# Capital Heterogeneity and Investment Prices

## How much are investment prices declining?

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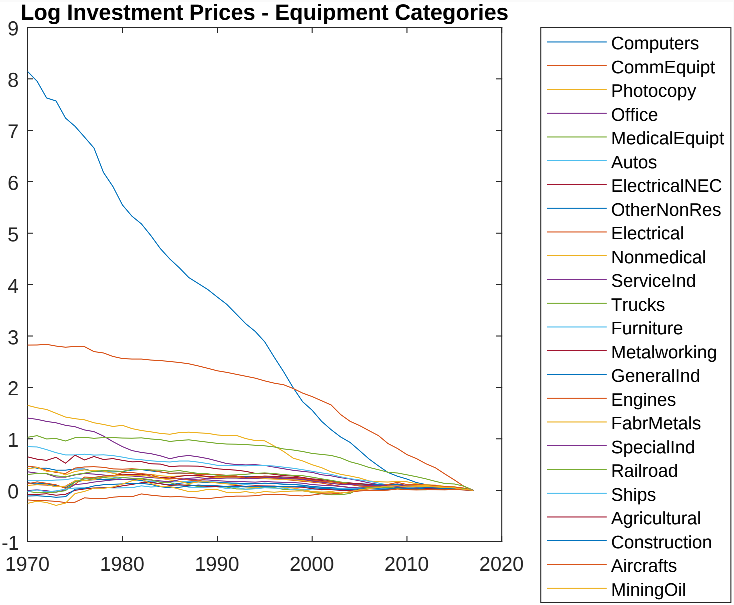
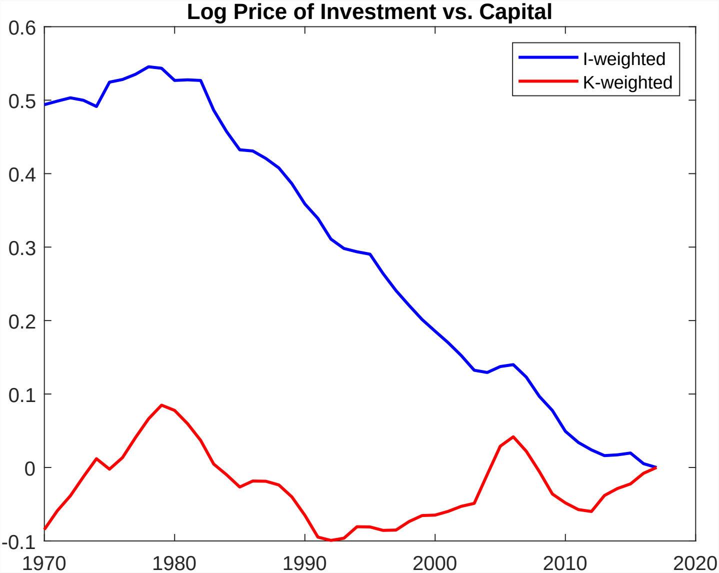
# Motivation

- Role of investment-specific technological change (ISTC)
    - Growth ( e.g., Greenwood, Hercovitz and Krusell 1997)
    - Business cycles (e.g., Fisher 2006)
    - Labor Share (e.g., Karabarbounis and Neiman 2012)
    - Decline of $r^*$ (e.g, Sajedi and Thwaites 2016)
    - Evolution of big "ratios" (e.g., Philippon, Eggertsson, ..)
- ISTC measured using price of new investment goods
- But: huge heterogeneity in price trends - aggregation?

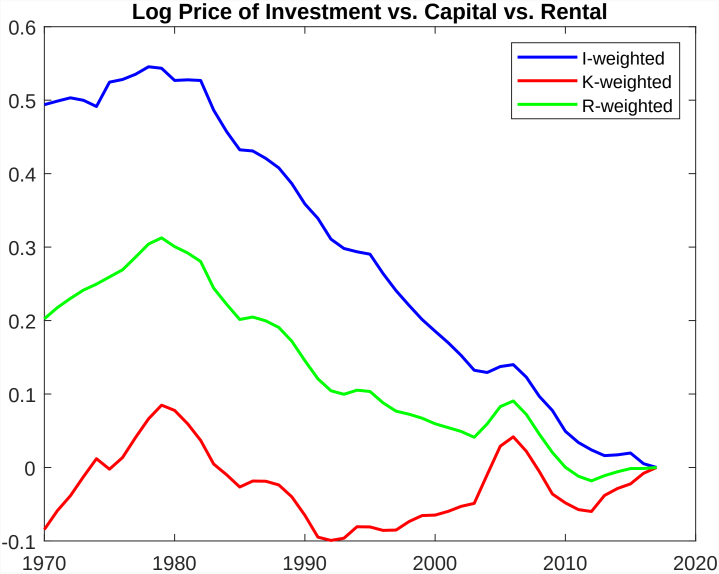
# Heterogeneity in Equipment Price Trends



# Flow- and Stock-weighted Prices



# Flow- and Stock-weighted Prices



# Outline

1. Simple framework
2. Role of ISTC for growth
3. Role of ISTC for big ratios
4. Role of ISTC for business cycles
5. Role of ISTC for labor share
6. Role of ISTC for $r^*$

# Simple Framework

# Simple Model

Utility function:

$$U = \int_0^\infty e^{-\rho t} \frac{C_t^{1-\sigma}}{1-\sigma} dt$$

Production function:

$$Y_t = A_t L_t^{\alpha_L} K_{1t}^{\alpha_{K_1}} .. K_{nt}^{\alpha_{K_n}}$$

Capital accumulation for each type:

$$\dot{K}_{it} = I_{it} - \delta_i K_{it}$$

Resource constraint:

$$Y_t = C_t + \sum_{i=1}^{n} p_{it} I_{it}$$

Exogenous: $A_t, L_t, p_{it}$

# Equilibrium

Euler equation:

$$\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\sigma},$$

Perfect competition capital demand:

$$\alpha_{K_i} \frac{Y_t}{K_{it}} = R_{it},$$

User cost equation:

$$R_{it} = p_{it} \left( r_t + \delta_i - \frac{\dot{p}_{it}}{p_{it}} \right),$$

Combining:

$$\frac{p_{it} K_{it}}{Y_t} = \frac{\alpha_{K_i}}{r_t + \delta_i - g_{p_i}}.$$

# Balanced growth path

- Capital demand:

$$\frac{p_{it}K_{it}}{Y_t} = \frac{\alpha_{K_i}}{r_t + \delta_i - g_{p_i}} \implies g_{K_i} = g_Y - g_{p_i},$$

