

Limited Household Risk Sharing: General Equilibrium Implications for the Term Structure of Interest Rates^{*}

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September 23, 2020

Abstract

We present a theory in which limited risk sharing of idiosyncratic labor income risk plays a key role in determining the dynamics of interest rates. Our production-based model relates the cross-sectional distribution of labor income risk to observable aggregate labor market variables. Our model makes two key predictions. First, it predicts positive risk premia for long-term bonds while simultaneously matching key macroeconomic moments. Second, it predicts a negative correlation between current labor market conditions (as measured by labor market tightness or the job-finding rate) and future bond excess returns. We provide evidence for these predictions.

(JEL numbers: E24, E43, E44, G12, J64.)

^{*}We thank Jack Bao, George Constantinides, Andres Donangelo, Jack Favilukis, Mike Gallmeyer (discussant), Shiyang Huang, Mete Kilic (discussant), Leonid Kogan, Yang Liu, Yukun Liu (discussant), Sydney Ludvigson, Andrey Malenko, Thomas Maurer, Jianjun Miao, Francisco Palomino, Andres Schneider (discussant), Andrea Tamoni, Haoxiang Zhu, and seminar participants at the AFA (scheduled), AFR Summer Institute, Atlanta Fed, BI Annual Workshop on Investment and Production-Based Asset Pricing (scheduled), City Univ. of Hong Kong, Cornell Univ., EEA-ESEM, European Finance Association, Federal Reserve Board, Florida State Univ., Labor and Finance Group, MFA, NASMES, New York Fed, NFA, Office of Financial Research, Pennsylvania State Univ., Temple Univ., Texas A&M Univ., Univ. of Delaware, Univ. of Georgia, Univ. of Hong Kong, Univ. of Houston, Univ. of Maryland, Univ. of Michigan, Univ. of Rochester, and the WFA for helpful comments and discussions. The views expressed here are ours and not necessarily those of the Federal Reserve Bank of Atlanta or the Federal Reserve System.

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1 Introduction

Empirically, it is well-established that the risk premium of a default-free bond (over a holding period shorter than its maturity) is positive and covaries with macroeconomic conditions. However, it has been challenging to simultaneously rationalize a large, positive bond risk premium with the observed dynamics of macroeconomic variables, particularly aggregate consumption growth ([Backus, Gregory, and Zin, 1989](#)). We provide a theory which addresses this puzzle. Our model makes two predictions. First, it predicts positive risk premia for long-term bonds, while matching moments of macroeconomic variables, especially aggregate consumption growth. Second, it predicts a negative correlation between current labor market conditions (as measured by labor market tightness or the job finding rate) and future bond excess returns.¹ We find support for these predictions in the data.

In our model, a representative firm hires labor supplied by a cross-section of individuals who face idiosyncratic labor income risk. The key friction that we focus on is the inability of individuals to diversify away this risk. In contrast to prior literature that takes the dynamics of labor income risk as given, income risk in our model arises as a consequence of the firm's labor policies. In particular, our production-based model relates the cross-sectional distribution of labor income risk to observable aggregate labor market variables using a labor search model.

Limited risk sharing is key for our model's first result, namely, predicting a positive risk premium for a long-term bond, while matching the dynamics of aggregate consumption growth. In our model, limited risk sharing implies an equilibrium stochastic discount factor (SDF) that is a consumption-weighted average of individual marginal utilities. Long-term

¹Labor market tightness is defined as the ratio of job vacancies to the unemployment rate.

bonds are a positive beta asset and earn a positive risk premium because the SDF is mean reverting.² The mean reversion is largely driven by the dynamics of the marginal utility process of a small fraction of individuals who experience large increases in marginal utility from income loss, rather than the dynamics of aggregate consumption growth. This allows bonds to earn a positive risk premium without counterfactual implications for the dynamics of aggregate consumption growth, thereby avoiding the [Backus, Gregory, and Zin \(1989\)](#) puzzle.³

The dynamics of labor income risk across individuals is endogenously determined by labor adjustment costs which we model as arising from search frictions in the labor market. The labor search setting has the advantage of allowing us to map the observed processes for labor market tightness and job-flow rates into an otherwise unobservable adjustment cost process. This disciplines our model-implied bond risk premia by restricting the process for the SDF. In our calibrated model, we find that limited risk sharing captures 40% and 55% of the average nominal and real bond risk premium, respectively, of an equal weighted portfolio of two-through five-year bonds in the data.

The intuition for our second result, namely, a negative correlation between current labor market conditions and future bond excess returns, is as follows. Firms reduce hiring during downturns which leads to an increase in the income risk of a larger than average fraction of individuals.⁴ This makes the SDF more volatile, thereby raising the market price of risk in

²For a mean reverting SDF, high realizations are associated with a low expected future value. Since the latter is equal to the bond price, the bond returns are negatively correlated with the SDF. The bond therefore earns a positive risk premium.

³They point out that, in an economy with perfect risk sharing and CRRA preferences, the required mean reversion in the SDF necessary to generate realistic bond risk premia would imply a counterfactually large mean reversion in aggregate consumption growth.

⁴This implication of our model, namely an increase in income risk during recessions, is in line with the findings of [Guvenen, Ozkan, and Song \(2014\)](#). These authors find that the income of top earners in the U.S. is highly cyclical.

recessions. Since a long-term bond is a positive beta asset in our model, this increases its risk premium. Since such periods are also associated with a decline in labor market tightness (due to a lower number of job vacancies and a higher unemployment rate) and the job finding rate, we obtain the negative relation between labor market conditions and bond risk premium.

We test our model’s predictions using both nominal bonds and Treasury Inflation Protected Securities (TIPS). We find empirical support for our model’s predictions for both types of bonds.

Literature review. Our paper contributes to two literatures and relates them. First, we contribute to the literature that analyzes the asset-pricing implications of non-diversifiable idiosyncratic labor income risk. The idea that non-diversifiable labor income risk can have a first-order effect on equities goes back to at least [Constantinides and Duffie \(1996\)](#) (see [Schmidt 2016](#) and [Constantinides and Ghosh 2017](#) for more recent examples). While labor income risk is exogenously specified in the prior literature, to the best of our knowledge, we are the first to consider the asset pricing implications of nondiversifiable labor income risk where this risk is derived from firms’ labor market policies. Therefore, our theory provides an explanation for the findings of [Ludvigson and Ng \(2009\)](#), [Joslin, Pribsch, and Singleton \(2014\)](#), [Huang and Shi \(2016, 2019\)](#), and [Bianchi, Büchner, and Tamoni \(2019\)](#) that bond excess returns are predicted by principal components of macroeconomic variables, which load on labor market variables (including employment, unemployment, and vacancies, amongst others). We focus on the implications of limited risk sharing for default-free bonds; in contrast to other asset classes (e.g., equities), Treasury bonds do not require us to make additional assumptions regarding cash flow risk.

Second, we contribute to the literature on the predictability of Treasury bond returns (see

Duffee 2013 for a review). Fama and Bliss (1987), Campbell and Shiller (1991), and Cochrane and Piazzesi (2005)), among others, document evidence for time-variation in the risk premia of nominal bonds. Pflueger and Viceira (2011, 2016) summarize evidence for time-variation in the risk premia of real bonds. Backus, Gregory, and Zin (1989) highlight the difficulty for a consumption-based model with perfect risk sharing and CRRA preferences to capture interest rate dynamics. This challenge has also been documented in workhorse macroeconomic models featuring production (e.g., Donaldson, Johnsen, and Mehra 1990, den Haan 1995, Rudebusch and Swanson 2008, and van Binsbergen, Fernandez-Villaverde, Koijen, and Rubio-Ramirez 2012). As a result, consumption- and production-based models that successfully capture time-varying bond risk premia use richer specifications for preferences, such as habit formation (e.g., Wachter 2006, Chen 2017, and Hsu, Li, and Palomino 2019), recursive preferences (e.g., Gallmeyer, Hollifield, Palomino, and Zin 2007, Piazzesi and Schneider 2007, Le and Singleton 2010, Rudebusch and Swanson 2012, Bansal and Shaliastovich 2013, and Kung 2015), and heterogeneity in investors’ preferences (Schneider, 2019). In contrast, we rationalize time-varying bond risk premia in a production-based model with non-diversifiable labor income risk playing a key role. A related paper is Kogan, Papanikolaou, and Stoffman (2020), who obtain positive bond risk premia through a different form of imperfect risk sharing—the inability of investors to share displacement risks associated with future technological innovations.

Other approaches to rationalizing bond risk premia emphasize the supply of bonds (Greenwood and Vayanos, 2014), banks’ interest rate exposure (Haddad and Sraer, 2020), disagreement and speculation regarding inflation (e.g., Hong, Sraer, and Yu 2016, and Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch 2018), fiscal policy (Bretscher, Hsu, and Tamoni, forthcoming), as well as using firms’ marginal rate of transformation for capital to directly price bonds (Cochrane, 1988; Jermann, 2013). In addition, there is a vast literature on affine

models of the term structure (see [Singleton 2006](#) for a textbook treatment).

The importance of our model’s source of adjustment costs, namely, search and matching frictions in the labor market has been explored by [Petrosky-Nadeau, Zhang, and Kuehn \(2018\)](#) for the aggregate stock market, and by [Kuehn, Simutin, and Wang \(2017\)](#) for the cross-section of stock returns. More generally, the importance of labor market frictions has been highlighted by [Belo, Lin, and Bazdresch \(2014\)](#), [Donangelo \(2014\)](#), [Favilukis and Lin \(2015\)](#), [Belo, Donangelo, Lin, and Luo \(2017\)](#), and [Liu \(2018\)](#) in the context of equity-returns, and by [Favilukis, Lin, and Zhao \(2019\)](#) in the context of defaultable corporate bonds.

The rest of the paper is organized as follows. Section [2](#) describes the model. Section [3](#) presents the quantitative implications of our model. Section [4](#) presents evidence for our model’s predictions. Section [5](#) concludes.

2 Model

This section presents our general equilibrium model of real interest rates. The key determinant of the stochastic discount factor (SDF) in our model is nondiversifiable idiosyncratic labor income risk, which we relate to aggregate labor market variables through an off-the-shelf Diamond-Mortensen-Pissarides ([Diamond, 1982](#); [Mortensen and Pissarides, 1994](#); [Pissarides, 1985](#)) labor search model.

2.1 The economy

The economy is set in discrete time, with the horizon being infinite. There is a single aggregate productivity shock whose value z_t evolves according to a first-order Markov chain with transition probabilities $p_{zz'} = \mathbb{P}(z_{t+1} = z' | z_t = z)$.

2.1.1. The household. There is a single household consisting of a unit mass of ex-ante identical individuals indexed by $i \in [0, 1]$. All decisions of the household are made by a single entity, which we term the “head of household,” whose preferences are given by

$$J_t = \mathbb{E}_t \left[\sum_{k=0}^{\infty} \beta^k \frac{\bar{C}_{t+k}^{1-\gamma}}{1-\gamma} \right], \quad (1)$$

where β is the time-preference parameter and γ is the coefficient of relative risk aversion. The utility of the head of household is defined over the consumption index

$$\bar{C}_t \equiv \left(\int_0^1 C_{it}^{1-\chi} di \right)^{\frac{1}{1-\chi}}, \quad (2)$$

where C_{it} denotes the consumption of individual i in period t , and χ is the head of household’s elasticity of substitution across individuals’ consumption.

