes. Proofs are delegated to the Appendix.

2 Model

We develop a parsimonious model to study how different types of collateral (i.e., initial margins and default funds) affect members' risk-taking incentives. There are two dates, $t = 0, 1$, a continuum of risk-averse protection buyers, N risk-neutral dealers who sell protection contracts, and a platform for central clearing (the CCP). At $t = 0$, the parties design and enter the contract; risk-management decisions are made. At $t = 1$, payoffs are realized.

Agents and Assets. Protection buyers are identical, with mean-variance preferences and risk-aversion parameter $\gamma > 0$. They are exposed to an adverse event at $t = 1$ which can be interpreted as a credit event: The return is $-D$ if the adverse event occurs, and zero otherwise. The adverse event happens with probability $p_c \in (0, 1)$.

Protection buyers seek insurance via a derivative contract from dealers, who are risk-neutral and have limited liability. The derivative contract mandates that the buyer pays the amount P to the dealer at $t = 0$; in exchange, the dealer promises to pay D to the buyer at

may redistribute, rather than reduce, risk; and Biais et al. (2012) and Kubitza et al. (2019) show that central clearing is not effective to insure aggregate risk.

$t = 1$ upon the occurrence of the adverse event. The derivative contract resembles a CDS: It swaps an upfront payment today for a promise to receive a random amount at $t = 1$. We use the institutional features of a CDS for concreteness, but our model is general to accommodate any class of derivative contracts in which the counterparty risk is one-sided. We model the adverse event as being systematic, so that the contracts effectively resemble index CDSs, which account for more than 99% of centrally cleared CDS contracts based on the report by Financial Stability Board (2018).[8]

Each dealer has an initial asset amount $A_0 > D$. Upon selling one unit of the derivative contract and receiving P from the buyer, the dealer posts part of his balance sheet as collateral to the buyer, and invests the residual amount in a risky and productive asset.[9] Dealers have unique skills to manage the risky asset and to extract excess return; protection buyers and the clearinghouse are not endowed with those skills. The risky asset has an expected return greater than one; it generates a per-unit return of zero in the bad state, which occurs with probability q_r, and of R_r in the good state which occurs with probability $1 - q_r$. The return of the risky asset is assumed to be independent across dealers, and also independent of the aggregate credit event.[10]

The dealer may be required to post collateral because of a moral hazard problem. Specifically, the dealer takes an unobservable risk-management action $a \in \{s, r\}$ at $t = 0$ to partially hedge the riskiness of his assets. When the dealer chooses $a = s$ (which can be interpreted as prudent risk management to make the asset safer), the probability of the bad state realization is reduced to q_s, at the cost of a lower return R_s in the good state, i.e.,

$$0 < q_s < q_r, \quad R_s < R_r. \tag{1}$$

[8] We focus our analysis on systematic credit risk because the continuum of protection buyers can diversify any idiosyncratic component of the credit risk among themselves. However, protection buyers cannot self-insure or trade alternative contracts to hedge systematic credit risk. This assumption grants an economically meaningful role to the derivative contracts, thereby ensuring that buyers value them.

[9] For analytical tractability, we assume dealers have the same size by normalizing the number of derivative contracts sold to one. Nonetheless, we consider an extension where sizes are heterogeneous in Section 4.

[10] The risky return is related to the asset side of the balance sheet, and not directly to the credit event underlying the derivative contract. While one could introduce a systematic component in the return outcome of the risky assets, the idiosyncratic risk component remains the necessary ingredient to make loss-sharing meaningful.

The risk-management process reflects the unique skills of the dealer and is therefore difficult for outside parties to observe and monitor. Combined with limited liability of the dealer, the upside potential of the risky asset generates moral hazard in terms of insufficient hedging.[11] The expected return of the asset is thus $(1-q_a)R_a$, which we denote as μ_a. We assume the expected return is higher when hedging is conducted than otherwise, so *risk management is efficient*, i.e.,

$$\mu_s > \mu_r > 1. \tag{2}$$

We thus refer to the risk-management choice $a_i = s, \forall i$, as the *first-best* benchmark, and refer to $a_i = r$ as dealer i engaging in *risk-shifting*.

Collateral and Central Clearing. Under central clearing, dealers establish direct contractual relationships with the CCP.[12] Acting as a public utility, the CCP does not have access to risky assets and thus has no profit-making incentive. Its role is to guarantee the promised payments to buyers at $t = 1$, and to design collateral rules that maximize the value of its members (i.e., it acts as a benevolent social planner).

At $t = 0$, dealers post two types of collateral—initial margin and default fund.[13] Both types of collateral are deposited in the form of safe assets (cash or Treasuries) with a gross return of one. Posting collateral reduces a dealer's scale of investment in the risky asset, thereby inducing a deadweight cost. Compared with initial margins, default fund posting generates an additional unit cost, $\delta > 0$, to the dealer by carrying a higher risk weight in the capital requirements than other forms of collateral; see Basel Committee's publication on Banking Supervision (2014).[14]

[11] Because of the upside potential of the risky asset, $R_r > R_s$, our model does not require costly effort to generate moral hazard as in e.g., Bolton and Oehmke (2015) and Biais et al. (2016).

[12] While the protection buyers could also be members of the CCP, we focus on the risk-shifting incentives of derivatives dealers because their risk-management choice is non-contractible, and they are subject to default. Consequently, the CCP has risk exposure only to dealers, and not to buyers.

[13] Clearing members maintain both proprietary positions, for which they post margins directly, and client accounts to clear trades on behalf of their clients. In the latter case, initial margins are posted by clients. The gross notional and number of contracts traded on the members' proprietary accounts are typically larger than the corresponding quantities for the client positions. Capponi et al. (2020) provide supporting empirical evidence for ICE Clear Credit.

[14] The capital requirements by the Basel Committee for bank exposures to central counterparties apply a

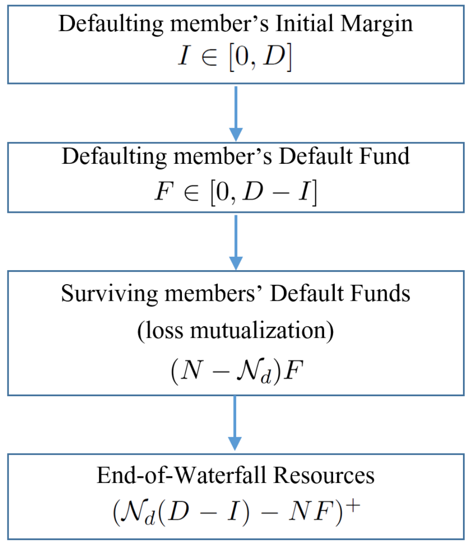

Figure 1. Default Waterfall Structure.

The pre-funded collateral resources and potential ex-post contribution by dealers allow mutualizing losses caused by defaults of clearing members. The mechanism behind the loss-mutualization is the clearinghouse default waterfall; see Figure 1. Initial margins, denoted by $I \in [0, D]$, serve as the first line of defense against losses when the posting dealer cannot fulfill the promise D to the buyer. The default fund of the defaulting dealer, $F \in [0, D - I]$, constitutes the second loss-absorption layer of the default waterfall. Unlike initial margins which are used to cover losses of the posting member only, default funds are shared among clearing members, which leads to the third layer of the default waterfall. Losses exceeding the defaulting members' collateral are allocated proportionally to the default funds of surviving members.[15] Hence, a surviving dealer might incur a loss when other members default, creating an externality among members.

The last layer consists of end-of-waterfall resources. Multiple clearing members can de-

risk-weight of 2% to collateral held at the CCP (excluding default fund contributions) for a clearing member bank. The capital requirements apply a different formula for the risk weight of default fund contributions. Determined comprehensively by the financial soundness of the CCP, the approach puts a floor of 2% on the risk weight of default fund exposure and a risk weight higher than 2% when the exposure amount of the CCP is not fully collateralized.

[15]There is not yet a universally agreed-upon loss allocation rule. Major clearinghouses, such as the ICE Clear Credit, adopt a pro-rata basis for futures, options, and CDS contracts (ICE, 2016).

fault. If the cumulative loss generated by all defaults reach the threshold $l := \lceil \frac{NF}{D-I} \rceil$, all resources till end of the waterfall are depleted. Let $\mathcal{N}_d$ be the number of defaulting members, i.e., $\mathcal{N}_d := \sum_i \mathbb{I}_{i \text{ defaults}} = \sum_i \mathbb{I}_{\text{credit event} \cap \text{bad state for } i}$, then $(\mathcal{N}_d(D-I) - NF)^+$ denotes losses that are not covered by the total collateral resources posted by members. To absorb these additional losses, surviving members contribute to the end-of-waterfall resources (CPMI-IOSCO, 2014; Cont, 2015; Duffie, 2015). Such additional contributions are financed at a unit cost α. We assume that[16]

$$0 < \alpha < \frac{\mu_s - 1}{p_c q_s}. \tag{3}$$

We make two simplifying assumptions in our model of the default waterfall structure. First, the CCP can always retrieve enough resources (at a cost) to honor the obligations towards buyers. This assumption is supported by the current stringent end-of-waterfall risk-management practices, and the regulatory progress of CCPs' failure resolution (Cunliffe, 2018).[17] Second, we do not consider the so-called "CCP's skin-in-the-game." Although in practice most CCPs absorb losses exceeding the collateral of a defaulting member using their own equity before using funds of surviving members, the CCP's equity contribution to the default waterfall is typically small relative to the collateral posted by members.[18] Moreover, the seniority structure in practice is intended to reduce moral hazard on the part of the CCP (Duffie, 2015; Huang, 2019). In our model and as Cunliffe (2018) points out, "CCPs are not risk-taking entities. Rather, they are mechanisms for the management and for the mutualization of their members' counterparty risk. Their greatest, though not only, vector of risk is the default of one or more of their members."

[16] If α is too large (violating condition (3)), dealers prefer a position fully collateralized by initial margins, which is an economically trivial case.

[17] From Cunliffe (2018): "Unlike banks, if prefunded resources are insufficient to absorb losses, the CCP does not become insolvent. CCP's rulebooks, to which all members agree in advance, allow a further series of recovery actions, in a prescribed sequence. A CCP could require its members to contribute more resources, it could reduce its liabilities to some of its members, and ultimately it could "tear-up" (i.e. cancel) all or some of its contracts–in effect the closure of the clearing service."

[18] The filings of 60 CCPs, in compliance with the Principles for Financial Market Infrastructures (CPMI-IOSCO, 2015), reveal that the equity contribution of an average CCP clearing interest rate swaps is only 1.6% of its prefunded resources, while the corresponding figure for a CCP clearing credit default swaps is on the order of 2% (Paddrik and Zhang, 2019).

Pricing of Bilateral and Centrally Cleared Contracts. The price of the derivative contract is determined by the seller's default risk and buyer's risk-aversion. We make the following assumption on protection buyers.

Assumption 1 *Protection buyers have zero bargaining power in the negotiation of the derivative contract. Moreover, their risk-aversion parameter is sufficiently high, i.e., it satisfies* $\gamma > \underline{\gamma}$ *where the explicit expression of* $\underline{\gamma}$ *is given in Eq.* (A18) *of the Appendix.*

Under Assumption 1, a dealer has full bargaining power and is able to extract monopoly rent (so protection buyers are held to their reservation utility). When matched with a protection buyer, each dealer makes an exclusive take-it-or-leave-it offer.[19] We also assume that the protection buyers are sufficiently risk averse, in that they attach a high enough value to a centrally cleared contract.

Let P_{BT} be the price of a bilaterally traded derivative contract, and P_{CCP} be the price of a centrally cleared derivative contract. A protection buyer in autarky has a payoff of zero in absence of the adverse event and a payoff of $-D$ otherwise, so her mean-variance utility is

$$-p_c D - \gamma p_c (1 - p_c) D^2. \tag{4}$$

Entering into a bilaterally traded derivative contract, a protection buyer reduces her credit risk exposure to $p_c q_a$. The buyer has a payoff of $-P_{BT}$, unless the dealer defaults in a credit event, which occurs with probability $p_c q_a$ and leaves the buyer with a payoff of $-D - P_{BT}$. Hence, the utility of the protection buyer becomes

$$-p_c q_a D - \gamma p_c q_a (1 - p_c q_a) D^2 - P_{BT}. \tag{5}$$

A protection buyer whose derivative contract is centrally cleared has a constant payoff of

$$-P_{CCP}. \tag{6}$$

[19]This assumption is supported by the empirical observation that derivatives markets tend to be concentrated on a small number of dealers who can then charge markups from end-users (Brunnermeier et al., 2013; Peltonen et al., 2014; Li and Schurhoff, 2019; Siriwardane, 2019). Consistent with this assumption, the CCP (acting as a social planner) places no weight on protection buyers.

We solve for P_{BT} and P_{CCP}. In a bilateral setting and supposing that the dealer chooses to hedge, the buyer is offered a price P_{BT} such that her utility in (5) is at least as large as her reservation value in (4). In a centrally cleared setting, instead, the buyer is offered a price P_{CCP} that makes her utility in (6) equal to her reservation value. This yields the prices

$$P_{BT} = p_c D(1 - q_s)\left(1 + \gamma D(1 - p_c - p_c q_s)\right), \tag{7}$$

$$P_{CCP} = p_c D\left(1 + \gamma D(1 - p_c)\right). \tag{8}$$

When a CCP guarantees the trade, the payment obligation of the derivative contract is honored with certainty, so the buyers are fully hedged against counterparty risk. Buyers value this guarantee because they are risk averse. Hence, they are willing to pay a positive premium $P_{CCP} - P_{BT} = p_c q_s D + \gamma p_c q_s\left(1 - p_c q_s\right) D^2$ for a centrally cleared contract.[20] Our model thus implies that central clearing carries economic value because it reduces counterparty risk.[21]

Given the contract prices (7) and (8), the balance sheet size of a dealer who trades bilaterally is $A_{BT} = A_0 + P_{BT}$, and that of a dealer who centrally clears his contract is $A_{CCP} = A_0 + P_{CCP}$. To generate a nontrivial role of collateral, we define the pledgeablility of a dealer's asset and make the following assumption.

Assumption 2

$$\mathcal{P} := \frac{\mu_s - \mu_r}{p_c(q_r - q_s)} < \frac{D}{A_{CCP}}. \tag{9}$$

Condition (9) implies that, without posting collateral, a dealer does not hedge risk regardless of whether he is trading bilaterally or through a clearinghouse; see Section 3.1 for more discussion. Since risk management is efficient from condition (2), the asset pledgeablility is positive, i.e., $\mathcal{P} > 0$. Because the dealer's balance sheet size is greater than his liability ($A_{CCP} > A_0 > D$), this further implies that $\mathcal{P} < 1$.

[20]This premium consists of two parts: the expected loss due to default of the counterparty under the adverse event, $p_c q_s D$, and the compensation for the reduction in the variance of payoffs, $\gamma p_c q_s(1 - p_c q_s)D^2$. The higher the risk aversion γ, the higher the premium earned by dealers.

[21]In the context of single-name CDSs, Loon and Zhong (2014) find that the spreads of centrally cleared derivative contracts increase around the initiation of central clearing.

3 The Optimal Collateral Requirements

In this section, we study members' incentives in undertaking risk management and the optimal design of collateral requirements. Section 3.1 studies the value of a dealer in a market where contracts are traded bilaterally. Sections 3.2 and 3.3 consider the setting in which traded contracts are centrally cleared, focusing respectively on the first-best benchmark where a benevolent social planner imposes members' risk-management actions and the second-best outcome where members strategically decide on risk management. Section 3.4 characterizes the optimal collateral requirements under the second-best outcome.

3.1 Dealer's Value in a Bilateral Market

By our assumption (2), hedging improves the expected return of a dealer. Next we show that private and social incentives are misaligned if dealers trade bilaterally without posting collateral to protection buyers.

A dealer who takes the risk-management action $a \in \{s, r\}$ achieves a payoff from the risky asset equal to $A_{BT}R_a$ in the good state, except when the credit event occurs and the dealer fully honors promise D to the buyer. In the bad state, which occurs with probability q_a, the dealer receives a zero payoff. The expected profit of a dealer in a bilateral market without posting collateral is given by

$$\max_{a \in \{s,r\}} (1 - q_a)(A_{BT}R_a - p_c D) = A_{BT}\mu_r - (1 - q_r)p_c D, \tag{10}$$

where the last equality follows directly from condition (9) and the inequality $A_{CCP} > A_{BT}$, which jointly imply that $A_{BT}\mathcal{P} < D$. Without posting collateral, dealers have insufficient pledgeable assets; thus, dealers prefer to engage in risk-shifting. Limited liability undermines a dealer's incentive to hedge at the cost of losing the upside potential of the risky asset, whereas the benefit of staying solvent accrues in part to protection buyers. Collateral, C, can be used to disincentivize the dealer from engaging in risk-shifting:

$$(A_{BT} - C)\mu_s + (1 - p_c)C - (1 - q_s)p_c(D - C) \geq (A_{BT} - C)\mu_r + (1 - p_c)C - (1 - q_r)p_c(D - C).$$

The above condition may be rewritten as $(A_{BT}-C)\mathcal{P} \geq D-C$. Hence, the minimum amount of collateral needed to induce efficient risk management, denoted by C_{BT}, is given by

$$C_{BT} = D - \frac{(A_{BT}-D)\mathcal{P}}{1-\mathcal{P}} = \frac{D - A_{BT}\mathcal{P}}{1-\mathcal{P}} \in (0, D). \tag{11}$$

Let V_{BT} be the expected profit of a dealer who posts the incentive-compatible collateral C_{BT} in a bilateral market; V_{BT} is given by

$$V_{BT} = (A_{BT} - C_{BT})\mu_s + (1-p_c)C_{BT} - (1-q_s)p_c(D - C_{BT}). \tag{12}$$

3.2 Central Clearing: the First-Best Benchmark

Value of a Clearing Member. A dealer participating in central clearing posts initial margin I and default fund F as collateral. The marginal deadweight loss is μ_a associated with posting initial margin and is $\mu_a + \delta$ with posting default fund. Given the collateral requirements and actions of other dealers, dealer i takes a risk-management action to maximize his expected profit. Let $V(a_i, a_{-i}; I, F)$ denote the expected profit of dealer i in a centrally cleared market. This profit depends on the dealer's action $a_i \in \{r, s\}$, the action of other dealers a_{-i}, and the collateral (I, F), i.e.,

$$\begin{aligned} V(a_i, a_{-i}; I, F) = -\delta F + (1-p_c)(I+F) + \\ (1-q_{a_i})\left[(A_{CCP} - I - F)\, R_{a_i} - p_c\left(D - I - F + X(a_{-i}; I, F)\right)\right]. \end{aligned} \tag{13}$$

The term $X(a_{-i}; I, F)$ denotes the expected contribution to loss-sharing and the associated financing costs for a dealer whose asset has a good state realization in a credit event, i.e.,

$$X(a_{-i}; I, F) := \mathbb{E}^{a_{-i}}\left[\frac{\mathcal{N}_d(D-I-F)}{N-\mathcal{N}_d} + \alpha\left(\frac{\mathcal{N}_d(D-I-F)}{N-\mathcal{N}_d} - F\right)^+\right], \tag{14}$$

where $\mathbb{E}^{a_{-i}}[\cdot]$ denotes the expectation under the probability distribution induced by the action profile a_{-i}, the occurrence of the credit event, and a good state realization for dealer i; $\mathcal{N}_d$ denotes the number of members, excluding i, who default in a credit event.