- Production function:

$$g_Y = g_A + \alpha_L g_L + \sum_{i=1}^{n} \alpha_{K_i} g_{K_i},$$

- Substitute:

$$g_Y - g_L = \frac{g_A - \alpha_K g_{p^R}}{\alpha_L}$$

$$g_{p^R} \equiv \frac{\sum_{i=1}^{n} \alpha_{K_i} g_{p_i}}{\sum_{i=1}^{n} \alpha_{K_i}}$$

$\implies$ Aggregate invt prices using **rental weights**

# Rental-, Stock-, and Flow-weighted indices

General (Divisia) index for given shares $s$:

$$\frac{\dot{p}_t^s}{p_t^s} = \sum_{i=1}^{n} s_{it} \frac{\dot{p}_{it}}{p_{it}}$$

Flow-weighted: invt price index (NIPA) used in ISTC research:

$$s_{it}^I \propto p_{it} I_{it}$$

Stock-weighted: capital price index (FAT)

$$s_{it}^K \propto p_{it} K_{it}$$

Rental-weighted index:

$$s_{it}^R \propto R_{it} K_{it}$$

# Rental-shares, Stock-shares, Flow-shares

Rental weights:

$$s_{it}^{R} = \frac{R_{it}K_{it}}{\sum_{j=1}^{n} R_{jt}K_{jt}} \propto \alpha_{K_i}$$

Stock weights:

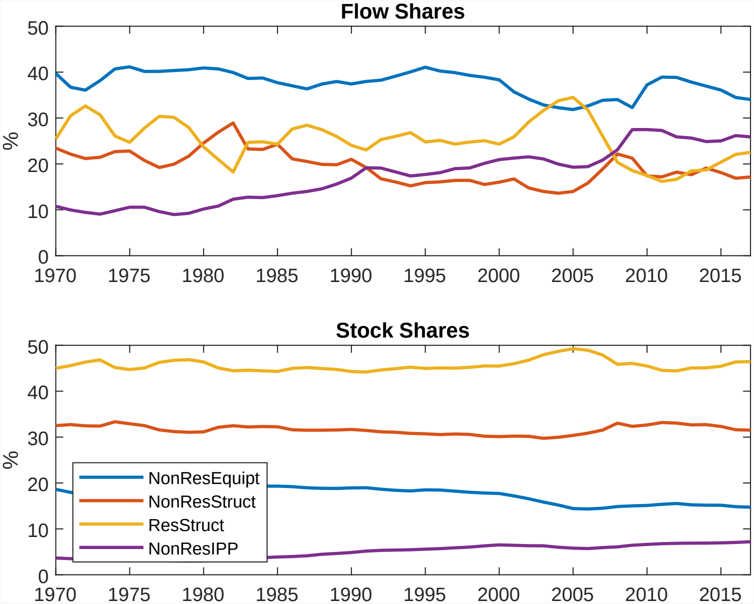
$$s_{it}^{K} = \frac{p_{it}K_{it}}{\sum_{j=1}^{n} p_{jt}K_{jt}} \propto \frac{\alpha_{K_i}}{r_t + \delta_i - g_{p_i}}$$

Investment weights on the BGP:

$$s_{it}^{I} = \frac{p_{it}I_{it}}{\sum_{j=1}^{n} p_{jt}I_{jt}} \propto \alpha_{K_i}\frac{g_Y + \delta_i - g_{p_i}}{r + \delta_i - g_{p_i}}$$

These shares are **very** different!

# I-share and K-share



# Relation between shares along BGP

On the balanced growth path:

$$s_i^R = \frac{s_I}{\alpha_K} s_i^I + \left(1 - \frac{s_I}{\alpha_K}\right) s_i^K$$

where:

- $s_I$ is the investment share of output
- $\alpha_K$ is aggregate capital share

Hence relation between price indices:

$$g_{p^R} = \frac{s_I}{\alpha_K} g_{p^I} + \left(1 - \frac{s_I}{\alpha_K}\right) g_{p^K}$$

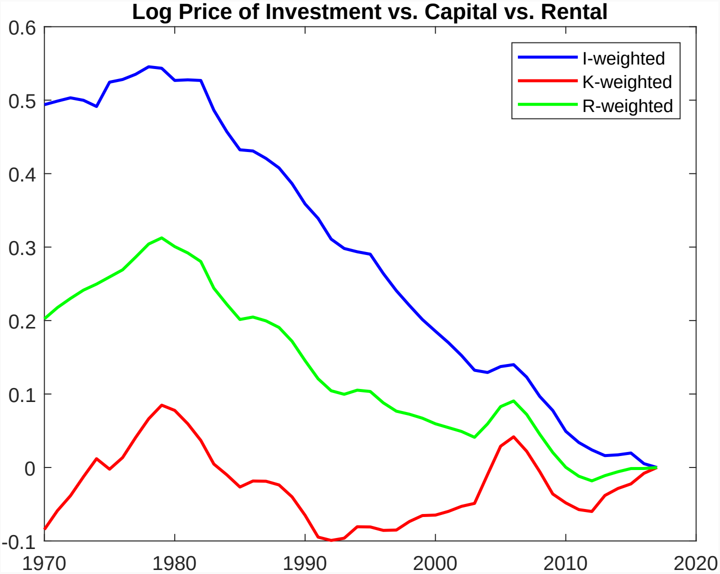
$\Longrightarrow$ Can infer $g_{p^R}$ from observables