The “head of household” is a modeling device that captures the essence of imperfect risk sharing in a heterogenous agent production-based setting, while still preserving the tractability of the representative agent framework. We further discuss this preference assumption in Section 2.3.

Idiosyncratic labor income risk. Individuals can be either employed or unemployed in each period t , with the i th individual’s employment outcome being given by

$$e_{it} = \begin{cases} 1 & \text{with probability } N_t, \\ 0 & \text{with probability } U_t. \end{cases} \quad (3)$$

To make the model tractable, we assume that, after conditioning on N_t and U_t , the idiosyncratic employment shocks e_{it} are independent both across individuals and over time.⁵ As a result, the employment and unemployment probabilities, N_t and U_t , also correspond to the

⁵This assumption avoids the need to track the history of cross-sectional realizations of idiosyncratic employment shocks.

equilibrium aggregate employment and unemployment rates, respectively, whose dynamics are described Section 2.1.2.

While unemployment risk (3) is the sole form of idiosyncratic labor income risk in our setting, investors face additional idiosyncratic wage risk in reality. We choose to abstract from the latter as it allows us to illustrate our mechanism more cleanly. This is because our labor search setting automatically implies unemployment risk which, as we show later, is already sufficient to generate non-trivial asset pricing implications when risk sharing is limited.

Limited risk sharing. The head of household chooses each individual's consumption depending on the realization of idiosyncratic labor income shocks:

$$C_{it} = \begin{cases} C_{e,t} & \text{if } e_{it} = 1, \\ C_{u,t} & \text{if } e_{it} = 0, \end{cases} \quad (4)$$

where $C_{e,t}$ and $C_{u,t}$ denote the consumption of employed and unemployed individuals, respectively. We assume that the idiosyncratic labor income shocks (3) are non-diversifiable and subject to limited risk sharing. We capture this by restricting the consumption policies,

$$C_{u,t} \leq \phi_t C_{e,t}, \quad (5)$$

so that the consumption of the unemployed is at most a fraction $\phi_t \in (0, 1]$ of that of the employed. The process ϕ_t captures the degree of market incompleteness for insuring idiosyncratic labor income risk. The full risk sharing benchmark (Merz, 1995; Andolfatto, 1996), in which all individuals consume the same amount (i.e., $C_{u,t} = C_{e,t}$) and therefore have the same marginal utility, corresponds to the case where $\phi_t = 1$ for all t . Deviations away from this benchmark are captured by letting $\phi_t < 1$ in some states, which results in

cross-sectional differences in marginal utilities. We further discuss equation (5) in Section 2.3.

We model risk sharing capacity to be a function of productivity,

$$\phi_t = \phi(z_t), \quad (6)$$

with $\phi(\cdot)$ being an increasing function so that risk sharing capacity is larger during good times when productivity is high.

Other than idiosyncratic labor income shocks, we assume that the head of household can trade a complete menu of state-contingent payoffs for aggregate risks. From the Fundamental Theorem of Asset Pricing (see, e.g., [Dybvig and Ross 1987, 2003](#)), the absence of arbitrage implies the existence of a SDF, $M_{t,t+n}$, which prices returns between t and $t+n$, $R_{t,t+n}$, according to the asset pricing relationship

$$1 = \mathbb{E}_t [M_{t,t+n} R_{t,t+n}]. \quad (7)$$

We characterize the equilibrium SDF in Section 2.2.5.

2.1.2. Labor market search frictions. There is a representative firm which produces output Y_t using a linear production technology with labor as the only input:

$$Y_t = z_t N_t, \quad (8)$$

where z_t is the current productivity, and $N_t \in [0, 1]$ is the total number of individuals who are employed in that period.

The representative firm posts V_t vacancies in period t at a cost of κ per vacancy. A total of $U_t = 1 - N_t$ unemployed individuals apply for these jobs. Due to search frictions, it takes time to fill vacancies. In particular, a total of $m(U_t, V_t)$ matches are successfully formed in period t . Following [den Haan, Ramey, and Watson \(2000\)](#), we parameterize the matching

function as

$$m(U_t, V_t) = \frac{U_t V_t}{(U_t^\iota + V_t^\iota)^{\frac{1}{\iota}}}, \quad (9)$$

with $\iota > 0$. The contact rate between the unemployed and vacancies depends on the ratio of the number of vacancies posted to the number of job-seekers. This quantity, known as labor market tightness,

$$\Theta_t \equiv V_t/U_t, \quad (10)$$

captures current labor market conditions. In particular, the probability of an unemployed individual successfully finding a job is

$$f(\Theta_t) = m(U_t, V_t)/U_t = (1 + \Theta_t^\iota)^{-\frac{1}{\iota}}, \quad (11)$$

while the probability of the firm filling a vacancy is

$$g(\Theta_t) = m(U_t, V_t)/V_t = (1 + \Theta_t^\iota)^{-\frac{1}{\iota}}. \quad (12)$$

Each period, a fraction s of the employed lose their jobs and become unemployed. Employed individuals are paid wages w_t determined using a generalized Nash-bargaining protocol, described in more detail in Section 2.2.3, in which employees obtain a fraction η of the surplus. Unemployed individuals are paid the amount b in each period of unemployment; these unemployment benefits are funded by lump sum taxes.

2.1.3. Timing of events. Figure 1 illustrates the timing of events within each period, which is as follows:

- (a) At the start of period t , there is a mass $N_t \in [0, 1]$ of employed individuals, and $U_t = 1 - N_t$ unemployed individuals. Nature draws aggregate productivity z_t according to its law of motion. Nature also draws the idiosyncratic employment shocks e_{it} .

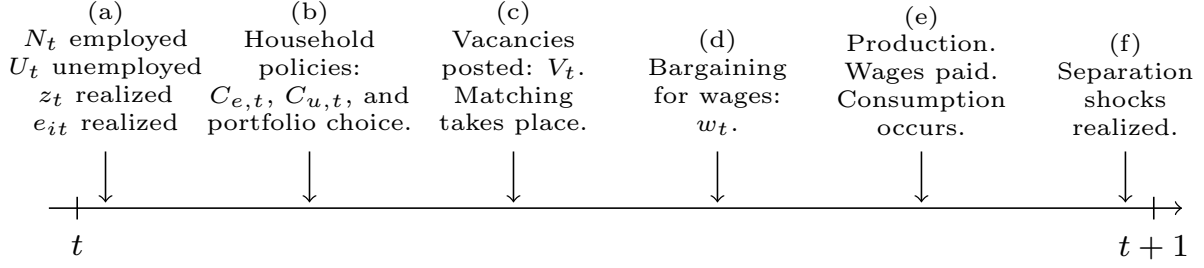


Figure 1: Timing of events within each period.

- (b) The head of household chooses its policies. This includes (1) the consumption of the employed, $C_{e,t}$, and the unemployed, $C_{u,t}$, and (2) portfolio choices.
- (c) The representative firm posts vacancies V_t , and labor market matching takes place. Matched individuals become employed at the start of the next period.
- (d) Wages are set via a generalized Nash bargaining rule with employed individuals capturing a fraction η of the surplus.
- (e) Production takes place and output Y_t is realized. Wages are then paid, unemployment benefits are collected, and consumption takes place.
- (f) Existing matches (excluding newly formed ones) exogenously separate with probability s .

2.2 Equilibrium

2.2.1. Firm's problem. The representative firm chooses the number of vacancies to post each period to maximize the present value of dividends,

$$F_t = \mathbb{E}_t \left[\sum_{k=0}^{\infty} M_{t,t+k} D_{t+k} \right]. \quad (13)$$

The dividend in period t is

$$D_t = Y_t - w_t N_t - \kappa V_t, \quad (14)$$

where output is given by (8), w_t is the wage paid to each employed individual and $V_t \geq 0$ is the number of vacancies posted by the firm. From the perspective of the firm, vacancy posting alters its labor force size through the law of motion

$$N_{t+1} = (1 - s)N_t + g(\Theta_t)V_t, \quad (15)$$

which reflects the fact that a fraction s of individuals separate from existing matches each period, while a fraction $g(\Theta_t)$ of newly posted vacancies are matched to individuals who begin working the next period. In recursive form, the firm's problem (13) is

$$F(z_t, N_t) = \max_{V_t \geq 0} D_t + \mathbb{E}_t [M_{t,t+1} F(z_{t+1}, N_{t+1})], \quad (16)$$

where future dividends are discounted using the SDF $M_{t,t+1}$, and vacancies are chosen subject to the law of motion (15) taking labor market tightness Θ_t as given.

The firm posts vacancies as long as the vacancy posting cost κ is less than the marginal benefit of posting a vacancy. The latter is the product of the vacancy-filling probability $g(\Theta_t)$ and the marginal value of a vacancy conditional on it being filled, $\mathbb{E}_t \left[M_{t,t+1} \frac{\partial F(z_{t+1}, N_{t+1})}{\partial N_{t+1}} \right]$ (an unfilled vacancy is worthless). The equilibrium amount of vacancies posted is then determined as the solution to the following complementary slackness problem:

$$\kappa \geq g(\Theta_t) \mathbb{E}_t \left[M_{t,t+1} \frac{\partial F(z_{t+1}, N_{t+1})}{\partial N_{t+1}} \right], \quad (17)$$

with equality if and only if total vacancies V_t are strictly positive.

2.2.2. Household's problem. The head of household maximizes utility (1). This problem can be expressed recursively as

$$J_t = \max_{C_{e,t}, C_{u,t}, \varphi_t^B, \varphi_t^S} \frac{\bar{C}_t^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}_t [J_{t+1}], \quad (18)$$

where the consumption index (2) is equal to

$$\bar{C}_t = \left(N_t C_{e,t}^{1-\chi^{-1}} + U_t C_{u,t}^{1-\chi^{-1}} \right)^{\frac{1}{1-\chi^{-1}}} \quad (19)$$

after aggregating individuals' idiosyncratic employment shocks (3). The choice variables consist of consumption for employed ($C_{e,t}$) and unemployed ($C_{u,t}$) individuals, and portfolio choices for the number of shares of the aggregate stock market (φ_t^S) and single-period risk-free bonds (φ_t^B) to hold. The aggregate stock market is a claim on the dividends of the representative firm; its cum-dividend value is given by equation (16). The choices are subject to the risk sharing constraint (5), and to the budget constraint

$$N_t C_{e,t} + U_t C_{u,t} + \varphi_t^S P_t^S + \varphi_t^B P_t^{(1)} \leq w_t N_t + b U_t - T_t + \varphi_{t-1}^S (D_t + P_t^S) + \varphi_{t-1}^B. \quad (20)$$

The left hand side of equation (20) is the sum of consumption and portfolio expenditures, with P_t^S and $P_t^{(1)}$ denoting the ex-dividend price of the aggregate stock market and the price of the single-period risk-free bond, respectively. The right hand side consists of wage income, unemployment benefits less lump sum taxes T_t , and the payoff from portfolio choices made in the previous period. From the perspective of the head of household, the value function (18) is subject to the laws of motion

$$N_{t+1} = (1-s)N_t + f(\Theta_t)U_t, \quad \text{and} \quad U_{t+1} = sN_t + (1-f(\Theta_t))U_t, \quad (21)$$

for number of the employed and the unemployed individuals, respectively.

