The Cyclicality of Job Creation with Multiple Offers

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November 19, 2020

Abstract

In search of a job, a worker may receive more than one offer. This paper studies the interaction between multiple offers and job creation over the business cycle. Theoretically, I endogenize multiple offers and the wage offer distribution in a Diamond-Mortensen-Pissarides framework. The model predicts that both the fraction of new hires with multiple offers and the share of high-wage vacancies posted by firms are procyclical, the latter implies recessions reduce the quality of vacancies available to job seekers. Empirically, I present evidence consistent with both predictions. The findings contribute to the understanding of the sullying effect of recessions.

Keywords: Business Cycles; Job Search; Unemployment; Vacancies; Wages

JEL codes: E24; E32; J31; J63; J64

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1 Introduction

In search of a job, a worker may receive more than one offer. Summarizing four surveys of employers, Barron et al. (1997) find the average number of offers made per hire is 1.1, suggesting that around 10 percent of job offers are rejected.[1] Wolthoff (2017) provides suggestive evidence that "the cause often lies in the simultaneous arrival of a financially more attractive offer from a different firm". Guo (2020) documents that around a third of new hires from nonemployment chose their current job over another employment opportunity available at the same time.

This paper studies the interaction between multiple offers and job creation over the business cycle. Theoretically, section 2 endogenizes multiple offers in a Diamond-Mortensen-Pissarides (DMP) framework. Following Wolthoff (2014) who documents that transitions between employment states are highly clustered around the first day of each workweek or month, I assume meetings between unemployed workers and vacancies occur randomly in continuous time but decisions on job creation and job destruction are made in discrete time at the end of a period. This allows an unemployed worker to potentially meet multiple vacancies and receive multiple offers before making a decision, the probability of which depends on the offer arrival rate and, in turn, labor market tightness defined as the number of vacancies over unemployment. Via market tightness, the number of vacancies has a positive effect on not only the job finding probability, which is equal to the probability of receiving at least one offer since all offers are acceptable in equilibrium, but also the probability of multiple offers conditional on having at least one. As a result, in addition to the standard predictions that both the number of vacancies and the job finding probability are procyclical, the model also predicts that the fraction of new hires with multiple offers is procyclical, where new hires are defined as unemployed workers with at least one offer.

Instead of wage bargaining as in the standard DMP model, I assume wages are posted by firms. Because the probability that an unemployed worker receives multiple offers in a period is strictly positive, a well-known result from Burdett and Judd (1983) is that all firms cannot post the same wage in equilibrium. So the wage offer distribution is nondegenerate, and it depends on the probability of multiple offers for unemployed workers. Assuming unemployed workers with multiple offers only accept the one with the highest wage, an increase in the probability of multiple offers reduces the chance that low-wage offers are accepted. In response, firms have to post a larger share of high-wage vacancies in equilibrium. That is, the probability of multiple offers has a positive impact on the share of high-wage vacancies posted by firms. As with the probability of multiple offers, the model predicts that the share of high-wage vacancies is also procyclical.

In short, by endogenizing the probability of multiple offers, the model also endogenizes the wage offer distribution, allowing us to study not only the quantity but also the quality (wage

[1] The four surveys are the Employment Opportunity Pilot Projects in 1980 and 1982, the 1992 Small Business Administration survey, and the 1993 Upjohn Institute Survey. The number of offers made per hire in the four surveys are 1.02, 1.08, 1.14 and 1.16, respectively. See Column 6 of Table 7.1 on page 163 for details.

composition) of vacancies over the business cycle. Relative to the standard DMP model where recessions reduce both the job finding probability and the quantity of vacancies, the model in this paper predicts recessions also reduce the fraction of new hires with multiple offers and the quality of vacancies available to job seekers. With a smaller probability of multiple offers and a smaller share of high-wage vacancies, new hires are less likely to have a high-wage offer and end up in a high-wage job during recessions. That is, the model predicts that the fraction of new hires in high-wage jobs is also procyclical.

Section 3 provides some evidence consistent with the model predictions. Using a unique variable on whether a new hire chose the current job over another employment opportunity, section 3.1 documents a positive association between aggregate productivity and the fraction of new hires with multiple offers, suggesting that the latter is procyclical. Section 3.2 shows that an increase in aggregate productivity is associated with an increase in the share of vacancies posted by high-wage industries. Under the assumption that high-wage industries post high-wage vacancies on average, this is consistent with the prediction that the share of high-wage vacancies is procyclical. Finally, section 3.3 obtains the residual wages of new hires by controlling for individual characteristics and estimates their responses to changes in aggregate productivity. The estimates suggest an increase in aggregate productivity is associated with a larger increase in the upper than lower end of the residual wage distribution of new hires. This is consistent with the model's prediction that an increase in aggregate productivity raises the fraction of new hires in high-wage jobs.

Section 4 concludes the paper by discussing its contributions and directions for future work. In short, this paper provides two novel channels for the sullying effect of recessions: the cyclicalities of multiple offers and the wage offer distribution. Future work could explore additional implications of multiple offers both empirically and theoretically by extending the model to allow for other factors such as worker heterogeneity and on-the-job search.

2 The Model

This section endogenizes multiple offers and, in turn, the wage offer distribution in a DMP framework. To highlight the role of multiple offers, the model abstracts from both firm and worker heterogeneity and on-the-job search. For simplicity, I focus on the steady state and use comparative statics to describe the response of the equilibrium to changes in productivity. It is easy to show that the predictions discussed below hold in a dynamic setting with stochastic productivity.

2.1 The Environment

Consider a labor market populated by a unit measure of homogeneous workers and a positive measure of homogeneous firms. Time is discrete and goes on forever. All agents are risk neutral, live forever, and use the same discount factor ρ.

In each period, a worker is either employed or unemployed and searching. Employed workers cannot engage in job search. An employed worker, however, becomes unemployed when the job is hit by a destruction shock, which occurs exogenously with probability δ in any period. Let u and $1-u$ be the measures of unemployed and employed workers, respectively. The unemployment payoff (or the value of non-market activity) per period is z. The productivity (output per period) of any firm-worker match is y.

At the beginning of each period, firms choose whether to enter the market by posting a vacancy. The cost of a vacancy per period is c. In posting a vacancy, a firm also chooses a wage w that will be paid to the worker upon hiring one. Let v be the endogenous measure of vacancies.

During a period, unemployed workers and vacancies meet each other randomly. The total number of meetings is deterministic and described by a meeting function $m(u,v)$. The number of meetings at the individual level, however, is random. More precisely, the number of vacancies that an unemployed worker meets in a period follows a Poisson distribution with mean $\lambda_u = \frac{m(u,v)}{u}$, and the number of unemployed workers that a vacancy meets in a period follows a Poisson distribution with mean $\lambda_v = \frac{m(u,v)}{v}$.

This is the key deviation from the standard DMP model that generates multiple offers and wage dispersion in the spirit of Burdett and Judd (1983). One way to understand this approach from the worker's perspective is that although meetings with vacancies occur in continuous time following a Poisson process with arrival rate λ_u, decisions like job offers are made only at the end of each period.[2] Wolthoff (2014) motivates this assumption with empirical evidence suggesting that transitions between employment states are highly clustered around the first day of each workweek or month. Different from Wolthoff (2014) who takes the vacancy arrival rate λ_u as given, this paper endogenizes λ_u through vacancy creation.

Let $p_u(j)$ be the probability that an unemployed worker meets j vacancies in a period, and $p_v(j)$ be the probability that a vacancy meets j unemployed workers in a period.

At the end of a period, a firm makes a wage offer w to one of the unemployed workers it meets, if any. Following Albrecht et al. (2006) and Galenianos and Kircher (2009), I assume a firm can make only one offer even if it meets multiple unemployed workers. One interpretation is that firms outsource the hiring process to a third party which promises to provide them with one candidate with probability $1-p_v(0)$ in each period at a cost of c. A natural extension is to follow Kircher (2009) and allow a firm that meets j unemployed workers to make a ith offer if the first $i-1$ offers are all rejected as long as $i \leq j$. This is left for future work.