# Contribution of ISTC to Growth

# Data

- Fixed Asset Tables: All private fixed assets
- Disaggregation in 57 categories
- Ex.: Invt::NonRes Equipt::Info Processing::Computers
- We use the BEA deflators (not Gordon-Violante-Cummins)

# Flow- and Stock-weighted Prices



# Contribution of ISTC to growth

- GHK: "ISTC contributes 58% to growth"
- Our approach (similar to theirs)
  1. Observe $\alpha_L, \alpha_K, g_{p^R}, g_Y - g_L$
  2. Infer TFP $g_A$ from:

$$g_Y - g_L = \frac{g_A - \alpha_K g_{p^R}}{\alpha_L}$$

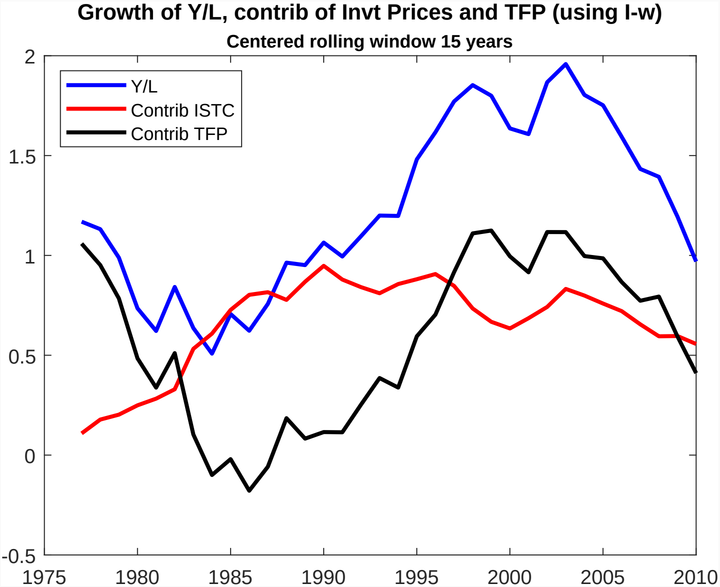
  3. Calculate counterfactual growth if $g_{p^R}$=0
  4. What if use $g_{p^I}$ instead of $g_{p^R}$

# Smaller ISTC contribution with R-weighting

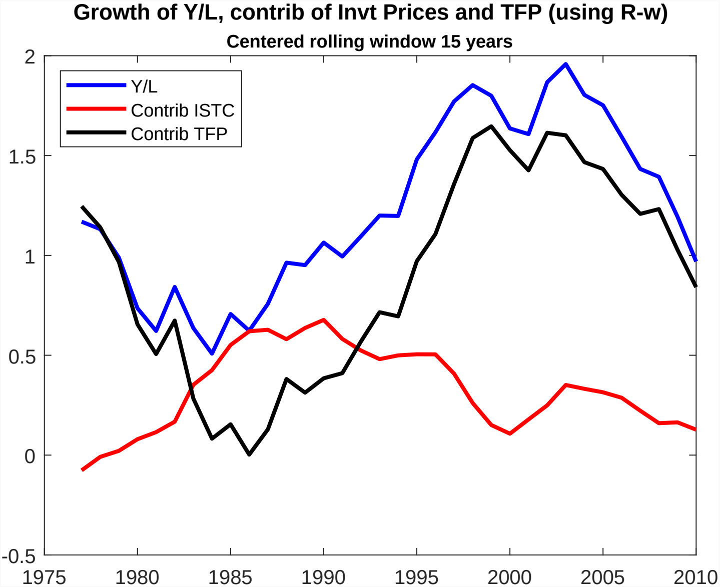
| | Data | Iw:ITC | Iw:TFP | Rw:ITC | Rw:TFP |
|---|---|---|---|---|---|
| **1970-2017** | 1.19 | 0.52 | 0.66 | 0.21 | 0.98 |
| **(%)** | 100.00 | 43.80 | 55.91 | 17.46 | 82.37 |

Avg. growth of Y/L and contributions of ISTC and TFP using either I-w or R-w to infer ISTC.

# Contributions to Growth: I-w (GHK)



# Contributions to Growth: R-w



# ISTC and the Big Ratios

# Aggregation

Result: along the BGP,

$$\frac{I}{K} = g_Y + \delta^K - g_{p^K}$$
$$\frac{\Pi}{K} = r + \delta^K - g_{p^K}$$
$$\frac{K}{Y} = \frac{\alpha_K}{r + \delta^K - g_{p^K}}$$

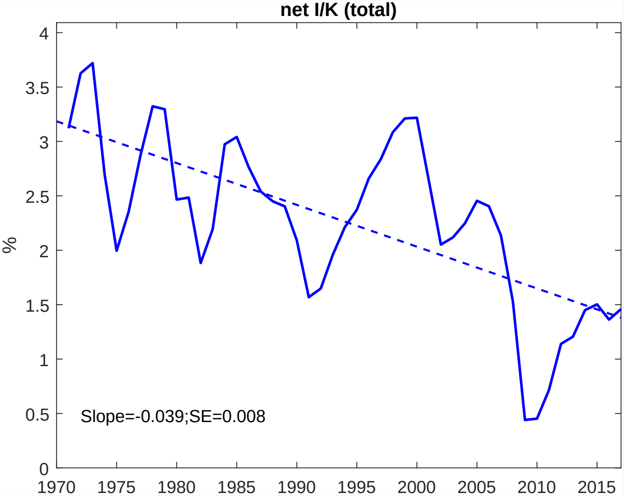
where

$$I = \sum p_i I_i, K = \sum p_i K_i, \Pi = \sum R_i K_i$$

are the (current-cost nominal) aggregates.

$\implies$ To calibrate one-capital model, use **stock-weighted** $\delta$ and price growth.