In equilibrium, the household owns the representative firm and single-period bonds are in

zero net supply so that

$$\varphi_t^S = 1 \quad \text{and} \quad \varphi_t^B = 0 \quad \text{for all } t. \quad (22)$$

Finally, lump sum taxes are levied to exactly finance unemployment benefits,

$$T_t = bU_t \quad \text{for all } t. \quad (23)$$

2.2.3. Wages. Wages are determined by Nash bargaining. The presence of search frictions generates a positive surplus whenever individuals are matched to firms. This surplus is then split via a generalized Nash bargaining rule, with the employed individual receiving a share $\eta \in [0, 1]$ of the surplus, and the representative firm receiving the remaining share $1 - \eta$ of the surplus. The resulting equilibrium wage rule is given by

$$w_t = \eta z_t + (1 - \eta)b + \eta\kappa\Theta_t. \quad (24)$$

The derivation of this result is in our Internet Appendix.

2.2.4. Equilibrium. The notion of equilibrium for the economy is standard: all agents solve their respective optimization problems and all markets clear. That is, the representative firm and head of household solve their respective value functions, (16) and (18). Wages are set according to the Nash bargaining rule (24). Labor market tightness is determined according to condition (17). The head of household owns the firm and bonds are in zero net supply (22). Furthermore, goods market clearing implies that equilibrium aggregate consumption,

$$C_t \equiv \int_0^1 C_{it} di = N_t C_{e,t} + U_t C_{u,t}, \quad (25)$$

is equal to

$$C_t = Y_t - \kappa V_t. \quad (26)$$

In equilibrium, all policies and value functions are a function of state variables z_t and N_t . For example, $\Theta_t = \Theta(z_t, N_t)$, $w_t = w(z_t, N_t)$, and so on.

2.2.5. Equilibrium SDF. Consider the optimality conditions for the head of household's problem (18). First, the risk sharing constraint (5) is binding in equilibrium,

$$C_{u,t} = \phi_t C_{e,t}, \quad (27)$$

so that unemployed individuals' consumption is exactly a fraction ϕ_t of employed individuals' consumption.

Next, the inter-temporal household optimality condition implies that the SDF is equal to

$$M_{t,t+n} = \beta^n \Lambda_{t+n} / \Lambda_t, \quad (28)$$

where Λ_t denotes the shadow price on the budget constraint (20). We show in Appendix A.1 that, in equilibrium, the shadow price equals

$$\Lambda_t = C_t^{-\gamma} \times \zeta_t. \quad (29)$$

The first term, $C_t^{-\gamma}$, is the marginal utility to the head of household from an increase in aggregate consumption C_t , which is determined from the goods market clearing condition (26). The second term is equal to

$$\zeta_t \equiv \left(\int_0^1 \frac{C_{it}}{C_t} \times \left(\frac{C_{it}}{C_t} \right)^{-\chi^{-1}} di \right)^{\frac{1-\gamma}{1-\chi^{-1}}}. \quad (30)$$

It is a consumption-weighted (C_{it}/C_t) average of individuals' marginal utilities $(C_{it}^{-\chi^{-1}})$,⁶ relative to the marginal utility of an individual consuming aggregate consumption $(C_t^{-\chi^{-1}})$.

⁶This weighted average interpretation for ζ_t follows from writing the consumption index (2) as the certainty equivalent consumption $\bar{C}_t = u^{-1} \left(\int_0^1 u(C_{it}) di \right)$, where $u(C_{it}) \equiv C_{it}^{1-\chi^{-1}} / (1-\chi^{-1})$ is interpreted as the utility of an individual.

From conditions (25) and (27), and noting that $U_t = 1 - N_t$, we see that this term is equal to

$$\zeta_t = (N_t + \phi_t(1 - N_t))^{\gamma-1} \left(N_t + \phi_t^{1-\chi^{-1}}(1 - N_t) \right)^{\frac{1-\gamma}{1-\chi^{-1}}} \quad (31)$$

in equilibrium.

To relate the SDF (28) to that in a representative household economy with perfect risk sharing, substitute equation (29) into equation (28) to obtain

$$M_{t,t+n} = \beta^n \left(\frac{C_{t+n}}{C_t} \right)^{-\gamma} \frac{\zeta_{t+n}}{\zeta_t}. \quad (32)$$

The first component of the SDF (32), $\beta^n(C_{t+n}/C_t)^{-\gamma}$, depends only on aggregate consumption growth and is the only term that would appear under perfect risk sharing (see, e.g., Breeden 1979). This property is illustrated by the dashed line in Figure 2: when $\phi_t = 1$ for all t , ζ_t is always a constant (equal to 1) so that idiosyncratic labor income risk does not affect the SDF.

The second component of the SDF (32), ζ_{t+n}/ζ_t , arises as a result of limited risk sharing (i.e., $\phi_t < 1$ in some states). In our model, the dispersion in non-diversifiable employment outcomes across individuals becomes larger at lower levels of employment, which raises the marginal utility of the head of household. This property is illustrated by the solid line in Figure 2, which shows that ζ_t is a decreasing function of employment N_t under imperfect risk sharing. Furthermore, the persistence in equilibrium employment N_t , which influences the distribution of employment shocks (3), implies that nondiversifiable labor income shocks have persistence effects on the SDF (32), which is necessary to generate non-trivial asset pricing implications.⁷

⁷For example, Constantinides and Duffie (1996, p. 231) remark that a “[l]ack of persistence renders the pricing implications of the heterogeneous-consumer model similar to those of a homogeneous-consumer model.”

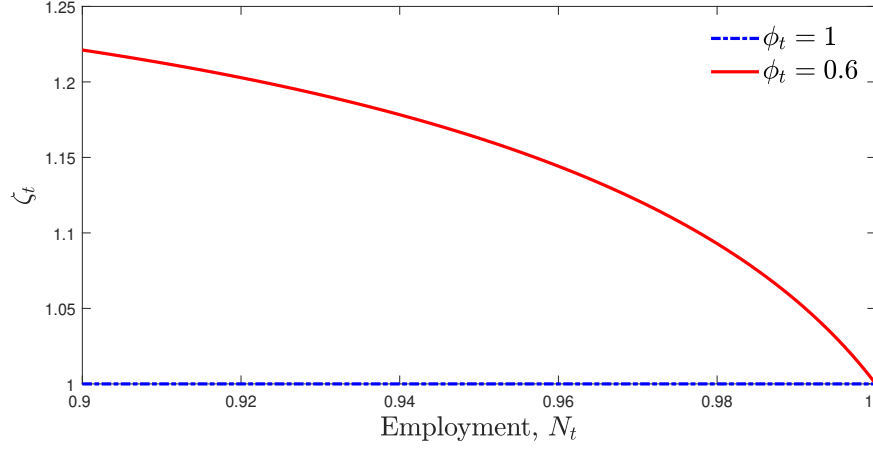


Figure 2: The function $\zeta(N)$. This figure plots the function $\zeta(N)$, defined in equation (31), with $\chi = 1/9$ and $\gamma = 2$ for various levels of employment N and risk sharing capacity ϕ .

2.2.6. Real bonds. The time t price of a default-free zero coupon bond which matures in n periods time and pays off a unit in real terms at maturity is given by

$$P_t^{(n)} = \mathbb{E}_t[M_{t,t+n}]. \quad (33)$$

The corresponding yield to maturity is $y_t^{(n)} = -\frac{1}{n} \log P_t^{(n)}$.

Consider the investment strategy of buying a T -period zero coupon bond at time t at a price of $P_t^{(T)}$, holding the bond for H periods, and selling the bond for $P_{t+H}^{(T-H)}$ at time $t+H$. This investments' realized log excess holding period return (i.e., its realized log holding period return, in excess of the risk free return from buying and holding the H period bond to maturity) is

$$rx_{t+H}^{(T)} \equiv \log \left(P_{t+H}^{(T-H)} / P_t^{(T)} \right) - \log \left(1 / P_t^{(H)} \right). \quad (34)$$

The corresponding ex-ante bond risk premium for this investment is

$$hpxr_t^{H,T} \equiv \mathbb{E}_t \left[rx_{t+H}^{(T)} \right]. \quad (35)$$

Decomposition of real bond risk premia. To gauge the contribution of limited risk sharing to the bond risk premium (35), it is convenient to study the conditional entropy of the SDF (Backus, Chernov, and Zin, 2014), defined as

$$L_t(M_{t,t+H}) \equiv \log \mathbb{E}_t [M_{t,t+H}] - \mathbb{E}_t [\log M_{t,t+H}]. \quad (36)$$

We can then write the risk premium (35) as

$$hpxr_t^{H,T} = \mathbb{E}_t [L_{t+H}(M_{t+H,t+T})] - L_t(M_{t,t+T}) + L_t(M_{t,t+H}), \quad (37)$$

which we decompose into three terms:

$$hpxr_t^{H,T} = hpxr_t^{H,T,C} + hpxr_t^{H,T,\zeta} + hpxr_t^{H,T,cross}. \quad (38)$$

The first and second terms,

$$hpxr_t^{H,T,C} \equiv \mathbb{E}_t [L_{t+H} ((C_{t+T}/C_{t+H})^{-\gamma})] - L_t ((C_{t+T}/C_t)^{-\gamma}) + L_t ((C_{t+H}/C_t)^{-\gamma}), \quad (39)$$

$$hpxr_t^{H,T,\zeta} \equiv \mathbb{E}_t [L_{t+H} (\zeta_{t+T}/\zeta_{t+H})] - L_t (\zeta_{t+T}/\zeta_t) + L_t (\zeta_{t+H}/\zeta_t), \quad (40)$$

summarize the contribution of the aggregate consumption growth and the ζ_{t+n}/ζ_t components of the SDF (32) to the bond risk premium, respectively. The third term summarizes the contribution of interaction effects between the two components of the SDF to the bond risk premium, and is defined as

$$\begin{aligned} hpxr_t^{H,T,cross} &\equiv \mathbb{E}_t [CL_{t+H} ((C_{t+T}/C_{t+H})^{-\gamma}, \zeta_{t+T}/\zeta_{t+H})] \\ &\quad - CL_t ((C_{t+T}/C_t)^{-\gamma}, \zeta_{t+T}/\zeta_t) + CL_t ((C_{t+H}/C_t)^{-\gamma}, \zeta_{t+H}/\zeta_t), \end{aligned} \quad (41)$$

where

$$CL_t ((C_{t+n}/C_t)^{-\gamma}, \zeta_{t+n}/\zeta_t) \equiv L_t ((C_{t+n}/C_t)^{-\gamma} \zeta_{t+n}/\zeta_t) - L_t ((C_{t+n}/C_t)^{-\gamma}) - L_t (\zeta_{t+n}/\zeta_t) \quad (42)$$

denotes the conditional coentropy of the two components of the SDF (Backus, Boyarchenko, and Chernov, 2018).

