Structuring, Adverse Selection and Financial Instability

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Abstract

We develop a model of financial fragility under asymmetric information in the context of structured securities. Equilibrium comes from balancing higher holding costs for sellers with agency costs associated with adverse selection against buyers. Structuring can cause fragility by introducing debt-like pieces into the structure. This produces two types of equilibria, one at low market share for selling and one with 100% market share. Movements between the two produce fragility that can lead to the equivalent of bank runs. Not taking account of these results can lead to underestimation of tail-risk and capital adequacy.

Keywords: Securitization; Structuring; Convexity; Optionality; Fragility

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1. Introduction

The defining characteristic of financial panics is abrupt changes in investor or depositor behavior in response to small changes in parameters, aka fragility. The recent Financial Crisis started in 2007 with deteriorating quality of mortgage-backed securities, and it became the Great Recession when it morphed into runs at major financial institutions (see Gorton (2009) and FCIC (2011)). The key stylized fact of the crash is that there was a surge in the market share of securitization, which looked like a debt bubble, followed by runs on shadow banks. The bubble and crash were associated with the rise and fall of structured deals, such as Collateralized Debt Obligations (CDOs). We suggest that this was not a coincidence. Our conclusion is that selling pools of loans or securities by breaking them up into separate tranches can cause fragility in some of the tranches even if the pools as a whole are not fragile. This unintended consequence is a negative externality that is manifested in instability.

Our results come from the interaction of adverse selection (with attendant agency costs) and holding costs in structured deals like CDOs with debt-like tranches. Our contribution is in bringing out the causal role of tranching and optionality in generating fragility. Optionality can produce two equilibria. The first is at low volume for the usual lemons type reasons, as agency costs increase until they equal holding cost. The other is at high volume due to optionality leading to an eventual decline in agency costs, as safer loans that are added to the pool being sold and loans become increasingly similar and easier to evaluate. This leads to a decline in agency costs, pushing market share to a corner solution, at 100% market share. Abrupt switches from low volume to high volume equilibria, and vice versa, are the source of the fragility. This can look like a bubble and bank run. The model can be applied to a range of structures, such as repurchase agreements (repos) and shadow banks. Fragility can be exacerbated by layering pieces of deals upon one another, such as CDOs made up of subprime pieces of debt tranches from other deals, and perhaps resecuritizing them. Each additional layer adds optionality and convexity.

History and Literature

Our models are variations on classic works by Akerlof (1970) and Diamond and Dibvig (1983). The former is the basis of the adverse selection model we use, and the latter is similar to our model of abrupt changes in behavior, and panics and bank runs. Both are concerned with market failure rather than market structure. Our empirical point of departure is Gorton and Metrick (2012), which shows how smooth increases in the ABX index (a proxy for risk in the subprime mortgage market) in 2008 were associated with a sudden jump in the LIBOR-OIS spread (a proxy for counterparty risk in the repo market). This is consistent with our model of a growing market in collateralized debt coming to a sudden halt after a seemingly small and continuous change in information from the ABX index.[1]

More broadly, Caballero and Krishnamurthy (2008) explains the recent 2008 crash with a model of a liquidity shortage and Knightian uncertainty. It emphasizes uncertainty aversion on top of increased risk exposure. In particular, uncertainty aversion can result in an unanticipated and large increase in bid-ask spread, which ends in reduction of liquidity (see also Easley and O'Hara (2005), (2008)). Routledge and Zin (2004) also show that the widening bid-ask spread can be a result of uncertainty aversion, which ultimately causes market illiquidity. Loayza and Rancière (2006) address financial fragility as a short-run outcome of financial liberalization. Banks in good times tend to be less concerned with the quality of mortgagors, and tend to over-lend as demand becomes high, given financial liberalization. Beltran *et al* (2017) present a model of trading CDOs with adverse selection on the part of sellers, and they document the large amount of cross-referencing that occurred in CDOs and resecuritization, so that correlations among items in deals were higher than thought at origination. Van Order (2006) uses a model of fragility that is the basis of our models. However, it does not cover the role of structured deals as causal, and it treats the adverse selection equilibrium differently than we do.

Acharya *et al* (2009) model how financial institutions owning high quality assets were not able to roll over their short-term debt as Repo depositors lost confidence in the quality of the assets securing their deposits. They point out in their theoretical model that the existence of a market freeze depends on whether the banks' expectations on the rollover risk are optimistic or pessimistic. We add to this line of research by focusing explicitly on optionality as a cause to roll

[1] See also Stanton and Wallace (2011) on ABX

over their short-term debt. More broadly, Gorton (2009) discusses the breakdown of liquidity in the Repo market in 2007.

There is a considerable literature on security design and structure. Allen and Gale (1988) is a widely cited early example. Several papers discuss tranching as a tool in selling securities. Riddiough (1997) discusses security design with tranche retention and the benefits of a senior/subordinated structure. Hartman- Glaser *et al* (2012) discusses the efficiency of pooling and optimal contracts with underwriters. Boot and Thakor (1993) show the benefits of dividing a pool into informationally sensitive and insensitive pieces, so that they be sold to different investor types. Demarzo (2005) analyzes ways of selling securities when there is asymmetric information, in a model that consistent with ours. If the seller has superior information, then pooling and selling separate tranches might still be optimal by balancing the tradeoff between diversification and liquidity. Demarzo and Duffie (1999) present a model with an Akerlof asymmetric information problem, which provides a downward sloping demand curve that is similar to our marginal cost curve. In their model holding a piece of the deal can give positive signals to investors. Firla-Cuchr, and Jenkinson (2005) test various hypotheses about why there is tranching and find support for asymmetric information and market segmentation as explanations. These papers provide the framework for our structured deals

Summary

We take tranching as a given, and we develop models with adverse selection and a cost to sellers for holding securities as the reason for loan sales. Our main result is that debt tranches, e.g., in CDOs, can trigger fragility because at high levels of sales they unravel agency costs in a destabilizing manner. There are several different types of structures to which it applies, including repurchase agreements (repos), CDOs and shadow banks. The fragility, which looks like a Poisson blip, is not likely to be detected from data, which can lead to underestimation of "tail risk," capital adequacy and instability. Section 2 develops models of risk ordering, adverse selection and default. Section 3 models securitization and structuring and how that can produce fragility. Section 4 develops three similar models that have the same results. Section 5 adds two complications. Section 6 concludes

2. Adverse Selection, Dumping and Default

We develop a model of securitization in which identical owners of assets put them into *pools* and sell pieces or *tranches* of them to identical investors. In what follows, *deals* are made up of *loans* that are taken from *portfolios* of loans owned by sellers. The sum of these identical portfolios is the potential *market*. The size of the representative seller portfolio is given by M and the size of the pool by N. N is endogenous; M is exogenous, and N/M is market share. Sales from portfolios into pools are assumed to have known asset classes with fixed sizes and two tranches, debt and equity. *Shares* in the deal are the total value of securities in the deal or tranche of the deal divided by N, and they are represented by lower case letters. Traders are all risk-neutral. Because of the use of representative individual and market results are interchangeable, and seller portfolios are the same as the *market*.

We employ an Akerlof-type adverse selection model: Sellers (primary market or PM), have loan by loan information about the loans that form pools, and they sell to less informed buyers (secondary/securitization market or SM) who know only average default cost of each pool, so they cannot do loan level pricing. PM has higher holding costs, which is what causes SM to exist. The loans are one period zero coupon bonds with default risk at the end of the period. We assume an infinitely elastic supply of funds from SM for pieces of deals at a price commensurate with average cost. So the action resides with choices made by identical sellers (in the PM) who place loans into pools as long as it is profitable. This choice determines equilibrium.

2.1 The Cost Structure of Loans

Let $V(x)$ be the distribution function that gives the level of the characteristics (e.g., default probabilities) as the characteristics vary from worst to best for the loans in the seller's portfolio. Let $V^{-1}(n)$ be the inverse of V, and $D(n)$ ($n \in (0, M)$) is the segment of $V^{-1}(n)$ that includes only the loans in the pool that are taken from the portfolio. Marginal default cost, $c(n)$, is the cost of the last loan put into the pool by PM, and it is given by

(1) $c(n) = c(D(n))$

with $c'(n) = c'D'$

and $c'' = c'D'' + D'c''$.

By construction $c'<0$. We assume initially that that $D(n)$ is linear, in which case the dumping function depends only on the shape of the default cost function.[2] We will for most of the paper assume that $D(n)$ is linear and let D'=1. Then we can focus on default modeling alone.

Then each PM seller has a "Dumping Function," which ranks loans in its portfolio from worst to best. It is given by $c(n)$. We assume that n is continuous and represents the distance between the nth best loan and the worst loan.