# Application: the decline of investment

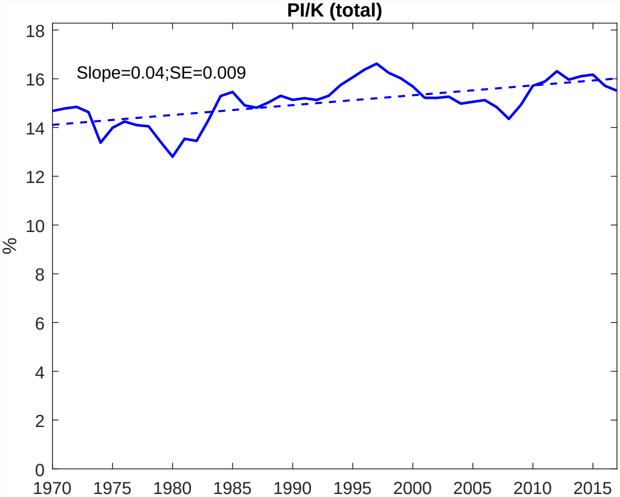


# Application: the decline of investment

$$\frac{I}{K} - \delta^K = g_Y - g_{p^K}$$

| | **Net I/K** | **Contrib $g_y$** | **Contrib $g_{pk}$** | **Residual** |
|---|---|---|---|---|
| **1990-2004** | 2.39 | 2.51 | -0.21 | 0.10 |
| **2003-2017** | 1.51 | 1.58 | -0.37 | 0.30 |
| **Change** | -0.89 | -0.93 | -0.16 | 0.20 |
| **Change If use PI** | -0.89 | -0.93 | -0.69 | 0.74 |

# Application: the stability of MPK (despite low Rf)



# Application: the stability of MPK (despite low Rf)

$$\frac{\Pi}{K} = r + \delta^K - g_{p^K}$$

We use this equation to infer $r$

| | $\Pi$/K | Contrib $\delta$ | Contrib $g_{pk}$ | Contrib r |
|---|---|---|---|---|
| **1990-2004** | 15.61 | 5.69 | -0.21 | 10.14 |
| **2003-2017** | 15.46 | 5.68 | -0.37 | 10.15 |
| **Change** | -0.15 | -0.01 | -0.16 | 0.02 |
| **Change If use PI** | -0.15 | -0.01 | -0.69 | 0.55 |

# ISTC and Business Cycles

# Transitional Dynamics (w elastic labor)

$$\begin{aligned}
\max_{C_t, I_{it}, K_{it}} U &= \int_0^\infty e^{-\rho t} \frac{C_t^{1-\sigma}}{1-\sigma} v(L_t) dt \\
s.t. &: \\
\dot{K}_{it} &= I_{it} - \delta_i K_{it} \\
Y_t &= C_t + \sum_{i=1}^{N} P_{it} I_{it} \\
Y_t &= A_t L_t^{\alpha_L} K_{1t}^{\alpha_{K_1}} .. K_{nt}^{\alpha_{K_n}}
\end{aligned}$$

for given $(K_{i0})$, and $(A_t, P_{it})$

# Proposition

Consider a small, permanent, unexpected shock to vector $p_{i0}$,
Then, the *full path* of aggregates $(Y_t, L_t, C_t, I_t)_{t\geq 0}$
(in deviation from BGP) depends only on:

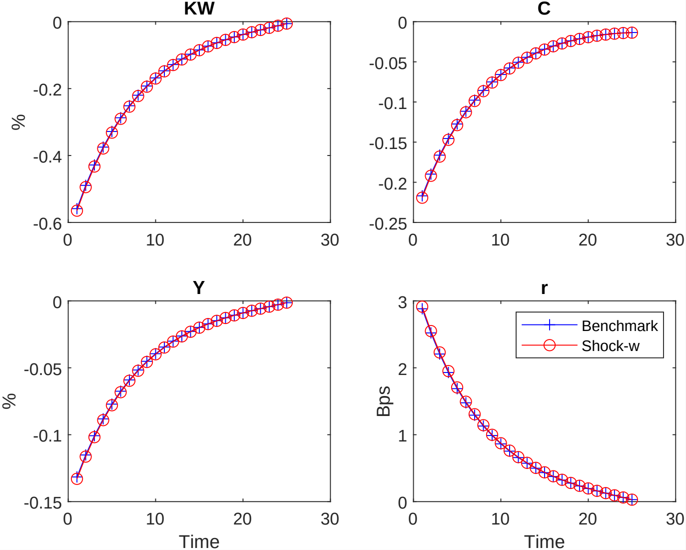
$$\widehat{p} = (1 - s_I)\widehat{p}^K + s_I \widehat{p}^I$$

where $s_I$ is the aggregate investment share

Intuition:

- State variable = total capital relative to BGP
- The shock shifts BGP to a parallel path
- Shock also shifts total capital at $t = 0$
- Overall effect on deviation depends
  only on its effect on state variable at $t = 0$

# Result



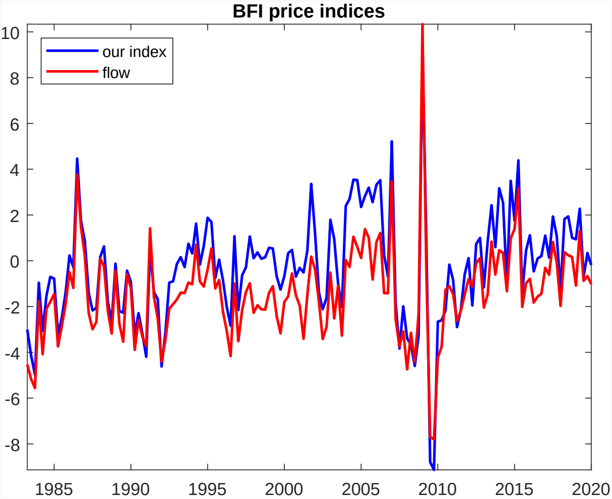
KW
%
0
-0.2
-0.4
-0.6
0
10
20
30
C
0
-0.05
-0.1
-0.15
-0.2
-0.25
Y
%
-0.15
Time
r
Bps
3
2
1
Benchmark
Shock-w