Appendix A.2 shows that a log-normal approximation for each of the three terms of the decomposition (38) is given by

$$hpxr_t^{H,T,C} \approx \underbrace{-\gamma^2 Cov_t(\Delta c_{t:t+H}, \Delta c_{t+H:t+T})}_{\text{“Aggregate consumption persistence”}}, \quad (43a)$$

$$hpxr_t^{H,T,\zeta} \approx \underbrace{-Cov_t(\Delta \log \zeta_{t:t+H}, \Delta \log \zeta_{t+H:t+T})}_{\text{“Labor income dispersion persistence”}}, \quad (43b)$$

$$hpxr_t^{H,T,cross} \approx \underbrace{\gamma Cov_t(\Delta c_{t:t+H}, \Delta \log \zeta_{t+H:t+T}) + \gamma Cov_t(\Delta \log \zeta_{t:t+H}, \Delta c_{t+H:t+T})}_{\text{“Cross-covariance”}}, \quad (43c)$$

where $c_t \equiv \log C_t$ denotes log aggregate consumption, and the notation $\Delta X_{t:t+k}$ denotes the difference $X_{t+k} - X_t$. The log-normal approximation (43) shows that the $hpxr_t^{H,T,C}$ and $hpxr_t^{H,T,\zeta}$ terms depend on the persistence of growth in aggregate consumption and labor income dispersion, respectively, while the $hpxr_t^{H,T,cross}$ term depends on the cross-covariance between current and future growth in aggregate consumption and labor income dispersion. For this reason, we refer to $hpxr_t^{H,T,C}$, $hpxr_t^{H,T,\zeta}$, and $hpxr_t^{H,T,cross}$ as the “Aggregate consumption persistence”, “Labor income dispersion persistence”, and “Cross-covariance” terms, respectively.

Role of non-diversifiable labor income risk. A positive risk premium for a long-term bond requires mean reversion of the SDF (see footnote 2). This holds in models with and without perfect risk sharing. However, the implications for the dynamics of aggregate consumption growth are quite different between these two models.

Under perfect risk sharing (i.e., $\phi_t = 1$ for all t), only the “Aggregate consumption persistence” term, $hpxr_t^{H,T,C}$, shows up in equation (38). This leads to the bond risk premium

puzzle highlighted by [Backus, Gregory, and Zin \(1989\)](#). They show that matching the observed (positive) bond risk premium would then require aggregate consumption growth to have a counterfactually large and negative autocorrelation.

Instead, when risk sharing is imperfect, labor income dispersion influences asset prices, and bond risk premia additionally depends on the terms in equation (38) involving labor income dispersion, namely, $hpxr_t^{H,T,\zeta}$ and $hpxr_t^{H,T,cross}$, or the “Labor income dispersion persistence” and “Cross-covariance” terms, respectively. In our calibrated model, these additional terms account for more than 94% of the risk premium of a long-term bond in our model (see [Table 4](#)). Our model is therefore able to generate large and positive risk premia for long-term bonds without counterfactual implications for the persistence of aggregate consumption growth.

Role of labor market search frictions. The log-normal approximation (43) shows that bond risk premia crucially depend on the time-series properties of labor income dispersion risk, as summarized by the variable ζ_t . For example, equation (43b) shows that the “Labor income dispersion persistence” term positively contributes to bond risk premia only if $\Delta \log \zeta_t$ displays negative autocorrelation. Since ζ_t is a function of N_t in equilibrium (see equation (31)), the time series properties of ζ_t depends on that of the employment rate N_t which, in turn, depends on labor adjustment costs. Rather than specify an ad hoc process for labor adjustment costs, we use an off-the-shelf labor search model which generates labor adjustment costs as an equilibrium outcome. This allows us to map the observed processes for labor market tightness and job-flow rates into an otherwise unobservable adjustment cost process (see [Section 3.1](#) for details).

2.2.7. Nominal bonds. In this section, we introduce inflation dynamics in order to obtain nominal bond prices. This allows us to (1) test our model’s predictions over a longer sample

(US treasuries data has a substantially longer history compared to US TIPS data), and (2) avoid having to estimate a liquidity premium for TIPS, which has been found to be non-negligible (see, e.g., [Pflueger and Viceira 2016](#)).

The time t price of a n period default-free zero coupon bond, which pays out a unit amount in nominal terms at maturity, is given by

$$P_t^{(\$,n)} = \mathbb{E}_t [M_{t,t+n} \exp (\pi_t - \pi_{t+n})], \quad (44)$$

where π_t is the log price level at time t . Following [Wachter \(2006\)](#) and [Piazzesi and Schneider \(2007\)](#), we model inflation as an ARMA(1,1) process:

$$\Delta \pi_{t+1} = \mu_\pi (1 - \rho_\pi) + \rho_\pi \Delta \pi_t + \xi_\pi \varepsilon_{\Delta c,t+1} + \nu_\pi \varepsilon_{\pi,t} + \varepsilon_{\pi,t+1}. \quad (45)$$

Here, μ_π is average inflation, and ρ_π is the autoregressive coefficient for inflation. Innovations to inflation consists of two components. The first component is correlated with innovations to aggregate consumption growth,

$$\varepsilon_{\Delta c,t+1} \equiv \Delta \log C_{t+1} - \mathbb{E}_t [\Delta \log C_{t+1}], \quad (46)$$

with ξ_π parameterizing the strength of this correlation. The second component is independent from the first and consists of *iid* shocks $\varepsilon_{\pi,t} \sim N(0, \sigma_\pi^2)$ with ν_π being the moving-average coefficient. The $\varepsilon_{\pi,t}$ shocks do not appear in the SDF [\(32\)](#) and are therefore not priced. Nevertheless, inflation risk is priced through its dependence on aggregate consumption growth shocks, $\varepsilon_{\Delta c,t+1}$, which are priced by the SDF.

We provide details for the computation of the nominal bond price [\(44\)](#) in [Appendix A.3](#).

2.3 Discussion of assumptions

We assume that the head of the household is the only decision maker in our economy and that her preferences are an aggregate of individual utilities as defined in equation (1). We discuss below how these two assumptions make our model as tractable as a representative agent model while capturing the effect of non-diversifiable idiosyncratic labor income risk on bond prices. We also discuss the simple consumption allocation rule used by the head in equation (5).

Assuming a single decision maker helps us avoid having to solve for consumption and portfolio choices of individuals in the cross-section. This is especially convenient in our production based model. Because, unlike an endowment economy where one is able to specify income processes that make the model tractable (e.g., an income process that ensures no-trade among investors as in [Constantinides and Duffie 1996](#)), the income process of individual agents in our model is endogenously determined and varies with macroeconomic conditions. By assuming a single decision maker, we avoid the need to keep track of the cross-sectional distribution of individual wealth which would otherwise have been a relevant state variable in our model.

Our assumption of the head's preference as an aggregate of individual utilities implies that non-diversifiable income risk affects the equilibrium SDF. For example, the increase in this SDF in regimes with imperfect risk sharing (i.e., $\phi_t < 1$) is largely driven by the increase in marginal utility of a small fraction of individuals who experience income loss, since the head allocates lower consumption to these individuals according to equation (5). This drives up the consumption-weighted average marginal utility, and hence ζ_t through equation (30).

We do not derive the optimal consumption response of individuals. Instead, we assume

a simple consumption allocation rule used by the head of household in equation (5), which directly maps individual income shocks into their consumption shocks.⁸ This simplifying assumption is sufficient to illustrate the asset pricing implications of limited risk sharing in our production-based setting. Together with the assumption that an individual’s income risk is independent of his/her current income level, we avoid the need to track the cross-sectional distribution of individual wealth.

3 Quantitative analysis

In this section, we quantify and provide intuition for our model’s two key predictions: (a) a yield curve that is upward sloping on average, and (b) a negative correlation between current labor market conditions (as measured by labor market tightness or the job finding rate) and future bond excess returns.

3.1 Calibration

We solve our model numerically using global methods (see our Internet Appendix for details). We simulate our model at monthly frequency using the parameters shown in Table 1. We report our model-implied moments over a horizon of a quarter in Table 2, together with their data counterparts. Details for the data used in our calibration procedure are available in Appendix B.

For our baseline analysis, we model the exogenous productivity process z_t as a two-regime

⁸Further endogenizing the degree of risk sharing is outside the scope of our analysis. Ai and Bhandari (2018) consider a setting in which contracting frictions endogenously determine the extent to which labor income risks can be shared.

Table 1: Parameter values. We simulate our model at a monthly frequency using the parameters shown in the table below.

Parameter	Symbol	Model
Persistence of productivity regimes	λ	0.9
Long-run probability, low productivity regime	p_1	0.167
Long-run probability, high productivity regime	p_2	0.833
Log-productivity, low productivity regime	$\log z_1$	-0.022
Log-productivity, high productivity regime	$\log z_2$	0
Time preference parameter	β	0.9984
Coefficient of relative risk aversion	γ	2
Elasticity of substitution in consumption index	χ	1/9
Employed individuals' bargaining power	η	0.025
Vacancy creation cost	κ	0.881
Curvature of matching function	ι	1.24
Job separation probability	s	0.034
Unemployment benefit parameter	b	0.923
Degree of risk-sharing in low productivity regime	$\phi_L \equiv \phi(z_1)$	0.6
Degree of risk-sharing in high productivity regime	$\phi(z_2)$	1.0
Average inflation	μ_π	0.00324
Autocorrelation coefficient, inflation	ρ_π	0.81
Loading on aggregate consumption growth shock, inflation	ξ_π	-0.035
Moving-average coefficient, inflation	ν_π	-0.338
Volatility of $\varepsilon_{\pi,t}$ shocks, inflation	σ_π	0.00245

Markov chain.⁹ Having a simple two-regime specification for productivity makes it easier to explain the intuition of our model, and all results presented in our main text are for this two-regime specification. We show in the Internet Appendix that our results are robust to increasing the number of states for z_t . Specifically, the Internet Appendix presents results for the case in which aggregate productivity follows a discretized AR(1) process with eleven productivity states. The AR(1) model produces near identical results for the yield curve and

⁹Note that even though there are only two regimes in our baseline specification, our model nevertheless features many states (z_t, N_t) due to the presence of the endogenous state variable for employment N_t .

Table 2: Moments of real variables and asset prices. This table reports model-implied moments along with its data counterparts. Yields and the equity premium are annualized; all other quantities are for a quarterly horizon. The average mean (volatility) of the yield curve is the average of the means (volatilities) of the one- through five-year nominal bonds. The data is for the period 1964Q1-2016Q4 (see Appendix B for details).

Moment	Data	Model
Target moments		
U : mean (%)	6.09	6.08
volatility (%)	0.78	0.75
log C growth: Autocorrelation	0.32	0.35
Volatility (%)	0.67	0.57
Ave. nominal yield curve: mean	5.66	5.61
volatility	3.17	3.37
$\sigma(\text{Wage bill})/\sigma(\text{Output})$	0.87	0.76
Nontarget moments		
Θ : Mean	0.58	0.89
Volatility	0.14	0.15
Correlation (U , Θ)	-0.85	-0.91
Equity premium, annualized (%)	6.36	2.61

macroeconomic moments, and is additionally able to generate a higher equity premium of 4.34%.