The first term in $V(a_i, a_{-i}; I, F)$ represents the additional cost of posting default fund collateral relative to initial margins, due to the increased risk weight in capital requirements. To understand the remaining terms, we list all possible outcomes at $t = 1$. In the good state (which occurs with probability $1 - q_{a_i}$), dealer i obtains investment proceeds $(A_{CCP} - I - F)\, R_{a_i}$. Moreover, the following happens depending on the occurrence of the adverse event:

(i) The adverse event occurs. Dealer i delivers D to the protection buyer and retrieves his initial margin I. The default fund posted by dealer i is used to absorb the shortfalls of defaulting members, reflecting loss mutualization. The total shortfall beyond the defaulting members' collateral, $\mathcal{N}_d(D - I - F)$, is shared equally among the surviving members, with each bearing the portion $\frac{\mathcal{N}_d(D-I-F)}{N-\mathcal{N}_d}$. If the share of shortfall allocated to dealer i is lower than his posted default fund, the residual is returned to dealer i; however, if the share of shortfall exceeds F (or equivalently if $\mathcal{N}_d \geq \lceil \frac{NF}{D-I} \rceil$), dealer i contributes an additional amount of $\left(\frac{\mathcal{N}_d(D-I-F)}{N-\mathcal{N}_d} - F\right)$ beyond the prefunded default fund as part of the end-of-waterfall mechanism. In such case, dealer i bears a proportional financing cost at rate α for the ex-post contribution.

(ii) The adverse event does not occur. Dealer i recovers all segregated collateral, $I + F$.

If, instead, the bad state realizes (with probability q_{a_i}), dealer i obtains zero return from the risky investment, and the following outcomes are realized:

(i) The adverse event occurs. The dealer's initial margin and default fund cover partially the promised payment D to the protection buyer. Dealer i gets a zero payoff.

(ii) The adverse event does not occur. Dealer i recovers the entire collateral.

First-Best Benchmark. Having defined dealers' expected profit in a centrally cleared market, we consider the case in which their risk-management actions are observable. There is no risk-shifting and the first-best is achieved. This setting offers a benchmark against which we can identify the inefficiencies that arise when dealers' risk-management actions are not observable.

In the first-best benchmark, the clearinghouse (who acts as the social planner) dictates clearing members' hedging actions, a^{FB}, and collateral requirements, (I^{FB}, F^{FB}), to maximize the sum of all clearing members' expected profits given by Eq. (13).[22] Let $W^{FB}(a; I, F)$ be the objective function of the social planner; the social planner solves

$$\max_{(a;I,F)} W^{FB}(a; I, F) := \max_{(a;I,F)} \frac{1}{N} \left\{ \sum_i V(a_i, a_{-i}; I, F) \right\}. \tag{15}$$

Proposition 1 characterizes the first-best outcome in terms of risk management actions and collateral requirements.

Proposition 1 *The first-best risk-management action is $a_i^{FB} = s$, $\forall i$. If the marginal financing cost at the end of the waterfall is not too large, i.e., $\alpha \leq \frac{\mu_s - 1 + \delta}{p_c(1-(1-q_s)^N)}$, the first-best collateral is zero, $\left(I^{FB}, F^{FB}\right) = (0, 0)$; otherwise, $\left(I^{FB}, F^{FB}\right) = (0, \frac{l^{FB}}{N} D)$ where l^{FB} is the minimum number of defaults that exhaust all prefunded collateral and given by Eq. (A1).*

In the first-best benchmark, the social planner internalizes any externality caused by the loss-mutualization mechanism among members. Hence, dealers engage in risk management because doing so increases their expected profits (see condition (2)) and reduces the total financing cost for the end-of-waterfall resources. The default risk of each member is kept at a minimum but remains positive and equal to $p_c q_s$. In the case of member defaults, losses are mutualized among the surviving members, and protection buyers are fully guaranteed with the promise D.

When the marginal financing cost for end-of-waterfall resources is not too high, collateral of neither type is used. This is because without the benefit of imposing incentives, posting collateral ex-ante is more costly than financing the shortfall ex-post. In this case, costly collateral would only be collected as an incentive device. If instead, the financing cost for end-of-waterfall is high, it is optimal to post collateral ex-ante to cover the shortfall partially. Compared with margin, default fund is more effective in mutualizing losses and reducing the

[22] Recall from Assumption 1 that protection buyers have zero bargaining power in the pricing of the derivative contracts; they are held to their reservation utility, which is independent of the clearinghouse's choice variables. For this reason, the clearinghouse places no weight on protection buyers when maximizing the total value of market participants.

end-of-waterfall burden for members, thus is the preferred collateral instrument. Notably, in both cases, initial margin is never used, and a fully collateralized position is never optimal.

3.3 Central Clearing: Members' Risk-shifting Incentives

In what follows, we consider the case in which dealers' risk-management actions are not observable. Like in the bilateral market without collateral discussed in Section 3.1, dealers choose not to hedge if collateral requirements are absent in the centrally cleared market. Posting collateral increases the *pledgeable income* and allows members to commit to hedging. Consider, for instance, the extreme case of a fully collateralized position: members would never default, and protection buyers are fully insured. Nevertheless, a fully collateralized position is costly. Central clearing, by mutualizing default losses, saves on collateral cost while providing full insurance to buyers. Two key questions arise: How much collateral is sufficient? Are the two types of collateral—initial margin and default fund—*perfect substitutes*?

Our results reveal that an additional unit of collateral posted as default fund is more effective in aligning members' incentives than if posted as initial margin. The intuition is as follows. According to the clearinghouse's recovery mechanisms, surviving members are responsible for losses at the end of the waterfall. Such additional ex-post contribution generates financing costs, the amount of which depends on the composition of collateral posted by members. More specifically, the clearinghouse can use the default funds of all members, but the initial margin of only defaulting members. An additional unit of default fund is thus more versatile than initial margin in avoiding the ex-post financing cost at the end of the waterfall. Consequently, default fund is more productive in ex-ante incentive provision. In sum, the difference between initial margin and default fund in providing incentives is closely tied to the loss-sharing mechanism and the financing cost of end-of-waterfall resources, which we now discuss in detail.

Suppose the credit event occurs, the good state realizes for the risky asset of dealer i, and g out of the other members hedge. From equation (14), the expected contribution to loss-sharing and the associated financing costs for a surviving dealer i is given by

$$X^g(I,F) := X(a_{i_1} = \cdots = a_{i_g} = s, a_{i_{g+1}} = \cdots = a_{i_{N-1}} = r; I, F), \tag{16}$$

where $i_1, i_2, \ldots, i_{N-1} \in \{1, \ldots, i-1, i+1, \ldots, N\}$ are arbitrary. This means that $X^g(I,F)$ only depends on the number of members other than i who hedge and not on the identity of those members.[23] Lemma IA.1 of the online Appendix establishes that the function $X^g(I,F)$ is piecewise linear in both I and F, and strictly decreasing in g, I, F. Hence, an increase in collateral posting means a higher pledgeable income, a lower expected contribution to mutualizing defaulting members' shortfall, and a lower ex-post financing cost for a surviving member. Similarly, both the expected contribution to loss-mutualization and the ex-post financing cost for a surviving member decline as more members choose to hedge. Proposition 2 summarizes our main results on the risk-management strategy of clearing members.

Proposition 2 *Given collateral requirements I and F, members' risk-management action in the Pareto-dominant Nash equilibrium satisfies*

$$a(I,F) = \begin{cases} r, \forall i & 0 \le F < \hat{F}(I) \\ s, \forall i & \hat{F}(I) \le F \le D - I \end{cases}. \tag{17}$$

For a given I, the function $\hat{F}(I)$ is uniquely determined by the following equation

$$D - I - \hat{F}(I) + X^{N-1}(I, \hat{F}(I)) = (A_{CCP} - I - \hat{F}(I))\mathcal{P}, \tag{18}$$

where $0 \le \hat{F}(I) < D - I$. The function $\hat{F}(I)$ is piecewise linear and strictly decreasing in I with slope greater than -1. At the set of kinks, $\{I_l : \frac{N\hat{F}(I_l)}{D-I_l} = l, l = 0, 1, ..., \lceil \frac{N\hat{F}(I=0)}{D} \rceil - 1\}$, $\hat{F}(I)$ is continuous and convex.

[23] The explicit expression is $X^g(I,F) = \sum_{\mathcal{N}_d=0}^{N-1} f^g(\mathcal{N}_d)\frac{\mathcal{N}_d(D-I-F)}{N-\mathcal{N}_d} + \sum_{\mathcal{N}_d=\lceil \frac{NF}{D-I} \rceil}^{N-1} f^g(\mathcal{N}_d)\alpha\frac{\mathcal{N}_d(D-I)-NF}{N-\mathcal{N}_d}$, where $\lceil \cdot \rceil$ is the ceiling function and $f^g(\mathcal{N}_d)$ is the probability that $\mathcal{N}_d$ of the $N-1$ members default when g of the $N-1$ members hedge. $\frac{\mathcal{N}_d(D-I-F)}{N-\mathcal{N}_d}$ is the contribution by member i to sharing losses of defaulting members and $\alpha\frac{\mathcal{N}_d(D-I)-NF}{N-\mathcal{N}_d}$ is the cost to finance the ex-post contribution beyond the prefunded default fund collateral. When $\mathcal{N}_d < \lceil \frac{NF}{D-I} \rceil$, the pre-funded collateral is sufficient to meet the aggregate shortfall, so the ex-post contribution by member i is zero.

Proposition 2 characterizes how collateral requirements affect members' risk-management action in equilibrium; see Figure 2 for an illustration. For $F < \hat{F}(I)$ (the pink region), "all not hedging" is the unique equilibrium because each member chooses not to hedge regardless of others' action. For $F \geq \hat{F}(I)$ (the blue region), "all hedging" is the Pareto-dominant Nash equilibrium. As elaborated in the Appendix, two cases are relevant in the blue region. These two cases are characterized by a cutoff function $\bar{F}(I)$ (the dashed line) which is determined by equation (A3). For $F > \bar{F}(I)$, "all hedging" is the unique equilibrium because each member chooses to hedge regardless of others' action. For $\hat{F}(I) \leq F \leq \bar{F}(I)$, both "all hedging" and "all not hedging" are equilibria, and "all hedging" is the Pareto-dominant equilibrium as each member achieves a higher expected profit when conducting risk-management than not. Observe that all members hedge under a pair of collateral requirement $(I, \bar{F}(I))$. If it is common knowledge that collateral requirements are designed to induce risk-management actions, no member would unilaterally deviate from hedging until the combination of collateral types falls below the curve $\hat{F}(I)$. For this reason, we adopt the Pareto-dominant equilibrium and consider $(I, \hat{F}(I))$ as the minimum combinations of incentive-compatible collateral that induces the "all hedging" equilibrium.

Importantly, we show that there exists no other equilibrium, such as some members hedge and others do not hedge. Intuitively, as more members hedge, the expected contribution and losses due to loss-mutualization for a surviving member decreases, and hedging incentives are enhanced. To see this, suppose g members choose to hedge and $(N - g)$ members choose not to hedge, for some $g = 1, \ldots, N - 1$. Then, any hedging member faces $g - 1$ other hedging members and finds it more profitable to hedge; by contrast, any member choosing not to hedge faces g hedging members (thus a stronger incentive to hedge), yet finds it more profitable to not to hedge. Hence, we obtain a contradiction. Essentially, members' risk-management actions display strategic complementarity, in line with e.g., Cooper and John (1988) and Farhi and Tirole (2012).

Proposition 2 conveys several key insights. First, members' incentives for risk management strengthen as collateral increases. Like in the bilateral market, posting collateral

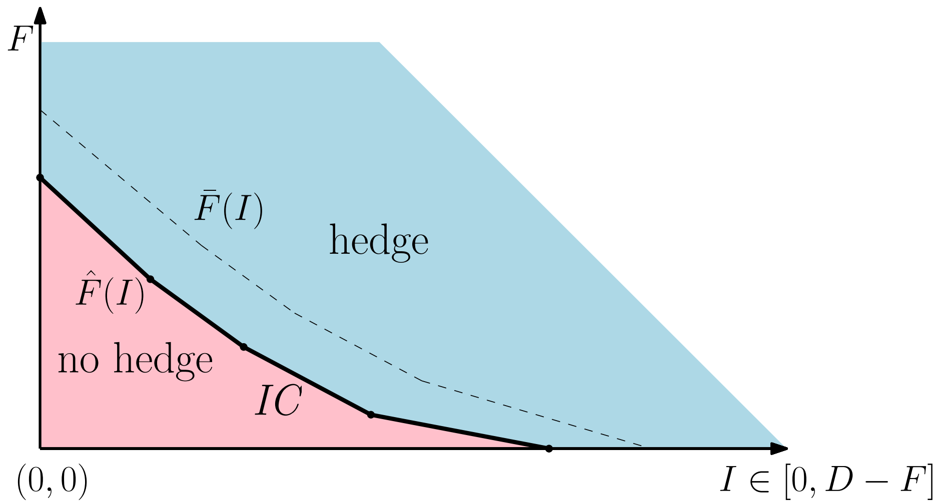

Figure 2. Members' Equilibrium Risk-management Action. This figure illustrates how collateral requirements affect members' risk-management action in equilibrium. The vertical and horizontal axes represent, respectively, default fund F and initial margin I. In the pink region where $0 \leq F < \hat{F}(I)$, "all not hedging" is the unique equilibrium; In the blue region where $\bar{F}(I) < F \leq D - I$, "all hedging" is the unique equilibrium; In the blue region where $\hat{F}(I) \leq F \leq \bar{F}(I)$, both "all hedging" and "all not hedging" are equilibria, and "all hedging" is the Pareto-dominant equilibrium. The bolded piecewise linear curve labeled with IC traces the minimum combinations of incentive-compatible collateral, $(I, \hat{F}(I))$, along which efficient risk-management action of all members is induced.

increases asset pledgeability. Yet, in the centrally cleared market collateral additionally reduces the expected contribution and financing costs through loss-mutualization (given by $X(a_{-i}; I, F)$ in Eq. (14)). As such, collateral helps to align members' incentives toward the first-best benchmark. To be "incentive-compatible," collateral needs to be sufficiently high. The left-hand side of Eq. (18) gives the maximum ex-post liability for a surviving member to induce hedging. For a given initial margin I, the default fund needs to reach the cutoff value $\hat{F}(I)$. A higher default fund contribution means a lower liability for a surviving member, which, in turn, makes survival more attractive and encourages members to hedge.

Second, while both types of collateral are instruments to avert risk-shifting, they are not perfect substitutes. This is illustrated in Figure 2. The bolded curve, given by Eq. (18), represents the minimum combinations of incentive-compatible collateral types along which risk management among all members is induced. The required default fund $\hat{F}(I)$ is linearly decreasing in I. Importantly, Proposition 2 states that $d\hat{F}(I)/dI \in (-1, 0)$: When the initial margin decreases by one unit, the incentive-compatible default fund increases by less than one unit. This is because default fund is shared across members to mutualize losses and is thus more effective than initial margin in reducing the ex-post financing cost at the end of the waterfall. Hence, when substituting default fund for initial margin, members post a lower amount of total collateral to prevent risk-shifting. This result has important implications: (1) Initial margin and default fund are substitutes, albeit imperfect ones, to induce efficient risk management; and (2) a unit of default fund is more useful than a unit of initial margin in aligning members' incentives.

3.4 Optimal Collateral Requirements

In this section, we proceed to characterize the optimal collateral requirements in the second-best allocation when dealers' risk-management actions are unobservable. Proposition 2 shows that collateral requirements can be set to induce efficient risk management. To pin down the optimal mix of the two collateral types, the social planner balances the relative effectiveness of default fund and initial margin in providing incentives with their relative opportunity cost.

Definition 1 *The optimal collateral requirements, (I^{SB}, F^{SB}), maximize the clearing members' expected profits in a centrally cleared market, taking as given the members' equilibrium risk-management action profile. Formally,*

$$(I^{SB}, F^{SB}) = \underset{(I,F)}{\arg\max}\, V(a_i, a_{-i}; I, F) \tag{19}$$

subject to

> *(i) the incentive-compatibility constraint: all members choosing to hedge is consistent with members' equilibrium risk-management action profile, $a(I, F)$, given by Eq. (17).*
>
> *(ii) the participation constraint: the expected profit of a dealer in a centrally cleared market given by Eq. (13) must not be lower than the corresponding profit in a bilateral*

market, V_{BT}, given by Eq. (12).

Different from the first-best benchmark, the social planner designs collateral requirements taking the members' equilibrium risk-management action profile $a(I, F)$ as given. We refer to the optimal collateral requirements (I^{SB}, F^{SB}) as the *incentive-constrained optimal* collateral. To solve for (I^{SB}, F^{SB}), first note that Assumption 1 guarantees the participation constraint to be slack when a clearing member's expected profit is maximized along the incentive-compatibility curve. This result is established by Lemma 1.

Lemma 1 *The maximum expected profit of a clearing member who posts incentive-compatible collateral,* $\max_I V(a = s; I, \hat{F}(I))$*, is greater than the expected profit of a dealer who posts incentive-compatible collateral in a bilateral market, given by Eq.* (12)*, and is greater than the value of a dealer whose position is fully collateralized with initial margin.*

Lemma 1 is intuitive. When the clearinghouse fully guarantees the promise to protection buyers, the dealer earns an additional premium, $A_{CCP} - A_{BT}$. This premium increases the asset size of the dealer. If protection buyers are sufficiently risk averse (per Assumption 1), they pay a sufficiently high premium to dealers, who are thus better off joining central clearing, despite the required contribution to loss-sharing and the associated financing costs.

Lemma 1 suggests that the optimization problem (19) can be recast into finding the most cost-effective combination of I and F along the incentive-compatibility curve outlined in Figure 2. All members hedge, and the optimal collateral requirements solve $\max_I V(a = s; I, \hat{F}(I))$ where $\hat{F}(I)$ satisfies Eq. (18). Plugging $a = s$ and the IC constraint in Eq. (18) into the expression of $V(a; I, F)$ given by Eq. (13), we can rewrite the expected profit of a clearing member under incentive-compatible collateral as

$$V(a = s; I, \hat{F}(I)) = A_{CCP} \left(\mu_s - (1 - q_s)p_c\mathcal{P}\right) - \Theta I - (\delta + \Theta)\hat{F}(I), \tag{20}$$

where we introduce $\Theta := \mu_s - 1 + p_c - p_c(1 - q_s)\mathcal{P} > 0$. Note that Θ and $\delta + \Theta$ can be interpreted as the effective unit cost of initial margin and default fund, respectively. The components of Θ include the opportunity cost of posting collateral $(\mu_s - 1)$ and the loss of collateral during an adverse event $p_c(1-(1-q_s)\mathcal{P})$; the lower the asset pledgeability $\mathcal{P}$, the higher the effective cost of collateral. Finally, δ models the stricter capital requirement for collateral posted as default fund. The social planner aims to minimize members' opportunity cost of collateral by selecting the most cost-effective combination of the two types. We state the formal result in Proposition 3.