As the probability that a vacancy meets no unemployed worker in a period is $p_v(0) = e^{-\lambda_v}$, the total number of offers is $v\left(1-e^{-\lambda_v}\right)$. With a total of $m(u,v)$ meetings per period, the

[2]Similar assumptions are made in directed search models of multiple applications like Albrecht et al. (2006), Galenianos and Kircher (2009) and Kircher (2009).

probability that a meeting results in an offer is

$$p_{mo} = \frac{v\left(1-e^{-\lambda_v}\right)}{m\left(u,v\right)} = \frac{1-e^{-\lambda_v}}{\lambda_v}$$

An offer is rejected if it gives the unemployed worker a lower value than unemployment or if the unemployed worker receives a better offer from another firm. The probability that an unemployed worker receives k offers is

$$p_o\left(k\right) = \sum_{j\geq k} p_u\left(j\right) B\left(k,j,p_{mo}\right) = e^{-\lambda_u p_{mo}} \frac{\left(\lambda_u p_{mo}\right)^k}{k!}$$

where $B\left(k,j,p\right) = \frac{j!}{k!(j-k)!} p^k \left(1-p\right)^{j-k}$ is the Binomial probability mass function which gives the probability that k out of the j vacancies that an unemployed worker meets result in an offer.

The number of offers received by an unemployed worker follows a Poisson distribution with mean $\lambda_u p_{mo}$. An unemployed worker with multiple offers chooses the one with the highest wage w if it gives the worker a larger value than unemployment. If an offer is accepted, a new job is created and production starts from the next period. Unemployed workers with no accepted offer and employed workers whose jobs are destroyed will search for a job in the next period.

For a worker earning wage w, the value is

$$W\left(w\right) = w + \rho\left[\left(1-\delta\right)W\left(w\right) + \delta U\right] = \frac{w+\rho\delta U}{1-\rho+\rho\delta}$$

where the continuation value is either $W\left(w\right)$ with probability $1-\delta$ or the value of unemployment U with probability δ. The value of unemployment U is given by

$$U = z + \rho\left[U + \sum_{k\geq 1} p_o\left(k\right) \int \max\left\{W\left(w\right) - U, 0\right\} dF\left(w\right)^k\right]$$

where $F\left(w\right)$ is the wage offer distribution, and the term in the summation operation is the expected gain from receiving k offers.

As $W\left(w\right)$ is strictly increasing in w, the solution to the worker's problem involves a reservation wage R such that an unemployed worker accepts the offer with the highest wage w as long as $w \geq R$. Let $\mathscr{F}$ be the support of F. In equilibrium, we must have $w \geq R$ for any $w \in \mathscr{F}$.

When a firm makes a wage offer $w \geq R$ to a worker, the offer is rejected if the worker has another offer paying a higher wage. Let $p_a\left(w\right)$ be the probability that a wage offer w is accepted, the expected value of a vacancy offering w is

$$V\left(w\right) = -c + \rho\left[1 - p_v\left(0\right)\right] p_a\left(w\right) J\left(w\right) \tag{1}$$

where $1-p_v(0)$ is the probability that the vacancy meets at least one unemployed worker, and $J(w)$ is the value to the firm of a job with wage w given by

$$J(w) = y - w + \rho(1-\delta)J(w) = \frac{y-w}{1-\rho+\rho\delta}$$

where the periodical payoff is $y-w$ and the continuation value is $J(w)$ with probability $1-\delta$ and zero otherwise.

Equation (1) also defines the maximum possible wage that a firm can offer without suffering a loss

$$w^{max} = y - \frac{1-\rho+\rho\delta}{\rho}c$$

which is obtained by setting $V(w^{max}) = 0$, $p_v(0) = 0$ and $p_a(w^{max}) = 1$. That is, w^{max} is the break-even wage if a vacancy can be filled in a period with probability one. Any offer with $w > w^{max}$ would result in a loss. Additionally, with $p_v(0) = e^{-\lambda_v}$ and $\lambda_v = \frac{m(u,v)}{v}$, $p_v(0) = 0$ implies $v = 0$ for any reasonable meeting function satisfying $m(u,v) < \infty$ for $v > 0$ and $u \in [0,1]$. That is, w^{max} cannot be offered in an equilibrium with positive entry $v > 0$. As a result, in any equilibrium with positive entry $v > 0$, which is what we are interested in, we must have $w < w^{max}$ for any $w \in \mathscr{F}$.

Now the steady state equilibrium of the market can be defined as follows.

Definition. A steady state equilibrium ("equilibrium") is a tuple $\{v, F, R, u\}$ such that (1) Profit maximization: $V(w) = \max_{w'} V(w')$ for all $w \in \mathscr{F}$; (2) Free entry: $V(w) = 0$ for all $w \in \mathscr{F}$; (3) Optimal reservation wage: $W(R) = U$; and (4) Steady state: u is consistent with labor market flows.

With $w \geq R$ and $w < w^{max}$ for any $w \in \mathscr{F}$, an equilibrium with positive entry must have $R < w^{max}$. In the following, I will start by assuming $R < w^{max}$, and then show that this assumption does hold in the resulting equilibrium with $v > 0$.

2.2 The Equilibrium

Consider a vacancy that offers wage $w \in \mathscr{F}$ to an unemployed worker. As the offer will be rejected if the worker has a better one, the firm faces a trade-off: a higher wage w increases the probability $p_a(w)$ that the offer will be accepted but reduces the value of the job to the firm conditional on acceptance $J(w)$. The probability of acceptance $p_a(w)$ depends on the number and the wages of competing offers that a worker has received. If a firm makes a wage offer w to an unemployed worker, the conditional probability that the worker has k offers is $\frac{kp_o(k)}{\sum_{k'\geq 1} k' p_o(k')} = \frac{kp_o(k)}{\lambda_u p_{mo}} = p_o(k-1)$, and the probability that all other $k-1$ offers have a wage lower than w is $F(w)^{k-1}$. This implies

$$p_a(w) = \sum_{k\geq 1} p_o(k-1) F(w)^{k-1} = e^{-\lambda_u p_{mo}[1-F(w)]}$$

where the product term in the summation operation gives the probability of a worker receiving k offers among which the best one pays a wage w, and the second equality uses the expression of $p_o(k)$ given above and the property of the Poisson distribution.

Substituting $p_a(w)$ and $J(w)$ into equation (1), we obtain

$$V(w) = -c + \frac{\rho\left[1 - p_v(0)\right]}{1-\rho+\rho\delta} e^{-\lambda_u p_{mo}[1-F(w)]}(y-w) \tag{2}$$

A firm maximizes equation (2) with respect to w, taking as given the distribution F which summarizes the choices of other firms. A well known result from Burdett and Judd (1983) is that firms must randomize their wage offers w as they don't know whether a worker has received competing offers from other firms.[3] In the absence of a competing offer, the optimal strategy is for a firm to offer the reservation wage $w = R$. With competing offers, however, a firm would want to offer a higher wage to increase the acceptance probability $p_a(w)$. This trade-off leads to a nondegenerate wage offer distribution F in equilibrium. Burdett and Judd (1983) also demonstrate that the infimum of $\mathscr{F}$ must be the reservation wage R, and there should be no mass point in the distribution F. The following lemma summarizes these results formally. All proofs are in Appendix A.

Lemma 1. *In any equilibrium with $R < w^{max}$, the support $\mathscr{F}$ is not a singleton but a connected set of positive measure with the reservation wage R being its infimum, and the wage offer distribution F is continuous in w.*

Naturally, firms must be indifferent to any $w \in \mathscr{F}$. Additionally, free entry implies that the expected value of a vacancy must be zero for all $w \in \mathscr{F}$. Together, the indifference and free entry conditions imply the following wage offer distribution F in equilibrium.

Lemma 2. *In any equilibrium with $R < w^{max}$, firms post wages according to*

$$F(w) = \begin{cases} 0 & w < R \\ \frac{1}{\lambda_u p_{mo}} \log \frac{y-R}{y-w} & w \in [R, \bar{w}] \\ 1 & w > \bar{w} \end{cases}$$

with $\bar{w} = e^{-\lambda_u p_{mo}} R + \left(1 - e^{-\lambda_u p_{mo}}\right) y$ and $R = y - \frac{1-\rho+\rho\delta}{\rho[1-p_v(0)]} e^{\lambda_u p_{mo}} c$.

With $c > 0$ and $p_v(0) \in (0,1)$, it's easy to show that $R < \bar{w} < w^{max}$. That is, the condition $R < w^{max}$ for an equilibrium with positive entry holds as long as the equilibrium exists.