We follow [Barton, David, and Fix \(1962\)](#) and, without loss of generality, parameterize the transition probabilities for the two regimes to be $p_{ij} = (1 - \lambda)p_j + \lambda$ if $i = j$, and $p_{ij} = (1 - \lambda)p_j$ if $i \neq j$, where p_j is the long-run probability of regime j , and the parameter λ captures the persistence of aggregate regimes ($\lambda = 0$ corresponds to i.i.d regimes). We choose $p_1 = 1/6$, $p_2 = 5/6$, and $\lambda = 0.9$. These choices imply an average duration of one and five years for the low and high productivity regimes, respectively, which are in line with the durations of NBER recessions and expansions over the period 1964Q1-2016Q4.¹⁰

¹⁰Our sample stops at the end of 2016 because the updated [Barnichon \(2010\)](#) Help-Wanted-Index, which

We normalize log productivity in the expansionary regime to $\log z_2 = 0$. We then set the difference in log-productivity between the two regimes to be $\log z_2 - \log z_1 = 0.022$ to match the unconditional volatility of the HP-filtered quarterly series for log US GDP per capita over the period 1964Q1-2016Q4 (using a smoothing parameter of 1600) which is 1.5%.

There are five labor market parameters. We estimate the monthly job-separation probability s and the curvature of the matching function ι directly from the data over the period 1964Q1—2016Q4. We set $s = 3.4\%$ in order to match the average monthly job-separation rate (constructed following [Elsby, Michaels, and Solon 2009](#)). We obtain a curvature of $\iota = 1.24$ for the matching function (9) by minimizing the sum square error of the difference between the empirically observed job-finding rate (constructed following [Elsby, Michaels, and Solon 2009](#)) and its model counterpart. The latter is obtained by feeding the observed times-series for labor market tightness into the model’s specification for the job-finding probability (11). We choose the vacancy posting cost $\kappa = 0.881$ and the unemployment benefit parameter $b = 0.923$ to match the first two moments of the unemployment rate. The mean unemployment rate is 6.08% in our model compared to 6.09% in the U.S. over the period 1964Q1—2016Q4. The volatility of the HP filtered series for the unemployment rate is 0.78% in the data and 0.75% in our model. Our choice of the value for b is within the range of estimates of b used in the literature which ranges between 0.4 used by [Shimer \(2005\)](#) to 0.955 used by [Hagedorn and Manovskii \(2008\)](#). Finally, we choose the bargaining power of employed individuals to be $\eta = 0.025$ to target the ratio of the volatility of the total wage bill to the volatility of output. The value of this ratio is 0.87 in the data ([Favilukis and Lin, 2015](#), Table 3) and 0.76 when we simulate our model.

Although not target moments, our model implied moments for labor market-tightness and

we use to construct labor market tightness, stops then.

its correlation with the unemployment rate are close to their data counterparts. Mean labor market tightness is 0.58 in the data and 0.89 in our model. The volatility of the HP filtered series for labor market tightness is 0.14 in the data and 0.15 in our model. The correlation of the unemployment rate and labor market tightness is -0.85 in the data and -0.91 in our model.

For the reasons discussed at the start of Section 2.2.7, our yield-curve-based calibration targets focus on moments of the nominal yield curve instead of the real yield curve. Our model’s predictions for the nominal term structure are based on the monthly ARMA(1,1) inflation process described in equation (45). We obtain parameter estimates for this process using maximum likelihood estimation over the monthly sample 1964m1—2016m12. As inputs into our estimation procedure, we measure inflation using the monthly Consumption Price Index, and obtain the data counterpart to the consumption growth shocks (46) using residuals from an AR(1) model fitted to log aggregate consumption growth data. The resulting parameter estimates are: an average monthly inflation of $\mu_\pi = 0.0034$, an autocorrelation coefficient of $\rho_\pi = 0.81$, a loading on the innovation to aggregate consumption of $\zeta_\pi = -0.035$, a moving-average coefficient of $\nu_\pi = -0.338$, and a volatility of $\sigma_\pi = 0.00245$ for the residuals $\epsilon_{\pi,t}$.

We choose the head of household’s time preference parameter, $\beta = 0.9984$, to target the average level of the nominal yield curve (defined as the equal-weighted average of one, two, three, four, and five year nominal yields). The average level of the nominal yield curve is 5.66% in the data and 5.61% in the model.

We choose the head of household’s coefficient of relative risk aversion $\gamma = 2$, the elasticity of substitution across individuals’ consumption $\chi = 1/9$, and degree of risk sharing in the low productivity regime $\phi_L \equiv \phi(z_1) = 0.6$, to jointly match the average volatility of nominal yields

(computed as the equal-weighted average of the mean volatility of the one, two, three, four, and five year nominal yields), and the autocorrelation and volatility of aggregate consumption growth. For simplicity, we fix the degree of risk sharing in the high productivity regime to be $\phi(z_2) = 1$ so that risk sharing worsens during downturns. The average volatility of nominal yields is 3.37% in our model compared to 3.17% in the data. The autocorrelation and volatility of quarterly consumption growth in our model is 0.35 and 0.57%, respectively; these values are close to their data counterparts which are 0.32% and 0.67%, respectively.¹¹

The degree of risk sharing in the low productivity regime ϕ_L is an important parameter of our model. In the Internet Appendix, we report results for a calibration with $\phi_L = 0.8$ and where κ , b , and γ are recalibrated so that the model-implied moments for the mean and volatility of the unemployment rate, and the average volatility of the nominal term structure once again match their respective data targets. We obtain near identical results for our model-implied yield curve and macroeconomic moments; the main effect is to lower the model-implied value for the equity premium.

In a separate exercise, we show that our results for the term structure are robust to considering an equal weighted average of individuals' utilities, $J_t = \mathbb{E}_t \left[\sum_{k=0}^{\infty} \beta^k \int_0^1 \frac{C_{it}^{1-\gamma}}{1-\gamma} di \right]$, which arises as a special case of the head of household's preferences (1) in which we restrict $\chi = \gamma^{-1}$ in the CES aggregator (2). We find that, compared to our baseline model, this restricted model generates near identical results for the term structure and macroeconomic moments, but produces a lower equity premium. The details for this exercise are available in the Internet Appendix.

¹¹Note that the seemingly large positive autocorrelation value for quarterly consumption growth in the data arises from aggregating the monthly series to a quarter (see, e.g., equation (42) of the review article [Breedon, Litzberger, and Jia 2015](#)).

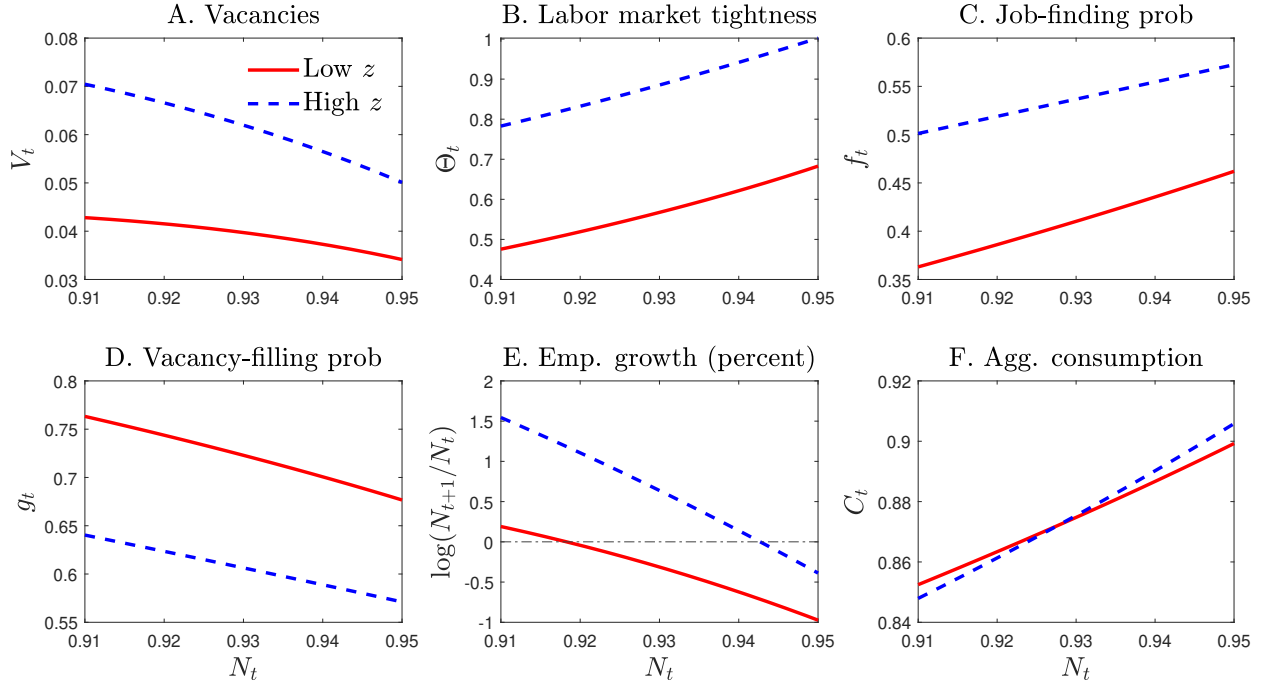


Figure 3: Hiring policies and aggregate quantities. Panels A through F plot the vacancy posting policy of the representative firm, labor market tightness, job-finding probability, vacancy-filling probability, employment growth, and aggregate consumption, respectively. The plots are for both the low (solid line) and high (dashed line) productivity regimes, and are all as a function of the current employment rate N_t .

Firm policies and real variables. In order to provide intuition for the dynamics of the equilibrium marginal utility process (29), we discuss the hiring policy of the representative firm and the resulting implications for aggregate variables.

Panel A of Figure 3 shows that firms post more vacancies in the high productivity regime compared to when productivity is low (the benefit to the firm from hiring a new individual is higher in the high productivity regime). This procyclicality in vacancy posting then leads to a procyclical labor market tightness (Panel B) which, in turn, results in a procyclical job-finding probability (Panel C) and a countercyclical vacancy-filling probability (Panel D). This is because the job-finding probability (11) and the vacancy-filling probability (12) are

increasing and decreasing functions of labor market tightness, respectively. Panel E shows that the resulting equilibrium employment growth rate is procyclical and is positive (negative) in the high (low) productivity regime.¹²

The first component of the marginal utility process (29) depends on aggregate consumption. Panel F of Figure 3 shows that aggregate consumption positively depends on current employment N . However, the dependence of aggregate consumption on productivity is ambiguous. This is because an increase in productivity results in both a higher output as well as an increase in expenses resulting from the firm posting a larger number of vacancies (i.e., κV).

The second component of the marginal utility process (29), ζ_t , behaves as follows. First, the dashed line in Figure 2 shows that $\zeta_t = 1$ is a constant in the high productivity regime where risk sharing is assumed to be perfect ($\phi(z_2) = 1$). When productivity switches to the low regime, ζ_t increases due to a worsening of risk sharing (see the solid line in Figure 2). Furthermore, ζ_t becomes larger the longer the economy stays in the low productivity regime. This is because ζ_t is a decreasing function of the employment rate (see equation (31)), and employment growth is negative in the low productivity regime (as illustrated by the solid line in Panel E of Figure 3).