Now we turn to the determinants of $c(n)$ from default models

2.3 Default Models and *c(n)*

Default on the nth loan happens at the end of the period if an indicator variable, $x(n)$, takes on a value less than some critical value, $X(n)$ (e.g., see Hough (2016)). The probability of default is the probability that $x(n) \leq X(n)$. X and x can vary across loans.[3] For instance, $x(n)$ might be the value of the nth property securing the loan and $X(n)$ the mortgage balance. Alternatively, $X(n)$ might be a variable, like a credit shock, that we do not observe directly. The trigger variable x happens at the end of the period, and may be conditional on its initial value $x(0)$, which is known to sellers and is the vehicle for adverse selection.

Let $H^n(x((n))$ be the distribution function of $x(n)$ conditional on $x(n(0))$, where $H^n(n)$ is the probability, $p(n)$, that $x(n) \leq X(n)$, and let $h^n(n)$ be the corresponding density function. We assume a loss rate $l^n(x(n))$, conditional on default. As above, we use n to order loans from worst to best. The nth loan is the nth best loan in the pool

[2] We relax this assumption in Section 4.4.

[3] For notational simplicity we do not characterize x or X by subscript or superscript until later.

Under risk neutral pricing, the expected cost of default is

$$(2) \qquad c(n) = \int_{-\infty}^{X(n)} (l^n(x(\theta))h^n\big(x(\theta|x(0)\big)d\theta)$$

We consider two cases.

Case 1: The first version of our model is one in which *x* refers to exogenous default probabilities. So for the nth loan, with constant loss severity fixed at *l*. Then *i*

$$(3) \qquad c(n) = p(n)l$$

where *p(n)* is the default rate of the *n*th best loan. This model employs direct estimates of probability of default and loss; the indicator variable, *x*, is not needed. It is a simple version of models that have been used for losses in mortgage pools (see Hull (2006)). We pursue this as Model 1 in the next Section and later sections.

Case 2: The loan is collateralized by property, with *h(n)* the value of the nth property, which is lognormally distributed with variance $\sigma^2 T$. Loss severity is the difference between mortgage value and house value. The value of *c*(*n*)) in this case is given by the Black-Scholes formula for a put option; and *c*(*n*) is the value of put option value of the *n*th best loan. We use this approach in Model 5 in 4.3.

Next we apply default models and the structure of selling to the two market equilibria described above

3. Structuring and Optionality

Many financial contracts and institutions have embedded options whose values depend on the value of some state variable. Optionality makes the relationship between the contract value and the state variable driving value nonlinear. Most securitization deals are broken into pieces (tranches) to be sold to investors. In order to focus on tranching as the cause of convexity, we first model deals where underlying assets do not exhibit convexity and have fixed default rates (Case 1 above), which can vary across loans, and we show how structuring can induce fragility. Then we analyze variations on that model, and how they contribute to fragility in essentially the same way.

3.1 Securitization: Model 1

Model 1 uses the fixed loss rate model in Case 1 above. We assume that the exogenous variables are default rates, $p(n)$, which is uniformly distributed, so that the last loan sold, $c(n)$, is a linear and downward sloping function of n.

Income and Behavior

We define $a(n)$ as the average cost, associated with $c(n)$, of the assets dumped to the buyers in SM up to the nth asset. That is

$$(4) \quad a(n) = \frac{1}{n}\int_0^n c(\theta)d\theta$$

This also represents the zero profit condition for PM. Figure 1 depicts the two curves if $c(n)$ is linear. We assume that sellers in PM are price takers, but they have monopoly power over information because they know $c(n)$. However, SM knows only $a(n)$ and supplies funds elastically at that cost. Equilibrium price, $r(n)$, is equal to average cost. As depicted in the figure, marginal and average costs intersect at the origin, which leads to an Ackerlof type solution where the information advantages of sellers prevent a market from forming. We obtain a solution by assuming that holding costs are larger for sellers than they are for buyers, or that there is a benefit (e.g. liquidity and/or diversification) to buyers, as a reason for buying loans despite an information advantage for sellers We call this cost difference b. For simplicity, we assume that SM has no costs, so b is sellers' holding costs.

Sellers' business is selling loans. Their income is the holding costs they shed by selling minus their costs of selling, which are the agency costs that come from buyers understanding the lemons problem. We let

5) *A(n) = a(n)-c(n)*

Then *A(n)* is a measure of the marginal agency cost of putting the *n*th, loan into the pool. That is, the last (marginal) loan costs only *c(n)* to the seller, but the buyers will only buy it at, *a(n)*. The total agency cost is the sum of all marginal agency costs from 0 to n. This is given by

(6) $s(n) = \int_0^n ((a(\theta) - c(\theta))d\theta = \int_0^n (A(\theta)\, d\theta$

Seller income is given by

(7) $y(n) == bn - s(n) = \int_0^n (b - (a(\theta) - c(\theta))d\theta = bn - \int_0^n (A(\theta)\, d\theta$

The seller choses a pool size, *N*, that maximizes (7), taking account sales price equaling average cost, which is also determined by sales, and that *N* is limited by the size of the market (seller portfolios) size), *M*.

The first order condition is

(8) *a(N)-c(N)=A(n) = b*

or

if *N* < *M*

Otherwise *N* =*M*.

The second order condition i

(9) $A'(n) \leq 0$

This requires that b equal marginal agency costs. This is the equilibrium depicted in Figure 2. The first order condition puts equilibrium at A. However if the market is given by M^l the solution is constrained, the N=M.

Model 1 is a parameterized example of this. We can write the model as

(10a) $c(n) = c(0) - \alpha n$

Then

(10b) $a(n) = \frac{1}{n}\int_0^n c(k)dk = c(0) - 0.5\alpha n$

with $\alpha = (c(n)-c(0))n$, $c(0)$ is the cost of the worst loan in the portfolio (dumped first).

Then equating average and marginal costs we have the equilibrium size of the pool, N, given by:

(11) $c(N) = c(0) - \alpha N = -\frac{1}{2}(c(0) - \alpha n) + b = r(N)$

Then if $M>N$ enough, equilibrium level of market size,

(12) $N=2b/\alpha$

If we let p^* be the mean default rate for the pool, assuming zero interest rates, then the average cost of the deal is p^*l.

The equilibrium does not exhibit discontinuities. That is, continuous changes in b or α or market size do not lead to discontinuous changes in market size. Note that market share does not depend on the level of default costs, but rather on the slope of $c(n)$, which measures the range of adverse selection. We use this model as a lead in to the structuring model in order to emphasize that the fragility results do not depend directly on the level of risk or the slope of the marginal cost curve, but rather on its curvature, as is described next.

3.2 Structuring: Model 2

We consider the same representative pool of loans that were chosen as in Model I, but they are now put into structured deals that sell off debt and equity tranches of the pools. This could be for efficiency reasons as was discussed above. There are also some regulatory advantages to highly rated debt pieces, so a strong equity share of the pool can generate a relatively safe debt tranches that become AAA and AA bonds and sell at a premium. We assume that the equity tranche level in the structure is the minimal amounts necessary to get the desired rating, conditional on what the rating agencies know. In any event we take the structure as given.

The cost advantage, *b*, is only for the debt piece. The equity piece of the structure, given by *e,* is exogenous and is kept by the PM (seller); the rest is the debt piece is sold to SM investors. PM dumps loans into the securitized pool from the original portfolio from worst (highest default rate) to best as in Figure 1, except that the shapes of the curves are now different. A version of this is a repo deal where the seller keeps a piece of the deal, the "haircut," but can select against the other side of the deal.

The value of a pool as a whole, under risk-neutral pricing, depends only on the mean default rate and not the distribution of default rates. The sales, however, are structured deals with debt and equity pieces. Their values will depend on the distribution of default outcomes. We assume that log *p(n),* denoted by (π(*n*)), follows a one period binomial distribution with constant loss severity per loan, *l*.[4] For independent draws the pools *N* loans have a variance about the mean given by $\pi(1-\pi)/N$. For large enough *N* the distribution of *p(n)* can be taken to be a lognormal.