Given the wage offer distribution F, we can solve for the value of unemployment U and use the condition $W(R) = U$ to obtain the reservation wage R given by the following lemma.

[3] Burdett and Judd (1983) consider a product market with identical buyers and identical sellers. This paper reinterprets their results to a labor market with identical firms and identical workers.

Lemma 3. *In any equilibrium with $R < w^{max}$, the reservation wage is given by*

$$R = \frac{1-\rho+\rho\delta}{1+\rho\delta-\rho p_o(0)} z + \frac{\rho-\rho p_o(0)}{1+\rho\delta-\rho p_o(0)} y - \frac{(1-\rho+\rho\delta)\lambda_u p_{mo}}{[1+\rho\delta-\rho p_o(0)][1-p_v(0)]} c.$$

Workers choose their reservation wage R by taking into account both the unemployment payoff z and the value of search that depends on productivity y and the vacancy cost c.

Combining the two expressions of R in the two lemmas, we obtain, after simplification

$$\frac{\rho\left(1-e^{-\lambda_v}\right)}{(1+\rho\delta)e^{\lambda_u p_{mo}} - \rho(1+\lambda_u p_{mo})}(y-z) = c \tag{3}$$

where the left hand side represents the expected benefit of a vacancy. In addition to the total surplus of a job $y-z$, the expected benefit accounts for two other factors: the probability of meeting at least one unemployed worker $1-e^{-\lambda_v}$ and the competition from other firms captured by the terms involving the offer arrival rate $\lambda_u p_{mo}$. In equilibrium, the expected benefit on the left hand side must be equal to the cost on the right hand side.

To proceed, I assume $m(u,v)$ is a Cobb-Douglas function.

Assumption 1. $m(u,v) = \alpha_m u^{\alpha_u} v^{1-\alpha_u}$ *with* $\alpha_m > 0$ *and* $\alpha_u \in (0,1)$.

This is a standard assumption in the job search literature. This paper simply reinterprets $m(u,v)$ as the meeting instead of the matching function. A Cobb-Douglas meeting function $m(u,v)$ is sufficient but not necessary for the results of this paper. Assumption 1 could be replaced with more general restrictions such as constant returns to scale combined with some monotonicity and Inada conditions. In particular, by allowing for multiple meetings at the individual level, we don't have to impose $m(u,v) \le \min\{u,v\}$ by either truncating the Cobb-Douglas function or resorting to other functional forms like $m(u,v) = \frac{uv}{(u^\alpha+v^\alpha)^{1/\alpha}}$ introduced by den Haan et al. (2000) and adopted in other studies such as Hagedorn and Manovskii (2008).

Let $\theta = \frac{v}{u}$ be the market tightness. As $m(u,v)$ is homogeneous of degree one, the meeting arrival rate for vacancies λ_v and the offer arrival rate for unemployed workers $\lambda_u p_{mo}$ are now functions of θ given by

$$\lambda_v = \frac{m(u,v)}{v} = \frac{m(1,\theta)}{\theta}$$

$$\lambda_u p_{mo} = \frac{v}{u}\left(1-e^{-\lambda_v}\right) = \theta\left[1-e^{-\frac{m(1,\theta)}{\theta}}\right] \equiv g(\theta)$$

and equation (3) can be rewritten as

$$\frac{\rho\left[1-e^{-\frac{m(1,\theta)}{\theta}}\right]}{(1+\rho\delta)e^{g(\theta)} - \rho[1+g(\theta)]}(y-z) = c \tag{4}$$

which is an implicit function of θ. The following lemma establishes the properties of this function.

Lemma 4. *Under assumption 1, as θ increases from zero to infinity, $\lambda_v = \frac{m(1,\theta)}{\theta}$ decreases from infinity to zero, $\lambda_u p_{mo} = g(\theta)$ increases from zero to infinity, and, consequently, the left hand side of equation (4) decreases from $\frac{\rho}{1-\rho+\rho\delta}(y-z)$ to* 0.

Intuitively, the expected benefit of a vacancy captured by the left hand side of equation (4) is decreasing in θ because a tight labor market makes it harder for a vacancy to meet and win the competition for an unemployed worker. With a constant cost c, there is a unique $\theta > 0$ that solves equation (4) under the following assumption.

Assumption 2. $\frac{\rho}{1-\rho+\rho\delta}(y-z) > c$.

The present discounted value of the total surplus of a job must be larger than the vacancy cost c. Otherwise no vacancy will be posted and the market would collapse.

With a solution for θ from equation (4), λ_u and λ_v are pinned down, so are other endogenous objects like p_u, p_v, p_{mo}, p_o, R, F and u. The following proposition makes this clear.

Proposition 1. *Under assumptions 1 and 2, there is a unique equilibrium where θ is given by equation (4) , R is given by Lemma 3, F is given by Lemma 2, and u is given by*

$$(1-u)\,\delta = u\left[1-p_o(0)\right] \Rightarrow u = \frac{\delta}{\delta+1-e^{-g(\theta)}}. \tag{5}$$

2.3 Comparative Statics

With equation (4) and Lemma 4, it is easy to obtain $\frac{\partial\theta}{\partial y} > 0$. That is, the market tightness θ is increasing in productivity y. Intuitively, an increase in productivity y raises the value of a job and, in turn, the expected benefit of a vacancy. More vacancies are posted, which leads to an increase in the market tightness θ. As the offer arrival rate $g(\theta)$ is increasing in the market tightness (Lemma4), $\frac{\partial\theta}{\partial y} > 0$ also implies a positive impact of productivity y on the job finding probability $1-e^{-g(\theta)}$ and, in turn, a negative impact of y on unemployment u (equation (5)).

Loosely speaking, these standard predictions can be viewed as the impacts of productivity on the extensive margins of whether a firm should post a vacancy and whether an unemployed worker can find a job. Next we consider the model's key predictions on some intensive margins absent from the standard DMP model.

Let $P_M = \frac{\sum_{k\geq 2} p_o(k)}{\sum_{k\geq 1} p_o(k)} = 1 - \frac{g(\theta)}{e^{g(\theta)}-1}$ be the probability of having multiple offers conditional on receiving at least one offer. Alternatively, it can be interpreted as the fraction of new hires with multiple offers, where new hires are defined as unemployed workers with at least one offer. With $\frac{\partial\theta}{\partial y} > 0$ and $\frac{\partial g(\theta)}{\partial\theta} > 0$, it is easy to obtain

Proposition 2. $\frac{\partial P_M}{\partial y} > 0$.

That is, the fraction of new hires with multiple offers is increasing in productivity.

For any $q \in [0,1]$, let w_F^q be the 100qth percentile of the wage offer distribution defined by $F\left(w_F^q\right) = q$. We have

Proposition 3. $\frac{\partial w_F^{q_2}}{\partial y} > \frac{\partial w_F^{q_1}}{\partial y} > 0$ *for any* $0 \leq q_1 < q_2 \leq 1$.

An increase in productivity y has two effects on the wage offer distribution F. First, it shifts the distribution to the right such that $\frac{\partial w_F^q}{\partial y} > 0$ for any $q \in [0,1]$. That is, vacancies across the whole distribution offer a higher wage when productivity is high. This is intuitive because a higher productivity allows firms to offer a higher wage while at the same time pushes them to do so through increased competition that raises the reservation wage R.

Secondly, an increase in productivity has a larger effect on the upper than lower end of the distribution as reflected by $\frac{\partial w_F^{q_2}}{\partial y} > \frac{\partial w_F^{q_1}}{\partial y}$ for any $0 \leq q_1 < q_2 \leq 1$. Intuitively, as the market tightness θ increases with productivity y, unemployed workers receive more offers on average. Vacancies at the lower end of the wage offer distribution are less likely to be filled because their offers are rejected with a higher probability. A larger increase in $J(w)$ is required for them to break even, which means the wage increase for vacancies at the lower end of the wage offer distribution must be smaller than it is for vacancies at the upper end of the distribution.

To put it another way, when productivity is high and the labor market is tight, unemployed workers receive more offers on average. This makes low-wage vacancies less attractive as their offers are more likely to be rejected. In response, firms post more high- relative to low-wage vacancies. This compositional shift leads to a clockwise rotation of the wage offer distribution.