# Business cycle analysis

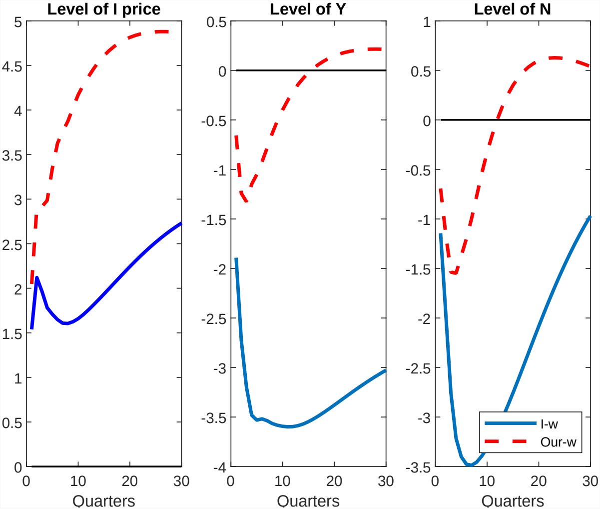
Run Fisher-style VAR:

- 3 variables: dlog(Invt Price), dlog(Y/L), log(L/Pop)
- Long-run restrictions to identify ISTC shock, TFP shock
- quarterly data, 4 lags, 1982IV-2019IV
- now only 14 categories of goods (e.g. info processing)

# Price indices

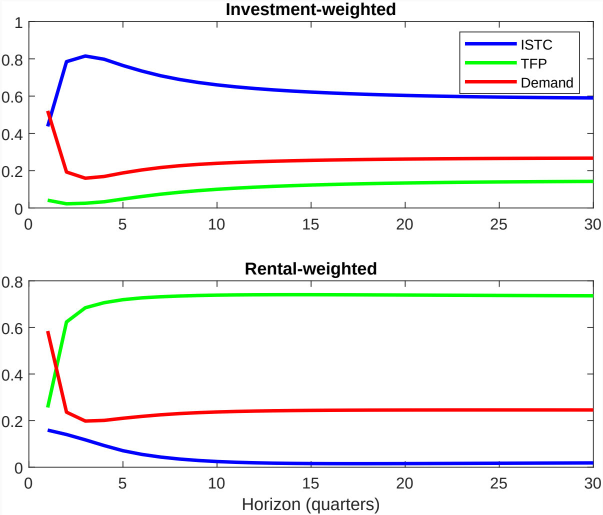


# VAR comparison



# Variance Decomposition BFI

Share of variance of hours due to ISTC / TFP / demand



# ISTC and the Labor share

# Labor Share

- If EOS K/L $\sigma \neq 1$, chg invt prices affect labor share
- Model extension:

$$Y = (b_K K^{\frac{\sigma-1}{\sigma}} + b_L L^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$$

$$K = K_1^{\gamma_{K_1}} \ldots K_n^{\gamma_{K_n}}.$$

- Note: nonstationary shares if ISTC
- Consider a permanent small shock to vector $p_i$.
- Then change in gross labor share is:

$$(\sigma - 1)\alpha_K \widehat{p}^R$$

$\Longrightarrow$ Relevant price for labor share is **R-weighted**

# Illustration

Implied change in labor share since 1970
given observed prices changes and assumed EOS:

| | **Iw** | **Rw** |
|---|---|---|
| $\sigma = 1.5$ | -0.17 | -0.07 |
| $\sigma = 1.25$ | -0.09 | -0.03 |
| $\sigma = 0.75$ | 0.09 | 0.03 |
| $\sigma = 0.5$ | 0.17 | 0.07 |

# ISTC and the decline of $r^*$

# Decline of $r^*$

- Lower investment price may reduce eqm interest rate
- Model extension: upward-sloping savings $W_t L_t S(r_t)$
  - e.g., OLG or Aiyagari
  - Otherwise, $r^*$ pined down by preferences
- Equilibrium in asset market:

$$\sum_{i=1}^{n} p_{it} K_{it} = W_t L_t S(r_t)$$

- Consider a permanent small shock to vector $p_{i0}$.
- Then change in $r^*$ is $\zeta \widehat{p}^R$
- Correct aggregation for $r^*$ is **R-weighted**