Proposition 3 *The incentive-constrained optimal collateral posted as initial margin and default fund* (I^{SB}, F^{SB}) *is given by*

$$\left(I^{SB}, F^{SB}\right) = \begin{cases} \left(\hat{I}(0), 0\right) & \mathcal{H}(1) < \frac{\delta}{\delta+\Theta} \\ \left(I_l, \hat{F}(I_l)\right) & \mathcal{H}(l+1) < \frac{\delta}{\delta+\Theta} \le \mathcal{H}(l), \quad l = 1, ..., \bar{l}-1 \\ \left(0, \max\left\{\hat{F}(0), F^{FB}\right\}\right) & \frac{\delta}{\delta+\Theta} \le \mathcal{H}(\bar{l}), \quad \bar{l} = \lceil \frac{N\hat{F}(0)}{D} \rceil \end{cases} \tag{21}$$

where $\mathcal{H}(l) := \frac{\alpha \mathbb{E}^s\left[\mathbb{I}_{\mathcal{N}_d \ge l}\right]}{\frac{1-q_s^N}{1-q_s} - \mathcal{P} + \alpha \mathbb{E}^s\left[\frac{N}{N-\mathcal{N}_d}\mathbb{I}_{\mathcal{N}_d \ge l}\right]} \in (0,1)$ *is a step function of* l, *and* $\left(I_l, \hat{F}(I_l)\right)$ *is the collateral pair that satisfies Eq. (18) and* $N\hat{F}(I_l) = l(D - I_l)$. $\hat{I}(0)$ *(given by Eq. (A17)) is the value of initial margin when setting* $F = 0$ *in Eq. (18), and* $\hat{F}(0)$ *(given by Eq. (A16)) is the value of default fund when setting* $I = 0$ *in Eq. (18).*

According to Proposition 3, the optimal mix of initial margin and default fund could feature the exclusive use of default fund, the exclusive use of initial margin, or a combination of the two collateral resources. To understand this result, consider the impact of an additional unit of initial margin by taking the first-order derivative of Eq. (20) with respect to I

$$\frac{\partial}{\partial I} V(a = s; I, \hat{F}(I)) = (\delta + \Theta)\left[\underbrace{\frac{\delta}{\delta+\Theta}}_{\text{cost-saving effect}} - \underbrace{\frac{d}{dI}\left(\hat{F}(I) + I\right)}_{\text{incentive effect}}\right].$$

Substituting one unit of initial margin for default fund reduces the opportunity cost by $\frac{\delta}{\delta+\Theta}$ (the "cost-saving effect"), but increases the total amount of collateral required from a member to prevent risk-shifting by $\frac{d}{dI}\left(\hat{F}(I) + I\right)$ (the "incentive effect"). The exact proportion of

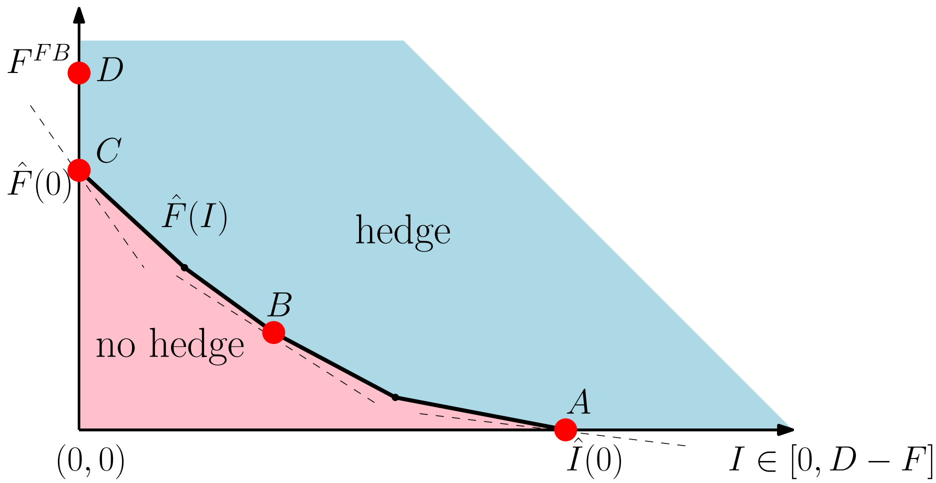

Figure 3. Optimal Collateral Requirements. This figure illustrates all possible cases for the optimal collateral requirements (I^{SB}, F^{SB}). The dashed line has slope $\frac{\delta}{\delta+\Theta} - 1$. The bolded piecewise linear curve labeled with IC traces the minimum combinations of incentive-compatible collateral, $(I, \hat{F}(I))$. Point A illustrates the exclusive use of initial margin where the cost-saving effect dominates the incentive effect; point B illustrates the use of both initial margin and default fund where the cost-saving effect balances the incentive effect; point C illustrates the exclusive use of default fund where the incentive effect dominates the cost-saving effect; point D illustrates the exclusive use of default fund where the incentive-compatibility constraint is slack at the first best default fund.

collateral posted as initial margin depends on the interaction of the two effects. We know from Proposition 2 that default fund is more productive in ex-ante incentive provision than initial margin because it achieves loss sharing among members. Formally, $\frac{d}{dI}\left(\hat{F}(I) + I\right) := \mathcal{H}(l) > 0$, where we recall $l := \lceil \frac{N\hat{F}(I)}{D-I} \rceil$ is the minimum number of defaults that will exhaust all prefunded collateral. Moreover, $\frac{d}{dI}\left(\hat{F}(I) + I\right)$ is a step function that is increasing in I, decreasing in l, and increasing in α. Hence, the objective function $V(a = s; I, \hat{F}(I))$ is continuous, piecewise linear, and concave in I. We next discuss how the relative magnitudes of δ (the additional opportunity cost of default fund) and α (the ex-post marginal financing cost) affect the composition of collateral.

When δ is high and α low—for example when the capital requirement is stringent—the cost-saving effect outweighs the incentive effect, making posting initial margin the preferred choice. For the exclusive use of initial margin to be optimal, the cost-saving effect needs to dominate even when the incentive effect is at its peak. This happens when the clearinghouse lacks tools for loss sharing so that it reaches the end of the waterfall whenever a default occurs. This scenario corresponds to setting $F^{SB} = 0$ (see point A in Figure 3).

For intermediate values of δ relative to α, the incentive-compatible collateral requirements comprise a mix of initial margin and default fund. The precise combination balances the cost-saving with the incentive effect. As the cost-saving needs become stronger (δ increases), the collateral composition tilts more towards initial margin. Notably, the optimal mix of the two collateral types covers losses triggered by an integer number of defaults $l = \frac{N\hat{F}(I_l)}{D-I_l}$. Observe that the incentive effect is characterized by a step function of l. This implies it is never optimal to cover losses associated with a non-integer number l' ($l < l' < l + 1$) of defaults because the extra opportunity cost associated with a larger default fund requirement would not be compensated by a higher number of covered defaults, i.e., the collateral resources would still cover l and not $l + 1$ of defaults. The interior optimal mix of initial margin and default fund thus corresponds to $\left(I_l, \hat{F}(I_l)\right)$ illustrated by point B in Figure 3.

When δ is relatively low and α high—for example during periods of systemic distress when liquidity is in shortage—end-of-waterfall resources are expensive to collect from members. The incentive effect then dominates the cost-saving effect, making posting default fund the preferred choice. For the exclusive use of default fund to be optimal, the incentive effect needs to dominate even at its minimum. This happens when the number of defaults covered is the highest at $\bar{l}$, so the clearinghouse has access to the most adequate resources for loss-mutualization, and faces the minimal risk of reaching the end of the waterfall. This scenario corresponds to setting $I^{SB} = 0$ at point C in Figure 3.

Finally, it may be the case that if δ is very low and α very high, the first-best collateral in Proposition 1 requires a default fund exceeding the minimum incentive-compatible level visualized by point C in Figure 3. In this case, financing the end-of-waterfall is so costly that demanding default fund collateral increases members' profit even absent of moral hazard. The incentive-compatibility constraint, therefore, is already satisfied at the first-best collateral. This outcome is visualized by point D in Figure 3.

To summarize, distinct from existing studies on central clearing that focus either on initial margin or default funds, our model solves the optimal mix of these two collateral resources *jointly* while accounting for how end-of-waterfall obligations feed back into members' incentives. Our analysis shows that the optimal mix minimizes members' opportunity cost of collateral while providing sufficient incentive for them to engage in risk management. Our model, therefore, sheds new light on the cost and benefit of these two fundamental layers of the CCP default waterfall structure.

Optimal Cover Rule for Default Funds. Proposition 2 highlights the incentive role of default fund relative to initial margin. Next, we compare the incentive-compatible default fund level implied by our model with the CPSS-IOSCO (2012) international regulatory guideline known as the "Cover 2" rule. According to this rule, the total default funds should cover at least the default losses of the two largest clearing members, i.e., $NF \geq 2(D - I)$.

We define a *generalized "Cover x fraction"* for a given number N of members and initial margin I,

$$x(I; N) = \frac{\hat{F}(I; N)}{D - I}, \tag{22}$$

where $\hat{F}(I; N)$ is the default fund required to induce risk management, and characterized by Eq. (18). The implied optimal cover number is $Nx(I; N)$. Proposition 3 shows that when a positive default fund is required from members, the optimal mix between default fund and initial margin covers losses triggered by an integer number of defaults, given by $\frac{NF^{SB}}{D-I^{SB}}$. The key insight of Proposition 3 is that the number of covered defaults should respond to the prevailing macroeconomic conditions, such as capital requirements and cost of liquidity during market distress. When $\frac{NF^{SB}}{D-I^{SB}} > 2$, our model provides a rationale to charge a default fund larger than the current regulatory requirement. Proposition 4 characterizes the asymptotic behavior of the "Cover x fraction" as the number of members grows large.

Proposition 4 *If the number of clearing members is large—as $N \to \infty$—for a given initial margin I, the optimal "Cover x fraction" which satisfies the incentive-compatibility con-*

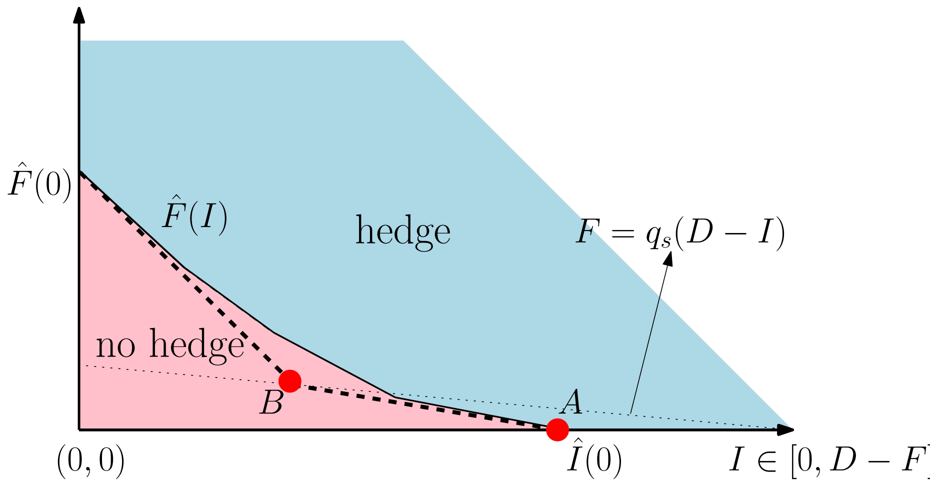

Figure 4. Optimal Collateral Requirements in the Limit. This figure illustrates the IC constraint and the optimal collateral requirements in the limiting case of a large clearing network. The solid piecewise linear curve traces the IC curve under a finite N. The bolded dashed line traces the IC curve as $N \to \infty$. The dotted line represents $\frac{F}{D-I} = q_s$. The bolded dashed IC curve has two linear pieces, one piece where $F \geq q_s(D - I)$ with a slope of -1, and one piece where $F < q_s(D-I)$ with a slope of $-1 + \frac{\alpha}{\frac{1+\alpha}{1-q_s} - \mathcal{P}}$. Points A and B correspond respectively to the two collateral requirements characterized in Eq. (24).

straint (18) is given by

$$x(I;N) \to \begin{cases} 1 - \dfrac{\frac{(A_{CCP}-D)\mathcal{P}}{D-I}}{\frac{1}{1-q_s} - \mathcal{P}}, & \text{if } D - I \geq \dfrac{(A_{CCP}-D)\mathcal{P}}{1-(1-q_s)\mathcal{P}} \\ 1 - \dfrac{\frac{(A_{CCP}-D)\mathcal{P}}{D-I} + \alpha}{\frac{1+\alpha}{1-q_s} - \mathcal{P}}, & \text{otherwise} \end{cases}. \tag{23}$$

At the incentive-constrained optimal collateral (I^{SB}, F^{SB}) characterized in Eq. (21), we have the following limiting result,

$$\left(I^{SB}, F^{SB}, x^{SB}\right) \to \begin{cases} \left(D - \frac{(A_{CCP}-D)\mathcal{P}}{1-(1-q_s)\mathcal{P}}, \frac{q_s(A_{CCP}-D)\mathcal{P}}{1-(1-q_s)\mathcal{P}}, q_s\right) & \frac{\delta}{\delta+\Theta} \leq \frac{\alpha}{\frac{1+\alpha}{1-q_s}-\mathcal{P}} \\ \left(D - \frac{(A_{CCP}-D)\mathcal{P}}{\frac{1+\alpha q_s}{1-q_s}-\mathcal{P}}, 0, 0\right) & \frac{\delta}{\delta+\Theta} > \frac{\alpha}{\frac{1+\alpha}{1-q_s}-\mathcal{P}} \end{cases}. \tag{24}$$

As the number of clearing members increases, the optimal collateral requirements take a simple form—rather than covering a fixed number of members as prescribed by "Cover 2", the total default funds should cover a fixed fraction of members. Using the closed-form expression given in (23), we can assess the sensitivity of the optimal cover fraction to various model parameters. First, “Cover x fraction” decreases with buyers’ risk aversion. As protection buyers become increasingly risk averse, they value more the benefits of central clearing and are willing to pay a higher price to the dealer. As a result, the dealer’s investment scales up and survival becomes more attractive, reducing the risk-shifting incentive. Second, “Cover x fraction” decreases with asset pledgeability. Members’ incentives for risk-shifting fall as asset pledgeability increases; thus, a lower default fund is required to align the incentives. Third, “Cover x” increases with ex-post financing cost, consistent with the notion that default fund is more effective than initial margin in reducing financing cost at the end of the waterfall.

If the clearing membership base is sufficiently large, the fraction of members that default is deterministic ($\mathcal{N}_d/N \to q_s$). To understand the limiting result of the incentive-constrained optimal collateral characterized in Eq. (24), let us consider two possible situations. The first is when “Cover x fraction” is at least q_s, which means that the end of the waterfall is never reached. Accordingly, ex-post financing cost is irrelevant, and default fund has no advantage over initial margin in providing incentives. In this extreme case, initial margin and default fund are perfect substitutes. The second situation is when “Cover x fraction” is bounded from above by q_s. The expected shortfall surely exceeds the prefunded collateral, which makes the ex-post financing cost a critical factor in incentive provision and default fund a more useful tool. If the cost-saving effect dominates, the exclusive use of initial margin is optimal; if the incentive effect dominates, maximum use of default fund is optimal, which corresponds to setting “Cover x fraction” at q_s.

Convex Financing Cost for End-of-Waterfall Resources. In the model, surviving members contribute to the end-of-waterfall resources while incurring a unit financing cost α. Nevertheless, when multiple defaults deplete all collateral resources, the market is likely distressed, making access to end-of-waterfall resources increasingly costly. To capture this state of the market, we can adjust the functional form of X in Eq. (14) and assume a convex quadratic cost for financing end-of-waterfall resources:

$$X(a_{-i}; I, F) = \mathbb{E}^{a_{-i}} \left[\frac{\mathcal{N}_d(D - I - F)}{N - \mathcal{N}_d} + \frac{\kappa}{2} \left(\frac{(\mathcal{N}_d(D - I) - NF)^+}{N - \mathcal{N}_d} \right)^2 \right]. \tag{25}$$

The following corollary shows that the trade-off between initial margin and default fund highlighted in Proposition 2 continues to hold, and the incentive-constrained optimal collateral requirements take a similar form as Eq. (21) in Proposition 3.

Corollary 1 *Assume a convex quadratic cost for financing end-of-waterfall resources as in Eq. (25), and assume that* $0 < \kappa < \frac{\mu_s - 1}{p_c q_s^2 D}$. *The results in Propositions 1–3 continue to hold qualitatively: (i) The first-best allocation satisfies* $a_i^{FB} = s, \forall i$; $\left(I^{FB}, F^{FB}\right) = (0, 0)$ *if* $\kappa \leq \frac{\mu_s - 1 + \delta}{p_c q_s D}$; *otherwise,* $\left(I^{FB}, F^{FB}\right) = (0, F^*)$ *where* F^* *satisfies* $\kappa = \frac{(\mu_s - 1 + \delta)}{p_c \mathbb{E}^s\left[\left(\frac{\mathcal{N}_d}{N} D - F^*\right)^+\right]}$; *(ii) Given collateral requirements* I *and* F, *members' risk-management action in the Pareto-dominant Nash equilibrium satisfies Eq. (17)–(18). The function* $\hat{F}(I)$ *is continuous, convex, and decreasing in* I *with first-order derivative greater than* -1. *(iii) The incentive-constrained optimal collateral,* $\left(I^{SB}, F^{SB}\right)$ *is given by*

$$\left(I^{SB}, F^{SB}\right) = \begin{cases} \left(\hat{I}(0), 0\right) & \frac{d}{dI}\hat{F}(\hat{I}(0)) + 1 < \frac{\delta}{\delta + \Theta} \\ \left(I^* : \frac{d}{dI}\hat{F}(I^*) + 1 = \frac{\delta}{\delta + \Theta}, \hat{F}(I^*)\right) & \frac{d}{dI}\hat{F}(0) + 1 < \frac{\delta}{\delta + \Theta} \leq \frac{d}{dI}\hat{F}(\hat{I}(0)) + 1 \\ \left(0, \max\left\{\hat{F}(0), F^{FB}\right\}\right) & \frac{\delta}{\delta + \Theta} \leq \frac{d}{dI}\hat{F}(0) + 1 \end{cases} \tag{26}$$

where $\frac{d}{dI}\hat{F}(I) + 1 = \frac{\kappa \mathbb{E}^s\left[\frac{\left(\mathcal{N}_d(D - I) - N\hat{F}(I)\right)^+}{N - \mathcal{N}_d}\right]}{\frac{1 - q_s^N}{1 - q_s} - \mathcal{P} + \kappa \mathbb{E}^s\left[\frac{N\left(\mathcal{N}_d(D - I) - N\hat{F}(I)\right)^+}{(N - \mathcal{N}_d)^2}\right]} \in (0, 1)$ *is continuous and increasing in* I. $\hat{I}(0)$ *is the value of initial margin that satisfies Eq. (18) when setting* $F = 0$.