Let the wage distribution of new hires be

$$G(w) = \frac{\sum_{k\geq 1} p_o(k) F^k(w)}{\sum_{k\geq 1} p_o(k)}$$

which first-order stochastically dominates the wage offer distribution F because unemployed workers with multiple offers only accept the one with the highest wage. For any $q \in [0,1]$, let w_G^q be the $100q$th percentile of G defined by $G\left(w_G^q\right) = q$. We have

Proposition 4. $\frac{\partial w_G^{q_2}}{\partial y} > \frac{\partial w_G^{q_1}}{\partial y} > 0$ *for any* $0 \leq q_1 < q_2 \leq 1$.

Similar to the effects on the wage offer distribution F, an increase in productivity y leads to both a rightward shift and a clockwise rotation of the wage distribution of new hires G. Part of this is a direct consequence of the effects of productivity on the wage offer distribution F. There is, however, an additional channel that works through P_M, or more generally, $p_o(k)$, the probability of receiving k offers. More precisely, when productivity is high and the labor market is tight, unemployed workers receive more offers on average. Since only the offer with the highest wage is accepted, more offers lead to a higher accepted wage even in the absence of any change in the wage offer distribution F.

In summary, the model predicts a positive impact of productivity on both the fraction of new hires with multiple offers and the share of high-wage vacancies posted by firms. Both channels contribute to a positive impact of productivity on the fraction of new hires who find high-wage jobs.

It should be noted that, although we focus on the intensive margins concerning new hires, the same predictions apply more generally to the larger group of unemployed workers. In particular, given the positive impact of productivity on the job finding probability, a positive impact of productivity on the fraction of new hires with multiple offers implies a positive impact of productivity on the fraction of unemployed workers with multiple offers, and a positive impact of productivity on the fraction of new hires in high-wage jobs implies a positive impact of productivity on the fraction of unemployed workers who find high-wage jobs.

3 Evidence

This section provides evidence consistent with the model's predictions in Propositions 2 to 4. Throughout this section, aggregate productivity y is measured at the quarterly frequency by the real output per hour of the private non-farm business sector from the Bureau of Labor Statistics (BLS).[4] Details about other data are in Appendix B.

3.1 The Fraction of New Hires with Multiple Offers

From 1984 to 1987, the Panel Study of Income Dynamics asked respondents who were working for money at the time of the survey the following question: In which month and year "*did you start working in your present (position/work situation)*?" If the answer is no earlier than the previous calendar year, e.g., 1983 for the 1984 survey, there were two more questions: (Q1) "*At the time you ... started in your present (position/work situation), was it the only job opportunity you had, or did you choose it over something else?*" and (Q2) "*... just before you started your current (position/work situation). Did you have another position with the same employer, were you unemployed and looking for work, temporarily laid off, working for another employer, self employed, or what?*"

Let new hires be those who were working at the time of the survey but were not working (nor temporarily laid off) just before starting that job (Q2). Among them, let those who "*choose it over something else*" (Q1) as new hires with multiple offers. Guo (2020) documents that (1) around one third of new hires had multiple offers; (2) relative to new hires with only one offer, comparable new hires with multiple offers enjoy a significant wage premium, suggesting that the empirical distinction between one and multiple offers is informative; and (3) conditional on individual characteristics, the probability that a new hire had multiple offers is negatively correlated with the unemployment rate at the time when the job started, suggesting that the fraction of new hires with multiple offers is procyclical as predicted by Proposition 2.

For more direct evidence, I assign each new hire to a calendar quarter based on the time when the job started. For each quarter from the first quarter of 1983 (1983Q1) to the second quarter of 1987 (1987Q2), figure 1 plots the fraction with multiple offers among new hires in

[4] Results are essentially the same when a similar variable based on the real output per *person* is used.

that quarter against aggregate productivity. Consistent with Proposition 2, the two are positively correlated: the slope of the fitted line is 2.11 with a standard error of 0.822.

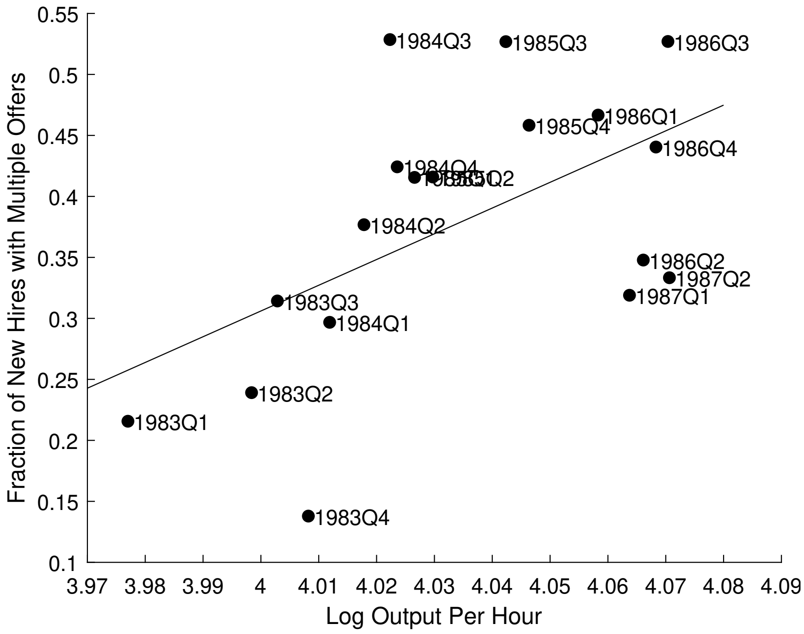

Figure 1: Productivity and The Fraction of New Hires with Multiple Offers
Notes: The slope of the fitted line is 2.11 with a standard error of 0.822.

3.2 The Distribution of Vacancies Across Industries

Although the model abstracts from firm heterogeneity for simplicity, intuitively, its predictions should apply more generally to labor markets with heterogeneous firms as long as they are competing for the same workers. This subsection uses the distribution of vacancies across industries to provide some evidence for Proposition 3 that the share of high-wage vacancies is procyclical.

Let $v_{j,t}$ be the number of vacancies posted by industry j in quarter t obtained from the Job Openings and Labor Turnover Survey, and $v_t \equiv \sum_j v_{j,t}$ be the total number of vacancies in quarter t. I estimate the following equation industry by industry

$$\Delta \log\left(\frac{v_{j,t}}{v_t}\right) = \eta_j \Delta \log y_t + \varphi_j + Q_t \beta_j + \zeta_{j,t} \tag{6}$$

where Δ is the first-difference operator such that $\Delta x_t = x_t - x_{t-1}$ for any x, φ is the constant term, Q_t is a vector of seasonal dummies (Summer, Fall and Winter), and $\zeta_{j,t}$ is the error term.[5]

Estimates of η_j, the industry-specific elasticity of the vacancy share with respect to productivity, are plotted on the vertical axis of figure 2. The horizontal axis plots a measure of the relative wage rw_j for each industry j. Using data from the Current Employment Statistics, rw_j is the percentage difference between the real average hourly earnings of production and non-

[5]Haefke et al. (2013) use a similar specification to estimate the cyclicality of the wages of new hires.

supervisory employees in industry j and the corresponding measure for the aggregate private non-farm sector.