3.2 Term structure of interest rates

In this section, we analyze the shape of the term structure of yields. We explain how our model with imperfect risk sharing generates a yield curve that is upward sloping on average (both nominal and real) without counterfactual implications for the dynamics of aggregate

¹²The range of values visited by N_t lies between 0.918 and 0.943, which are the values for which employment growth is equal to zero in the low and high productivity regimes, respectively.

Table 3: Term structure of bond yields. Bond yields are annualized and in percentage terms. Moments of nominal bond yields are estimated from the monthly, nominal zero coupon bond yields from the Fama-Bliss dataset over the period 1964m1—2016m12.

	Maturity (years)				
	1	2	3	4	5
A. Yields, mean					
Nominal: data	5.28	5.50	5.69	5.85	5.97
Nominal: model	5.43	5.58	5.65	5.68	5.70
Real: model	1.57	1.72	1.79	1.83	1.85
B. Yields, volatility					
Nominal: data	3.32	3.26	3.17	3.09	3.01
Nominal: model	5.89	4.04	2.91	2.23	1.79
Real: model	5.81	4.01	2.89	2.21	1.78

consumption growth.

In panel A of [Table 3](#) we report average yields for maturities of one through five years, as observed in the data and implied by our model. The data numbers are computed from the monthly, nominal zero coupon bond yields from the Fama-Bliss dataset between the period 1964m1—2016m12. We see that our model captures the observed upward slope of the nominal term structure; the slope, as measured by the difference in yields between the five and one year bond, is 0.69% in the data and 0.27% in our model. In addition, our model implies an upward sloping term structure of the real yield curve. This is in line with the findings in [Haubrich, Pennacchi, and Ritchken \(2012\)](#) who estimate the difference between the five and one year real yields to be 0.51% over the period January 1982 to May 2010.^{[13](#),[14](#)}

¹³We thank the authors of [Haubrich, Pennacchi, and Ritchken \(2012\)](#) for providing us with the estimates used in producing Figure 3 of their paper.

¹⁴[Haubrich, Pennacchi, and Ritchken \(2012\)](#) estimate the real yield curve using inflation swaps, survey data, and nominal bonds. We also obtain an upward sloping yield curve when we use TIPS data directly to

In comparison, our model predicts this slope to be 0.28%. Our channel, therefore, captures 40% and 55% of the observed slopes of the nominal and real term structures, respectively.

In Panel B of [Table 3](#), we report volatilities for long-term yields for maturities one through five years, as observed in the data and as implied by our model. We see that although our model matches the average volatility of the nominal term structure (since this is a calibration target), our model-implied volatility for the one year bond is higher than that in the data (5.89% vs. 3.32%).

Next, we use the decomposition [\(38\)](#) to study the contribution of limited risk sharing to our positive model-implied slope. The five minus one year unconditional real slope is related to the unconditional risk premium of an equal-weighted portfolio of two- through five-year real bonds through the identity

$$\mathbb{E}[y_t^{(60)} - y_t^{(12)}] = \frac{1}{5} \mathbb{E} [hpxr_t^{12,24} + hpxr_t^{12,36} + hpxr_t^{12,48} + hpxr_t^{12,60}] , \quad (47)$$

where $hpxr_t^{H,T}$ is the holding period bond risk premium [\(35\)](#).

[Table 4](#) reports the decomposition for the risk premium for the equal-weighted portfolio in both productivity regimes (over a holding period of a year and conditional on the mean employment level in each of the two productivity regimes). The mean conditional bond risk premium is 0.72% and 0.28% in the low and high-productivity regimes, respectively. More than 94% of the bond risk premium is due to the presence of labor income dispersion (i.e., the “Labor income dispersion persistence” [\(40\)](#) and the “Cross-covariance” [\(41\)](#) terms). Aggregate consumption risk (i.e., the “Aggregate consumption persistence” term [\(39\)](#)) contributes less

measure the slope of the real yield curve over the period 1999m1—2016m12 (see the Internet Appendix for details). In fact, the slope (five minus one year yield) was positive throughout this sample period. However, the slope of the TIPS curve is much larger than the estimate in [Haubrich, Pennacchi, and Ritchken \(2012\)](#). This is likely because of a liquidity premium for TIPS and because of differences in the sample period—for instance, the Great Recession constitutes a non-negligible portion of the sample period 1999m1—2016m12.

Table 4: Decomposition of real bond risk premium. This table shows the contribution of each term in the decomposition (38) for an equal-weighted portfolio of two- through 5-year real bonds with a 1-year holding period, $\frac{1}{4}\mathbb{E}\left[hpxr_t^{12,24} + hp xr_t^{12,36} + hp xr_t^{12,48} + hp xr_t^{12,60}\right]$. For each of the two aggregate productivity regimes, N is set to be equal its mean conditional on the regime. All numbers are in percent.

Term	Symbol	Low productivity	High productivity
Aggregate consumption persistence	$hp xr^C$	0.054	0.015
Income dispersion persistence	$hp xr^\zeta$	0.404	0.169
Cross-covariance	$hp xr^{cross}$	0.262	0.097
Bond risk premium	$hp xr$	0.721	0.280

than 6% of the total risk premium. Because a significant portion of the risk premium arises due to non-diversifiable idiosyncratic risk, our model provides a potential resolution of the puzzle highlighted by [Backus, Gregory, and Zin \(1989\)](#). In other words, our model predicts long-term bonds to have a positive and empirically realistic bond risk premium without predicting a counterfactually large negative autocorrelation of aggregate consumption growth.

[Table 2](#) shows that the equity premium in our two-regime model is 2.61%. In computing this premium, we defined equity as a levered claim on the unlevered value of the firm with a leverage-factor of 3 ([Abel, 1999](#)).¹⁵ The average stock market excess return is 6.36% over our sample period. Our model generates a higher equity premium once we increase the number of productivity states: the AR(1) model has an equity premium of 4.34% (see our Internet Appendix).

3.3 Labor market conditions and bond excess returns

Our model’s second key prediction is that there is a negative correlation between current labor market conditions (as measured by labor market tightness or the job finding rate) and

¹⁵More precisely, the ex-dividend price for a levered equity claim in period t is defined as $P_t \equiv \mathbb{E}_t[M_{t,t+1}Payoff_{t+1}]$, with the payoff being given by $Payoff_{t+1} = F(z_{t+1}, N_{t+1})^{LEV}$ where $F(z_{t+1}, N_{t+1})$ is the cum-dividend value for the unlevered firm (16) in period $t + 1$, and LEV is the leverage factor.

Table 5: Predictive regressions in the model. This table reports the model-implied slope coefficients, $\beta^{(n)}$, from the return predictability regression (48). Panels A and B report the results when labor market tightness ($X_t = \Theta$) and the job-finding rate ($X_t = f_t$) is used as the predictive variable, respectively.

	\overline{rx}_{t+12}	$rx_{t+12}^{(24)}$	$rx_{t+12}^{(36)}$	$rx_{t+12}^{(48)}$	$rx_{t+12}^{(60)}$
A. Labor Market Tightness					
$\beta^{(n)}$, Data	-4.05	-1.79	-3.19	-4.85	-6.36
$\beta^{(n)}$, Model	-1.05	-0.96	-1.07	-1.08	-1.09
B. Job-finding rate					
$\beta^{(n)}$, Data	-6.31	-2.51	-4.75	-7.64	-10.34
$\beta^{(n)}$, Model	-2.69	-2.46	-2.75	-2.77	-2.77

future bond excess returns. We discuss this result and its intuition next.

We run the predictive regression

$$rx_{t+12}^{(n)} = \alpha^{(n)} + \beta^{(n)} X_t + \varepsilon_{t+12}^{(n)} \quad (48)$$

using simulated data from our model. The left hand side variable is either the log excess return of a $n \in \{24, 36, 48, 60\}$ month nominal bond, over a holding period of one year, or the log excess return for an equal-weighted portfolio of two through five year bonds, $\overline{rx}_{t+12} \equiv \frac{1}{4} \left(rx_{t+12}^{(24)} + rx_{t+12}^{(36)} + rx_{t+12}^{(48)} + rx_{t+12}^{(60)} \right)$. The predictive variable is either labor market tightness ($X_t = \Theta_t$) or the job-finding rate ($X_t = f_t$).

Table 5 reports the model-implied slope coefficient for regression (48) along with its data counterpart (taken from Section 4). Panel A shows that our model predicts lower future bond excess returns when labor market tightness is high. This is in line with the data. The magnitude of the model-implied slope coefficient on labor market tightness are about a quarter of that in the data when predicting excess returns for the equal-weighted portfolio

(-4.05 in the data vs. -1.05 in the model). Similarly, Panel B shows that current job-finding rates negatively predict future bond excess returns both in our model and in the data, with the slope coefficient for the equal-weighted portfolio being -2.69 in our model versus -6.31 in the data.

Our model predicts a negative relationship between each of the two labor market variables and bond risk premia because the former behaves in a procyclical fashion (see [Figure 3](#)) while the latter behaves in a countercyclical fashion (see [Table 4](#)). To show why bond risk premia are countercyclical, we focus on the ζ_{t+n}/ζ_t component of the SDF (32), since it is the dominant determinant of bond risk premia (see [Table 4](#)). Countercyclical bond risk premia (for a holding period of a year) results from countercyclicity in the volatility of ζ_{t+12}/ζ_t . Two reasons contribute to a more volatile ζ_{t+12}/ζ_t when the economy is in the low productivity regime at time t . First, a switch to the high productivity regime over the holding period of the bond leads to a large drop in ζ_t from its current value to a value of 1.0 (see [Figure 2](#)). In our model, which is calibrated to NBER business cycles, the low regime lasts for one year on average so that the likelihood of such a regime change is reasonably large. Second, should the economy remain in the low productivity state, ζ_t will increase further due to a decline in employment. In contrast, when the economy is in the high productivity regime at time t , the volatility of ζ_{t+12}/ζ_t is low because the high productivity regime is more persistent (with an average duration of five years) and the value of ζ_t remains constant (equal to one) in this regime.

4 Empirical evidence for bond return predictability

Our model predicts a negative correlation between current labor market conditions and future bond excess returns. In this section, we provide evidence of our model’s prediction using four variables each of which capture current labor market conditions: labor market tightness, the job-finding rate, and the job-separation rate. To the best of our knowledge, our bond predictability results for labor market tightness and the flow rates are new.

We run the predictive regression (48) using data for nominal bonds of maturities ranging between two to five years. Our sample consists of monthly observations of annual returns for the period 1964m1—2016m12 (see Appendix B for details regarding the data). Table 6 reports the summary statistics for all variables appearing in regression (48).

We begin by showing that labor market tightness negatively predicts bond excess returns, with the effect being stronger for bonds of longer maturities. Tightness determines the law of motion for (un)employment (21) and is the key variable driving labor market volatility in search models (see, e.g., Hall and Schulhofer-Wohl 2017 for recent evidence). Panel A of Table 7 reports the results for regression (48) with labor market tightness as the predictive variable (i.e., $X_t = \Theta_t$). We report Newey-West standard errors with twelve lags to account for the overlapping windows in the regressions. The estimated coefficients imply that a one standard deviation decrease in labor market tightness is associated with an increase of 1.05% in excess returns for the equal weighted portfolio; the corresponding R^2 is 8.3%. Across maturities, the associated increase ranges between 0.47% for the two year bond to 1.65% for the five year bond; the associated R^2 ranges between 7.3% for the two year bond and 9.3% for the five year bond. This negative relation between labor market tightness and bond excess returns is in line with our model’s predictions in Section 3.3.