Observe that the incentive-compatibility curve $\hat{F}(I)$ is smooth and convex rather than piecewise linear as in Proposition 3. Accordingly, the incentive effect—captured by the term $\frac{d}{dI}\left(\hat{F}(I) + I\right)$—is continuous and increasing in I rather than a step function of I. When $\frac{\delta}{\delta + \Theta} \in \left(\frac{d}{dI}\hat{F}(0) + 1, \frac{d}{dI}\hat{F}(\hat{I}(0)) + 1\right]$, the incentive-constrained optimal initial margin I^* features a tangency condition, $\frac{d}{dI}\hat{F}(I^*) + 1 = \frac{\delta}{\delta + \Theta}$, which equates the incentive effect with the cost-saving effect; see Figure 5. To provide economic intuition, observe that the incentive

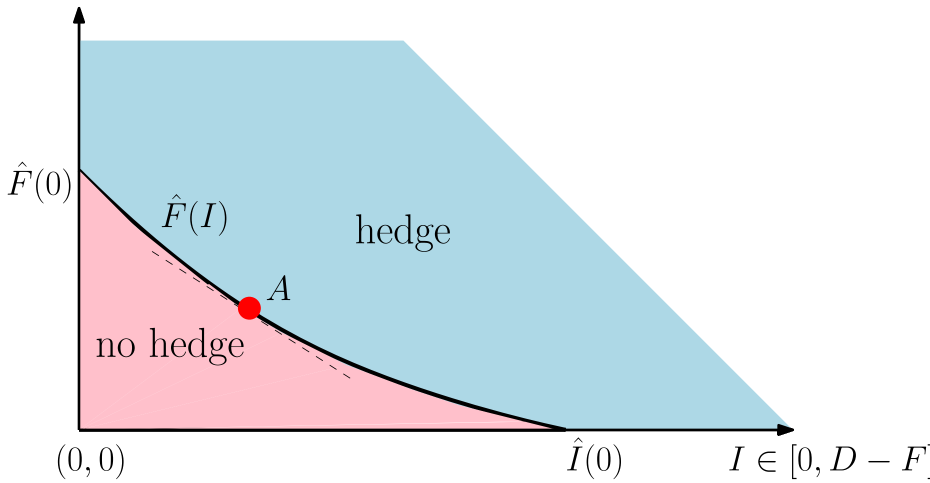

Figure 5. Convex Financing Cost for End-of-Waterfall Resources. This figure illustrates the optimal collateral requirements when the cost of financing end-of-waterfall resources takes a convex quadratic form. At point A, the optimal collateral features interior levels of initial margin and default fund at which the cost-saving effect equals the incentive effect.

effects dominates at $I > I^*$. Thus, increasing the fraction of collateral allocated as default fund improves the expected profit of members. An increase in the share of default fund collateral reduces the probability of resorting to end-of-waterfall resources, which in turn, diminishes the incentive effect of default fund collateral. The increase of default fund collateral continues until the cost-saving effect equals the incentive effect, at which the optimal collateral mix is achieved.

4 Heterogeneity in Member Size

In our baseline model, we assume that the clearinghouse has the same exposure, D, to each clearing member. In practice, CCPs' outstanding exposures are typically concentrated on a few large clearing members. For example, according to the public quantitative disclosures by ICE Clear Europe in the year 2019, the largest five clearing members account for 52.61% and the largest ten account for 84.34% of the average daily open CDS positions. The fractions of margin and default fund contributions by the largest members are also substantial.

Motivated by these empirical observations, we extend our model to account for heterogeneity in exposures among members. We adapt our setup as follows. We assume the size of dealer $i = 1, 2, \ldots, N$ to be $K_i > 0$ times the size of those ex-ante identical dealers in the baseline model. Specifically, dealer i holds an initial asset in the amount $K_i A_0$ and sells K_i units of the CDS contract to protection buyers, which results in an initial balance sheet of $K_i A_{CCP}$ (recall $A_{CCP} = A_0 + P_{CCP}$ where P_{CCP} is the unit price of a CDS contract given in Eq. (8)). This dealer therefore has an obligation to CDS buyers equal to $K_i D$. To investigate whether members should post collateral disproportionately to their size, let us denote the initial margin and default fund of dealer i *scaled by its size* as I_i and F_i, respectively.

We show that the economic forces highlighted in the baseline model remain robust in this new setting (albeit the analysis becomes more tedious). We start by analyzing the members' risk-management actions. The results are summarized in the following proposition.

Proposition 5 *Suppose clearing member $i = 1, 2, \ldots, N$ is $K_i > 0$ times the size of those ex-ante identical dealers in the baseline model. Given collateral requirements I and F and other members risk-management action $a_{-i} = s$, the risk-management action of member i satisfies*

$$a_i(a_{-i} = s; I, F) = \begin{cases} r & 0 \leq F < \hat{F}_i(I) \\ s & \hat{F}_i(I) \leq F \leq D - I \end{cases}.$$

For a given I, the function $\hat{F}_i(I)$ is uniquely determined by Eq. (A26), and is piecewise linear, continuous, convex, and strictly decreasing in I with slope greater than -1. In addition, for arbitrary i_1, $i_2 \in 1, 2, \ldots, N$, such that $K_{i_1} > K_{i_2}$, we have that $0 \leq \hat{F}_{i_1}(I) < \hat{F}_{i_2}(I) < D - I$.

Like in the baseline model, members have incentives to take excessive risk; posting collateral increases members' pledgeable income and enables them to commit credibly to hedging. By allowing for loss mutualization, a unit of default fund is more effective than initial margin in reducing the ex-post financing cost at the end of the waterfall. For this reason, replacing default fund with initial margin would increase the total amount of collateral that members need to post to prevent risk-shifting. This insight holds for members across all sizes, as seen from the result that $\hat{F}_i(I)$ is strictly decreasing in I with slope greater than -1 for all i.

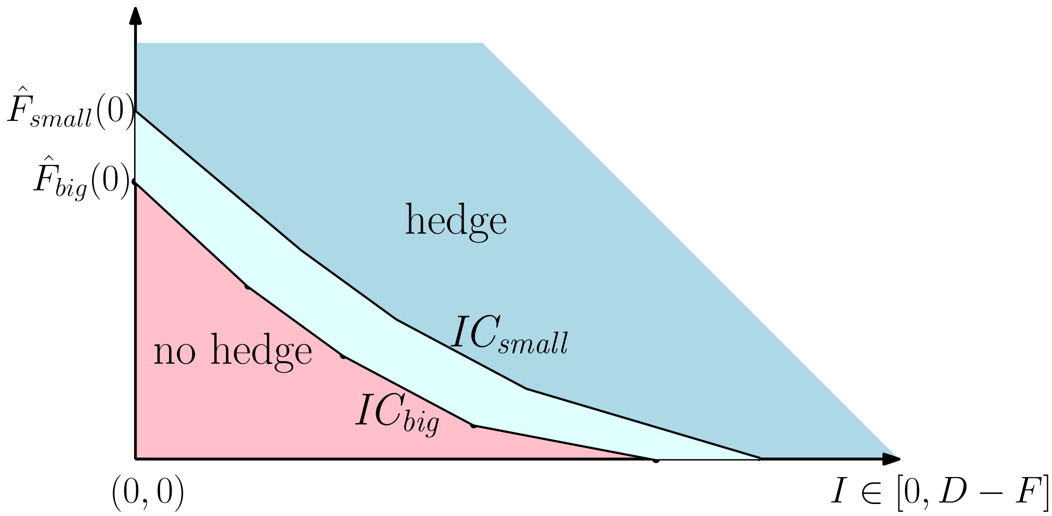

Figure 6. Heterogeneity in Size. This figure illustrates the optimal collateral requirements when the clearinghouse has heterogeneous exposures to members. The minimum combinations of incentive-compatible collateral vary with member size. Consider two members, one big and the other small. Their minimum combinations of incentive-compatible collateral are labeled by IC_{big} for the big member, and by IC_{small} for the small member.

Unlike the baseline model, however, the minimum combinations of incentive-compatible collateral differ by member size. Notably, our results show that small members should be given stronger incentives than big members on a per-unit notional basis. According to Proposition 5, for a given level of initial margin, the default fund required to induce risk management is disproportionately lower for the big member than for the small member; we graphically illustrate this outcome in Figure 6.

The intuition for why the size-scaled incentive-compatible default fund collateral decreases with member size is as follows. The end-of-waterfall mechanism requires that surviving members contribute pro rata to resources needed at the end of the waterfall. For a small surviving member, other members' defaults cause a significant shortfall; but for a big surviving member who constitutes a significant share among the clearing membership base, defaults of other members have a less significant impact. Hence, the expected contribution to loss-sharing is disproportionately smaller for a bigger surviving member.[24] Accordingly, a bigger member

[24]Consider, for example, the extreme case of a single big member and a single small member. If the big member fails, the small surviving member has to contribute a substantial amount, which makes surviving less

finds surviving more attractive, thereby facing a less severe incentive problem than a smaller member. This mechanism explains why big members contribute less collateral relative to small members on a per-unit notional basis.

Based on these results, our next corollary characterizes the incentive-constrained optimal collateral posted as initial margin and default fund for member i.

Corollary 2 *The incentive-constrained optimal collateral that member i posts as initial margin and default fund,* $\left(I_i^{SB}, F_i^{SB}\right)$, *is given by*

$$\left(I_i^{SB}, F_i^{SB}\right) = \begin{cases} \left(\hat{I}_i(0), 0\right) & \frac{d\hat{F}_i\left(\hat{I}_i(0)\right)}{dI} + 1 < \frac{\delta}{\delta+\Theta} \\ \left(I_i^* = \inf\left\{I : \frac{\delta}{\delta+\Theta} \leq \frac{d\hat{F}_i(I)}{dI} + 1\right\}, \hat{F}_i(I_i^*)\right) & \frac{d\hat{F}_i(0)}{dI} + 1 < \frac{\delta}{\delta+\Theta} \leq \frac{d\hat{F}_i\left(\hat{I}_i(0)\right)}{dI} + 1 \\ \left(0, \max\left\{F^{FB}, \hat{F}_i(0)\right\}\right) & 0 < \frac{\delta}{\delta+\Theta} \leq \frac{d\hat{F}_i(0)}{dI} + 1 \end{cases} \tag{27}$$

where $\hat{I}_i(0)$ *is the value of initial margin that satisfies Eq. (A26) when setting* $F = 0$, *and* $\hat{F}_i(0)$ *is the value of default fund that satisfies Eq. (A26) when setting* $I = 0$, *and* $F^{FB} = \inf\left\{F : 1 - \mu_s - \delta + p_c\alpha\mathbb{E}^s\left[\mathbb{I}_{\sum_i \mathbb{I}_{i\ defaults}\frac{K_i}{\sum_j K_j} > \frac{F}{D}}\right] \leq 0\right\}$.

Our prediction on the relation between member size and collateral appears to be consistent with empirical patterns observed in major clearing services. Figure 7 shows the percentage of open positions, initial margin, and default fund collateral held by the largest five and the largest ten clearing members at ICE Clear Europe for CDS clearing. Across all four quarters in 2019, the top clearing members take a larger share of open positions than their collateral contributions. In other words, these top members post less collateral on a per-unit notional basis compared to the other members.

Our finding stands in contrast with that from the banking literature, which has shown that big institutions tend to take excessive risk. For example, Davila and Walther (2020) show that larger banks have higher leverage and default more frequently, because they internalize that their actions directly affect the bailout by taxpayers. By contrast, clearing members

attractive. This small member therefore has, ex-ante, a weaker incentive for risk management.

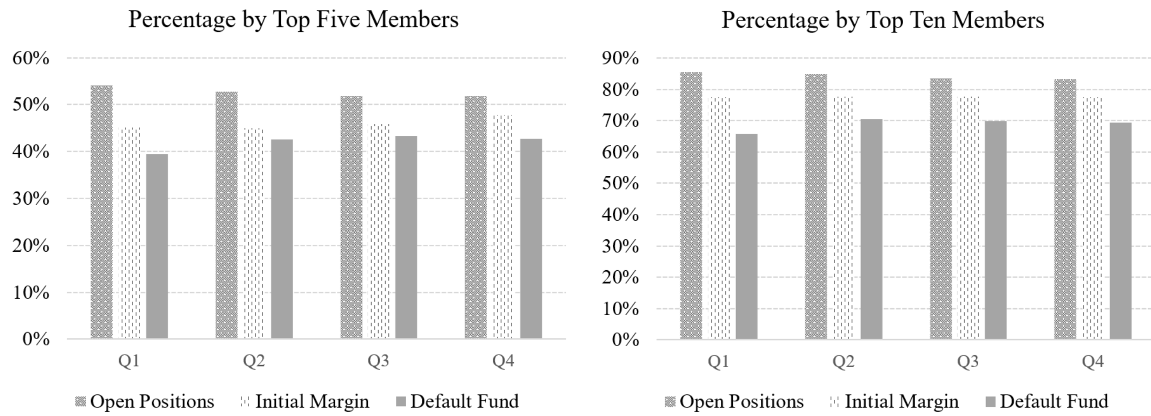

Figure 7. Percentage of Open Positions and Collateral by Top Clearing Members. This figure illustrates the percentage of open positions, initial margin, and default fund collateral held by the largest five and the largest ten clearing members at ICE Clear Europe for CDS clearing in the four quarters of 2019. The source of data is the CCP quantitative disclosures.

bail out each other and their risk management actions impose externalities on each other. A big member effectively acts as an *internally coordinated group* of small members, and thus finds it easier to internalize the externality and undertakes efficient risk management.

5 Policy and Empirical Implications

We discuss policy implications of our results, and list empirical predictions implied by our analysis.

Collateral requirements and CCP resilience. As the volume of centrally cleared derivative contracts grows, so does the importance of clearinghouse resilience. The new international guidelines adopted since 2016 require CCPs to make quarterly disclosures of their default waterfalls (CPMI-IOSCO, 2015), making it feasible to closely monitor CCP collateral requirements (Office of Financial Research, 2017). While comparable waterfall structures are adopted across the board, the quarterly filings of CCPs following these recent international guidelines reveal significant variation in how resources are allocated in initial margin and default funds.[25]

Our framework allows assessing the trade-off between different collateral layers of the default waterfall. Fundamental considerations in designing collateral requirements include: the financing cost for resources at the end of the waterfall, the opportunity cost of posting different types of collateral, and their effectiveness in incentive provision. When the opportunity cost of default fund is the primary concern, the collateral policy should rely more on initial margin. In contrast, when concerns about funding resources at the end of the waterfall dominate, the collateral policy should rely more on default fund. Note that the end-of-waterfall resources are invoked during extreme market events when all prefunded measures are exhausted. Hence, one can alternatively view the CCP having to use end-of-waterfall resources as a severe challenge to its resilience. In the context of our model, a high funding cost in stressed scenarios (α) can thus be interpreted as a strong regulatory weight on preventing systemic distress. Hence, our results suggest that default fund may also be viewed as a tool to enhance CCP resilience due to the loss-mutualization feature.

Our analysis establishes a rationale for collecting default funds in distressed market scenarios and prescribes a generalized "*Cover x fraction*" rule: The total default funds collected should cover the shortfalls of a fraction of clearing members. In addition, our findings highlight the importance of accounting for members' size when allocating collateral requirements. Our model predicts that big members should contribute disproportionately lower levels of default fund relative to small members, which is consistent with the current protocol of ICE Clear Europe for its CDS clearing business.

The value of central clearing. Our study highlights the value of central clearing. In the constrained-optimal allocation, dealers are better-off clearing with the CCP because they earn higher profits than in bilateral trades. The underlying reason is that, by pooling resources against defaults, the CCP fully guarantees the promised payments to protection

[25]Using proprietary data from CCPView, Paddrik and Zhang (2019) document that the average ratio between default fund to initial margin varies from 17% to 30% across different derivative classes and from 0 to 34% across regions. For instance, the ratio of default fund to initial margin for CDS is 40.9% at CME and 7.1% at ICE Clear Credit.

buyers. Consequently, dealers earn an additional insurance premium. However, if the collateral requirements set by the CCP deviate from the constrained-optimal allocation, or if the dealers are not able to receive an additional premium for eliminating counterparty risk, then dealers might prefer not to join the CCP. These findings offer regulatory guidance on how to encourage clearing participation. First, the clearing mandate should be accompanied by dealers' ability to charge a higher spread for a centrally cleared contract. For this to happen, counterparty risk needs to be significantly reduced in a centrally cleared environment, which requires the CCP to collect enough resources to guarantee the transactions. Second, the collateral requirements should be thoughtfully designed. While providing incentives to encourage risk management is essential, it must be done in a way that accounts for members' opportunity cost of collateral.

Our model has empirical implications regarding collateral in centrally cleared markets. According to our theory, posting collateral increases members' pledgeable income and provides them incentives for risk management. To be "incentive-compatible," collateral needs to be sufficiently high. Based on Proposition 2, the minimum incentive-compatible collateral is given by $\hat{F}(I) + I$. This term is decreasing with $A_{CCP} = A_0 + P_{CCP}$ (recall P_{CCP} is the price of a centrally cleared CDS) and decreasing with asset pledgeability $\mathcal{P} := \frac{\mu_s - \mu_r}{p_c(q_r - q_s)}$ (a direct conclusion from the explicit expression for $\hat{F}(I)$ in Eq. (A15)). We therefore conclude that the total amount of collateral needed is increasing with the probability of the credit event (p_c), and decreasing with the CDS price (P_{CCP}) as well as with the effectiveness of risk-management procedures ($\frac{\mu_s - \mu_r}{q_r - q_s}$).

Empirical Implication 1 *Other things equal, the clearinghouse collects a higher amount of collateral when the reference entities underlying the CDS contracts impose a higher credit risk, and collects a lower amount of collateral when the spread of a centrally cleared CDS contract is high. Clearing members who demonstrate more effective risk-management procedures post lower amounts of collateral.*

The following predictions are supported by Proposition 3 and Corollary 1.

Empirical Implication 2 *Other things equal, the fraction of collateral allocated as default fund increases with the perceived financing cost of resources at the end of the waterfall, and decreases with the risk weight associated with default fund collateral in capital requirements.*

The following empirical predictions follow from Proposition 3 and Proposition 5.

Empirical Implication 3 *The total pool of default funds at a CCP increases with the entry of clearing members. On a per-unit notional basis, bigger members put in less collateral than smaller members.*

All the empirical implications above share a common caveat: the CCP acts as a benevolent social planner who sets the optimal collateral requirements to maximize the total value of market participants. However, this may not always be the case in practice. In fact, as Huang (2019) points out, some CCPs are for-profit publicly listed financial firms and thus may have misaligned incentives. They may, for instance, act strategically in setting collateral to maximize fee income, hence the resulting requirements would deviate from the social optimum.

6 Conclusion

Reforms after the financial crisis of 2007–09 have promoted the use of central counterparties (CCPs) to reduce counterparty risk. To efficiently manage counterparty risk, a CCP collects two types of collateral from its clearing members: initial margin and default funds. Despite extensive debate on current clearing practices, there is little work aimed at understanding the design and regulation of collateral at CCPs, and especially the joint determination of initial margins and default fund.