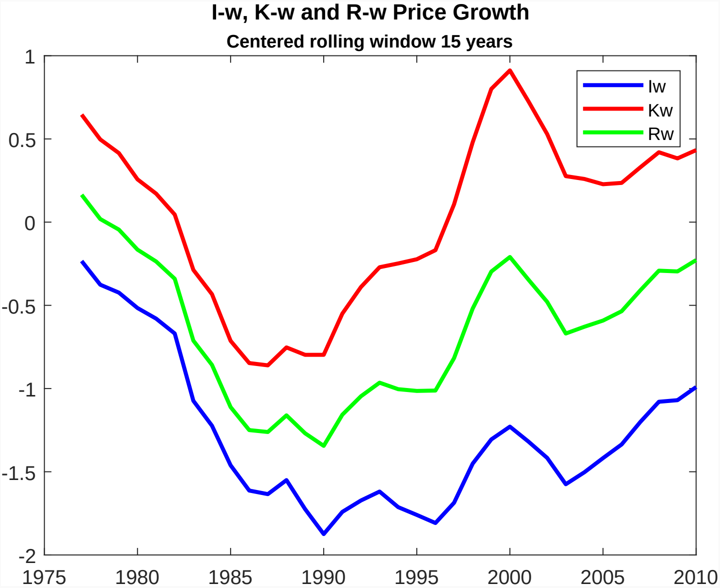
# Conclusion

# Conclusion

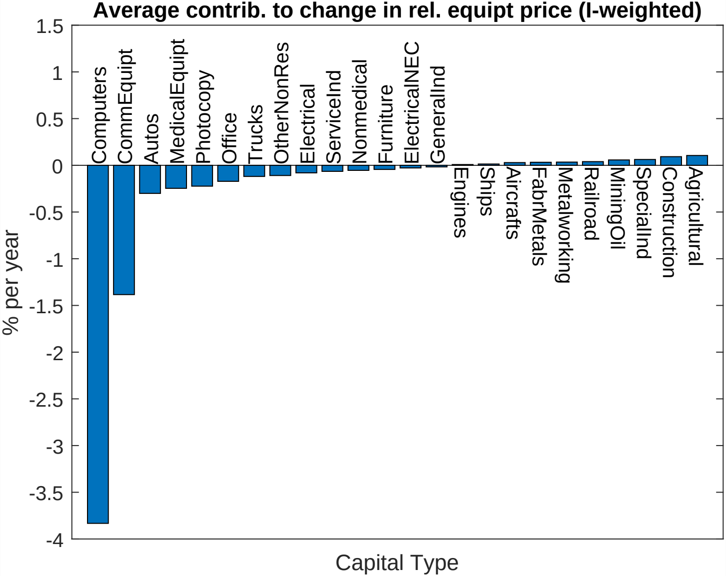
- Methodology: appropriate aggregation
  depends on question at hand! I-w, K-w, R-w, Stock-w ...
- Simple calculations illustrate this can matter
- In progress: relax some simplifying assumptions
  (BGP, perfect competition, Cobb-Douglas, ...)

# Backup

# Rolling windows: Price Growth



# Contributions to Equipment Deflator



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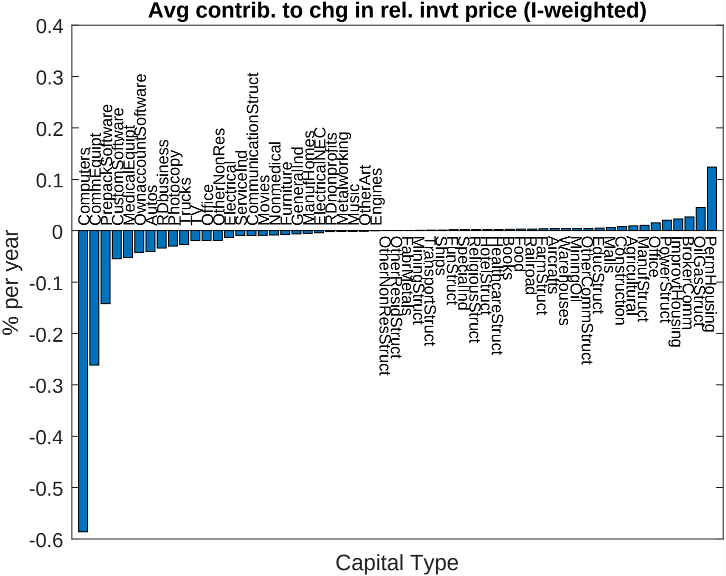
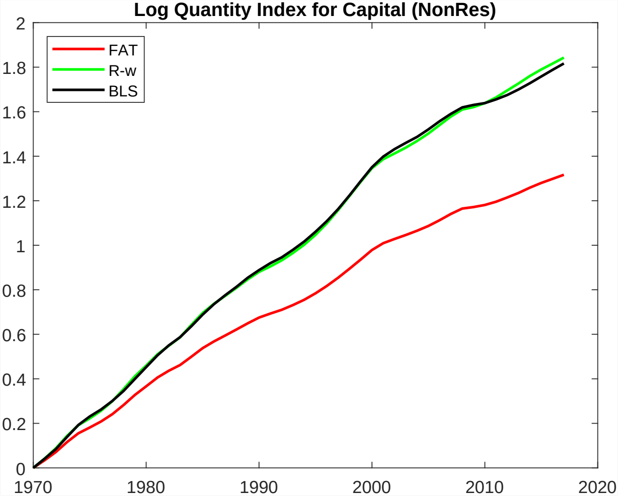
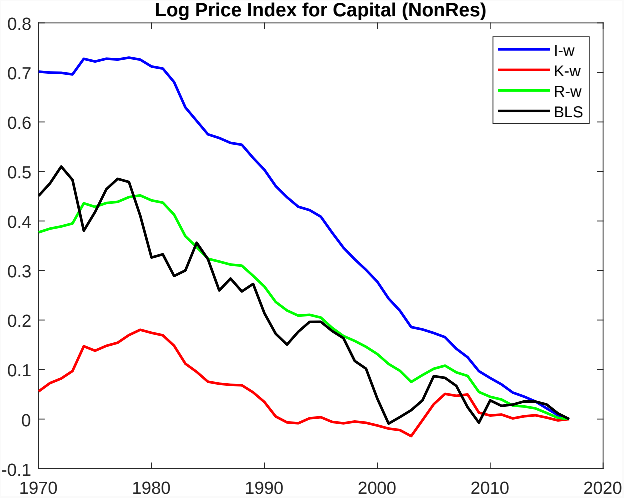