However, we should not expect the draws across loans to be independent. For instance they might be correlated with a common market-wide factor. We assume that is the case and that there are

[4] We chose this distribution because it allow the probabilities to be bunched at low level ands

two independent factors that affect $\sigma(n)$: a market one, m, and the idiosyncratic one, given by $\frac{\pi(1-\pi)}{N}$. The marginal contribution of m to $\sigma(n)$is given by $\rho(n)$. Then the variance of the nth loan is

(13) $\sigma(n)^2 = \frac{\pi(1-\pi)}{N} + \rho(n)^2\sigma_m^2$

and

(14) $\frac{\partial\sigma(n)}{\partial\rho(n)} = \sigma_m \equiv q$

A convenient special case is where the pool is sufficiently large that the first term in **(**13) can be ignored and

(15) $\sigma(n) = \rho(n)\,\sigma_m$

Then $\sigma(n)$ is the standard error of the loan with the nth highest level of diversification. Selection can be modeled as based on $\rho(n)$, and in turn $\sigma(n)$. Again, we can approximate the distribution of *p(n)* with a lognormal distribution with density function: $f\big(p(n), \sigma(\rho(n))\big)$

The value of a share in the pool into which the N mortgages or loans are put is given by

(16) $v_p = \left(1 - l\int_0^N f\big(n, \sigma(\rho n)\big) n dn\right) = (1 - p^* l).$

The density function, $f\big(p, \sigma(\rho)\big)$, is known by all, but loan by loan details are known only to sellers. This corresponds to Model 1.

Optionality and in turn convexity comes from creation of a debt tranche along with an equity tranche with limited liability for PM, who holds the equity tranche. We assume that $p^*l > e$, that is, the debt piece is initially "out of the money." Then, assuming interest rates are zero, the value of a share in the debt piece is given by

$$(17) \qquad v_d = \left((1-e) - N^{-1}\int_e^N f(n,\sigma(n))ndn\right)$$

The last term is the expected level of default costs beyond the equity piece, so asset price equals initial balance minus expected cost, and the value of an equity share is 1- v_d , which is the difference between the pool value and the debt value.[5] Note the simple negative relationship between cost and price. We use the terms interchangeably, being careful to keep track of the sign difference.

3.3 Properties of Model 2

The second term inside the brackets in (17) is *c(n)* for this model. The value of this can be formulated (assuming that having all or nearly all of the loans default is very unlikely), as if it were a call option on a lognormally distributed underlying security, *pl*, with exercise price *e*. The value of a share of that is given by the Black-Scholes formula:

$$(18) \quad c(n) = c(p(n)l\,,\sigma(n)) = plF(d(n)) + eF(d(n)-\sigma(n))$$

where $d(n) = \frac{ln\frac{pl}{e}+\frac{1}{2}\sigma(n)^2}{\sigma(n)}$, $F(\cdot)$ is cumulative normal, and σ is the volatility of π; the length of the period is set to equal to one. This is expected loss to debt-holders from adding a loan to the pool.

In Model 2 we assume that $\sigma(n)$ is constant, and we focus on default rates as the selection vehicle. We will use $\sigma(n)$ as a selection instrument in section 4.

[5] It is straightforward to extend this to the case of pool wide insurance, which is a straightforward debt/equity structure that has the same general properties as *c*(*n*) in (24) and (25) below.

Then for Model 2

(19) $c'(n) = c'\big(p(n)\big) = \frac{\partial c}{\partial p}(-p') = -lF(d)p'(n) < 0,$

Convexity of $c(n)$ at n is given by (note $p'(n)$ is constant)

(20) $\frac{\partial^2 c(n,\sigma)}{\partial n^2} = -lp'(n)\left[\frac{l}{p(n)\sigma} f(d)\right] > 0$

So $c(n)$ is convex and approaches zero from above as n increases (more and more safe loans put into the pool)..

These properties are defining characteristics for our fragility results, and they are common to our other models. We provide some derivations in the appendix. However, much of our analysis applies standard text book theorems about marginal and average curves, so we describe solutions graphically.

3.4 Equilibrium in Model 2

Equilibrium comes form maximizing $y(n)$. Again, the first order condition is given by

(21) $b - s'(N) = = b - A(N) = 0$

and the second order condition is

(22) $A'(N) \leq 0$

Its properties are derived in the Appendix. $A(n, \sigma)$ goes through the origin and is rising at the origin, has a maximum, then declines and approaches the horizontal axis as n increases without limit. These are the key convexity properties. The equilibrium can both be described by variations on these. We next look at equilibrium from three views:

View 1: cost-market size space

A version of this equilibrium, View 1, is depicted in Figure 3. It is comparable to Figure 1 except that $c(n)$ is not linear, and decreases from above with increase in value of loans, n, at a decreasing rate. Note the curves given by $c(n)$, $a(n)$ and $c(n)+b$. Because, from standard microeconomics, $a(n)$ is average to the marginal of $c(n)$, it meets $c(n)$ at $c(0)$, as before in the linear case. It is steeper at first but flatter as n gets larger. The addition of b shifts the $c(n)$ curve up, guaranteeing the possibility of two solutions, as depicted. As b increases, it can be tangent to $a(n)$, and without an equilibrium above the tangency, in which case the cost advantages of SM are so large that it takes over the whole market, at M. When $c(n)+b$ moves from the tangent to just above it, there is a discrete shift in market share from a small n to the maximum market volume M. This creates fragility in the sense that the quick expansion looks like a debt bubble. The next view illustrates the equilibrium more clearly.

View 2: b-n space

We know that $A(n, \sigma)$ is upward sloping at the origin, continuous and approaches zero for large n. This means it must have a maximum. We show in the appendix that this maximum is unique and there is no minimum. Figure 4 depicts $A(n, \sigma)$ and intersection for values of b. The maximum of $A(n, \sigma)$ corresponds to the tangency in Figure 3. Equilibrium must be either at an intersection of b and $A(n, \sigma)$ or at the intersection of b and market size M, with price, $r(n)$, equal to $a(n)$. Along the segment OQ income is maximized, To left of it $y(n)$ is increasing and the right it is decreasing. The arrows show the direction of adjustment to maximum $y(n)$. The segment QH is shows minima. The arrows near it show adjustment away from it. For any level of b between b_1 and b^* if market size is large, M_h, the result is ambiguous because taking over the whole market could be better than staying along GH, This raises the question of discrete jump, which is our version of fragility. We turn to view 3

View 3: y-n space

Here we look at the model by looking at *y vs n,* given, *b* in figure 4. First, *y(n*) is maximized at A, which corresponds to and intersection of *b* and *A(n)* in figure 4. After reaching the maximum the value of *y(n)* declines, corresponding to the downward sloping part of *A(n)* in Figure 2. Then it turns upward and eventually approaches the curve *y=bn.* This corresponds to *c(n)* and *a(n)* both approaching the horizontal axis in Figure 4,. The solution to maximizing income can be at A, the tangency or by taking over the whole market at M^h. which is the level of *n* at which income is as high as the maximum income at A. Which one it is depends on the size of the market. If the market size is M^h or greater, then income is maximized at the corner solution (selling off everything). However if the market is smaller, then the solution is the interior solution at A unless the market size is less than M^l.

The critical part of this model, the change in slope of *y(n)* and approaching *bn* is due to optionality, which causes total agency cost, $\int_0^n (a(\theta) - c(\theta) d\theta$, to decrease after increasing and to approach zero, so that *y(n)* approaches *bn.* This implies that there is always a market size, *M*, large enough for the solution to be to sell everything.

3.5 Equilibrium and Fragility

Fragility can be seen in Figure 5 from the two equilibria at A and B. There is no equilibrium between M^l and M^h. As *y(n*) increases and reaches a maximum level of y* equilibrium jumps from A to B. This jump is the model's option-driven fragility. From B it increases continuously and approaches *bn.* The solution depends on how market size constrains the equilibrium. For a small market, M^i, fragility is not possible, but for any market size greater than M^h, there can be a jump from A to B and vice versa. This allows a discrete increase in SM size, followed by sharp decreases driven by small changes in underlying parameters. Note that this is not a model of two equally good equilibria (a sin Diamond-Dibvig), but of almost unique equilibria with distinct tipping/switch points.

Comparative Statics and fragility

Here we derive comparative statics of the model. In particular, we look at effects of some parameters on disctrete shifts in *y(n, σ)* in the neighbor hood of fragilty, at M^l and M^h . That is we ask: if we are close to fragilty, in which direction will parameter changes move us: toward fragility away? In Figure 5 the question is about the switch points at A and B.

We first note that point A is at a local maximum. Then the slope is zero, and being a little bit on either side of M^l will not affect eguilibirum. The action is at M^h where income is increasing in *n*. At B an upward shift in *y(n)* from a bit to the right of M^h to a bit to the left causes fragiity via a switch to M^h from M^l . Things that cause *y(n)* to shift up at M^h will tend to promote fragility.