Our paper fills this gap and provides a tractable framework to study the determinants of incentive-compatible optimal levels of initial margin and default fund requirements. Posting collateral increases members' pledgeable income, thereby reducing their risk-shifting incentives. However, different types of collateral have distinct implications for members' risk-shifting incentives and CCP's resilience. We have shown that, by allowing members to share losses ex-post, default funds are more effective than initial margin in enhancing the loss-absorption capacity of CCPs' default waterfall and in reducing surviving members' burden to make additional contributions ex-post. This, in turn, makes default fund a preferred instrument for incentive provision.

Our findings lead to novel implications on the regulation of clearinghouse collateral requirements, and suggest that they should be determined in accordance with prevailing economic conditions. The key insight of our analysis is that the optimal mix between initial margin and default fund depends on the trade-off between the marginal financing cost of resources at the end of the waterfall and the relative opportunity costs of default funds versus initial margins. This suggests that the allocation of resources in the default waterfall is sensitive to the prevailing macroeconomic conditions. For example, default funds should be higher in an environment characterized by flight-to-safety and a high probability of default clustering because it is challenging to retrieve resources to replenish the default waterfall under these circumstances.

The tractability of our model opens the door to several directions for future research. First, besides heterogeneity in size, it would be of interest to explore how the results generalize to a setting in which members differ ex-ante in balance sheet compositions, such as the percentage of liquid assets held. Second, we have assumed that investment outcomes are independent across dealers. One may introduce correlated investments caused, for example, by securitization. We conjecture that the presence of aggregate risk might change the importance of loss-sharing relative to collateral opportunity cost, and introduce an additional layer of trade-off between initial margin and default funds.

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Appendix: Proofs

Proof of Proposition 1

Plugging Eq. (13) into Eq. (15), we obtain the objective function of the first-best benchmark as

$$\max_{(a;I,F)} W^{FB}(a;I,F) := \max_{(a;I,F)} \frac{1}{N}\left\{\sum_i V(a_i,a_{-i};I,F)\right\}$$
$$= \max_{(a;I,F)} I + F - \delta F + (A_{CCP} - I - F)\mu_a - p_c D - p_c \alpha \mathbb{E}^a\left[\left(\frac{\mathcal{N}_d(D-I)}{N} - F\right)^+\right].$$

It follows from conditions (1)–(2) that $W^{FB}(s;I,F) > W^{FB}(r;I,F)$, so $a_i^{FB} = s$, $\forall i$. We take the left and right derivative of $W^{FB}(a = s; I, F)$ with respect to I and F as follows:

$$\frac{\partial}{\partial I-}W^{FB} = 1 - \mu_s + p_c\alpha\mathbb{E}^s\left[\frac{\mathcal{N}_d}{N}\mathbb{I}_{\mathcal{N}_d \geq \lceil \frac{NF}{D-I}\rceil}\right], \quad \frac{\partial}{\partial I+}W^{FB} = 1 - \mu_s + p_c\alpha\mathbb{E}^s\left[\frac{\mathcal{N}_d}{N}\mathbb{I}_{\mathcal{N}_d > \lceil \frac{NF}{D-I}\rceil}\right]$$
$$\frac{\partial}{\partial F-}W^{FB} = 1 - \mu_s - \delta + p_c\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{N}_d \geq \lceil \frac{NF}{D-I}\rceil}\right], \quad \frac{\partial}{\partial F+}W^{FB} = 1 - \mu_s - \delta + p_c\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{N}_d > \lceil \frac{NF}{D-I}\rceil}\right]$$

where $\mathbb{E}^s[\cdot]$ denotes the probability measure conditional on the occurrence of the credit event and the dealer's risk-management action $a = s$.

For given I, define $\left\{F_l : \frac{NF_l}{D-I} = l, l = 0, 1, ...N\right\}$; for given F, define $\left\{I_l : \frac{NF}{D-I_l} = l, l = 0, 1, ...N\right\}$. At a kink I_l, we have $\frac{\partial}{\partial I-}W^{FB}(a = s; I = I_l, F) = 1 - \mu_s + p_c\alpha\mathbb{E}^s\left[\frac{\mathcal{N}_d}{N}\mathbb{I}_{\mathcal{N}_d \geq \frac{NF}{D-I_l}}\right]$ and $\frac{\partial}{\partial I+}W^{FB}(a = s; I = I_l, F) = 1 - \mu_s + p_c\alpha\mathbb{E}^s\left[\frac{\mathcal{N}_d}{N}\mathbb{I}_{\mathcal{N}_d \geq \frac{NF}{D-I_l}+1}\right]$; similarly at a kink F_l, $\frac{\partial}{\partial F-}W^{FB}(a = s; I, F = F_l) = 1 - \mu_s - \delta + p_c\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{N}_d \geq \frac{NF_l}{D-I}}\right]$ and $\frac{\partial}{\partial F+}W^{FB}(a = s; I, F = F_l) = 1 - \mu_s - \delta + p_c\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{N}_d \geq \frac{NF_l}{D-I}+1}\right]$. Hence, both derivatives $\frac{\partial}{\partial I}W^{FB}$ and $\frac{\partial}{\partial F}W^{FB}$ are left-continuous and decreasing at the kinks. By the dominated convergence theorem, the function $W^{FB}(a = s; I, F)$ is continuous and concave at a set of kinks, and is piecewise linear in I and F.

To solve for $\left(I^{FB}, F^{FB}\right)$, we distinguish between the following cases based on the value taken by the parameter α, which we recall satisfies $\alpha < \frac{\mu_s - 1}{p_c q_s}$ per Assumption (3).

Case 1: $0 < \alpha \leq \frac{\mu_s - 1 + \delta}{p_c(1-(1-q_s)^N)}$. Then the derivatives at $(I, F) = (0, 0)$ satisfy

$$\frac{\partial}{\partial I+}W^{FB}(a = s; 0, 0) = 1 - \mu_s + p_c\alpha\mathbb{E}^s\left[\frac{\mathcal{N}_d}{N}\mathbb{I}_{\mathcal{N}_d \geq 1}\right] < 0,$$
$$\frac{\partial}{\partial F+}W^{FB}(a = s; 0, 0) = 1 - \mu_s - \delta + p_c\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{N}_d \geq 1}\right] \leq 0.$$

Since $W^{FB}(a = s; I, F)$ is concave in I and F, the above conditions imply that $I^{FB} = F^{FB} = 0$.

Case 2: $\frac{\mu_s - 1 + \delta}{p_c(1-(1-q_s)^N)} < \alpha < \frac{\mu_s - 1}{p_c q_s}$. Under this condition, $\frac{\partial}{\partial I+}W^{FB}(a = s; 0, 0) < 0$ and

$\frac{\partial}{\partial F+} W^{FB}(a=s;0,0)>0$. Then $I^{FB}=0$ and $F^{FB}=\frac{l^{FB}}{N}D$, where

$$l^{FB} := \inf\{l=1,...,N-1 : 1-\mu_s-\delta+p_c\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{N}_d\geq l^{FB}+1}\right]\leq 0\}. \tag{A1}$$

Proof of Proposition 2

Step 1: analyze members' equilibrium risk-management action

To understand the risk-management action of member i, we consider different action profiles of other members. Recall the expression for a member's value function given in Eq. (13) and (14). Given collateral requirements I and F, and assuming $g=0,\dots,N-1$ of the remaining $(N-1)$ members hedge, member i hedges if and only if

$$\begin{aligned}V^g(s;I,F) &:= V(s,a_{i_1}=\cdots=a_{i_g}=s,a_{i_{g+1}}=\cdots=a_{i_{N-1}}=r;I,F)\\ &\geq V(r,a_{i_1}=\cdots=a_{i_g}=s,a_{i_{g+1}}=\cdots=a_{i_{N-1}}=r;I,F) =: V^g(r;I,F)\end{aligned}$$

where $i_1,i_2,\dots,i_{N-1}\in\{1,\dots,i-1,i+1,\dots,N\}$ are arbitrary. This is because the expected profit of member i who chooses to hedge only depends on the number of members other than i who hedge and not on the identity of those members. The same holds for the expected profit of member i who chooses not to hedge. The inequality above reduces to

$$(A_{CCP}-I-F)\mathcal{P}\geq(D-I-F)+X^g(I,F), \tag{A2}$$

where $\mathcal{P}:=\frac{(\mu_s-\mu_r)}{p_c(q_r-q_s)}\in(0,1)$ is the asset pledgeability, and $X^g(I,F)$, specified in Eq. (16), is the expected contribution to loss-sharing and the associated financing costs for a surviving member i in a credit event.

For a given I, the cutoff function $\hat{F}(I)$ satisfies Eq. (18); and the cutoff function $\bar{F}(I)$ satisfies the following equation

$$D-I-\bar{F}(I)+X^0(I,\bar{F}(I))=(A_{CCP}-I-\bar{F}(I))\mathcal{P}. \tag{A3}$$

Recall from Lemma IA.1 in the online Appendix, $X^g(I,F)$ is strictly decreasing in g, piecewise linear, continuous, and strictly decreasing in both I and F. Then, we have that $\hat{F}(I)$ and $\bar{F}(I)$ are uniquely determined for a given I. It can also be immediately seen from equations (14) and (16) that $X^g(I;D-I)=0$, thus we have $0\leq\hat{F}(I)<\bar{F}(I)<D-I$. Since both $X^{N-1}(I,F)$ and $X^0(I,F)$ are continuous in I and F, it follows from the implicit function theorem that the functions $\hat{F}(I)$ and $\bar{F}(I)$ solving, respectively, Eq. (18) and Eq. (A3) are continuous in I.

We can distinguish the following cases:

1. $(A_{CCP}-I-F)\mathcal{P}-(D-I-F)<X^{N-1}(I,F)$, which is equivalent to $0\leq F<\hat{F}(I)$ by Lemma IA.1. In this case, each member chooses not to hedge regardless of other members' actions, so "all not hedging" is the unique equilibrium.

2. $(A_{CCP}-I-F)\mathcal{P}-(D-I-F)>X^0(I,F)$, which is equivalent to $\bar{F}(I)<F\leq D-I$ by Lemma IA.1. In this case, each member chooses to hedge regardless of other members' actions, so "all hedging" is the unique equilibrium.

3. $X^{N-1}(I,F)\leq(A_{CCP}-I-F)\mathcal{P}-(D-I-F)\leq X^0(I,F)$, which is equivalent to $\hat{F}(I)\leq F\leq\bar{F}(I)$. In this case, there are two Nash equilibria: "all hedging" and "all not hedging", and "all hedging" is the Pareto-dominant equilibrium under which each member achieves a higher expected profit (per Proposition 1).

We next show that under case 3 above, *there does not exist any other equilibrium, such as some members choosing to hedge and other members choosing not to hedge.* We argue by contradiction. Suppose g members choose to hedge and $(N-g)$ members choose not to hedge in equilibrium, for some $g = 1, \ldots, N-1$. Then, any member choosing not to hedge faces g members choosing to hedge. Using inequality (A2), this member chooses not to unilaterally deviate only if

$$(A_{CCP} - I - F)\mathcal{P} - (D - I - F) < X^g(I, F). \tag{A4}$$

Similarly, any member choosing to hedge faces $(g-1)$ other members choosing to hedge. Using inequality (A2), this member chooses not to unilaterally deviate only if

$$(A_{CCP} - I - F)\mathcal{P} - (D - I - F) \geq X^{g-1}(I, F). \tag{A5}$$

But conditions (A4) and (A5) cannot hold simultaneously since $X^g(I, F)$ is strictly decreasing in g. Hence, we obtain a contradiction. Combining the above three cases, we have that members' risk-management action in a Pareto-dominant Nash equilibrium satisfies Eq. (17).

Step 2: characterize $\hat{F}(I)$ as a function of I

To analyze the properties of $d\hat{F}(I)/dI$, we apply the implicit function theorem to Eq. (18) and solve for $d\hat{F}(I)/dI$ as follows:

$$\frac{d\hat{F}(I)}{dI} = \frac{\frac{\partial}{\partial F}X^{N-1}(I, F) - \frac{\partial}{\partial I}X^{N-1}(I, F)}{\frac{\partial}{\partial F}X^{N-1}(I, F) - (1 - \mathcal{P})} - 1. \tag{A6}$$

Using Eq. (14) and Eq. (16), we take partial derivatives and obtain

$$\frac{\partial}{\partial I}X^{N-1}(I, F) = -\mathbb{E}^s\left[\frac{\mathcal{N}_d}{N - \mathcal{N}_d}\right] - \alpha\mathbb{E}^s\left[\frac{\mathcal{N}_d}{N - \mathcal{N}_d}\mathbb{I}_{\mathcal{N}_d \geq \lceil \frac{NF}{D-I} \rceil}\right]$$
$$\frac{\partial}{\partial F}X^{N-1}(I, F) = -\mathbb{E}^s\left[\frac{\mathcal{N}_d}{N - \mathcal{N}_d}\right] - \alpha\mathbb{E}^s\left[\frac{N}{N - \mathcal{N}_d}\mathbb{I}_{\mathcal{N}_d \geq \lceil \frac{NF}{D-I} \rceil}\right]$$

where $\mathcal{N}_d \sim Binom(N-1, q_s)$ because $N-1$ members choose the "s" action, and thus $\mathbb{E}^s\left[\frac{\mathcal{N}_d}{N-\mathcal{N}_d}\right] = \frac{q_s - q_s^N}{1-q_s}$. Plugging $\frac{\partial}{\partial I}X^{N-1}(I, F)$ and $\frac{\partial}{\partial F}X^{N-1}(I, F)$ into Eq. (A6) and using that $\mathcal{P} \in (0, 1)$, we conclude that

$$\frac{d\hat{F}(I)}{dI} + 1 = \frac{\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{N}_d \geq \lceil \frac{NF}{D-I} \rceil}\right]}{\mathbb{E}^s\left[\frac{N}{N-\mathcal{N}_d}\right] - \mathcal{P} + \alpha\mathbb{E}^s\left[\frac{N}{N-\mathcal{N}_d}\mathbb{I}_{\mathcal{N}_d \geq \lceil \frac{NF}{D-I} \rceil}\right]} \in (0, 1), \tag{A7}$$

thus, $d\hat{F}(I)/dI \in (-1, 0)$. From (A7), $\frac{d\hat{F}(I)}{dI} + 1$ is a step function of I; we conclude that $\hat{F}(I)$ is a piecewise linear function of I, and is strictly decreasing in I with slope greater than -1.

Consider the set of kinks, $\{I_l : \frac{N\hat{F}(I_l)}{D-I_l} = l, l = 0, 1, ..., \lceil \frac{N\hat{F}(I=0)}{D} \rceil - 1\}$. Since $\hat{F}(I)$ is piecewise linear and strictly decreasing in I, for each l there exists a unique I_l that satisfies $N\hat{F}(I_l) = l(D - I_l)$. We next characterize the monotonicity of I_l. From Eq. (18), we have that $\left(I_l, \hat{F}(I_l)\right)$ satisfy

$$\left(\frac{1-q_s^N}{1-q_s}-\mathcal{P}\right)(D-I_l-\hat{F}(I_l))+\alpha\mathbb{E}^s\left[\frac{\left(\mathcal{N}_d(D-I_l)-N\hat{F}(I_l)\right)^+}{N-\mathcal{N}_d}\right]=(A_{CCP}-D)\mathcal{P}. \quad \text{(A8)}$$

Plugging $\hat{F}(I_l)=\frac{l(D-I_l)}{N}$ into Eq. (A8) and dividing $D-I_l$ from both sides, we obtain

$$\left(\frac{1-q_s^N}{1-q_s}-\mathcal{P}\right)\frac{N-l}{N}+\alpha\mathbb{E}^s\left[\frac{(\mathcal{N}_d-l)^+}{N-\mathcal{N}_d}\right]=\frac{(A_{CCP}-D)\mathcal{P}}{D-I_l}.$$

Since the left-hand side of the above expression is decreasing in l, the right-hand side $\frac{(A_{CCP}-D)\mathcal{P}}{D-I_l}$ is also decreasing in l. We conclude that $I_l > I_{l+1}$. It follows that around a kink I_l with sufficiently small $\epsilon > 0$, we have

$$\lceil\frac{N\hat{F}(I_l-\epsilon)}{D-(I_l-\epsilon)}\rceil=l+1, \qquad \lceil\frac{N\hat{F}(I_l+\epsilon)}{D-(I_l+\epsilon)}\rceil=l.$$

Next, we take the left and right derivatives of $\hat{F}(I)$ at kink $I=I_l$ to show that $\hat{F}(I)$ is convex at kink I_l.

$$\frac{d\hat{F}(I_l-\epsilon)}{dI}+1=\frac{\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{N}_d\geq l+1}\right]}{\frac{1-q_s^N}{1-q_s}-\mathcal{P}+\alpha\mathbb{E}^s\left[\frac{N}{N-\mathcal{N}_d}\mathbb{I}_{\mathcal{N}_d\geq l+1}\right]} \quad \text{(A9)}$$

$$\frac{d\hat{F}(I_l+\epsilon)}{dI}+1=\frac{\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{N}_d\geq l}\right]}{\frac{1-q_s^N}{1-q_s}-\mathcal{P}+\alpha\mathbb{E}^s\left[\frac{N}{N-\mathcal{N}_d}\mathbb{I}_{\mathcal{N}_d\geq l}\right]}. \quad \text{(A10)}$$

Given that

$$\frac{1}{\frac{N}{N-l}}=\frac{\alpha\mathbb{P}^s\left(\mathcal{N}_d=l\right)}{\alpha\frac{N}{N-l}\mathbb{P}^s\left(\mathcal{N}_d=l\right)}>\frac{\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{N}_d\geq l+1}\right]}{\alpha\mathbb{E}^s\left[\frac{N}{N-\mathcal{N}_d}\mathbb{I}_{\mathcal{N}_d\geq l+1}\right]}>\frac{\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{N}_d\geq l+1}\right]}{\frac{1-q_s^N}{1-q_s}-\mathcal{P}+\alpha\mathbb{E}^s\left[\frac{N}{N-\mathcal{N}_d}\mathbb{I}_{\mathcal{N}_d\geq l+1}\right]},$$

we conclude that $\hat{F}(I)$ is convex at kink I_l,[26] that is

$$\frac{d\hat{F}(I_l+\epsilon)}{dI}>\frac{d\hat{F}(I_l-\epsilon)}{dI}. \quad \text{(A11)}$$

Step 3: derive the explicit expression of $\hat{F}(I)$

[26]For $x_1, x_2, y_1, y_2 > 0$ such that $\frac{x_1}{y_1} > \frac{x_2}{y_2}$, we have $\frac{x_1+x_2}{y_1+y_2} > \frac{x_2}{y_2}$.