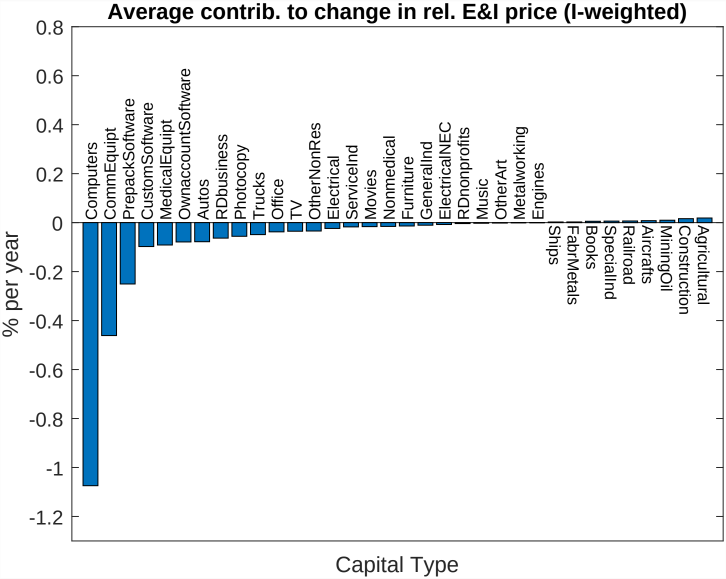
# Comparison with BLS



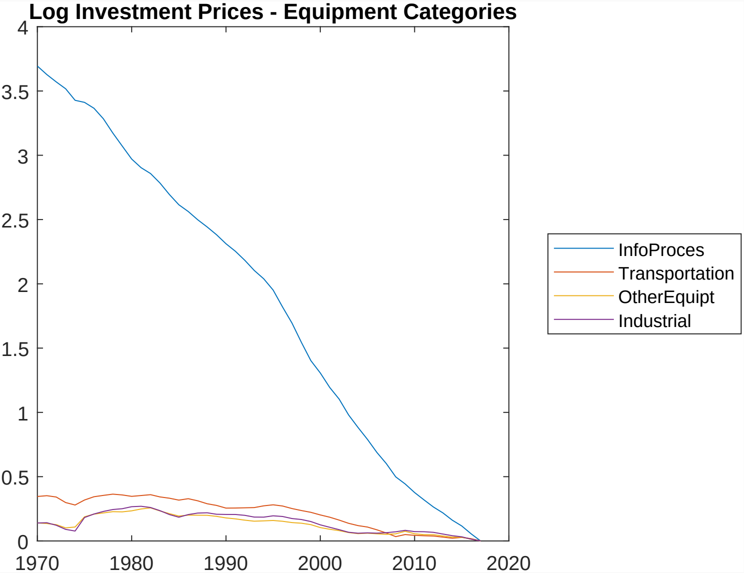
# Comparison with BLS



# Contributions to I-w E&I price

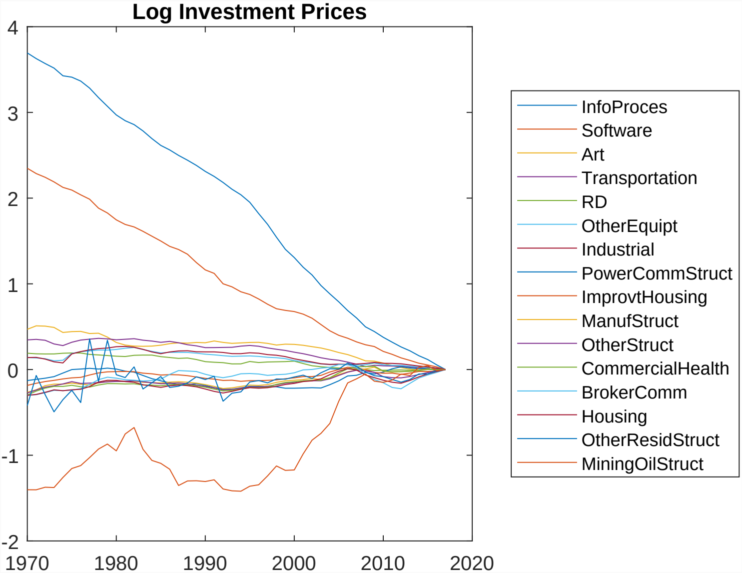


# Prices



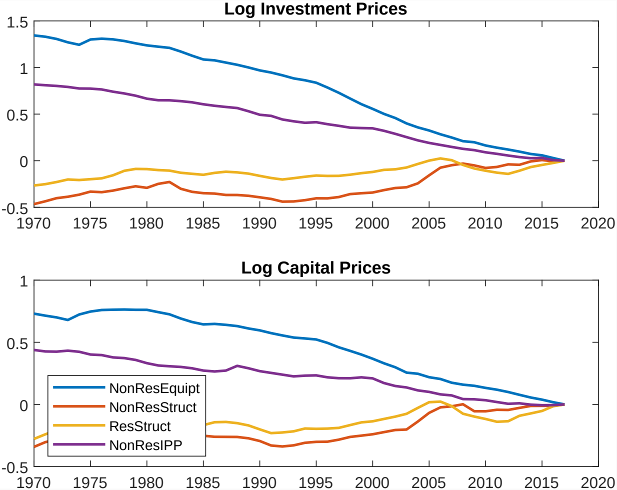
Log Investment Prices - Equipment Categories
4
3.5
3
2.5
2
1.5
1
0.5
0
1970
1980
1990
2000
2010
2020
InfoProces
Transportation
OtherEquipt
Industrial

# Prices



Log Investment Prices
4
3
2
1
0
-1
-2
1970
1980
1990
2000
2010
2020
InfoProces
Software
Art
Transportation
RD
OtherEquipt
Industrial
PowerCommStruct
ImprovtHousing
ManufStruct
OtherStruct
CommercialHealth
BrokerComm
Housing
OtherResidStruct
MiningOilStruct

# Prices



Log Investment Prices
1.5
1
0.5
0
-0.5
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
2020
Log Capital Prices
1
0.5
0
-0.5
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
2020
NonResEquipt
NonResStruct
ResStruct
NonResIPP