It is shown in the appendix that

(23) $$\frac{\partial y}{\partial \sigma} = -\int_0^n \frac{\partial A(k,\theta)}{\partial \sigma} d\theta \ < 0$$

(24) $$\frac{\partial y}{\partial e} = -\int_0^n \frac{\partial A(\theta,\sigma)}{\partial e} d\theta > 0$$

(25) $$\frac{\partial y}{\partial pl} = -\int_0^n \frac{\partial A(\theta,\sigma)}{\partial pl} dk < 0$$

and

(26) $$\frac{\partial y}{\partial b} = n > 0$$

Then decreases in σ or *pl* (debt) and increases in *e* or *b* in the neighborhood of M^h cause discrete increases in market size, and vice versa. So, for instance, an increase in diversification can cause a sharp increase pool size and then a sudden decrease, if for instance opinions about it change.

3.6 An example: Securitization boom and bust

Here we exploit the comparative statics in 3.5 to present a version of boom and bust. A stylized version of the crash in 2008 begins with a rise in the CDO market share, and it is followed by a sharp crash. In Figure 5 we assume an initial equilibrium with y(n) intersecting at a level just below that given by M^l. There is an increase in b (increased advantage of SM, which moves the equilibrium abruptly to M^h. This is the CDO and securitization boom.

The sharp increase in market share and higher price for tranche shares can look like a debt bubble. However, it is not really a bubble in the sense that there is nothing in the shift upward that suggests decline later, unless the process for b has something mean reversion. But the equilibrium can shift back, for instance if other parameters are stochastic. Assume for instance that σ is stochastic with a 50-50 chance of rising or falling. Then a small decline in σ shifts *y(n)* a bit to the left and the new new market size is too small land the market abruptly contracts. This looks like a bubble bursting, but it comes from σ falling, not b reverting, and σ falling is not likely to be predictable. This is the sense in which the equilibrium is fragile; the crash can come from any parameter change. This is consistent with and the observation, referred to above, by Gorton and Metric (2012), that smooth increases in the ABX in 2008 were associated with a sudden jump in the LIBOR-OIS spread (a proxy for counterparty risk in the repo market).

3.7 Comments

Multiple equilibria, as depicted in Figure 5, are not unusual in economics. They require important non-linearities. The key difference between models one and two is that marginal agency cost, *A(n)*, is quite nonlinear. In Model l there was a unique equilibrium because *A(n)* is always increasing in *n*. In model two that is not the case. Agency costs decline again as the market increases so that income increases later. This shows up Figure 2 vs figure 5.

The low volume equilibrium is stable, as in Figure 2, but the high volume minimum is not stable because selling more *lowers* agency costs, and the incentive is to keep expanding. The reason for this is that in the selection model increasing sales means putting safer loans into the pool, and the convergence of both marginal and average cost as this takes place makes the debt piece easier to understand and agency cost go to zero. Then the stable equilibrium happens when SM takes over the entire market. Fragility happens from switching from low volume to high volume and back. This is conditional on market size. Small markets (small seller portfolios) do not have enough room for fragility. From Figure 5 it is clear that there is always a market (portfolio size) large enough for fragility to be possible.

We have assumed that sellers have a monopoly on information and get to exploit it as they sell-by setting sales where marginal agency cost equals *b*. Another possibility is that they set price, perhaps do to entry or regulation, equal to cost with zero net income. In our model that is characterized by tipping points at D and E in Figure 5, and the model is essentially unchanged in terms of fragility.

4. Three Almost Isomorphic Models

The central characteristic of the optionality in Model 2 is that *c(n)* is convex and approaches the origin from above. We now look at three models that mimic this property and, hence, will appear as depicted Figures (4) and (5). The first model is insurance on individual loans. The second is structuring based on inside information about volatility. The last one incorporates borrowers' options at loan level. We go through these and show that the key characteristic of *c(n)* in each case is the same as in Model 2.

4.1 Model 3: Loan Level Mortgage Insurance

Here we introduce structuring and convexity in the form of insurance. It covers the first loss on each loan in a pool with a limit of e, representing the insurers' equity per loan. We assume that $pl > e$, meaning insurance is not expected to cover average losses. The value of a share in the insured piece is given by

$$v_d = \left((1-e) - \int_e^n f(n)(kl-e)dn\right) \quad (27)$$

and equity (the insurers position) is the difference between pool value and debt value.[6] This can also be applied to debt pieces of pools consisting of resecuritized debt pieces of other pools (rather than individual loans) that have the debt-equity structures taking the form of either the insurance on pools as a whole or with the equity piece.

Properties of Model 3

The second term in (27) is the value of the "put" that the insurer exercises against the insured investors. Mathematically, as with Model 2, (27) generates a payoff that looks like a Black-Scholes call option. The owner receives loan principal net of losses in the pool in excess of the insured losses, above *e*, and *pl* is the underlying "asset." Pool losses at the end of the period are approximately lognormally distributed. The value of debt, formulated as a call option on a lognormally distributed underlying security, is, again, given by

(28) $$c(pl\,,\sigma) = plF(d) + eF(d-\sigma)$$

It is the case that

(29) $$\frac{\partial c}{\partial p} = lF(d) > 0,$$

(30) $$\partial c^2/\partial p^2 = \frac{l}{\sigma} f(d) > 0.$$

Assuming uniform distribution and suitable choice of units, we have

(31) $$\frac{\partial c}{\partial n} = \frac{\partial c}{\partial p(n)} \frac{\partial p(n)}{\partial n} = lF(d)\frac{\partial p(n)}{\partial n} = -lF(d) < 0$$

(32) $$\frac{\partial^2 c}{\partial n^2} = -\left(\frac{1}{\sigma}\right) f(d)\frac{\partial p(n)}{\partial n} = \left(\frac{1}{\sigma}\right) f(d) > 0.$$

---

[6] It is straightforward to extend this to the case of pool wide insurance, which is a plain debt/equity structure that has the same general properties as $c(n)$ in expressions (18) and (19) below.

It can be seen from (31) and (32) that $c(n)$ is downward sloping, concave, and it approaches 0 as n expands (that is, cost of default on debt approaches zero as the probability of default declines). As a result, Figure 3 can represent Model 3 in the same way as with Model 2. Adding optionality by structuring in the form of insurance can also introduce fragility to pools that would not be fragile otherwise.[7]

4.2 Model 4: CDOs with Inside Information about Diversification

Here we consider a version of Model 2 above, but with asymmetric information about a different dimension, diversification. A complaint about structured deals during the recession was that investors overestimated their diversification benefits. For instance, Beltran *et al* (2017) document hidden correlations among CDO tranches. We show that asymmetric information can generate fragility via the same sort of convexity as Model 2.

We take the underlying securities in the CDOs (perhaps pieces of other deals) to be given by the same structures as the loans in Model 2. We assume that default probabilities across loans in the pool are the same, but their correlations are different, thus inducing different amounts of diversification in the pool, for instance because of regional or line-of-business similarities. We consider a model that is based on models used in pricing CDOs (see Hull and White (2004).

From 3.2 above it is clear that sellers can use asymmetric information about diversification as a selection variable in the same way as default rates in Model 2. Here we assume the simple version of diversification, and we use σ as the choice variable. We next show that using $\sigma(n)$ instead of $p(n)$ does not affect the basic convexity properties of the model.

[7] Note that we can easily handle multiple tranches. The senior or catastrophic piece is the same as above but with equity, e, and equal to that in all the previous tranches. Other (mezzanine) pieces with be like the catastrophic piece, but with a put when their balance is used up.

Properties of Model 4

From (28) above:

(33) $$\frac{\partial c}{\partial \sigma} = plf(d)\, q > 0$$

and

(34) $$\frac{\partial c}{\partial n} = \frac{\partial c}{\partial \sigma}\frac{\partial \sigma}{\partial n} =< 0$$

Because $c(n)$ is downward sloping and positive it approaches 0 as n increases it has the same convex shape as in Models 2 and 3. Hence, this model, with the sellers having inside information about diversification has the same properties, and therefore similar fragility, as depicted in Figure 3.

4.3 Model 5: Borrower Options without Structuring

Here we present a situation where default is modeled as a put option on property securing a mortgage at a strike price equal to the mortgage balance, and we call it Model 5. This uses the default model given by Case 2 in 2.3 above. The exogenous variables are initial ratios of house price to loan balance, which are uniformly distributed. The model here is adapted from Van Order (2006). An example of this sort of structures is the Private Label (PLS) market that securitized mortgages without guarantees on either the mortgages or the structure. That market rose and fell rapidly in the early stages of the millennium.