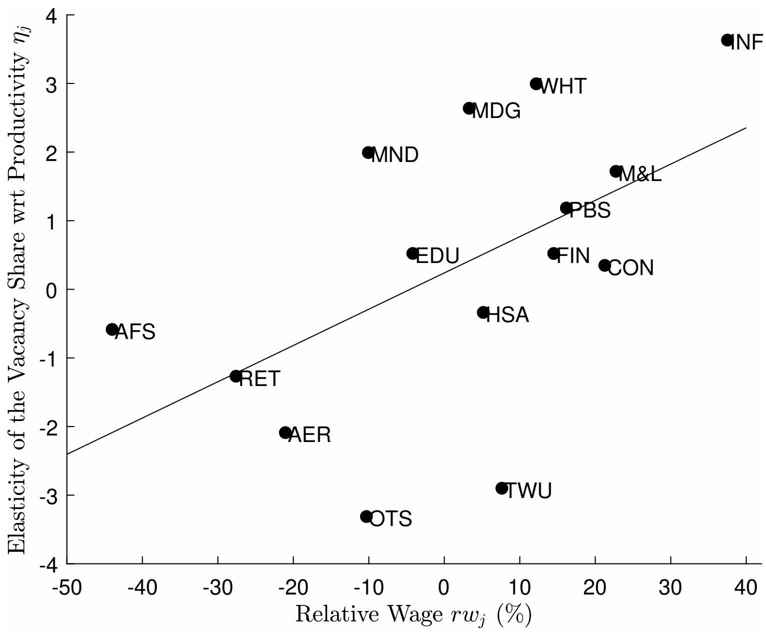

Figure 2: Relative Wage and Elasticity of the Vacancy Share by Industry
Notes: The slope of the fitted line is 0.053 with a standard error of 0.023. The 15 industries are mining and logging (M&L); construction (CON); durable goods manufacturing (MDG); nondurable goods manufacturing (MND); wholesale trade (WHT); retail trade (RET); transportation, warehousing and utilities (TWU); information (INF); financial activities (FIN); professional and business services (PBS); educational services (EDU); health care and social assistance (HSA); arts, entertainment, and recreation (AER); accommodation and food services (AFS); and other services (OTS).

Figure 2 suggests that high-wage industries have a larger elasticity of the vacancy share on average: the slope of the fitted line is 0.053 with a standard error of 0.023.[6] This implies an increase in aggregate productivity is associated with an increase in the share of vacancies posted by high-wage industries and, correspondingly, a decrease in the share of vacancies posted by low-wage industries. Under the reasonable assumption that high-wage industries post high-wage vacancies, figure 2 also suggests that an increase in aggregate productivity is associated with an increase in the share of high-wage vacancies posted by firms, consistent with Proposition 3.[7]

[6] Although the estimate of η_j is not statistically significant for all industries j, its slope with respect to rw_j is. Another way to see this is to estimate the following variant of equation (6) by pooling all industries together

$$\Delta \log \left(\frac{v_{j,t}}{v_t} \right) = \eta_0 \Delta \log y_t + \eta_1 \Delta \log y_t \times rw_j + \varphi_j + Q_t \beta_j + \zeta_{j,t}$$

where η_1 is the key parameter representing how the elasticity of the vacancy share varies with the relative wage of an industry. η_1 is estimated to be 0.053 with a standard error of 0.031 and a p-value of 0.091.

[7] As the wages of new hires are directly related to the wages posted by vacancies, we can use the former as a proxy for the latter, as having been done by Christensen et al. (2005), Jolivet et al. (2006), and Jolivet (2009), among others. As described in more detail in the next subsection, we can identify new hires from the Current

3.3 The Wage Distribution of New Hires

This subsection provides evidence for Proposition 4 that the upper end of the wage distribution of new hires is more responsive to changes in productivity than the lower end.

Let G_t be the wage distribution of new hires in quarter t, and $w_{q,t}$ be its $100q$th percentile defined by $G_t\left(w_{q,t}\right) = q$. For each $q \in [0,1]$, we can estimate the following equation

$$\Delta \log w_{q,t} = \eta_q \Delta \log y_t + \varphi_q + Q_t \beta_q + \varepsilon_{q,t} \tag{7}$$

which is the same as equation (6) except that we are now studying the wage distribution of new hires instead of the distribution of vacancies across industries. A sufficient condition for Proposition 4 is $\frac{\partial \eta_q}{\partial q} > 0$.

As the model is about how firms compete for homogeneous workers by offering different wages, it is critical to control for the part of the wage variation due to worker heterogeneity. This is done by estimating the following equation in advance

$$\log e_{i,t} = X_{i,t}\gamma + \log w_{i,t} \tag{8}$$

where, for each worker i in quarter t, $\log e$ is the log hourly wage, X is a vector of individual characteristics, and $\log w$ is the residual. One interpretation is that X captures worker heterogeneity in human capital, and w reflects the rental rate of human capital paid by the firm.

Equations (8) and (7) are estimated sequentially using data from the Current Population Survey (CPS). As the main labor force survey for the U.S., the CPS has a rotating panel structure, where households and their members are surveyed in four consecutive months, rotated out of the panel for eight months, and then surveyed again for another four consecutive months. Labor force status is recorded in each interview, whereas weekly hours and earnings are collected only in the fourth and the eighth interviews.

Mueller (2017) documents that in recessions the pool of the unemployed shifts toward workers with high wages in their previous jobs, and the finding is robust to the control of observable characteristics such as education and experience. This compositional change could lead to cyclical variations in the distribution of raw wages e among all new hires even if recessions have no impact on the distribution of wages w among new hires with the same human capital. One way to address this concern is to control for lagged wages $\log e_{i,s<t}$. Given the structure of the CPS, equation (8) is estimated using non-farm wage and salary workers in

Population Survey and obtain the residual wage $\log w_{i,t}$ of each new hire i in quarter t by controlling for individual characteristics. Using new hires from 2001 to 2018, I regress $\log w_{i,t}$ on a vector of 15 industry dummies and a vector of quarterly dummies spanning from the first quarter of 2001 to the last quarter of 2018. Let α_j be the estimated coefficient for industry j, which can be viewed as a measure of the relative wage of vacancies posted by the industry. I find the correlation between α_j and rw_j to be 0.84, suggesting that high-wage industries do post high-wage vacancies on average. Regressing η_j on $100\alpha_j$ where the factor 100 is used to make sure α_j and rw_j are of the same scale, I obtain an estimate of 0.207 with a standard error of 0.087, suggesting that the share of high-wage vacancies is indeed more cyclical.

their eighth interview. In addition to typical controls including years of schooling, a quartic of potential work experience, and dummies for gender, race and marital status, the vector X also includes $\log e_{i,t-4}$, the log hourly wage of the same worker one year before in the fourth interview.

Following Haefke et al. (2013), a non-farm wage and salary worker in the eighth interview is defined as a new hire if the worker was not employed in at least one of the three preceding months covered by the fifth to seventh interviews. As the sample of new hires is relatively small and not representative of all workers, to obtain a better estimate of γ and thus $\log w_{i,t}$, equation (8) is estimated using all non-farm wage and salary worker in the eighth interview instead of only new hires. With $\log w_{i,t}$ estimated and new hires defined, it is straightforward to obtain $\log w_{q,t}$ and estimate equation (7) for any given q.[8]

Figure 3 plots the estimates of η_q for $q \in \{0.05, 0.1, 0.15, ..., 0.95\}$. The estimates suggest that η_q is increasing in q: the slope of the fitted line is 0.013 with a standard error of 0.003.[9] That is, consistent with Proposition 4, the upper end of the (residual) wage distribution of new hires is more responsive to changes in productivity than the lower end.

[8]This two-stage strategy is similar to that of Haefke et al. (2013), with two main differences. First, Haefke et al. (2013) estimate equation (8) using workers in both the fourth and the eighth interviews without controlling for the lagged wage, which may be inadequate in addressing worker heterogeneity given the findings in Mueller (2017). Second, Haefke et al. (2013) are interested in the level of η in equation (7) where the dependent variable is either the mean or the median wage of new hires. In contrast, this paper is about the slope of η with respect to q.