Using the expression of $X^{N-1}(I,F)$ in Eq. (A28) of Lemma IA.1, we have:

$$
\begin{aligned}
X^{N-1}(I,F) &= \mathbb{E}^s\left[\frac{\mathcal{N}_d}{N-\mathcal{N}_d}\right](D-I-F) + \alpha\mathbb{E}^s\left[\frac{\mathcal{N}_d(D-I)-NF}{N-\mathcal{N}_d}\mathbb{I}_{\mathcal{N}_d \geq \lceil \frac{NF}{D-I} \rceil}\right] \\
&= \frac{q_s - q_s^N}{1-q_s}(D-I-F) + \sum_{\mathcal{N}_d=l}^{N-1}\binom{N-1}{\mathcal{N}_d} q_s^{\mathcal{N}_d}(1-q_s)^{N-1-\mathcal{N}_d}\alpha\left(\frac{N(D-I-F)}{N-\mathcal{N}_d} - (D-I)\right) \\
&= \frac{q_s - q_s^N}{1-q_s}(D-I-F) + \alpha y_l^{(N)}(q_s)(D-I-F) - \alpha z_l^{(N)}(q_s)(D-I), \qquad \text{(A12)}
\end{aligned}
$$

where $l := \lceil \frac{NF}{D-I} \rceil$ is the minimum number of defaults such that pre-funded resources are exhausted and end-of-waterfall resources are needed, and $y_l^{(N)}(q_s)$ and $z_l^{(N)}(q_s)$ are given by

$$
y_l^{(N)}(q_s) := \sum_{\mathcal{N}_d=l}^{N-1}\binom{N-1}{\mathcal{N}_d} q_s^{\mathcal{N}_d}(1-q_s)^{N-1-\mathcal{N}_d}\frac{N}{N-\mathcal{N}_d} = \frac{1}{1-q_s}\sum_{\mathcal{N}_d=l}^{N-1}\binom{N}{\mathcal{N}_d} q_s^{\mathcal{N}_d}(1-q_s)^{N-\mathcal{N}_d}; \qquad \text{(A13)}
$$

$$
z_l^{(N)}(q_s) := \sum_{\mathcal{N}_d=l}^{N-1}\binom{N-1}{\mathcal{N}_d} q_s^{\mathcal{N}_d}(1-q_s)^{N-1-\mathcal{N}_d}. \qquad \text{(A14)}
$$

To understand the terms $y_l^{(N)}(q_s)$ and $z_l^{(N)}(q_s)$, suppose Y follows a Binomial distribution with parameter (N, a) and Z follows a Binomial distribution with parameter $(N-1, a)$, then Eq. (A13) and (A14) imply that $y_l^{(N)}(a) = \frac{\mathbb{P}(l \leq Y \leq N-1)}{1-a}$ and $z_l^{(N)}(a) = \mathbb{P}(l \leq Z)$. Hence, both $y_l^{(N)}(q_s)$ and $z_l^{(N)}(q_s)$ are step functions of l, $(1-q_s)y_l^{(N)}(q_s) \in (0,1)$, and $z_l^{(N)}(q_s) \in (0,1)$.

Plugging Eq. (A12) into Eq. (18) and solving for $\hat{F}(I)$, we obtain,

$$
\hat{F}(I) = D - I - \frac{(A_{CCP}-D)\mathcal{P} + \alpha z_l^{(N)}(q_s)(D-I)}{\frac{1-q_s^N}{1-q_s} - \mathcal{P} + \alpha y_l^{(N)}(q_s)}. \qquad \text{(A15)}
$$

Let $\hat{F}(0)$ be the value of default fund when setting $I=0$ in Eq. (18), and $\hat{I}(0)$ be the value of initial margin when setting $F=0$ in Eq. (18), then

$$
\hat{F}(0) = D - \frac{(A_{CCP}-D)\mathcal{P} + \alpha z_l^{(N)}(q_s)D}{\frac{1-q_s^N}{1-q_s} - \mathcal{P} + \alpha y_l^{(N)}(q_s)}; \qquad \text{(A16)}
$$

$$
\hat{I}(0) = D - \frac{(A_{CCP}-D)\mathcal{P}}{(1+\alpha)\frac{q_s-q_s^N}{1-q_s} + 1 - \mathcal{P}}, \qquad \text{(A17)}
$$

where the last equality is because at $F=0$, $l = \lceil \frac{NF}{D-I} \rceil = 0$ and $\alpha y_{l=0}^{(N)}(q_s) - \alpha z_{l=0}^{(N)}(q_s) = \alpha\frac{1-q_s^N}{1-q_s} - \alpha = \alpha\frac{q_s-q_s^N}{1-q_s}$.

Finally, the properties of function $\bar{F}(I)$ solving Eq. (A3) can be examined analogously by replacing q_s with q_r in the above analysis.

Proof of Lemma 1

Using Eq. (13), (14), and (18), we obtain that the expected profit of a clearing member under the minimum combinations of incentive-compatible collateral $(I, \hat{F}(I))$ satisfies Eq. (20). Since $\hat{I}(0)$ is the value of initial margin when setting $F = 0$ in Eq. (18), the collateral combination, $\left(\hat{I}(0), F = 0\right)$, is one point on the IC curve, $(I, \hat{F}(I))$. Therefore, we must have that $\max_I V(a = s; I, \hat{F}(I)) \geq V(a = s; \hat{I}(0), F = 0)$. Using $\Theta := \mu_s - 1 + p_c - (1 - q_s)p_c\mathcal{P}$ and plugging $\hat{I}(0)$ given by Eq. (A17) and $F = 0$ into Eq. (20), we have that a clearing member's expected profit under $\left(\hat{I}(0), F = 0\right)$ is

$$\begin{aligned} V(a = s; \hat{I}(0), F = 0) &= A_{CCP}\left(\mu_s - (1 - q_s)p_c\mathcal{P}\right) - \Theta\hat{I}(0), \\ &= (1 - p_c + \frac{(\Delta + 1)\Theta}{\Delta + 1 - \mathcal{P}})A_{CCP} - \frac{(\Delta + 1)\Theta D}{\Delta + 1 - \mathcal{P}} \end{aligned}$$

where we introduce $\Delta := (1 + \alpha)\frac{q_s - q_s^N}{1 - q_s}$.

The expected profit of a dealer who posts incentive-compatible collateral in a bilateral market V_{BT} given by (12) can be reduced to

$$V_{BT} = (A_{BT} - D)\frac{\Theta}{1 - \mathcal{P}} + (1 - p_c)A_{BT} = (1 - p_c + \frac{\Theta}{1 - \mathcal{P}})A_{BT} - \frac{\Theta D}{1 - \mathcal{P}}.$$

Plugging in $A_{CCP} = A_0 + P_{CCP}$ and $A_{BT} = A_0 + P_{BT}$, under the condition that

$$\gamma > \underline{\gamma} = \frac{\left(\frac{1}{1-\mathcal{P}} - \frac{(\Delta+1)}{\Delta+1-\mathcal{P}}\right)\Theta(A_0 - D) - \left(\frac{(\Delta+1)\Theta}{\Delta+1-\mathcal{P}} - \frac{\Theta(1-q_s)}{1-\mathcal{P}} + (1 - p_c)q_s\right)p_c D}{\left[(1 - p_c + \frac{(\Delta+1)\Theta}{\Delta+1-\mathcal{P}})(1 - p_c) - (\frac{\Theta}{1-\mathcal{P}} + 1 - p_c)(1 - q_s)(1 - p_c - p_c q_s)\right]p_c D^2}, \quad \text{(A18)}$$

after some algebra we obtain,

$$\max_I V(a = s; I, \hat{F}(I)) \geq V(a = s; \hat{I}(0), F = 0) > V_{BT}.$$

A dealer with a fully collateralized margin position never defaults. Hence, the price of the derivative contract is P_{CCP} regardless of whether traded in a bilateral market or in a centrally cleared market. Consequently, the expected profit of a fully collateralized dealer is $(1 - p_c)D + (A_{CCP} - D)\mu_s$. Comparing the values we have,

$$\begin{aligned} &V(a = s; \hat{I}(0), F = 0) - ((1 - p_c)D + (A_{CCP} - D)\mu_s) \\ =&(A_{CCP} - D)\left(\frac{\Delta + 1}{\Delta + 1 - \mathcal{P}}\Theta + (1 - p_c) - \mu_s\right) \\ =&(A_{CCP} - D)\mathcal{P}\left(\frac{\Theta}{\Delta + 1 - \mathcal{P}} - (1 - q_s)p_c\right) > \frac{(A_{CCP} - D)\mathcal{P}}{\Delta + 1 - \mathcal{P}}\left(\mu_s - 1 - q_s\alpha p_c\right) > 0, \end{aligned}$$

where we have used $\Delta + 1 = (1 + \alpha)\frac{q_s - q_s^N}{1 - q_s} + 1 < \frac{(1+\alpha)q_s}{1 - q_s} + 1 = \frac{q_s\alpha + 1}{1 - q_s}$ and $\alpha < \frac{\mu_s - 1}{p_c q_s}$ using assumption (3).

Proof of Proposition 3

Lemma 1 reduces the constrained optimization problem (19) to an equivalent one where the clearing member's expected profit is maximized along the IC curve $\left(I, \hat{F}(I)\right)$; that is, the optimal collateral solves $\max_I V(a = s; I, \hat{F}(I))$ subject to the constraint specified in (18). Combining Eq. (13), (14), and (18), we obtain that if the demanded collateral pair is $\left(I, \hat{F}(I)\right)$, all members hedge and the expected profit of a clearing member satisfies Eq. (20), where we recall that $\Theta := \mu_s - 1 + p_c - (1 - q_s)p_c\mathcal{P}$. We take the partial derivative of the member's value function, given by (20), with respect to I and obtain[27]

$$\begin{aligned}
&\frac{\partial}{\partial I}V(a = s; I, \hat{F}(I)) = (\delta + \Theta)\left[\frac{\delta}{\delta + \Theta} - \frac{d}{dI}\left(\hat{F}(I) + I\right)\right] \\
=&(\delta + \Theta)\left[\frac{\delta}{\delta + \Theta} - \frac{\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{N}_d \geq l}\right]}{\frac{1-q_s^N}{1-q_s} - \mathcal{P} + \alpha\mathbb{E}^s\left[\frac{N}{N-\mathcal{N}_d}\mathbb{I}_{\mathcal{N}_d \geq l}\right]}\right] := (\delta + \Theta)\left[\frac{\delta}{\delta + \Theta} - \mathcal{H}(l)\right], \qquad \text{(A19)}
\end{aligned}$$

where the second equality follows from Eq. (A7) and $l := \lceil \frac{NF}{D-I} \rceil$, and the last equality follows from the definition of $\mathcal{H}(l)$.

We have established in Proposition 2 that $\hat{F}(I)$ is continuous in I and convex at the set of kinks $\left\{I_l : l = \frac{N\hat{F}(I_l)}{D-I_l} = 0, 1, ..., \lceil \frac{N\hat{F}(0)}{D} \rceil - 1\right\}$. From the proof of Proposition 2, we know that $I_l > I_{l+1}$; Given $I \in [0, \hat{I}(0)]$, the minimum value of I is $I_{\lceil \frac{N\hat{F}(0)}{D} \rceil} = 0$ (not necessarily a kink), and the maximum value of I is $I_0 = \hat{I}(0)$ (this also corresponds to the largest kink, obtained by setting $l = 0$). From (A9)–(A10), we also know that $\frac{d}{dI}\hat{F}(I_l + \epsilon) + 1 = \mathcal{H}(l)$ and $\frac{d}{dI}\hat{F}(I_l - \epsilon) + 1 = \mathcal{H}(l + 1)$. (A11) implies that $\mathcal{H}(l) > \mathcal{H}(l + 1)$. Therefore, for $l = 0, 1, ..., \bar{l} = \lceil \frac{N\hat{F}(0)}{D} \rceil$, the maximum value of $\mathcal{H}(l)$ is $\mathcal{H}(0)$ and the minimum value of $\mathcal{H}(l)$ is $\mathcal{H}(\bar{l})$. Moreover, it follows directly from Eq. (A7) and the definition of $\mathcal{H}(\cdot)$ that the step function $\frac{d}{dI}\left(\hat{F}(I) + I\right)$ admits the following explicit expression

$$\frac{d}{dI}\left(\hat{F}(I) + I\right) = \begin{cases} \mathcal{H}\left(\bar{l}\right), & I \in \left[0, I_{\bar{l}-1}\right) \\ \mathcal{H}\left(l\right), & I \in \left(I_l, I_{l-1}\right), \quad l = \bar{l} - 1, ..., 1 \\ \mathcal{H}\left(0\right), & I = I_{0+} \end{cases} \qquad \text{(A20)}$$

where we recall that $\bar{l} = \lceil \frac{N\hat{F}(0)}{D} \rceil$. Figure A1 gives a graphical illustration of $\frac{d}{dI}\left(\hat{F}(I) + I\right)$ as a step function of I.

To solve for the optimal initial margin and default fund pair, we can then discuss the following cases by comparing the relative magnitude of $\frac{\delta}{\delta+\Theta}$ and $\mathcal{H}(l)$ based on Eq. (A19).

Case 1: when $\mathcal{H}(0) < \frac{\delta}{\delta+\Theta}$, we have that $\max_l \mathcal{H}(l) < \frac{\delta}{\delta+\Theta}$. Accordingly, $\frac{\partial}{\partial I}V(a = s; I, \hat{F}(I)) > 0$ for $I \in [0, \hat{I}(0)]$, and so I^{SB} takes the maximum value of initial margin, which is $\hat{I}(0)$ defined by the value of initial margin that satisfies Eq. (18) by setting $F = 0$. This case features the exclusive use of initial margin.

[27] All the results can be obtained if we were to follow an alternative method by taking the partial derivative with respect to F instead.

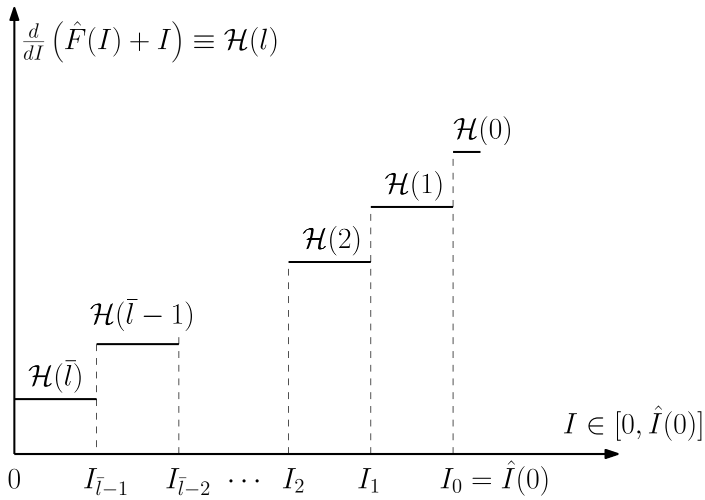

Figure A1. Plot of Step Function $\frac{d}{dI}\left(\hat{F}(I)+I\right)$. This figure illustrates $\frac{d}{dI}\left(\hat{F}(I)+I\right) := \mathcal{H}(l)$ as a step function of I.

Case 2: when $\mathcal{H}(1) < \frac{\delta}{\delta+\Theta} \leq \mathcal{H}(0)$, again we have $I^{SB} = \hat{I}(0)$ because $\frac{\partial}{\partial I}V(a = s; I, \hat{F}(I)) > 0$ for $I < \hat{I}(0)$. (Recall that $\frac{d}{dI}\hat{F}(I_0 - \epsilon) = \mathcal{H}(1)$.)

Case 3: when $\mathcal{H}(\bar{l}) < \frac{\delta}{\delta+\Theta} \leq \mathcal{H}(1)$, since $\mathcal{H}(l)$ is a step function, let us examine the case when $\mathcal{H}(l+1) < \frac{\delta}{\delta+\Theta} \leq \mathcal{H}(l)$ for $l = 1, ..., \bar{l}-1$. We have that $I^{SB} = I_l$ because for $I < I_l$, we have that $\frac{\partial}{\partial I}V(a = s; I, \hat{F}(I)) > 0$, so I_l is the minimum value of initial margin that satisfies $\frac{\partial}{\partial I}V(a = s; I, \hat{F}(I)) \leq 0$.

Case 4: when $\frac{\delta}{\delta+\Theta} \leq \mathcal{H}\left(\bar{l}\right)$, since $\mathcal{H}\left(\bar{l}\right) = \min_l \mathcal{H}(l)$, we have $\frac{\partial}{\partial I}V(a = s; I, \hat{F}(I)) \leq 0$ for $I \in [0, \hat{I}(0)]$, and so I^{SB} takes the minimum value of initial margin that is $I^{SB} = 0$. This case features the exclusive use of default fund. To solve for the default fund collateral, we need to compare $\hat{F}(0)$ which is the exclusive use of default fund on the IC curve by setting $I = 0$, and F^{FB}, which is the first-best collateral. If $1 - \mu_s + p_c\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{N}_d \geq \frac{N\hat{F}(0)}{D}}\right] < \delta$, we conclude that the first best default fund satisfies $F^{FB} < \hat{F}(0)$. In other words, (I^{FB}, F^{FB}) does not satisfy incentive compatibility. Hence, to achieve incentive-compatibility, we need to set $F^{SB} = \hat{F}(0)$. If, instead $1 - \mu_s + p_c\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{N}_d \geq \frac{N\hat{F}(0)}{D}}\right] \geq \delta$, we must have that $F^{FB} \geq \hat{F}(0)$. This relation implies that the incentive-compatibility constraint is satisfied at the first-best collateral; consequently, we have $F^{SB} = F^{FB}$. Altogether we have that under $\frac{\delta}{\delta+\Theta} \leq \mathcal{H}\left(\bar{l}\right)$, $I^{SB} = 0$ and $F^{SB} = \max\left\{\hat{F}(0), F^{FB}\right\}$.

Combining all above cases lead to the characterization of the second best collateral requirements given in (21), and concludes the proof of Proposition 3.

Proof of Proposition 4

By Eq. (18), the limit of $X^{N-1}(I, \hat{F}(I))$ directly yields the limit of $\hat{F}(I)$. Hence, we first compute the limit of $X^{N-1}(I, \hat{F}(I))$ as $N \to \infty$. Using the definitions of $y_l^{(N)}$ and $z_l^{(N)}$, given respectively in Eq. (A13) and Eq. (A14), we have:

$$y_l^{(N)}(q_s) = \frac{\mathbb{P}(l \le Y \le N-1)}{1-q_s}, \quad z_l^{(N)}(q_s) = \mathbb{P}(l \le Z),$$

where Y follows a Binomial distribution with parameters (N, q_s), and Z follows a Binomial distribution with parameters $(N-1, q_s)$. Combining $\lim_{N\to\infty} y_l^{(N)}(q_s) = \frac{\lim_{N\to\infty} z_l^{(N)}(q_s)}{1-q_s}$ with Eq. (A12), we obtain that

$$\lim_{N\to\infty} X^{N-1}(I,F) = \frac{q_s}{1-q_s}(D-I-F) + \alpha\frac{q_s(D-I)-F}{1-q_s} \lim_{N\to\infty} z_l^{(N)}(q_s). \tag{A21}$$

To compute $\lim_{N\to\infty} z_l^{(N)}(q_s)$, we rewrite the probability $\mathbb{P}(Z \ge l)$ as

$$\mathbb{P}(Z \ge l) = \mathbb{P}\left(\sqrt{N-1}\left(\frac{Z}{N-1} - q_s\right) \ge \sqrt{N-1}\left(\frac{l}{N-1} - q_s\right)\right).$$

By the central limit theorem, $\sqrt{N-1}\left(\frac{Z}{N-1} - q_s\right)$ converge in distribution to a Gaussian distribution with mean 0 and variance $q_s(1-q_s)$. Using the inequality $\frac{NF}{D-I} \le l = \lceil \frac{NF}{D-I} \rceil < \frac{NF}{D-I} + 1$ we conclude that

$$\lim_{N\to\infty} \sqrt{N-1}\left(\frac{l}{N-1} - q_s\right) = \infty \times \mathbb{I}_{\frac{F}{D-I} > q_s} + 0 \times \mathbb{I}_{\frac{F}{D-I} = q_s} + (-\infty) \times \mathbb{I}_{\frac{F}{D-I} < q_s}.$$

Using the fact that a zero-mean Gaussian distribution is greater than ∞ with probability 0, greater than 0 with probability 1/2, and greater than $-\infty$ with probability 1, we obtain that

$$\lim_{N\to\infty} z_l^{(N)}(q_s) = 0 \times \mathbb{I}_{\frac{F}{D-I} > q_s} + \frac{1}{2} \times \mathbb{I}_{\frac{F}{D-I} = q_s} + 1 \times \mathbb{I}_{\frac{F}{D-I} < q_s}. \tag{A22}$$

Plugging Eq. (A22) into Eq. (A21), we have

$$\lim_{N\to\infty} X^{N-1}(I,F) = \frac{q_s}{1-q_s}(D-I-F) + 0 \times \mathbb{I}_{\frac{F}{D-I} \ge q_s} + \alpha\frac{q_s(D-I)-F}{1-q_s}\mathbb{I}_{\frac{F}{D-I} < q_s},$$

where we observe that $\left(\frac{q_s(D-I)-F}{1-q_s}\right)\mathbb{I}_{\frac{F}{D-I} = q_s} = 0$ is a degenerate case.