We assume a particular version of the option model in which the change in the log of property values is normally distributed and traders are risk-neutral. Let $h(n)$ be the initial property value relative to mortgage balance for the nth loan. We assume that it is uniformly distributed across loans. The model is for one period after which the borrower either exercises the default option or pays off the debt and the risk-free rate is zero. The default cost function for a particular loan, in terms its rank, *n*, is like a put option on a non-dividend paying asset using Black-Scholes formula; that is

(35) $$c(h(n), \sigma) = F\,(-d + \sigma) - h(n)F\,(-d))$$

The function $c(h(n),\sigma)$ is smooth and convex in *h*, which approaches zero as *n* approaches infinity. The uniform distribution assumption for $h(n)$ means that $\frac{\partial h}{\partial n}$ is constant and positive (i.e. dumping is in the order of successively higher house price loans) and can be set equal to 1. Then

(36) $$\frac{\partial c}{\partial n} = \frac{\partial c}{\partial h}\frac{\partial h}{\partial n} = \frac{\partial c}{\partial h} = [F(-d) - 1] < 0$$

and

(372) $$\frac{\partial^2 c}{\partial h^2} > 0$$

Again, $c(n)$ is downward sloping and strongly convex in *n* (it approaches zero as *n* approaches infinity), and Figure 3 applies.

4.4 Two Extensions: Layering Convexity and nonlinear loan distributions

CDOs have often been composed of shares of debt pieces from other structured deals. Sometimes the debt tranches of a new/final CDO are in the debt tranches of previous deals, which makes the structures more complicated. Pieces that are dumped into the new CDOs are already convex to begin with, so there is layering of convexity. Here we use convexity (rather than optionality) properties of our models to show that resecuritizing debt tranches can make the new pieces more convex and therefore increase the likelihood of fragility.

We assume a pool of debt pieces made from debt pieces from other pools. This is the stage one deal, and we assume its cost and dumping function is given by

(38) $q'(n) < 0$ and $q''(n) > 0$.

Stage two takes restructures these debt tranches, breaking it up into debt and equity tranches like those in the first stage. It's costs are given by $g(q(n))$. Then let

$H(n)= g(q(n))$ be the valuation for the second stage debt tranche. Then

(39) $H'=g'q'$

Alternatively

$Log\ h'=log\ g' + log\ q'$

Taking derivatives of this:

(40) $H''/H' = g''/g' + q''/q'$

The items on the right had side of (40) are all positive. As a result, resecuritizing increases the elasticity of convexity relative to an increase in slope change. This is what we mean by layering convexity. It suggests that not only are resecuritized structures more complicated, they might be more prone to fragility, and fragility increases with increases in resecuritization of more layers of CDOs (e.g., "CDO-squared" deals that securitize CDO pieces). .

Nonlinear Loan Distributions

The assumptions of uniform distributions of probabilities of default or house prices are useful in bringing out the role of optionality and convexity in fragility. They are however restrictive because of the following. First, default rates or collateral values are generally clustered rather than evenly spread out. Second, it implies that fragility only happens when SM has taken over the entire market. Here we provide an alternative assumption by adjusting Model 2 above. [8]

In Model 2, $c(n)$ is convex throughout, and $A(n, \sigma)$ has no local minimum. However, $c(n)$ can be concave over some range without affecting the basic results of the model. Instead of being linear, we assume that $F(X(0))$ is *S*-shaped, for instance, because it is a segment of a lognormal distribution. This allows clustering of default rates. The highest value of $X(0)$, corresponding to the safest loan, is at a point on F where $F'' < 0$. Then $c(n)$ is still concave for low levels of n, but it may be convex for high levels, raising the possibility of multiple maxima in $A(n, \sigma)$. The main properties of $A(n, \sigma)$ still hold, but it can no longer be proven that $A(n, \sigma)$ does not have a local minimum. If there is a minimum it will be to the right of the maximum. Both average and marginal default costs still approach zero as n approaches infinity because the value of the put still goes to zero as asset value grows and $F(X(0))$ is bounded from above.

We cannot say much more without knowing how nonlinear the distribution function is. We present a version this by adapted Figure 5 for a "hump" in the distribution, so that $y(n)$ turns downward after the initial local maximum and then turns up again-because of the clustering loan types in the middle of the distribution that lead to the hump. This is depicted in Figure 6, which is the same as Figure 5 except that the distribution default probabilities in not linear. Equilibria can occur for markets along OA or DE or HI. Then there can be two sets of tipping points, from A to D and E to H (and vice versa), so there can be an intermediate jump before taking over the entire market. There are other possibilities, such the second bump being a local maximum with value less than y*, in which case the bump doesn't affect anything. In any case, while there can be more than one tipping point, it is still the case that that there is always a market size large enough for fragility.

[8] This section follows Van Order (2006)

5. Conclusions

We have developed models of securitization and structured deals. Equilibrium in the models is characterized by trading off holding cost that sellers shed against the agency cost from buyers understanding of adverse selection. In our first model (Model 1) this leads to a single equilibrium because agency costs rise as market increases. However, as we add structuring with a debt piece (in Model 2 and the rest) this changes. For instance in a debt tranche of a deal at low volume agency costs do rise with volume, but at high volume levels average and marginal costs converge, and agency costs (the difference between average and marginal cost) go to zero, making the pool more knowable and decreasing the lemons problem. This is unstable and pushes the secondary market toward taking over the entire market.

Then there can be two types of equilibria in larger markets-at a low level and at a high level. However, the second equilibrium is a corner solution because the optimizing (declining agency costs) requires expanding volume with increasing benefit, and that the answer is to sell everything to SM. This result is caused by the structuring that creates the debt tranche and is not inherent in the loans in the pool. Two side observations are that: fragility is likely to come from adding much safer loans to a deal, and that not only is adverse selection a source of inefficiency (first best is to transfer all loans to SM), it can also contribute to instability.

The fragility is inherently difficult to predict. Discrete shifts can, for instance, begin with a small continuation of declining holding costs or increasing liquidity that reduces costs in the securitization market, which cross a critical value, leading to a discrete jump in market share, raising bond prices and looking like a bond market bubble. This can be reversed in the neighborhood of the critical value, for instance, by a decrease in diversification (or a revision of the model that predictions correlations) among securities in the pool. This can be interpreted as a crude description of the crash in the CDO market in 2007. It is not like the usual bubble in that it is not triggered by an endogenous reversion of costs back below the critical level. A crash is not inevitable (diversification or cost advantage can increase and move the market further away from a critical point), and it can come from changes in unpredictable (to the source of the bubble) parameters that are urealted to the bubble trigger.

The model can be applied to a range of financial structures such as securitized banking and structures like repos. Resecuritization can produce layered convexity. So can re-hypothecation in the repo market, and shadow banks holding pieces of CDOs. An implication of the paper is that modeling pieces of structured pools (and equivalent pieces of banks and shadow banks) by using historical loss probabilities, but not taking account of possibility of fragility in the loans' structures, can lead to misspecification, underestimation of senior tranche risk, capital adequacy and overestimation of the optimality of structured deals.

References

Acharya, Viral, Douglas Gale, and Tanju Yorulmazer (2009). Rollover Risk and market Freezes, NYU Working Paper No. FIN-08-030N.

Akerlof, George A. (1970). The Market for “Lemons”: Quality Uncertainty and the Market Mechanism, *Quarterly Journal of Economics*, 84(3), 488-500.

Allen, F and D. Gale (1988). Optimal Security Design. Review of Financial Studies.1 pp. 229-263

Beltran, Daniel, Larry Cordell, and Charles Thomas (2017) Asymmetric Information and the Death of ABS CDOs, *Journal of Banking and Finance*, 76, 1-14.

Boot, A and A. Thakor (1993) Security Design**.** Journal Finance, 48 pp. 1349-1378

Caballero, Ricardo J. and Arvind Krishnamurthy (2008). Collective Risk management in a Flight to Quality Episode, *Journal of Finance*, 63(5), 2195-2230.

Deng, Yongheng, John M. Quigley, and Robert Van Order (2000). Mortgage terminations, heterogeneity and the exercise of mortgage options. *Econometrica* 68 (2) (03): 275-307.

DeMarzo, Peter, and Darrell Duffie. (1999) "A Liquidity-Based Model of Security Design." Econometrica 67.1 65-99 (

DeMarzo, Peter M. (2005) "The pooling and tranching of securities: A Model oOf Informed Intermediation.” The Review of Financial Studies 18.1: 1-35

Diamond DW and Dybvig PH (1983). Bank Runs, Deposit Insurance, and Liquidity. *Journal of Political Economy*, 91(3), 401–419

Easley, David and Maureen O’Hara (2005). Regulation and Return: The Role of Ambiguity, Working Paper, Cornell University.