[9]Without the two points on the right (the 90th and 95th percentiles), the slope would be 0.008 with a standard error of 0.002. The estimate for the median is $\eta_{0.5} = 0.469$, much smaller than the corresponding estimate of about 1 by Haefke et al. (2013). This is mainly due to the difference in data coverage: Figure 3 uses data from 1984 to 2018, while the data in Haefke et al. (2013) ends in 2006. Using data from 1984 to 2006, I obtain $\eta_{0.5} = 1.014$, and the slope of η_q with respect to $100q$ is 0.015 with a standard error of 0.004. This suggests that including the data since 2007 reduces the level of η_q without much impact on how it varies with q. Finally, when the following variant of equation (7) is estimated by pooling the data for each q, the estimate of η_1 is 0.013 with a standard error of 0.006. This suggests that the slope of η_q with respect to q is statistically significant even if some estimates of η_q from equation (7) are not.

$$\Delta \log w_{q,t} = \eta_0 \Delta \log y_t + \eta_1 \Delta \log y_t \times 100q + \varphi_q + Q_t \beta_q + \varepsilon_{q,t}$$

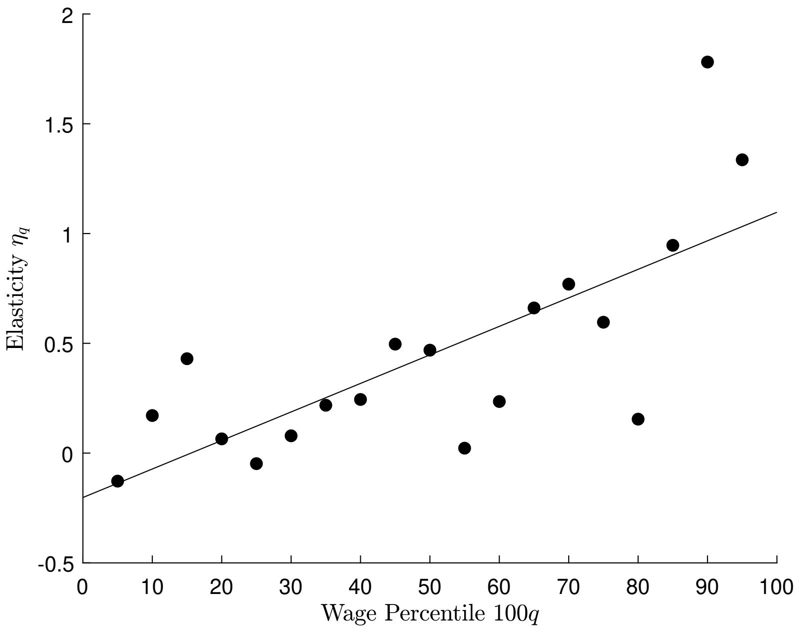

Figure 3: Elasticity of the Residual Wages of New Hires with Respect to Productivity
Notes: The slope of the fitted line is 0.013 with a standard error of 0.003. Without the two points on the right (the 90th and 95th percentiles), the slope would be 0.008 with a standard error of 0.002.

4 Conclusion

Through a simple modification of the standard DMP model, this paper endogenizes multiple offers and, in turn, the wage offer distribution. As a complement to existing theories focusing on the role of job-to-job transitions, e.g., Barlevy (2002), Moscarini and Postel-Vinay (2013) and Lise and Robin (2017), the model provides two novel channels for the sullying effect of recessions. First, by reducing the probability of multiple offers, recessions have a negative impact on the wages of new hires. Second, as firms respond to the decline in the probability of multiple offers by posting a smaller share of high-wage vacancies, recessions cause a deterioration in the wage offer distribution, which further reduces the wages of new hires. Both channels are consistent with the negative impact of recessions on labor market entrants, e.g., Kahn (2010), Oreopoulos et al. (2012) and Altonji et al. (2016).

For simplicity, the model abstracts from both firm and worker heterogeneity and on-the-job search. An interesting direction for future work is to explore additional implications of multiple offers by incorporating these factors into the model. When workers are heterogeneous, in addition to wage offers, firms may respond to cyclical changes in the probability of multiple offers by adjusting the hiring standard, as documented by Modestino et al. (2016, 2019) and Hershbein and Kahn (2018), among others. With on-the-job search and other factors such as occupation-specific human capital, we can have a better understanding of how comparable new hires with different numbers of offers could experience persistent differences in wages, e.g., Guo (2020).

A critical assumption of the model is that workers could receive and hold two or more offers simultaneously during job search, which is consistent with anecdotes and the limited amount of evidence available. Labor market surveys rarely collect information on the number of job offers received by a worker during job search, with the surveys documented by Krueger and Mueller (2011) and Faberman et al. (2017) as two recent exceptions, let alone the exact time when each offer is received and accepted/rejected which is critical for identifying simultaneous offers. This paper suggests future surveys should attempt to collect these information as they are critical for us to better understand the search and matching process that creates jobs and determines wages and how it functions over the business cycle.

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Online Appendix
The Cyclicality of Job Creation with Multiple Offers

Junjie Guo

A Proofs

Proof of Lemma 1. This proof uses equation (1) and the trade-off that firms face between the value of a match $J(w)$ and the offer acceptance probability $p_a(w)$. First, suppose all firms offer the same wage $w \in [R, w^{max})$. Then any firm could make a profitable deviation by offering $w+\varepsilon$ with $\varepsilon > 0$ but arbitrarily small. Because $J(w)$ is continuous in w, the deviation would result in a small decrease in $J(w)$. However, it would cause a discrete jump in $p_a(w)$ because the firm can now beat all competing offers. Overall, the deviation is profitable because the jump in $p_a(w)$ more than compensates the decrease in $J(w)$. This proves that the support $\mathscr{F}$ cannot be a singleton.

The same argument also rules out any mass point in the wage offer distribution F. If there is a mass of firms offering the same wage w, any of them could make a profitable deviation by offering a slightly higher wage. As a result, F must be continuous.

Second, assume the existence of $w_1 < w_2$ such that $w_1 \in \mathscr{F}$, $w_2 \in \mathscr{F}$ but $w \notin \mathscr{F}$ for any $w \in (w_1, w_2)$. Any firm offering w_2 could make a profitable deviation by offering w_1 instead. As F is continuous and the set (w_1, w_2) is not part of the support $\mathscr{F}$, switching from w_2 to w_1 has a minimal effect on $p_a(w)$. However, the switch would lead to a discrete jump in $J(w)$, making it a profitable deviation. Consequently, the support $\mathscr{F}$ must be connected.

Finally, the infimum of $\mathscr{F}$ cannot be larger than the reservation wage R. Otherwise, any firm offering the infimum could make a profitable deviation by switching to R. The argument is similar to the one used above for the switch from w_2 to w_1. As the infimum of $\mathscr{F}$ cannot be smaller than R either, the two must be equal to each other. This completes the proof.

Proof of Lemma 2. $F(w)$ is obtained from equation (2) by setting $V(w) = V(R)$ and $F(R) = 0$, where the former comes from the indifference condition and the latter holds because the wage offer distribution F has no mass point and R is the infimum of its support $\mathscr{F}$. With the expression for $F(w)$, $\bar{w}$ is obtained from $F(\bar{w}) = 1$. Finally, the expression for R is obtained from the zero-profit condition $V(R) = 0$.

Proof of Lemma 3. The value of unemployment is

$$
\begin{aligned}
U &= z+\rho\left[U+\sum_{k\geq 1} p_o(k)\int \max\{W(w)-U,0\}\,dF(w)^k\right]\\
&= z+\rho\left[U+\sum_{k\geq 1} p_o(k)\int_R^{\bar{w}} \frac{w+(\rho-1)U}{1-\rho+\rho\delta}dF(w)^k\right]\\
&= z+\frac{\rho\left[\rho\delta+(1-\rho)p_o(0)\right]}{1-\rho+\rho\delta}U+\frac{\rho}{1-\rho+\rho\delta}S
\end{aligned}
$$

where the second equality uses the expression for $W(w)$ and the fact that $W(w)\geq U$ for all $w\in\mathscr{F}=[R,\bar{w}]$, and S is given by

$$
\begin{aligned}
S &= \sum_{k\geq 1} p_o(k)\int_R^{\bar{w}} w dF(w)^k\\
&= \int_R^{\bar{w}} wd\left[\sum_{k\geq 1} p_o(k)F(w)^k\right]\\
&= \int_R^{\bar{w}} wd\left[e^{-\lambda_u p_{mo}[1-F(w)]}-e^{-\lambda_u p_{mo}}\right]\\
&= e^{-\lambda_u p_{mo}}(y-R)\int_R^{\bar{w}} wd\frac{1}{(y-w)}\\
&= \left(1-e^{-\lambda_u p_{mo}}\right)y-\lambda_u p_{mo}\frac{1-\rho+\rho\delta}{\rho\left[1-p_v(0)\right]}c
\end{aligned}
$$

where the second equality holds because the order of the summation and the integral are interchangeable, the third equality uses the expression for $p_o(k)$ and properties of the Poisson distribution, the fourth equality uses the expression for $F(w)$ from Lemma 2, and the last equality uses integration by parts and the expressions for R and $\bar{w}$ from Lemma 2.