It follows that the limit of Eq. (18) becomes

$$D - I - \hat{F}(I) + \frac{q_s}{1-q_s}(D-I-\hat{F}(I)) + \alpha\frac{q_s(D-I)-\hat{F}(I)}{1-q_s}\mathbb{I}_{\frac{\hat{F}}{D-\hat{I}} < q_s} = (A_{CCP} - I - \hat{F}(I))\mathcal{P}.$$

For a fixed I, the solution for $\hat{F}(I)$ is the unique zero of a piecewise linear, monotone, and continuous function. Solving for $\hat{F}(I)$ yields

$$\begin{cases} D - I - \dfrac{(A_{CCP} - D)\mathcal{P}}{\frac{1}{1-q_s} - \mathcal{P}}, & \text{if } D - I \geq \dfrac{(A_{CCP} - D)\mathcal{P}}{1 - (1-q_s)\mathcal{P}}; \\ D - I - \dfrac{(A_{CCP} - D)\mathcal{P} + \alpha(D-I)}{\frac{1+\alpha}{1-q_s} - \mathcal{P}}, & \text{otherwise.} \end{cases} \tag{A23}$$

It is straightforward to show that the expression in (A23) is continuous and strictly decreasing in I. Observe that for any finite N, Eq. (18) is uniformly bounded and monotone in I and $\hat{F}(I)$ for all nonnegative $I, \hat{F}(I)$ satisfying $0 \leq I + \hat{F}(I) < D$. We then have that taking the limit of Eq. (18) as $N \to \infty$ and then solving for $\hat{F}$ yields the same result as solving for $\hat{F}$ for fixed I and N, and then taking limit as $N \to \infty$. Therefore, the above expression in (A23) corresponds with $\lim_{N\to\infty} \hat{F}(I; N)$. Using the definition of $x(I; N)$ given in (22), we obtain that, as $N \to \infty$, the limiting result in (23) holds. Note that if $D - I < \frac{(A_{CCP}-D)\mathcal{P}}{1-(1-q_s)\mathcal{P}}$, $\lim_{N\to\infty} x(I; N) < q_s$, under which we have that $\frac{\partial}{\partial\alpha} \lim_{N\to\infty} x(I; N) > 0$.

Recall that $\frac{l}{N} = \frac{F^{SB}}{D-I^{SB}}$. Moreover, as $N \to \infty$, $\frac{\mathcal{N}_d}{N} \to q_s$ by law of large numbers. Hence, using the definition of $\mathcal{H}(l)$, we obtain that $\mathcal{H}(l) \to \begin{cases} 0 & \lim_{N\to\infty} x(I, N) > q_s \\ \frac{\alpha}{\frac{1+\alpha}{1-q_s} - \mathcal{P}} & \lim_{N\to\infty} x(I, N) \leq q_s \end{cases}$.

Consider the case $\lim_{N\to\infty} x(I, N) > q_s$, which leads to $\mathcal{H}(l) \to 0$. Using the characterization of the second best collateral given in Eq. (21), the optimal collateral features $\left(\hat{I}(0), 0\right)$; this implies that $\frac{F^{SB}}{D-I^{SB}} = 0$, which is inconsistent with $\lim_{N\to\infty} x(I, N) > q_s$. Hence, this case does not lead to a consistent optimal collateral combination in the limit.

Consider the case $\lim_{N\to\infty} x(I, N) \leq q_s$. Following the analysis in Proposition 3, if $\frac{\delta}{\delta+\Theta} > \lim_{N\to\infty} \mathcal{H}(l) = \frac{\alpha}{\frac{1+\alpha}{1-q_s} - \mathcal{P}}$, then $\left(I^{SB}, F^{SB}\right) \to \left(\lim_N \to \infty \hat{I}(0), 0\right) = \left(D - \frac{(A_{CCP}-D)\mathcal{P}}{\frac{1+\alpha q_s}{1-q_s} - \mathcal{P}}, 0\right)$, where we recall that the expression of $\hat{I}(0)$ is given in Eq. (A17); if $\frac{\delta}{\delta+\Theta} \leq \lim_{N\to\infty} \mathcal{H}(l) = \frac{\alpha}{\frac{1+\alpha}{1-q_s} - \mathcal{P}}$, then $\lim_{N\to\infty} x(I^{SB}, N) = q_s$; combining with Eq. (23), we obtain that $I^{SB} \to D - \frac{(A_{CCP}-D)\mathcal{P}}{1-\mathcal{P}(1-q_s)}$. This completes the proof of Proposition 4.

Proof of Proposition 5

Denote by $\mathcal{S}_d$ the size weighted share of defaulting members in the credit event; that is, $\mathcal{S}_d = \sum_i \mathbb{I}_{i \text{ defaults}} \frac{K_i}{\sum_j K_j}$. Here, $\frac{K_i}{\sum_j K_j}$ denotes the percentage size of member i with respect to the total size of clearing members.[28] Prefunded collateral posted by members is exhausted in the event that $\mathcal{S}_d \geq \frac{F}{D-I}$.

To understand the risk-management action of member i, we analyze the IC constraint of member i (which is analogous to Eq. (18)). Given collateral requirements I and F, and given that all other members hedge, $a_{-i} = s$, member i hedges if and only if $V_i(a_i = s, a_{-i} = s, I, F) \geq V_i(a_i = r, a_{-i} = s, I, F)$, which reduces to

$$(D - I - F)(1 - \mathcal{P}) + X_i(I, F) \leq (A_{CCP} - D)\mathcal{P}, \tag{A24}$$

[28] Note that if $K_i = 1, \forall i$, we recover the baseline model with size homogeneity, i.e., where $\mathcal{S}_d = \sum_i \mathbb{I}_{i \text{ defaults}} \frac{1}{N} = \frac{\mathcal{N}_d}{N}$.

where $X_i(I, F)$ denotes the expected contribution to loss-sharing and the associated financing cost incurred by the surviving member i during the credit event, i.e.,

$$\begin{aligned}
&X_i(I, F) \\
=&\mathbb{E}^s\left[\frac{\sum_{j\neq i}\mathbb{I}_{j\text{ defaults}}\frac{K_j}{\sum_j K_j}(D-I-F)}{1-\sum_{j\neq i}\mathbb{I}_{j\text{ defaults}}\frac{K_j}{\sum_j K_j}}+\alpha\left(\frac{\sum_{j\neq i}\mathbb{I}_{j\text{ defaults}}\frac{K_j}{\sum_j K_j}(D-I-F)}{1-\sum_{j\neq i}\mathbb{I}_{j\text{ defaults}}\frac{K_j}{\sum_j K_j}}-F\right)^+\right] \\
=&\mathbb{E}^s\left[\frac{\mathcal{S}_d-\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}}{1-\mathcal{S}_d+\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}}\right](D-I-F)+\alpha\mathbb{E}^s\left[\left(\frac{(\mathcal{S}_d-\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j})(D-I)-F}{1-\mathcal{S}_d+\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}}\right)^+\right],
\end{aligned} \tag{A25}$$

where in the last equality, we have used that $\sum_{j\neq i}\mathbb{I}_{j\text{ defaults}}\frac{K_j}{\sum_j K_j}=\mathcal{S}_d-\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}$.

We define the function $\hat{F}_i(I)$ as satisfying the following IC equation for a given I:

$$(D-I-\hat{F}_i(I))(1-\mathcal{P})+X_i(I,\hat{F}_i(I))=(A_{CCP}-D)\mathcal{P}, \tag{A26}$$

where $X_i(I, F)$ is given by Eq. (A25). Using a reasoning analogous to that in Lemma IA.1 of online Appendix IA.I, we apply the dominated convergence theorem and conclude that the function $X_i(I, F)$ is piecewise linear, continuous, and strictly decreasing in both I and F. Then we have that $\hat{F}_i(I)$ is uniquely determined for a given I. We also conclude that for $F \geq \hat{F}_i(I)$, condition (A24) holds, and so $V_i(s, a_{-i} = s, I, F) \geq V_i(r, a_{-i} = s, I, F)$; equivalently, we have

$$a_i(a_{-i}=s; I, F)=\begin{cases} r & 0\leq F<\hat{F}_i(I) \\ s & \hat{F}_i(I)\leq F\leq D-I\end{cases}.$$

Next, we characterize $\hat{F}_i(I)$ as a function of I. The function $X_i(I, F)$ is piecewise linear, continuous, and strictly decreasing in both I and F, and the first term on the left-hand side of Eq. (A26) is also linear and continuous in both I and F. Using the implicit function theorem, we conclude that $\hat{F}_i(I)$ is piecewise linear and continuous in I. We further apply the implicit function theorem to Eq. (A26) and solve for $d\hat{F}_i(I)/dI$ as follows:

$$\frac{d\hat{F}_i(I)}{dI}+1=\frac{\alpha\mathbb{E}^s\left[\mathbb{I}_{\left(\mathcal{S}_d-\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}\right)\geq\frac{\hat{F}_i(I)}{D-I}}\right]}{(1-\mathcal{P})+\mathbb{E}^s\left[\frac{\mathcal{S}_d-\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}}{1-\mathcal{S}_d+\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}}\right]+\alpha\mathbb{E}^s\left[\frac{1}{1-\mathcal{S}_d+\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}}\mathbb{I}_{\left(\mathcal{S}_d-\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}\right)\geq\frac{\hat{F}_i(I)}{D-I}}\right]}. \tag{A27}$$

Since $1-\mathcal{P}>0$, and both the numerator and the denominator of (A27) are positive, we must have $\frac{d\hat{F}_i(I)}{dI}+1>0$. Since $\frac{1}{1-\mathcal{S}_d+\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}}>1$, we have that $\mathbb{E}^s\left[\mathbb{I}_{\left(\mathcal{S}_d-\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}\right)\geq\frac{\hat{F}_i(I)}{D-I}}\right]<\mathbb{E}^s\left[\frac{1}{1-\mathcal{S}_d+\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}}\mathbb{I}_{\left(\mathcal{S}_d-\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}\right)\geq\frac{\hat{F}_i(I)}{D-I}}\right]$, which suggests that $\frac{d\hat{F}_i(I)}{dI}+1<1$. Together, these results imply that $\frac{d\hat{F}_i(I)}{dI}\in(-1,0)$, suggesting that the function $\hat{F}_i(I)$ is strictly decreasing in I with slope greater than -1.

Next, we show that $\hat{F}_i(I)$ is convex in I. From Eq. (A27), $\frac{d\hat{F}_i(I)}{dI}$ depends on I only through the term $\mathbb{I}_{\left(\mathcal{S}_d-\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}\right)\geq\frac{\hat{F}_i(I)}{D-I}}$. Since the set of possible values that $\mathcal{S}_d - \mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}$ can take is a discrete set (depending on the realizations of members' defaults and the given size distribution K),[29] $\frac{d\hat{F}_i(I)}{dI}$ is a step function of I. Consider a kink point of the function $\hat{F}_i(I)$ at $\left(I_x, \hat{F}_i(I_x)\right)$ where $\frac{d\hat{F}_i(I_x+\epsilon)}{dI} \neq \frac{d\hat{F}_i(I_x-\epsilon)}{dI}$. From Eq. (A27), it must be that $x := \frac{\hat{F}_i(I_x)}{D-I_x}$ is one of the possible values of $\mathcal{S}_d - \mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}$. Plugging $\hat{F}_i(I_x) = x(D - I_x)$ into Eq. (A26), we have

$$(1-x)\left(1-\mathcal{P}+\mathbb{E}^s\left[\frac{\sum_{j\neq i}\mathbb{I}_{j \text{ defaults}}\frac{K_j}{\sum_j K_j}}{1-\sum_{j\neq i}\mathbb{I}_{j \text{ defaults}}\frac{K_j}{\sum_j K_j}}\right]\right)+ \alpha\mathbb{E}^s\left[\left(\frac{\sum_{j\neq i}\mathbb{I}_{j \text{ defaults}}\frac{K_j}{\sum_j K_j}-x}{1-\sum_{j\neq i}\mathbb{I}_{j \text{ defaults}}\frac{K_j}{\sum_j K_j}}\right)^+\right] = \frac{(A_{CCP}-D)\mathcal{P}}{(D-I_x)}.$$

Since the left-hand side of the above equation is decreasing in x, the right-hand side must also decrease in x; hence I_x decreases with $x = \frac{\hat{F}_i(I_x)}{D-I_x}$, or equivalently $\frac{\hat{F}_i(I_x)}{D-I_x}$ is monotonically decreasing with I_x. Hence, $\frac{\hat{F}_i(I_x-\epsilon)}{D-(I_x-\epsilon)} > x$ and $\frac{\hat{F}_i(I_x+\epsilon)}{D-(I_x+\epsilon)} < x$, which gives

$$\frac{d\hat{F}_i(I_x-\epsilon)}{dI}+1 = \frac{\alpha\mathbb{E}^s\left[\mathbb{I}_{\left(\mathcal{S}_d-\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}\right)>x}\right]}{(1-\mathcal{P})+\mathbb{E}^s\left[\frac{\mathcal{S}_d-\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}}{1-\mathcal{S}_d+\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}}\right]+\alpha\mathbb{E}^s\left[\frac{1}{1-\mathcal{S}_d+\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}}\mathbb{I}_{\left(\mathcal{S}_d-\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}\right)>x}\right]}$$

$$\frac{d\hat{F}_i(I_x+\epsilon)}{dI}+1 = \frac{\alpha\mathbb{E}^s\left[\mathbb{I}_{\left(\mathcal{S}_d-\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}\right)\geq x}\right]}{(1-\mathcal{P})+\mathbb{E}^s\left[\frac{\mathcal{S}_d-\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}}{1-\mathcal{S}_d+\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}}\right]+\alpha\mathbb{E}^s\left[\frac{1}{1-\mathcal{S}_d+\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}}\mathbb{I}_{\left(\mathcal{S}_d-\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}\right)\geq x}\right]} =$$

$$\frac{\alpha\mathbb{E}^s\left[\mathbb{I}_{\left(\mathcal{S}_d-\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}\right)>x}\right]+\alpha\mathbb{E}^s\left[\mathbb{I}_{\left(\mathcal{S}_d-\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}\right)=x}\right]}{(1-\mathcal{P})+\mathbb{E}^s\left[\frac{\mathcal{S}_d-\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}}{1-\mathcal{S}_d+\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}}\right]+\alpha\mathbb{E}^s\left[\frac{1}{1-\mathcal{S}_d+\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}}\mathbb{I}_{\left(\mathcal{S}_d-\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}\right)>x}\right]+\alpha\mathbb{E}^s\left[\frac{1}{1-x}\mathbb{I}_{\left(\mathcal{S}_d-\mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}\right)=x}\right]}$$

[29] For instance, when $K_i = 1, \forall i$, we recover the baseline model where the set of possible values of $\mathcal{S}_d - \mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}$ is $\left\{0, \frac{1}{N}, \ldots, \frac{N-1}{N}\right\}$. If all other members have the same size and only member i is K_i times the size of others, then the set of possible values of $\mathcal{S}_d - \mathbb{I}_{i \text{ defaults}}\frac{K_i}{\sum_j K_j}$ is $\left\{0, \frac{1}{N-1+K_i}, \ldots, \frac{N-1}{N-1+K_i}\right\}$.

Since $\frac{1}{1-x}$ monotonically increases with x, we must have

$$\frac{\alpha\mathbb{E}^s\left[\mathbb{I}_{\left(\mathcal{S}_d-\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}\right)=x}\right]}{\alpha\mathbb{E}^s\left[\frac{1}{1-x}\mathbb{I}_{\left(\mathcal{S}_d-\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}\right)=x}\right]} > \frac{\alpha\mathbb{E}^s\left[\mathbb{I}_{\left(\mathcal{S}_d-\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}\right)>x}\right]}{\alpha\mathbb{E}^s\left[\frac{1}{1-\left(\mathcal{S}_d-\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}\right)}\mathbb{I}_{\left(\mathcal{S}_d-\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}\right)>x}\right]} > \frac{d\hat{F}_i(I_x-\epsilon)}{dI}+1.$$

Applying the same logic as in footnote 26, we conclude that $\frac{d\hat{F}_i(I_x+\epsilon)}{dI} > \frac{d\hat{F}_i(I_x-\epsilon)}{dI}$, so the function $\hat{F}_i(I)$ is convex at the kinks.

Finally, since both $\frac{\mathcal{S}_d-\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}}{1-\mathcal{S}_d+\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}}$ and $\left(\frac{(\mathcal{S}_d-\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j})(D-I)-F}{1-\mathcal{S}_d+\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_j K_j}}\right)^+$ are decreasing functions of K_i (the size of member i), it follows from Eq. (A25) that, for any realization of members' risky investments, we must have that $X_i(I,F)$ decreases with K_i. Equivalently, take arbitrary $i_1,\ i_2 \in 1,2\ldots,N$ with $K_{i_1} > K_{i_2}$, we have that $X_{i_1}(I,F) < X_{i_2}(I,F)$. Eq. (A26) for both i_1 and i_2 imply that

$$X_{i_1}(I,\hat{F}_{i_1}(I)) - X_{i_2}(I,\hat{F}_{i_2}(I)) = (1-\mathcal{P})\left(\hat{F}_{i_1}(I)-\hat{F}_{i_2}(I)\right).$$

Suppose for contradiction that $\hat{F}_{i_1}(I) \geq \hat{F}_{i_2}(I)$ for a given I, then the above equation must imply that $X_{i_1}(I,\hat{F}_{i_1}(I)) - X_{i_2}(I,\hat{F}_{i_2}(I)) \geq 0$. But using the earlier conclusion that $X_{i_1}(I,F) < X_{i_2}(I,F)$ and that both $X_{i_1}(I,F)$ and $X_{i_2}(I,F)$ are strictly decreasing in F, we must have that $X_{i_1}(I,\hat{F}_{i_1}(I)) \leq X_{i_1}(I,\hat{F}_{i_2}(I)) < X_{i_2}(I,\hat{F}_{i_2}(I))$, thus a contradiction. Therefore, we conclude that $\hat{F}_{i_1}(I) < \hat{F}_{i_2}(I)$ for a given I.