Firla-Cuchr, Maciej, and Tim Jenkinson (2005). Security Design in the Real World: Why Are Securitization Issues Tranched? SSRN Electronic Journal

Glaeser, Edward L and Hédi D.Kallal (1997). Thin Markets, Asymmetric Information, and Mortgage-Backed Securities. Journal of Financial Intermediation. Volume 6, Issue 1, Pages 64-86

Fortunato, Francesca (2018). Limit Theorems for the Multiplicative Binomial Distribution (MBD), Department of Statistical Sciences, University of Bologna, Italy.

Gorton, Gary (2009). Slapped in the Face by the Invisible Hand: Banking and the Panic of 2007, Presented at 2009 Financial Markets Conference: Financial Innovation and Crisis, Federal Reserve Bank of Atlanta, May 11-13.

Gorton, G and A Metrick. (2012). Securitized Banking and the Run on Repos. *Journal of Financial Economics*, 104(3), 425-451.

Habib, Elsayed Ali (2010). Estimation of Log-Linear-Binomial Distribution with Applications, *Journal of Probability and Statistics*, 2010, 1-13 doi:10.1155/2010/423654.

Hartman-Glaser, Barney, Tomasz Piskorski, and Alexei Tchistyi. "Optimal Securitization with Moral Hazard." *Journal of Financial Economics* 104, no. 1 (April 1, 2012): 186-202.

Hull, John C. (2006). Options, Futures, and Other Derivatives, 6th edition, Prentice Hall International Inc.

Hull, John V. and A. White (2004). Valuation of a CDO and an n-th to Default CDS without Monte Carlo Simulation. *Journal of Derivatives*, 12 (2), 8-23.

Loayza, Norman V. and Romain Rancière (2006). Financial Development, Financial Fragility, and Growth, *Journal of Money, Credit, and Banking*, 38(4), 1051-1076.

McKenzie, Joseph A. (2002). A Reconsideration of the Jumbo/Non-Jumbo Mortgage Rate Differential, *Journal of Real Estate Finance and Economics*, 25(2/3), 197-213.

Modigliani, F and M.H. Miller, (1958). The cost of Capital, Corporation Finance and the Theory of Investment, *American Economic Review*, 48(3), pp. 261-297.

RiddioughT., (1997) Optimal design and governance of asset-backed securities. Journal of Financial Intermediation 4/1

Stanton, Richard and Nancy Wallace (2011). The Bear's Lair: Index Credit Default Swaps and the Subprime Mortgage Crisis. *The Review of Financial Studies*, Volume 24, Issue 10, 3250–3280,

Van Order, Robert, (2006). A Model of Financial Structure and Financial Fragility. *Journal of Money, Credit and Banking*, 38(3), 565-585.

Appendix

Properties of marginal agency cost: the difference between the average and marginal costs

Recall that marginal agency cost is given by

$$A(n) = -\left(\frac{1}{n}\int_0^n C(\theta,\sigma)d\theta) - C(n)\right)$$

Then

(A-1) $\frac{\partial A}{\partial n} = -\frac{1}{n}\left(\frac{1}{n}\int_0^n C(\theta,\sigma)d\theta) - C(n)\right) - \frac{\partial C(n,\sigma)}{\partial n} = -\frac{1}{n}A(n,\sigma) - \frac{\partial C(n,\sigma)}{\partial n}$

(A-2) $\frac{\partial^2 A}{\partial n^2} = \frac{1}{n^2}\left(A(n,\sigma)\right) + \frac{1}{\mathrm{n}}\frac{\partial A}{\partial n} - \frac{\partial^2 C(n,\sigma)}{\partial n^2} = \frac{1}{n^2}(A(n)) - \frac{1}{\mathrm{n}}\left(\frac{\partial A}{\partial n}\right) - \frac{\partial^2 C(n,\sigma)}{\partial n^2}$

At a maximum (from (A-1)

(A-3) $\frac{1}{n}A(n,\sigma) = -\frac{\partial C(n,\sigma)}{\partial n}$

Then at an extremum

(A-4) $\frac{\partial^2 A}{\partial n^2} = -(\frac{1}{n}\frac{\partial C(n,\sigma)}{\partial n} + \frac{\partial^2 C(n,\sigma)}{\partial n^2})<0$

The first term inside the brackets on the right hand side is negative, and the second is positive. However, the relationship between the two is the same as the average and marginal curves above. The second is the marginal and the first average, and so the second is steeper. Therefore, $A(n,\sigma)$ is concave at the maximum, and there is no minimum.

Because the marginal and average costs are equal at $n = 0$,

(A-5) $A(0, 0) = 0$.

Then

(A-6) $\frac{\partial A}{\partial n} > 0$ at $n = 0,$

From the properties of the Black-Scholes model, it can be shown that

(A-6) $A(n,\sigma) \geq 0$ *for* $n \geq 0$, and $\sigma_s \geq 0$,

$A(n,\sigma) \to 0$ as $n \to \infty$,

$A(n,\sigma) \to \infty$ as $\sigma \to \infty$.

These assure that $A(n,\sigma)$ has a maximum and that it is concave in the neighborhood of *n* for which $\frac{\partial A}{\partial n} = 0$.

Comparative Statics

Any expression of the form

(A-7) $$\frac{1}{n}\int_0^n \frac{\partial C(x)}{\partial x} d\theta - \frac{\partial C(x)}{\partial x}$$

has an "average-marginal" interpretation, the first term being the average of *C(x)* starting at zero and the second term being the marginal. The sign can be positive of negative. If *C(x)* is downward sloping then the term will be positive and vise-versa if upward sloping.

Applying this:

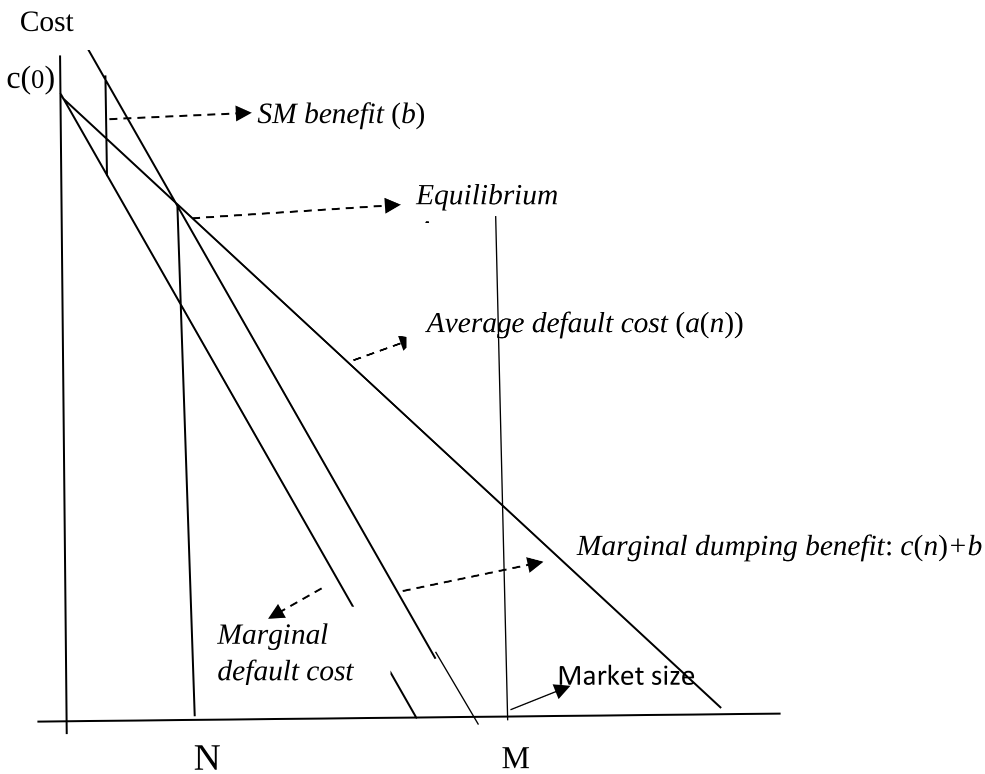
(A-8) $$\frac{\partial y}{\partial \sigma} = -\int_0^n \frac{\partial A(k,\sigma)}{\partial \sigma} dk < 0$$

(A-9) $$\frac{\partial y}{\partial e} = -\int_0^n \frac{\partial A(k,\sigma)}{\partial e} dk > 0$$

(A-10) $$\frac{\partial y}{\partial pl} = -\int_0^n \frac{\partial A(k,\sigma)}{\partial pl} dk < 0$$