Substituting the expression for S back into U, we obtain, after simplification

$$
\begin{aligned}
U = &\frac{1-\rho+\rho\delta}{(1-\rho)\left[1+\rho\delta-\rho p_o(0)\right]}z+\frac{\rho\left[1-p_o(0)\right]}{(1-\rho)\left[1+\rho\delta-\rho p_o(0)\right]}y\\
&-\frac{(1-\rho+\rho\delta)\lambda_u p_{mo}}{(1-\rho)\left[1+\rho\delta-\rho p_o(0)\right]\left[1-p_v(0)\right]}c.
\end{aligned}
$$

Finally, from $W(R)=U$ we obtain $U=\frac{R}{1-\rho}$. Substituting this into the above expression for U gives the expression for R in the lemma.

Proof of Lemma 4. First, $\lambda_v=\frac{m(1,\theta)}{\theta}$ is decreasing in θ because

$$
\frac{\partial\lambda_v}{\partial\theta}=\frac{m_2(1,\theta)\theta-m(1,\theta)}{\theta^2}=-\frac{m_1(1,\theta)}{\theta^2}<0
$$

where m_i is the derivative of function m with respect to its ith argument, the second equality follows from the assumption that $m(u,v)$ is homogeneous of degree one, and the inequality holds because $m_1 > 0$.

The limit of λ_v as θ approaches zero from above is

$$\lim_{\theta\to 0}\frac{m(1,\theta)}{\theta} = \lim_{\theta\to 0} m_2(1,\theta) = +\infty$$

where the first equality uses the L'Hospital's rule, and the second equality uses Assumption 1. Similarly, the limit of λ_v as θ approaches infinity from below is

$$\lim_{\theta\to +\infty}\frac{m(1,\theta)}{\theta} = \lim_{\theta\to +\infty} m_2(1,\theta) = 0.$$

Second, $g(\theta) = \theta\left[1-e^{-\frac{m(1,\theta)}{\theta}}\right]$ is increasing in θ because

$$g_1 = 1-e^{-\frac{m(1,\theta)}{\theta}}\left[1+\frac{m_1(1,\theta)}{\theta}\right] = 1-\frac{1+\alpha_m\alpha_u\theta^{-\alpha_u}}{e^{\alpha_m\theta^{-\alpha_u}}} > 0$$

where the first equality uses the assumption that $m(u,v)$ is homogeneous of degree one, the second equality uses the Cobb-Douglas specification of $m(u,v)$, and the inequality holds because $\frac{1+\alpha_u x}{e^x} < 1$ for any $x > 0$ and $\alpha_u \in (0,1)$.

As $\lim_{\theta\to 0}\frac{m(1,\theta)}{\theta} = +\infty$, it's easy to see that $\lim_{\theta\to 0} g(\theta) = 0$. On the other hand,

$$\lim_{\theta\to +\infty} g(\theta) = \lim_{\theta\to +\infty}\frac{1-e^{-\frac{m(1,\theta)}{\theta}}}{\frac{1}{\theta}} = \lim_{\theta\to +\infty} e^{-\frac{m(1,\theta)}{\theta}} m_1(1,\theta) = +\infty$$

where the second equality uses the L'Hospital's rule and the assumption that $m(u,v)$ is homogeneous of degree one, and the last equality holds because $\lim_{\theta\to +\infty}\frac{m(1,\theta)}{\theta} = 0$ and $\lim_{\theta\to +\infty} m_1(1,\theta) = +\infty$.

Third, it's easy to show that $f(x) = (1+\rho\delta)e^x - \rho(1+x)$ is increasing in x for $x > 0$, with $\lim_{x\to 0} f(x) = 1-\rho+\rho\delta$ and $\lim_{x\to +\infty} f(x) = +\infty$. Let $x = g(\theta)$ and use the properties of $g(\theta)$ shown above, we obtain that $f(g(\theta))$ is increasing in θ with $\lim_{\theta\to 0} f(g(\theta)) = 1-\rho+\rho\delta$ and $\lim_{\theta\to +\infty} f(g(\theta)) = +\infty$.

Finally, with the properties of $\frac{m(1,\theta)}{\theta}$ and $f(g(\theta))$ shown above, it's easy to obtain that the left hand side of equation (4), which can be rewritten as $\frac{\rho\left[1-e^{-\frac{m(1,\theta)}{\theta}}\right]}{f(g(\theta))}(y-z)$, decreases from $\frac{\rho}{1-\rho+\rho\delta}(y-z)$ to 0 as θ increases from zero to infinity.

Proof of Proposition 3. From $F\left(w_F^q\right) = q$ and Lemma 2 we obtain

$$w_F^q = y - A(\theta, q)$$

with

$$A(\theta,q) = c\frac{1-\rho+\rho\delta}{\rho}\frac{e^{g(\theta)(1-q)}}{1-e^{-\frac{m(1,\theta)}{\theta}}}.$$

Let $B(\theta) = c\frac{e^{g(\theta)}-g(\theta)-1}{1-e^{-\frac{m(1,\theta)}{\theta}}}$. Equation (4) can be rewritten as $y-z = A(\theta,0)+B(\theta)$. Taking derivative with respect to y, we obtain $1 = \frac{\partial A(\theta,0)}{\partial\theta}\frac{\partial\theta}{\partial y}+\frac{\partial B(\theta)}{\partial\theta}\frac{\partial\theta}{\partial y}$. As $\frac{\partial B(\theta)}{\partial\theta} > 0$ and $\frac{\partial\theta}{\partial y} > 0$, we have $\frac{\partial A(\theta,0)}{\partial\theta}\frac{\partial\theta}{\partial y} < 1$.

From the definition of $A(\theta,q)$, we have $\frac{\partial A(\theta,q)}{\partial q} = -g(\theta)A(\theta,q)$ and

$$\frac{\partial^2 A(\theta,q)}{\partial q\partial\theta} = -\frac{\partial g(\theta)}{\partial\theta}A(\theta,q)-g(\theta)\frac{\partial A(\theta,q)}{\partial\theta} < 0.$$

As a result, $\frac{\partial\left[\frac{\partial A(\theta,q)}{\partial\theta}\frac{\partial\theta}{\partial y}\right]}{\partial q} = \frac{\partial^2 A(\theta,q)}{\partial q\partial\theta}\frac{\partial\theta}{\partial y} < 0$. That is, for any $0 \le q_1 < q_2 \le 1$, we have

$$\frac{\partial A(\theta,q_2)}{\partial\theta}\frac{\partial\theta}{\partial y} < \frac{\partial A(\theta,q_1)}{\partial\theta}\frac{\partial\theta}{\partial y} \le \frac{\partial A(\theta,0)}{\partial\theta}\frac{\partial\theta}{\partial y} < 1$$

which implies $\frac{\partial w_F^{q_2}}{\partial y} > \frac{\partial w_F^{q_1}}{\partial y} > 0$ by noting that $\frac{\partial w_F^q}{\partial y} = 1-\frac{\partial A(\theta,q)}{\partial\theta}\frac{\partial\theta}{\partial y}$ for any $q\in[0,1]$.

Proof of Proposition 4. The wage distribution of new hires from unemployment is

$$G(w) = \frac{\sum_{k\ge1}p_o(k)F^k(w)}{\sum_{k\ge1}p_o(k)} = \frac{e^{g(\theta)F(w)}-1}{e^{g(\theta)}-1} = \frac{\frac{A(\theta,0)}{y-w}-1}{e^{g(\theta)}-1}$$

where the second equality uses properties of the Poisson distribution and the fact that the number of offers k follows a Poisson distribution with arrival rate $g(\theta)$, and the third equality uses Lemma 2 and the definition of $A(\theta,q)$ above.

From $G\left(w_G^q\right) = q$ we obtain

$$w_G^q = y-C(\theta,q)$$

with

$$C(\theta,q) = \frac{A(\theta,0)}{1+q\left[e^{g(\theta)}-1\right]}$$

Obviously, $C(\theta,0) = A(\theta,0)$, which implies $\frac{\partial C(\theta,0)}{\partial\theta}\frac{\partial\theta}{\partial y} = \frac{\partial A(\theta,0)}{\partial\theta}\frac{\partial\theta}{\partial y} < 1$.