Table 6: Summary statistics. This table reports summary statistics for the monthly period 1964m1-2016m12. Panels A and B report summary statistics for nominal bonds’ one-year holding period log excess returns and yields, respectively. All values are expressed in percentage terms. Panel C reports results for macroeconomic variables. See Appendix B for additional details regarding the data.

A. Excess returns			B. Yields		
	Mean	SD		Mean	SD
\overline{rx}	0.97	3.64	$y_{nom}^{(12)}$	5.28	3.32
$rx^{(24)}$	0.48	1.72	$y_{nom}^{(24)}$	5.50	3.26
$rx^{(36)}$	0.88	3.15	$y_{nom}^{(36)}$	5.69	3.17
$rx^{(48)}$	1.21	4.39	$y_{nom}^{(48)}$	5.85	3.09
$rx^{(60)}$	1.33	5.43	$y_{nom}^{(60)}$	5.97	3.01
C. Macroeconomic moments					
	Mean	SD		Mean	SD
Θ	0.59	0.26	FFR	5.35	3.75
f , monthly	0.55	0.15	INF , monthly	0.32	0.32
s , monthly	0.034	0.006	$MWDGDP$	2.28	1.11

Notes. The reported values for standard deviations are not HP-filtered, and are therefore different from their HP-filtered counterparts in Table 2.

Next, we report bond excess return predictability results for the job-finding rate f_t and the job-separation rate s_t , separately. We construct the two job flow rates following [Elsby, Michaels, and Solon \(2009\)](#). Panel B of [Table 7](#) shows that the job-finding rate negatively forecasts bond excess returns, with the regression coefficients increasing in magnitude as a function of the maturity of the bond. This is again in line with our model’s prediction from [Section 3.3](#). Panel C of [Table 7](#) shows that, in contrast to the job-finding rate, the job separation rate does not forecast bond returns—the estimated slopes for the job separation rate are all insignificant. Our modeling assumption of a constant job separation rate reflects this finding.

Table 7: Labor market variables and real bond excess returns. Panels A through C report results for the univariate return predictability regression (48) for nominal bonds with labor market tightness, the job-finding rate, and the job separation rate being the predictive variable, respectively. Observations are for annual returns at a monthly frequency over the period 1964m1-2016m12 (all regressions have 636 observations). Parenthesis enclose Newey-West t-statistics computed with 12 lags.

	\overline{rx}_{t+12}	$rx_{t+12}^{(24)}$	$rx_{t+12}^{(36)}$	$rx_{t+12}^{(48)}$	$rx_{t+12}^{(60)}$
A. Results for $X_t = \Theta_t$					
$\beta_{\Theta}^{(n)}$	-4.05*** (-3.33)	-1.79** (-2.97)	-3.19** (-2.95)	-4.85*** (-3.32)	-6.36*** (-3.60)
$\alpha_{\Theta}^{(n)}$	3.32*** (4.05)	1.51*** (3.82)	2.73*** (3.79)	4.02*** (4.05)	5.02*** (4.15)
R^2	.083	.073	.069	.082	.093
B. Results for $X_t = f_t$					
$\beta_f^{(n)}$	-6.31** (-3.19)	-2.51* (-2.50)	-4.75** (-2.72)	-7.64** (-3.22)	-10.34*** (-3.53)
$\alpha_f^{(n)}$	4.46*** (3.98)	1.86*** (3.35)	3.50*** (3.59)	5.43*** (4.02)	7.04*** (4.19)
R^2	.063	.045	.048	.063	.076
C. Results for $X_t = s_t$					
$\beta_s^{(n)}$	-27.92 (-0.36)	11.48 (0.32)	-14.77 (-0.22)	-40.18 (-0.43)	-68.20 (-0.59)
$\alpha_s^{(n)}$	1.92 (0.79)	0.09 (0.08)	1.38 (0.66)	2.57 (0.88)	3.65 (1.01)
R^2	.002	.002	.001	.003	.006
+ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$					

More broadly, Ludvigson and Ng (2009), Joslin, Priebsch, and Singleton (2014), Huang and Shi (2016, 2019), and Bianchi, Büchner, and Tamoni (2019) show a robust link between bond excess returns and principal components of a wide set of real macroeconomic variables,

which include labor market variables such as employment, unemployment, and vacancies, amongst others. In particular, [Ludvigson and Ng \(2009\)](#) find bond excess returns to be predicted by a principal component of real macroeconomic variables that has a large loading on employment and hours. Our theory provides an explanation for these findings.

We conclude our empirical analysis with robustness results in which we show that our results above remain unchanged even after we control for forces that we did not explicitly model. These include monetary policy and changes in the supply of outstanding government bonds. As a further robustness check, we also control for the level of inflation. For nominal bonds, we rerun regression (48) with additional controls for monetary policy (measured by the federal funds rate), inflation (measured using the CPI index), and bond supply (measured by the maturity-weighted debt to GDP series from [Greenwood and Vayanos 2014](#)). These results are shown in [Table B.1](#) in [Appendix B](#). Our findings remain unchanged: labor market tightness and the job-finding rate continue to negatively forecast excess returns, while the job-separation rate remains insignificant.

For real bonds, we rerun the univariate regression (48) using data on Treasury Inflation-Protected Securities (TIPS) as the left hand side variable over the sample period 1999m1–2016m12. Our findings remain unchanged from those without these additional controls. These results are available in our Internet Appendix.

5 Conclusion

We present a theory in which limited risk sharing of idiosyncratic labor income risk across individuals plays a key role in determining the dynamics of interest rates. In the general equilibrium, the income risk faced by individuals arises as a consequence of labor policies

of the firm. In our model, labor search frictions endogenously generate labor adjustment costs that play an important role in shaping the dynamics of the distribution of idiosyncratic income risk; this approach has the advantage of allowing us to map an unobserved adjustment cost process to observable labor market variables.

Our model makes two predictions, both of which are supported by the data. First, it predicts positive risk premia for long-term bonds, while matching moments of macroeconomic variables, especially aggregate consumption growth. As a result of limited risk sharing, the dynamics of the SDF is mainly driven by the marginal utility process of a small fraction of individuals who experience large increases in marginal utility from income loss. This allows bonds to earn a positive risk premium without counterfactual implications for the dynamics of aggregate consumption growth, thereby avoiding the [Backus, Gregory, and Zin \(1989\)](#) puzzle.

Second, consistent with the data, our model predicts a negative correlation between current labor market conditions (as measured by labor market tightness or the job finding rate) and future bond excess returns. This predictability in bond excess returns arises in our model because the SDF is more volatile during periods of deterioration in labor market conditions.

Appendix

A Model Appendix

A.1 Derivation of the SDF

The Lagrangian for the head of household's maximization problem (18) is given by

$$\begin{aligned} \mathcal{L}_t = & \frac{\left(N_t C_{e,t}^{1-\chi^{-1}} + U_t C_{u,t}^{1-\chi^{-1}}\right)^{\frac{1-\gamma}{1-\chi^{-1}}}}{1-\gamma} + \mathbb{E}_t[J_{t+1}] + \Psi_t (\phi_t C_{e,t} - C_{u,t}) \\ & + \Lambda_t \left[w_t N_t + b U_t - T_t + \varphi_{t-1}^S (D_t + P_t^S) + \varphi_{t-1}^B - N_t C_{e,t} - U_t C_{u,t} - \varphi_t^S P_t^S - \varphi_t^B P_t^{(1)} \right], \end{aligned} \quad (\text{A.1})$$

where Ψ_t and Λ_t are the multipliers on the risk sharing constraint (5) and the budget constraint (20), respectively, and the laws of motion (21) are implicitly assumed by the conditional expectation appearing in equation (A.1).

The characterization of the SDF in equation (28) can be obtained by combining the first order conditions characterizing optimal portfolio choice,

$$\Lambda_t P_t^S = \mathbb{E}_t \left[\frac{\partial J_{t+1}}{\partial \varphi_t^S} \right], \quad \text{and} \quad \Lambda_t P_t^{(1)} = \mathbb{E}_t \left[\frac{\partial J_{t+1}}{\partial \varphi_t^B} \right], \quad (\text{A.2})$$

with the envelope conditions

$$\frac{\partial J_{t+1}}{\partial \varphi_t^S} = \Lambda_{t+1} (D_{t+1} + P_{t+1}^S), \quad \text{and} \quad \frac{\partial J_{t+1}}{\partial \varphi_t^B} = \Lambda_{t+1}. \quad (\text{A.3})$$

To obtain equation (29) for the shadow price Λ_t , begin with the first order conditions for consumption choice:

$$0 = \left(N_t C_{e,t}^{1-\chi^{-1}} + U_t C_{u,t}^{1-\chi^{-1}} \right)^{\frac{\chi^{-1}-\gamma}{1-\chi^{-1}}} N_t C_{e,t}^{-\chi^{-1}} + \Psi_t \phi_t - \Lambda_t N_t, \quad (\text{A.4})$$

$$0 = \left(N_t C_{e,t}^{1-\chi^{-1}} + U_t C_{u,t}^{1-\chi^{-1}} \right)^{\frac{\chi^{-1}-\gamma}{1-\chi^{-1}}} U_t C_{u,t}^{-\chi^{-1}} - \Psi_t - \Lambda_t U_t. \quad (\text{A.5})$$

The complementary slackness condition $\Psi_t (\phi_t C_{e,t} - C_{u,t}) = 0$ implies that the sum of $C_{e,t}$ times (A.4) and $C_{u,t}$ times (A.5) is equal to

$$\Lambda_t (N_t C_{e,t} + U_t C_{u,t}) = \left(N_t C_{e,t}^{1-\chi^{-1}} + U_t C_{u,t}^{1-\chi^{-1}} \right)^{\frac{1-\gamma}{1-\chi^{-1}}}. \quad (\text{A.6})$$

Substituting in equation (25) for aggregate consumption, we then obtain

$$\Lambda_t = C_t^{-\gamma} \left(N_t \left(\frac{C_{e,t}}{C_t} \right)^{1-\chi^{-1}} + U_t \left(\frac{C_{u,t}}{C_t} \right)^{1-\chi^{-1}} \right)^{\frac{1-\gamma}{1-\chi^{-1}}}, \quad (\text{A.7})$$

which corresponds to equation (29).