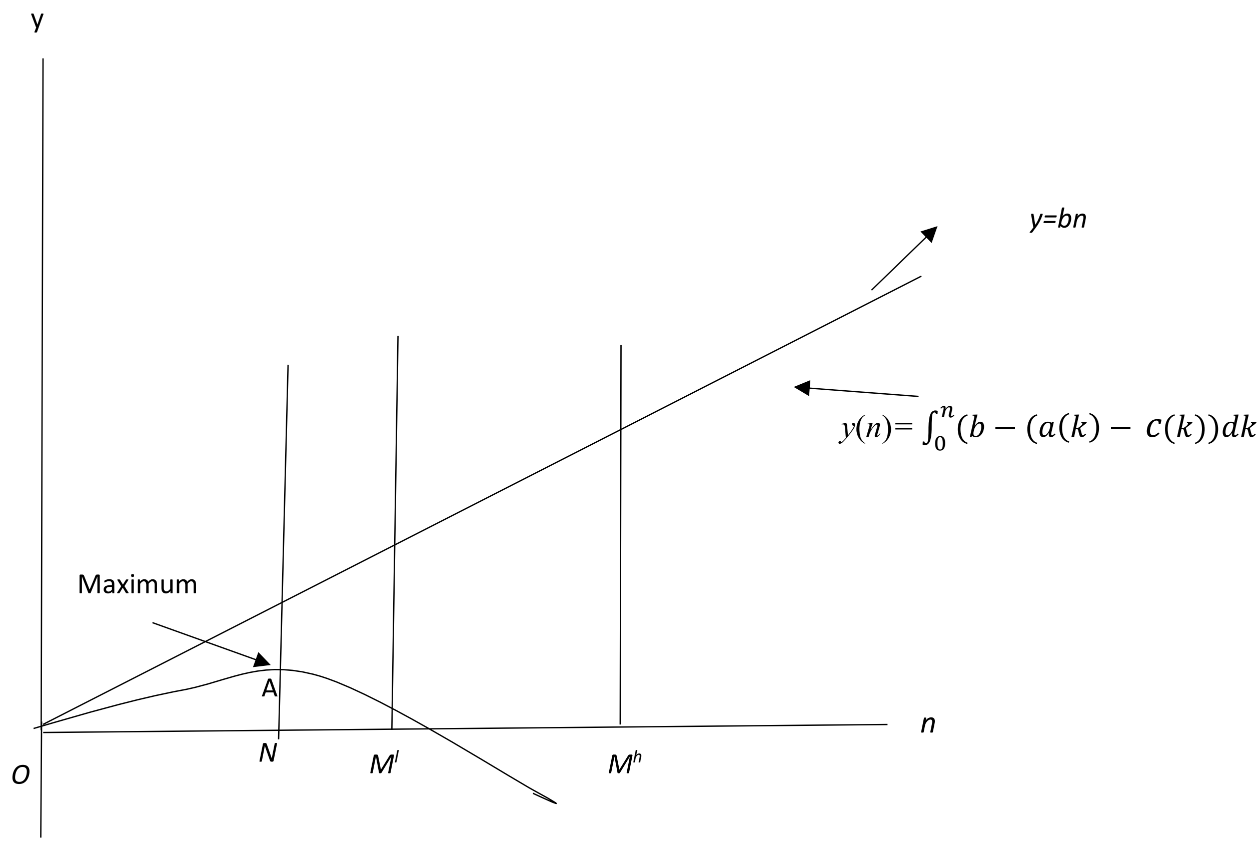
FIGURES

Figure 1 Equilibrium in Model 1 (Linear case)



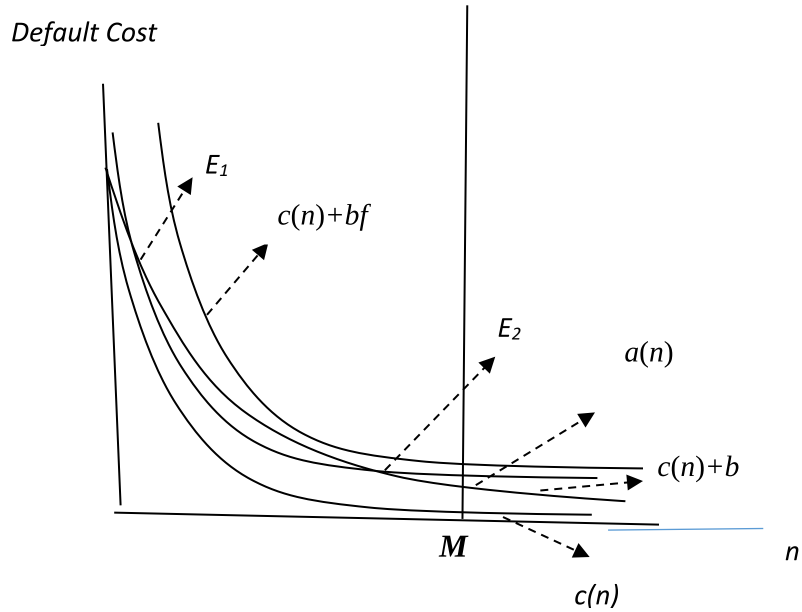
This shows marginal (for seller) and average (for buyer) curves. Equilibrium comes from the balance between buyers (primary market) and sellers (secondary market), given lower holding costs (b) for the secondary market. The size of the pools in the secondary market is N, and the entire market is M

Figure 2. Model 1: Both Monopoly Pricing and Free Entry (Linear Case)



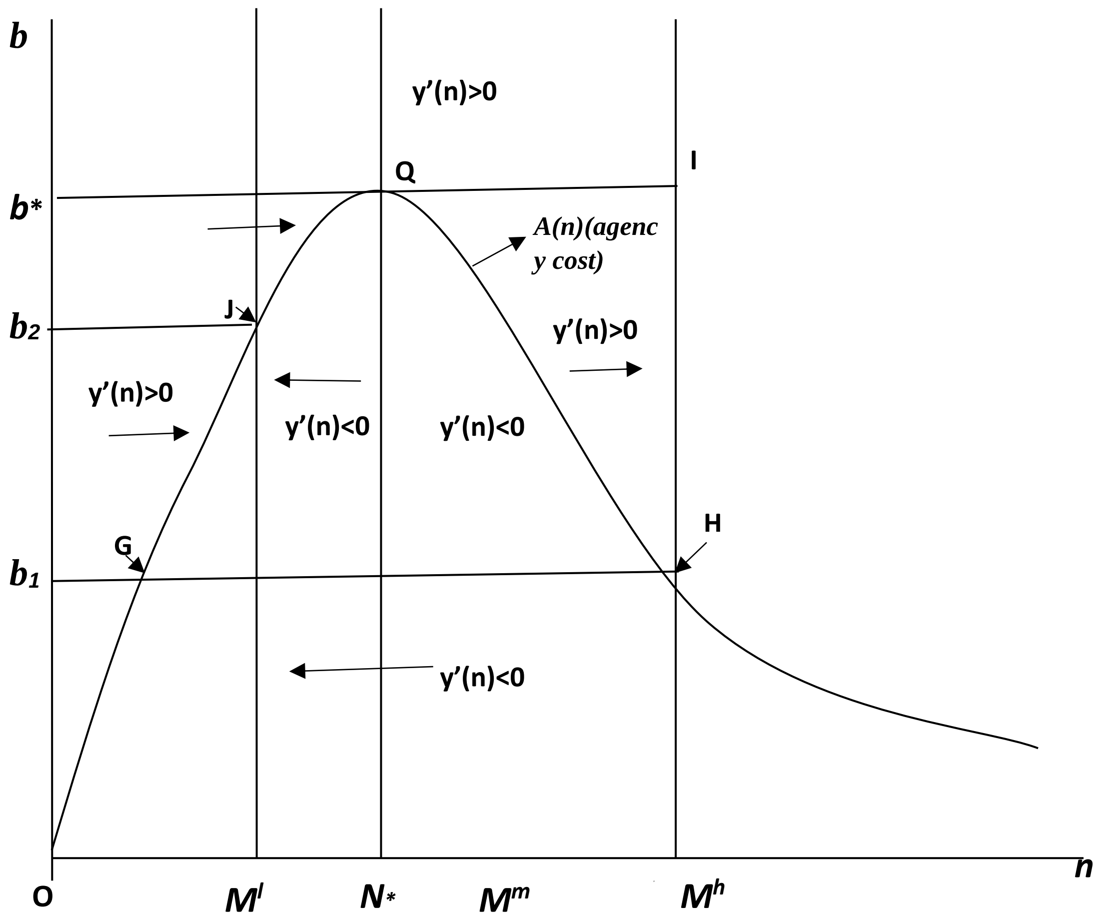
Seller income, y, as a function of Market Size. The line given by y(n) is income from PM sales, given b. The curve rises at first reaches a maximum and the falls below zero. The monopoly solution is at A, and the free entry solution is at B. If the market size is M^l then the market is smaller than "optimal." At M^l equilibrium is not constrained by market size.