After some tedious algebra, we can show that $\frac{\partial^2 C(\theta,q)}{\partial q\partial\theta} < 0$. As a result, for any $0 \le q_1 < q_2 \le 1$, we have

$$\frac{\partial C(\theta,q_2)}{\partial\theta}\frac{\partial\theta}{\partial y} < \frac{\partial C(\theta,q_1)}{\partial\theta}\frac{\partial\theta}{\partial y} \le \frac{\partial C(\theta,0)}{\partial\theta}\frac{\partial\theta}{\partial y} < 1$$

which implies $\frac{\partial w_G^{q_2}}{\partial y} > \frac{\partial w_G^{q_1}}{\partial y} > 0$ by noting that $\frac{\partial w_G^q}{\partial y} = 1-\frac{\partial C(\theta,q)}{\partial\theta}\frac{\partial\theta}{\partial y}$ for any $q\in[0,1]$.

B Data

B.1 The Panel Study of Income Dynamics (PSID)

The PSID is the longest running longitudinal household survey in the world. It began in 1968 with two independent samples of over 18,000 individuals living in 5,000 families in the United States: a cross-sectional national sample known as the SRC (Survey Research Center) sample and a national sample of low income families known as the SEO (Survey of Economic Opportunities) sample. Information on these individuals and their descendants has been collected continuously (annually until 1997 and biennially thereafter), including data covering employment, income, wealth, expenditures, health, marriage, childbearing, child development, philanthropy, education, and numerous other topics.

To obtain the fraction of new hires with multiple offers plotted on the vertical axis of figure 1, I use the SRC respondents who (1) answered the question used to define multiple offers mentioned in the paper, and (2) were not employed right before starting the job at the time of the survey. New hires are grouped into calendar quarters based on the month and year when they started the job at the time of the survey. There are more than 20 new hires for each quarter plotted in the figure. More details can be found in Guo (2020), where the vertical axis of figure 1 is obtained in the same way.

B.2 Vacancies and Wages by Industry

The Job Openings and Labor Turnover Survey is a monthly survey starting from December, 2000 which collects data on total employment, job openings, hires, quits, layoffs, discharges, and other separations from approximately 16,000 U.S. business establishments. It covers all nonagricultural industries in the public and private sectors for the 50 States and the District of Columbia.[10] I use the data on job openings by industry from January, 2001 to December, 2018, and aggregate the monthly data to quarterly by summing over the three months in each quarter. After excluding the government, I obtain a measure of $v_{j,t}$ for each private non-farm industry from the first quarter of 2001 to the last quarter of 2018. The first two columns of table B.1 report the mean (over time t) and the standard deviation of the vacancy share $\frac{v_{j,t}}{v_t}$ for each industry, where the 15 industries are mutually exclusive and collectively exhaustive of the private non-farm sector.

The Current Employment Statistics (CES) is a monthly survey of approximately 142,000 businesses and government agencies, representing approximately 689,000 individual worksites. Each month, the CES produces detailed industry estimates of non-farm employment, hours, and earnings of workers on payrolls.[11] For each industry j and month m, let $w_{j,m}$ be the CES measure of the real average hourly earnings of production and non-supervisory employees, and

[10] For more information, see https://www.bls.gov/jlt/home.htm.

[11] For more information, see https://www.bls.gov/ces/.

Table B.1: Vacancy Share and Relative Wage by Industry

	Vacancy share $100\frac{v_{j,t}}{v_t}$		Relative wage $rw_{j,m}$	
Industry	Mean	Std	Mean	Std
Mining and logging (M&L)	0.447	0.185	22.844	5.710
Construction (CON)	3.561	0.904	21.365	1.370
Manufacturing: Durable goods (MDG)	4.685	0.827	3.404	2.901
Manufacturing: Nondurable goods (MND)	2.841	0.304	-9.955	3.357
Wholesale trade (WHT)	3.731	0.590	12.279	0.980
Retail trade (RET)	11.514	0.997	-27.492	3.351
Transportation, warehousing and utilities (TWU)	3.589	0.640	7.728	3.221
Information (INF)	2.734	0.673	37.618	1.596
Financial activities (FIN)	7.230	0.818	14.604	4.151
Professional and business services (PBS)	20.474	1.874	16.276	2.909
Educational services (EDU)	1.874	0.323	-4.072	2.324
Health care and social assistance (HSA)	18.814	1.924	5.270	1.885
Arts, entertainment, and recreation (AER)	1.613	0.262	-20.950	1.962
Accommodation and food services (AFS)	12.457	1.275	-43.899	1.112
Other services (OTS)	4.437	0.828	-10.213	1.053

let w_m be the same measure for the aggregate private non-farm sector in month m. I define the relative wage of industry j in month m as $rw_{j,m} = 100\left(\frac{w_{j,m}}{w_m} - 1\right)$, and use its average from January, 2001 to December, 2018 as a measure of rw_j.[12] The last two columns of table B.1 report, for each industry j, the average and the standard deviation of $rw_{j,m}$, with the former being rw_j by definition. The within-industry standard deviations are generally small relative to the variation of the mean rw_j across industries, suggesting that the rank of industries is relatively stable and rw_j is a reasonable measure of the relative wage across industries during the sample period.

B.3 The Current Population Survey (CPS)

I use the monthly data in the IPUMS-CPS database from January 1984 to December 2018. As the IPUMS-CPS database does not have information on earnings and work hours before 1989, I supplement it with the NBER extracts of CPS Merged ORG files.[13] Following Haefke et al. (2013), 1984 is chosen as the first year in light of the so-called Great Moderation where various second moments of macroeconomic variables changed around that year. I match observations across interviews using the CPSIDP variable that uniquely identifies individuals across CPS samples (Drew et al. (2014)), and define a match to be successful if the observations in two

[12]Starting from March, 2006, the CES has been reporting a second wage measure, the real average hourly earnings of all employees by industry. The results from this alternative measure, not reported but available upon request, are extremely close to those reported below in the paper.

[13]For more information on the IPUMS-CPS database, see Sarah Flood, Miriam King, Renae Rodgers, Steven Ruggles, and J. Robert Warren. Integrated Public Use Microdata Series, Current Population Survey: Version 6.0 [dataset]. Minneapolis, MN: IPUMS, 2018. https://doi.org/10.18128/D030.V6.0. For more information on the NBER extracts, see https://www.nber.org/data/morg.html.

interviews have (1) the same CPSIDP, gender and race, and (2) an age differential of at most one.

I measure hourly earnings by the hourly wage for workers paid by the hour, and by the ratio of weekly earnings to usual hours worked per week for workers who report weekly earnings instead. For workers without an exact number of usual hours worked per week, including those who report varying hours and those with missing values, I impute their usual hours per week by running four separate regressions by gender and full-time/part-time status of usual hours worked per week on age, age squared, years of schooling, and dummies for race, ethnicity and marital status. I define (potential work) experience as age minus years of schooling minus 6, and deflate the nominal hourly earnings by the quarterly implicit price deflator of personal consumption expenditures from the U.S. Bureau of Economic Analysis.

Starting from non-farm wage and salary workers in the eighth interview who can be matched successfully to each of the four preceding interviews, I drop a few observations with negative experience. To limit the effect of outliers, I calculate the empirical distributions of hourly earnings and usual hours worked per week for each quarter, and drop observations in the bottom and top 1 percent of each distribution. The resulting sample is used to estimate equation (8), and the estimated $\log w_{i,t}$ of new hires are used to construct $\log w_{q,t}$ and estimate equation (7). With each t being a quarter, for each q, we obtain 131 observations of $\log w_{q,t}$, which is smaller than 140, the total number of quarters from 1984 to 2018, because changes in the CPS instruments make it impossible to match some monthly samples in 1985-1986 and 1995-1996 with other samples. The number of new hires based on which $\log w_{q,t}$ is calculated ranges from 117 to 562. The sample of workers used to estimate equation (8) is obviously much larger.

Following the literature, e.g., Mueller (2017), I use the product of the survey weight (earnwt) and usual hours worked per week as the basic weight, and adjust the basic weight to account for the fact that the sample is restricted to workers with a successful match to the four preceding interviews. More precisely, using workers in the eighth interview, I run a logit regression of a dummy indicating a successful match with all four preceding interviews on years of schooling, a quartic of experience, and dummies of gender, race and marital status, and define the adjusted weight as the product of the basic weight and the inverse of the predicted value of the logit regression. The adjusted weight is used to estimate equation (8) and construct the dependent variable $\log w_{q,t}$ for equation (7).