Online Appendix

In this online Appendix, we present auxiliary lemmas that are used to establish the technical results in the main paper; we also present proofs for the corollaries in the main paper.

IA.I Technical Lemma

Lemma IA.1 *The term $X^g(I,F)$ given in Eq.* (16) *admits the following explicit expression:*

$$X^g(I,F) = \sum_{\mathcal{N}_d=0}^{N-1} f^g(\mathcal{N}_d)\frac{\mathcal{N}_d(D-I-F)}{N-\mathcal{N}_d} + \sum_{\mathcal{N}_d=\lceil \frac{NF}{D-I}\rceil}^{N-1} f^g(\mathcal{N}_d)\alpha\frac{\mathcal{N}_d(D-I)-NF}{N-\mathcal{N}_d}, \tag{A28}$$

where $\lceil\cdot\rceil$ denotes the ceiling function, and for each $\mathcal{N}_d$, $f^g(\mathcal{N}_d)$ is the probability that $\mathcal{N}_d$ out of $N-1$ members default when $g=0,1,\ldots,N-1$ members hedge, i.e.,

$$f^g(\mathcal{N}_d) = \sum_{m=0}^{\mathcal{N}_d}\binom{g}{m}q_s^m(1-q_s)^{g-m}\binom{N-1-g}{\mathcal{N}_d-m}q_r^{\mathcal{N}_d-m}(1-q_r)^{N-1-g-(\mathcal{N}_d-m)}. \tag{A29}$$

For $I+F<D$, the function $X^g(I,F)>0$ is strictly decreasing in g, piecewise linear, continuous, and strictly decreasing in both I and F. Moreover, $X^g(I;D-I)=0$. For notational simplicity, set $\mathbb{E}^s[\cdot] := \mathbb{E}^{g=N-1}[\cdot]$ and $\mathbb{E}^r[\cdot] := \mathbb{E}^{g=0}[\cdot]$. We have $\mathbb{E}^s\left[\frac{\mathcal{N}_d}{N-\mathcal{N}_d}\right] = \frac{q_s-q_s^N}{1-q_s}$, $\mathbb{E}^s\left[\frac{N}{N-\mathcal{N}_d}\right] = \frac{1-q_s^N}{1-q_s}$, $\mathbb{E}^r\left[\frac{\mathcal{N}_d}{N-\mathcal{N}_d}\right] = \frac{q_r-q_r^N}{1-q_r}$, $\mathbb{E}^r\left[\frac{N}{N-\mathcal{N}_d}\right] = \frac{1-q_r^N}{1-q_r}$.

Proof. Notice that the function $X^g(I,F)$ is the sum of two terms. We start analyzing the first term $\mathbb{E}^g\left[\frac{\mathcal{N}_d}{N-\mathcal{N}_d}\right] = \sum_{\mathcal{N}_d=0}^{N-1} f^g(\mathcal{N}_d)\frac{\mathcal{N}_d}{N-\mathcal{N}_d} > 0$. Using the moment generating function of the binomial distribution, we have for any $t>0$,

$$\begin{aligned}&\mathbb{E}^g\left[t^{X+Y}\middle|X\sim Binom(g,1-q_s), Y\sim Binom(N-1-g,1-q_r), X\perp Y\right]\\ =&[(1-q_s)t+q_s]^g[(1-q_r)t+q_r]^{N-1-g},\end{aligned}$$

where the notation $X\perp Y$ means that X and Y are uncorrelated. By Fubini's theorem, it follows that

$$\begin{aligned}&\mathbb{E}^g\left[\frac{\mathcal{N}_d}{N-\mathcal{N}_d}\right] = \mathbb{E}^g\left[\frac{N}{N-\mathcal{N}_d}\right]-1\\ =&N\mathbb{E}\left[\frac{1}{1+X+Y}\middle|X\sim Binom(g,1-q_s), Y\sim Binom(N-1-g,1-q_r), X\perp Y\right]-1\\ =&N\int_0^1\mathbb{E}\left[t^{X+Y}\middle|X\sim Binom(g,1-q_s), Y\sim Binom(N-1-g,1-q_r), X\perp Y\right]dt-1\\ =&N\int_0^1[(1-q_s)t+q_s]^g[(1-q_r)t+q_r]^{N-1-g}dt-1.\end{aligned}$$

For any $t \in (0,1)$ and $g = 0, 1, \ldots, N-2$,

$$0 < \frac{[(1-q_s)t+q_s]^{g+1}[(1-q_r)t+q_r]^{N-1-(g+1)}}{[(1-q_s)t+q_s]^{g}[(1-q_r)t+q_r]^{N-1-g}} = \frac{(1-q_s)t+q_s}{(1-q_r)t+q_r} < 1.$$

This proves that $[(1-q_s)t+q_s]^g[(1-q_r)t+q_r]^{N-1-g}$ is strictly decreasing in g. Therefore, $\mathbb{E}^g\left[\frac{\mathcal{N}_d}{N-\mathcal{N}_d}\right] > 0$ is also strictly decreasing in g, and for $I+F<D$, $\mathbb{E}^g\left[\frac{\mathcal{N}_d(D-I-F)}{N-\mathcal{N}_d}\right] > 0$ is strictly decreasing in g, is linear, continuous, and strictly decreasing in both I and F. Moreover, $\mathbb{E}^{N-1}\left[\frac{\mathcal{N}_d}{N-\mathcal{N}_d}\right] = N\int_0^1[(1-q_s)t+q_s]^{N-1}dt - 1 = \frac{q_s-q_s^N}{1-q_s}$, and $\mathbb{E}^0\left[\frac{\mathcal{N}_d}{N-\mathcal{N}_d}\right] = N\int_0^1[(1-q_r)t+q_r]^{N-1}dt - 1 = \frac{q_r-q_r^N}{1-q_r}$.

We next analyze the second term in $X^g(I,F)$. Observe that $\sum_{\mathcal{N}_d=\lceil\frac{NF}{D-I}\rceil}^{N-1} f^g(\mathcal{N}_d)\alpha\frac{\mathcal{N}_d(D-I)-NF}{N-\mathcal{N}_d}$ depends on g only through $f^g(\mathcal{N}_d)$. Since $f^g(\mathcal{N}_d)$ is strictly decreasing in g (as shown above), $X^g(I,F)$ is strictly decreasing in g. Within any subinterval $F \in \left(\frac{(\mathcal{N}_d-1)(D-I)}{N}, \frac{\mathcal{N}_d(D-I)}{N}\right]$ for a given $\mathcal{N}_d = \lceil\frac{NF}{D-I}\rceil, \ldots, N-1$, $\sum_{\mathcal{N}_d=\lceil\frac{NF}{D-I}\rceil}^{N-1} f_g(\mathcal{N}_d)\alpha\frac{\mathcal{N}_d(D-I)-NF}{N-\mathcal{N}_d}$ is linear and strictly decreasing in I and F. Hence, we conclude that $X^g(I,F)$ is piecewise linear and strictly decreasing in I and F. The nonnegative random variable $\alpha\frac{(\mathcal{N}_d(D-I)-NF)^+}{N-\mathcal{N}_d}$ is almost surely continuous in I and F and bounded from above by $\alpha\frac{\mathcal{N}_d D}{N-\mathcal{N}_d}$. By the dominated convergence theorem, the expected value of this random variable is continuous in I and F. Therefore, the function $X^g(I,F)$, is piecewise linear, continuous, and strictly decreasing in I and F.

Finally, when $F = D-I$, both terms in Eq. (A28) are zero, and so $X^g(I, D-I) = 0$. ■

IA.II Proof of Corollary 1

As in Proposition 1, we take the first-order conditions with respect to I and F when X is given by Eq. (25).

$$\frac{\partial}{\partial I}W^{FB}(a=s;I,F) = 1-\mu_s+p_c\kappa\mathbb{E}^s\left[\frac{\mathcal{N}_d\left(\mathcal{N}_d(D-I)-NF\right)^+}{N^2}\right],$$
$$\frac{\partial}{\partial F}W^{FB}(a=s;I,F) = 1-\mu_s-\delta+p_c\kappa\mathbb{E}^s\left[\frac{\left(\mathcal{N}_d(D-I)-NF\right)^+}{N}\right].$$

Under the assumption that $\kappa < \frac{\mu_s-1}{p_c q_s^2 D}$, we have that $\frac{\partial}{\partial I}W^{FB}(a=s;0;0) = 1-\mu_s+p_c\kappa\mathbb{E}^s\left[\frac{\mathcal{N}_d}{N}\right]^2 D < 0$, so $I^{FB}=0$. Given that $W^{FB}(a=s;I,F)$ is concave in both I and F, we can then discuss two scenarios. First, under condition: $0 < \kappa \le \frac{\mu_s-1+\delta}{p_c q_s D}$, we have that $\frac{\partial}{\partial F}W^{FB}(a=s;0,0) = 1-\mu_s-\delta+p_c\kappa\mathbb{E}^s\left[\frac{\mathcal{N}_d}{N}\right]D \le 0$. So $I^{FB}=F^{FB}=0$. Otherwise, we have that $\frac{\partial}{\partial F}W^{FB}(a=s;0,0) > 0$, so $\left(I^{FB},F^{FB}\right) = (0,F^*)$ where F^* satisfies $\kappa = \frac{\mu_s-1+\delta}{p_c\mathbb{E}^s\left[\left(\frac{\mathcal{N}_d}{N}D-F^*\right)^+\right]}$.

When members' risk-management actions are not observable, $a(I,F)$ satisfies Eq. (17)–(A3). To analyze $\hat{F}(I)$, first notice that since $X(N-1;I,F)$ is continuous in I, $\hat{F}(I)$ is also continuous in I following Eq. (18). To further analyze the properties of $d\hat{F}(I)/dI$, we apply the implicit function theorem to Eq. (18) and solve for $d\hat{F}(I)/dI$ as follows:

$$\frac{d\hat{F}(I)}{dI}+1=\frac{\kappa\mathbb{E}^s\left[\frac{\left(\mathcal{N}_d(D-I)-N\hat{F}(I)\right)^+}{N-\mathcal{N}_d}\right]}{\frac{1-q_s^N}{1-q_s}-\mathcal{P}+\kappa\mathbb{E}^s\left[\frac{\left(\mathcal{N}_d(D-I)-N\hat{F}(I)\right)^+}{N-\mathcal{N}_d}\times\frac{N}{N-\mathcal{N}_d}\right]}\in(0,1).$$

We conclude that $\frac{d\hat{F}(I)}{dI}+1$ is continuous in I. To show that $\frac{d\hat{F}(I)}{dI}+1$ is increasing in I, we take the second-order partial derivative of Eq. (18) and obtain

$$\frac{\partial^2 X}{\partial I^2}+\frac{\partial^2 X}{\partial F^2}\left(\frac{d\hat{F}(I)}{dI}\right)^2+\left(-(1-\mathcal{P})+\frac{\partial X}{\partial I}\right)\times\frac{d^2\hat{F}(I)}{dI^2}=0.$$

By Jensen's inequality, $\frac{\partial^2 X}{\partial I^2}>0$ and $\frac{\partial^2 X}{\partial F^2}>0$. Since $\mathcal{P}\in(0,1)$ and $\frac{\partial X}{\partial I}<0$, we have $-(1-\mathcal{P})+\frac{\partial X}{\partial I}<0$; it follows that $\frac{d^2\hat{F}(I)}{dI^2}>0$, which implies that $\frac{d\hat{F}}{dI}+1$ is increasing in I. Function $\hat{F}(I)$ is thus continuous, smooth, piecewise convex, and decreasing in I with first-order derivative greater than -1. In the second-best constrained optimization problem, the objective function is concave with respect to I, which directly gives Eq. (26).

IA.III Proof of Corollary 2

We start by analyzing the first-best collateral requirement. As in Proposition 1, we take the right derivative of the first-best objective function with respect to I and F as follows:

$$\frac{\partial}{\partial I+}W^{FB}(a=s;I,F)=\sum_i K_i\left(1-\mu_s+p_c\alpha\mathbb{E}^s\left[\mathcal{S}_d\mathbb{I}_{\mathcal{S}_d>\frac{F}{D-I}}\right]\right),$$
$$\frac{\partial}{\partial F+}W^{FB}(a=s;I,F)=\sum_i K_i\left(1-\mu_s-\delta+p_c\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{S}_d>\frac{F}{D-I}}\right]\right).$$

First note that at $I=F=0$, we have $\mathbb{E}^s\left[\mathcal{S}_d\mathbb{I}_{\mathcal{S}_d>0}\right]=\mathbb{E}^s\left[\mathcal{S}_d\right]=\sum_i\mathbb{E}^s\left[\mathbb{I}_{i\text{ defaults}}\right]\frac{K_i}{\sum_i K_i}=q_s$, where the last two equalities use the definition $\mathcal{S}_d=\sum_i\mathbb{I}_{i\text{ defaults}}\frac{K_i}{\sum_i K_i}$ and that the default realizations are i.i.d. across members regardless of their sizes. Under Assumption (3) that $\alpha<\frac{\mu_s-1}{p_c q_s}$, we have that $\frac{\partial}{\partial I+}W^{FB}(a=s;I,F)\leq\frac{\partial}{\partial I+}W^{FB}(a=s;0;0)<0$. Hence, we conclude that $I^{FB}=0$. Observe that $\frac{\partial}{\partial F+}W^{FB}(a=s;0;0)=\sum_i K_i\left(1-\mu_s-\delta+p_c\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{S}_d>0}\right]\right)$, where $\mathbb{E}^s\left[\mathbb{I}_{\mathcal{S}_d>0}\right]=1-(1-q_s)^N$. We can then differentiate two scenarios as done in Proposition 1. Under the first scenario, $0<\alpha\leq\frac{\mu_s-1+\delta}{p_c(1-(1-q_s)^N)}$, we have $\frac{\partial}{\partial F+}W^{FB}(a=s;I,F)\leq\frac{\partial}{\partial F+}W^{FB}(a=s;0;0)\leq 0$. We conclude that $I^{FB}=F^{FB}=0$. The value $F^{FB}=0$ satisfies $F^{FB}=\inf\left\{F:1-\mu_s-\delta+p_c\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{S}_d>\frac{F}{D}}\right]\leq 0\right\}$. Under the second scenario, $\alpha>\frac{\mu_s-1+\delta}{p_c(1-(1-q_s)^N)}$, we have $\frac{\partial}{\partial F+}W^{FB}(a=s;0;0)>0$, and so $F^{FB}>0$. The first-best default fund collateral again satisfies $F^{FB}=\inf\left\{F:1-\mu_s-\delta+p_c\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{S}_d>\frac{F}{D}}\right]\leq 0\right\}$. Hence, combining both scenarios, we have $I^{FB}=0$ and $F^{FB}=\inf\left\{F:1-\mu_s-\delta+p_c\alpha\mathbb{E}^s\left[\mathbb{I}_{\mathcal{S}_d>\frac{F}{D}}\right]\leq 0\right\}$.

Next, we analyze the collateral requirements of each member $\left(I_i^{SB},F_i^{SB}\right)$ that maximize the member's expected profits subject to the incentive-compatibility constraint given in (A24). Using Eq. (A26), we conclude that at the pairs of minimum incentive-compatible collateral, the expected profit of member i, scaled by its size, is given by

$$\frac{1}{K_i} V_i(a_i = a_{-i} = s; I, \hat{F}_i(I)) = A_{CCP} \left(\mu_s - p_c(1 - q_s)\mathcal{P}\right) - \Theta I - (\delta + \Theta)\hat{F}_i(I).$$

Taking the first-order derivative of member i's expected profit with respect to I, we have

$$\frac{1}{K_i} \frac{\partial}{\partial I} V_i(a_i = a_{-i} = s; I, \hat{F}_i(I)) = (\delta+\Theta) \left[\frac{\delta}{\delta + \Theta} - \frac{d}{dI} \left(\hat{F}_i(I) + I \right) \right] = (\delta+\Theta) \left[\frac{\delta}{\delta + \Theta} - H_i(I) \right]$$

where for notational simplicity, we denote $H_i(I) := \frac{d\hat{F}_i(I)}{dI} + 1$, whose explicit expression is given in Eq. (A27). As shown in Proposition 5, $H_i(I) \in (0, 1)$, and $H_i(I)$ is a step function of I which is increasing in I. We can then discuss the following cases by comparing $\frac{\delta}{\delta+\Theta}$ and $H_i(I)$.

Case 1: $H_i\left(\hat{I}_i(0)\right) < \frac{\delta}{\delta+\Theta}$. We then have $\max H_i(I) < \frac{\delta}{\delta+\Theta}$. Accordingly, $\frac{\partial}{\partial I} V_i > 0$ and so I_i^{SB} takes the maximum value of initial margin, which is $\hat{I}_i(0)$ and defined by the value of initial margin that satisfies Eq. (A26) when $F = 0$. This case features the exclusive use of initial margin for member i, i.e., $I_i^{SB} = \hat{I}_i(0)$ and $F_i^{SB} = 0$.

Case 2: $H_i(0) < \frac{\delta}{\delta+\Theta} \leq H_i\left(\hat{I}_i(0)\right)$. We have $\min H_i(I) < \frac{\delta}{\delta+\Theta} \leq \max H_i(I)$. Then I_i^{SB} takes the minimum value of I such that $\frac{\partial}{\partial I} V_i \leq 0$, i.e., $I_i^{SB} = \inf\left\{ I : \frac{\delta}{\delta+\Theta} - H_i(I) \leq 0 \right\}$.

Case 3: $\frac{\delta}{\delta+\Theta} \leq H_i(0)$. We then have $\frac{\delta}{\delta+\Theta} \leq \min H_i(I)$, and so $\frac{\partial}{\partial I} V_i \leq 0$, which implies $I_i^{SB} = 0$. To solve for the default fund collateral, we need to compare $\hat{F}_i(0)$, which corresponds to the exclusive use of default fund on the IC curve, and F^{FB}, which is the first-best collateral. If $1 - \mu_s - \delta + p_c \alpha \mathbb{E}^s \left[\mathbb{I}_{\mathcal{S}_d > \frac{\hat{F}_i(0)}{D}} \right] < 0$, we must have $F^{FB} < \hat{F}_i(0)$; hence, to achieve incentive-compatibility for member i, we need to set $F^{SB} = \hat{F}_i(0)$. If, instead $1 - \mu_s - \delta + p_c \alpha \mathbb{E}^s \left[\mathbb{I}_{\mathcal{S}_d > \frac{\hat{F}_i(0)}{D}} \right] \geq 0$, we must have $F^{FB} \geq \hat{F}_i(0)$. This relation implies that the incentive-compatibility condition is already satisfied at the first-best collateral; consequently, we have $F^{SB} = F^{FB}$. Altogether, we have that when the condition $\frac{\delta}{\delta+\Theta} \leq H_i(0)$ holds, then $I_i^{SB} = 0$ and $F_i^{SB} = \max\left\{ F^{FB}, \hat{F}_i(0) \right\}$.