A.2 Approximate bond risk premia

The log of the SDF (32) is equal to $m_{t,t+n} \equiv \log M_{t,t+n} = n \log \beta - \gamma \Delta c_{t:t+n} + \Delta \log \zeta_{t:t+n}$. A log normal expression for the conditional entropy (36) yields

$$L_t(M_{t,t+n}) \approx \frac{1}{2} \text{Var}_t(m_{t,t+n}) = \frac{1}{2} \gamma^2 \text{Var}_t(\Delta c_{t:t+n}) + \frac{1}{2} \text{Var}_t(\Delta \log \zeta_{t:t+n}) - \gamma \text{Cov}_t(\Delta c_{t:t+n}, \Delta \log \zeta_{t:t+n}).$$

Plugging this expression into equation (37), we obtain

$$hpxr_t^{H,T} \approx hpxr_t^{H,T,\Delta c} + hpxr_t^{H,T,\Delta \log \zeta} + hpxr_t^{H,T,Cov}, \quad (\text{A.8})$$

where

$$\begin{aligned} hpxr_t^{H,T,\Delta c} &= -\gamma^2 \text{Cov}_t(\Delta c_{t:t+H}, \Delta c_{t+H:t+T}) \\ &\quad + \frac{1}{2} \gamma^2 \mathbb{E}_t [\text{Var}_{t+H}(\Delta c_{t+H:t+T}) - \text{Var}_t(\Delta c_{t+H:t+T})], \end{aligned} \quad (\text{A.9})$$

$$\begin{aligned} hpxr_t^{H,T,\Delta \log \zeta} &= -\text{Cov}_t(\Delta \log \zeta_{t:t+H}, \Delta \log \zeta_{t+H:t+T}) \\ &\quad + \frac{1}{2} \mathbb{E}_t [\text{Var}_{t+H}(\Delta \log \zeta_{t+H:t+T}) - \text{Var}_t(\Delta \log \zeta_{t+H:t+T})], \end{aligned} \quad (\text{A.10})$$

$$\begin{aligned} hpxr_t^{H,T,Cov} &= \gamma \text{Cov}_t(\Delta c_{t:t+H}, \Delta \log \zeta_{t+H:t+T}) + \gamma \text{Cov}_t(\Delta \log \zeta_{t:t+H}, \Delta c_{t+H:t+T}) \\ &\quad - \gamma \mathbb{E}_t [\text{Cov}_{t+H}(\Delta c_{t+H:t+T}, \Delta \log \zeta_{t+H:t+T}) - \text{Cov}_t(\Delta c_{t+H:t+T}, \Delta \log \zeta_{t+H:t+T})]. \end{aligned} \quad (\text{A.11})$$

Finally, we arrive at expression (43) by ignoring the stochastic volatility terms in equation (A.8). That is, by ignoring the last term in each of the equations (A.9), (A.10), and (A.11).

A.3 Computing nominal bond prices

The inflation process (45) is of the form

$$\Delta \pi_{t+1} = \mu_\pi (1 - \rho_\pi) + \rho_\pi \Delta \pi_t + \xi_\pi \varepsilon_{\Delta c}(z_t, N_t, z_{t+1}, N_{t+1}) + \nu_\pi \varepsilon_{\pi,t} + \varepsilon_{\pi,t+1}, \quad (\text{A.12})$$

with the aggregate consumption shock (46) being a function of z_t , N_t , z_{t+1} , and N_{t+1} . We begin by decomposing inflation into the sum of two orthogonal components:

$$\Delta\pi_{t+1} = \Delta\pi_{t+1}^{(1)} + \Delta\pi_{t+1}^{(2)} \quad (\text{A.13})$$

$$\Delta\pi_{t+1}^{(1)} = \mu_\pi(1 - \rho_\pi) + \rho_\pi\Delta\pi_t^{(1)} + \nu_\pi\varepsilon_{\pi,t} + \varepsilon_{\pi,t+1}, \quad (\text{A.14})$$

$$\Delta\pi_{t+1}^{(2)} = \rho_\pi\Delta\pi_t^{(2)} + \xi_\pi\varepsilon_{\Delta c}(z_t, N_t, z_{t+1}, N_{t+1}). \quad (\text{A.15})$$

The nominal bond price (44) then has functional form

$$P_t^{(\$, n)} = P^{(\$, A, n)}(z_t, N_t) P^{(\$, B, n)}(\Delta\pi_t^{(1)}, \varepsilon_{\pi,t}) P^{(\$, C, n)}(\Delta\pi_t^{(2)}). \quad (\text{A.16})$$

The first term on the right hand side of equation (A.16) can be computed recursively as follows:

$$P^{(\$, A, 0)}(z, N) = 1, \quad (\text{A.17})$$

$$P^{(\$, A, n)}(z, N) = \mathbb{E} \left[M(z, N, z', N') \exp \left\{ -\xi_\pi \left(1 + \frac{\rho_\pi(1 - \rho_\pi^{n-1})}{1 - \rho_\pi} \right) \varepsilon_{\Delta c}(z, N, z', N') \right\} \right. \\ \left. \times P^{(\$, A, n-1)}(z', N') | z, N \right]. \quad (\text{A.18})$$

Next, the second term on the right hand side of equation (A.16) is given by

$$P^{(\$, B, n)}(\Delta\pi^{(1)}, \varepsilon_\pi) = \exp \left(-a^{(\$, B, n)} - b^{(\$, B, n)}\Delta\pi^{(1)} - c^{(\$, B, n)}\varepsilon_\pi \right), \quad (\text{A.19})$$

where the coefficients can be computed recursively:

$$a^{(\$, B, n)} = a^{(\$, B, n-1)} + \mu_\pi(1 - \rho_\pi) \left(1 + b^{(\$, B, n-1)} \right) - \frac{\sigma_\pi^2}{2} \left(1 + b^{(\$, B, n-1)} + c^{(\$, B, n-1)} \right)^2, \quad (\text{A.20})$$

$$b^{(\$, B, n)} = \rho_\pi \left(1 + b^{(\$, B, n-1)} \right), \quad (\text{A.21})$$

$$c^{(\$, B, n)} = \nu_\pi \left(1 + b^{(\$, B, n-1)} \right), \quad (\text{A.22})$$

starting from the initial condition

$$a^{(\$, B, 0)} = b^{(\$, B, 0)} = c^{(\$, B, 0)} = 0. \quad (\text{A.23})$$

Finally, the third term on the right hand side of equation (A.16) is given by

$$P^{(\$, C, n)}(\Delta\pi^{(2)}) = \exp \left(-\frac{\rho_\pi(1 - \rho_\pi^n)}{1 - \rho_\pi} \Delta\pi^{(2)} \right). \quad (\text{A.24})$$

B Empirical Appendix

We use data from the sources described below.

Nominal bond returns. We use the Fama-Bliss Discount Bonds monthly series from the Center for Research in Security Prices.

Aggregate consumption. First, we use the quarterly real personal consumption expenditures per capita (A794RX0Q048SBEA) series, retrieved from Federal Reserve Economic Data (FRED), to compute the data moments for aggregate consumption growth reported in Table 2. Second, we construct a monthly series for real aggregate consumption by using the Personal Consumption Expenditures: Chain-type Price Index (PCEPI) series to deflate the Personal Consumption Expenditures (PCE) series. The resulting monthly series is used to construct the consumption growth innovations (46).

Inflation. We measure inflation based on growth in the Consumer Price Index (CPIAUCSL) series, which we downloaded from FRED.

Monetary policy. We measure monetary policy using the effective Federal Funds Rate (FEDFUNDS) series, which we downloaded from FRED.

Bond supply. We measure nominal bond supply using the maturity weighted debt to GDP series from the data appendix of Greenwood and Vayanos (2014).

Labor market tightness. Tightness is the ratio of (1) the composite Help Wanted Index from Barnichon (2010), downloaded from Regis Barnichon’s website <https://sites.google.com/site/regisbarnichon/data>; and (2) the seasonally adjusted civilian unemployment rate (UNRATE) series, downloaded from FRED.

Job-finding and separation rates. We construct job-finding and separation rates following the procedure in Elsby, Michaels, and Solon (2009). This procedure makes use of the following series from FRED: civilian labor force size (CLF16OV), unemployment level (UNEMPLOY), and number of civilians unemployed for less than 5 weeks (UEMPLT5).

Table B.1: Labor market search and bond risk premia. This table reports results for the return predictability regression (48) with the federal funds rate (FFR), inflation (INF), and maturity weighted debt to GDP ($MWDGDP$) included as additional controls. Panels A through C show the results when labor market tightness, the job-finding rate, and the job-separation rate are used as the predictive variable, respectively. Observations are monthly observations of annual returns over the period 1964m1-2007m12 (all regressions have 540 observations). Parenthesis enclose Newey-West t -statistics computed with 12 lags.

	$\bar{r}\bar{x}$	$r\bar{x}^{(24)}$	$r\bar{x}^{(36)}$	$r\bar{x}^{(48)}$	$r\bar{x}^{(60)}$		$\bar{r}\bar{x}$	$r\bar{x}^{(24)}$	$r\bar{x}^{(36)}$	$r\bar{x}^{(48)}$	$r\bar{x}^{(60)}$
A. $X_t = \Theta_t$						B. $X_t = f_t$					
X_t	-3.51* (-2.32)	-1.79* (-2.47)	-2.86* (-2.14)	-4.14* (-2.28)	-5.25* (-2.38)	X_t	-7.14* (-2.11)	-3.65* (-2.28)	-5.73+ (-1.93)	-8.42* (-2.07)	-10.75* (-2.17)
FFR_t	0.22 (1.28)	0.16* (2.12)	0.22 (1.50)	0.24 (1.19)	0.24 (0.97)	FFR_t	0.22 (1.28)	0.16* (2.11)	0.22 (1.49)	0.24 (1.19)	0.24 (0.97)
INF_t	-2.57* (-2.14)	-1.22* (-2.23)	-2.41* (-2.33)	-3.09* (-2.15)	-3.57* (-1.99)	INF_t	-2.71* (-2.22)	-1.29* (-2.31)	-2.53* (-2.40)	-3.25* (-2.22)	-3.77* (-2.06)
$MWDGDP_t$	0.57+ (1.66)	0.30+ (1.78)	0.49 (1.59)	0.71+ (1.70)	0.80 (1.60)	$MWDGDP_t$	0.58 (1.61)	0.30+ (1.72)	0.49 (1.55)	0.71+ (1.66)	0.80 (1.56)
$Const$	1.25 (0.89)	0.30 (0.43)	0.94 (0.74)	1.56 (0.93)	2.22 (1.11)	$Const$	3.33 (1.48)	1.37 (1.23)	2.58 (1.28)	4.01 (1.49)	5.37 (1.65)
R^2	.148	.181	.147	.147	.140	R^2	.141	.173	.139	.140	.134
C. $X_t = s_t$											
X_t	6.09 (0.07)	4.20 (0.10)	-6.30 (-0.08)	7.95 (0.07)	18.51 (0.14)	X_t					
FFR_t	0.20 (1.12)	0.15+ (1.79)	0.22 (1.39)	0.23 (1.04)	0.21 (0.80)	FFR_t					
INF_t	-2.67* (-2.13)	-1.27* (-2.19)	-2.50* (-2.31)	-3.21* (-2.13)	-3.71* (-1.98)	INF_t					
$MWDGDP_t$	0.75* (1.98)	0.39* (2.13)	0.62+ (1.89)	0.91* (2.00)	1.06+ (1.94)	$MWDGDP_t$					
$Const$	-1.42 (-0.43)	-1.09 (-0.73)	-0.9 (-0.32)	-1.62 (-0.40)	-2.06 (-0.42)	$Const$					
R^2	.100	.126	.104	.100	.090	R^2					

+ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

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