Figure 3 Equilibrium in Model 2: View 1 (Nonlinear case): cost-pool size space



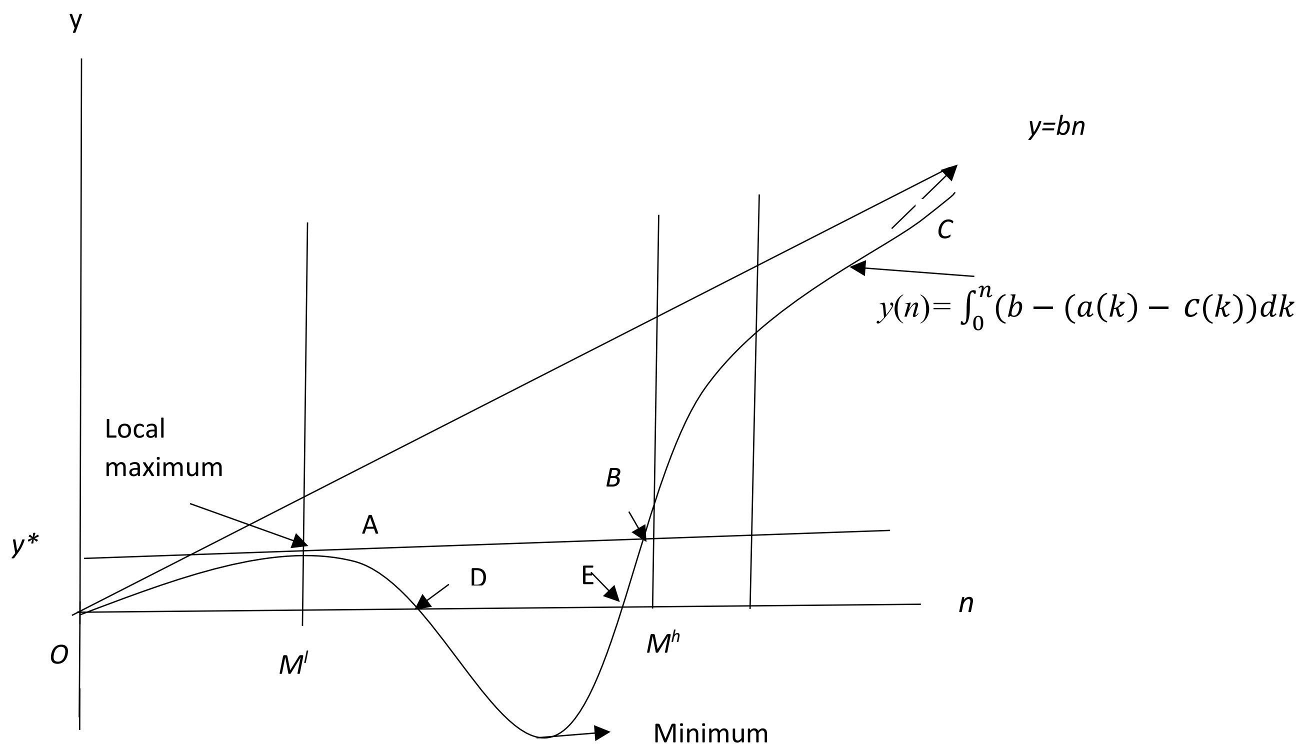
This is the same model as in Figure one, except that the marginal and average cost curves are non-linear and there can be a discontinuous shift in market share between buyers and sellers at some critical level of b, and the secondary market suddenly takes over the entire market.

Figure 4: Equilibrium in Model 2: View in *b*-*n* space with y constant



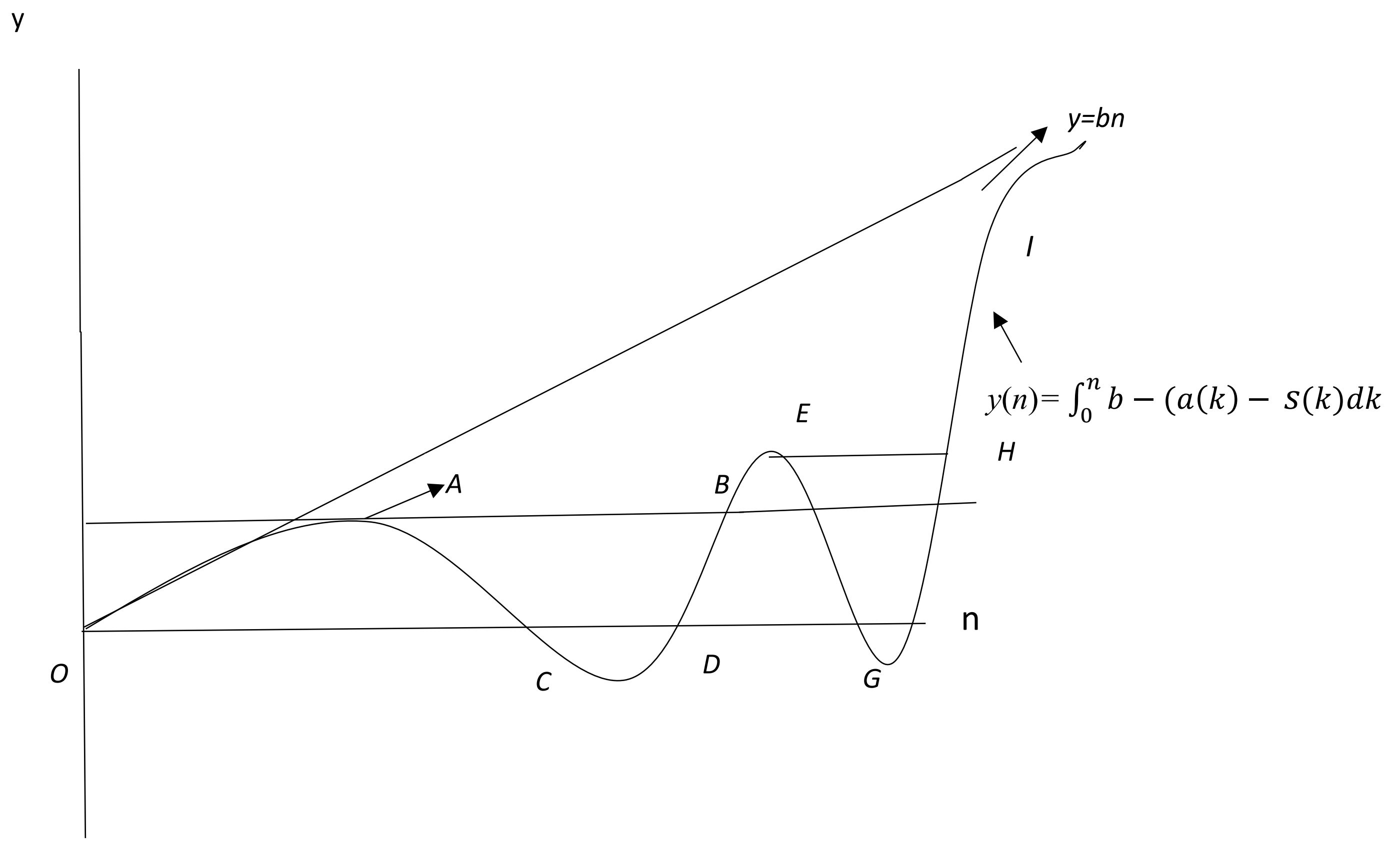
This is the same picture as in Figure 2 except that it depicts agency cost *(A(n)* its intersection with *b*. It shows multiple solutions, which generate fragility. In particular at $b=b_1$, at point G a small increase in b will shift the equilibrium to H. Possible equilibria are along POG and HI. Above b* y is always positive

Figure 5: View in *y*-*n* space holding *b* constant



The curvy line depicts income, *y(n)*, from PM sales, given *b*. The curve rises at first a reaches maximum at A and the falls below zero, then reaches a minimum and rises again. y* is the value of y at the local maximum. Equilibrium can be along OA or BE, and there is a discrete move possible from A to B (and vice versa). A key property is that there is always a market size that is large enough for sellers to sell everything. D and E are switch points if equilibrium is characterized by zero net income.

Figure 6. Equilibria with a Type of Nonlinear Distribution of Default Rates



This is the same as Figure 6 except that the distribution default probabilities in not linear, but rather has a peak on the middle. Equilibria can occur for markets of sizes along OA or- DE or HI two sets of tipping points. There are two discrete moves possible points, from A to B and E to H (and vice versa) so there can be an intermediate jump before taking over the entire market. As before there is always a market size large enough for fragility.