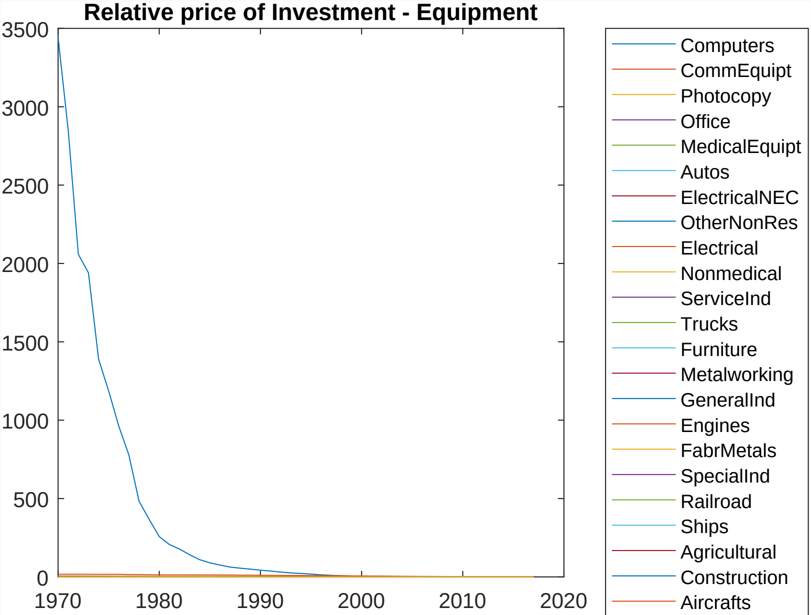
# Proof

$$
\begin{aligned}
R_i K_i &= \left(r + \delta_i - g_{p_i}\right) P_i K_i \\
&= \left(r - g_Y\right) P_i K_i + \left(g_Y + \delta_i - g_{p_i}\right) P_i K_i \\
&= \left(r - g_Y\right) P_i K_i + P_i I_i
\end{aligned}
$$

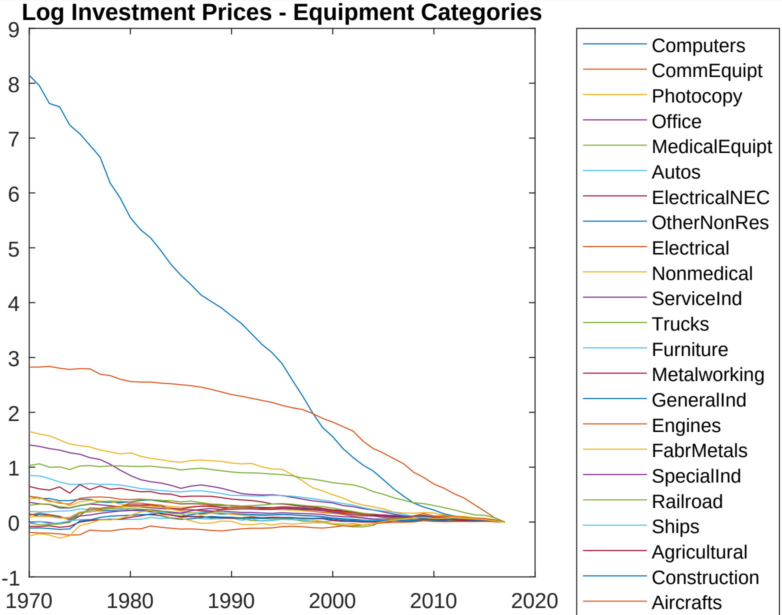
$$
\sum R_i K_i = \alpha_K Y = \left(r - g_Y\right) K + I
$$

$$
\begin{aligned}
s_i^R &= \frac{R_i K_i}{\sum R_j K_j} \\
&= \frac{\left(r - g_Y\right) P_i K_i + P_i I_i}{\left(r - g_Y\right) K + I} \\
&= \frac{P_i K_i}{K}\left(1 - \frac{s_I}{\alpha_K}\right) + \frac{P_i I_i}{I}\frac{s_I}{\alpha_K}
\end{aligned}
$$

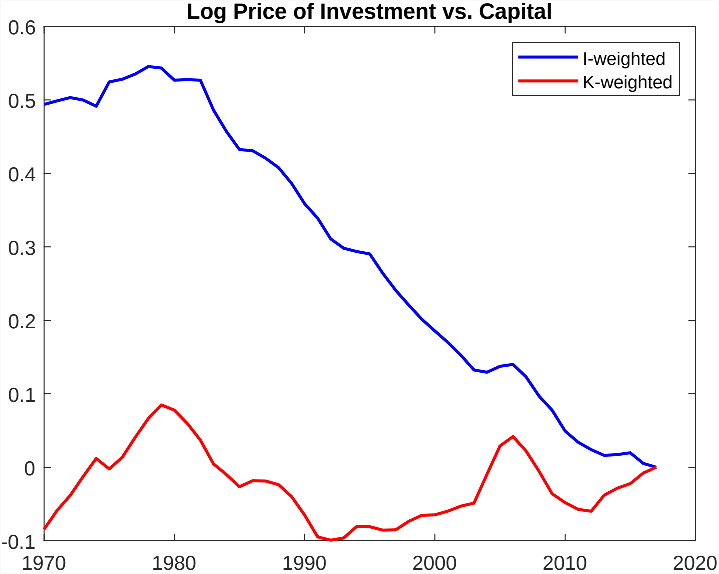
# Relative prices



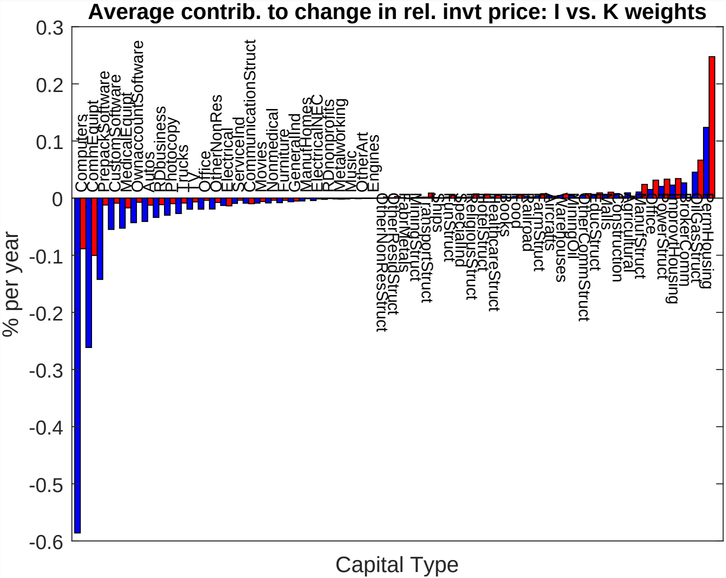
# Log relative prices



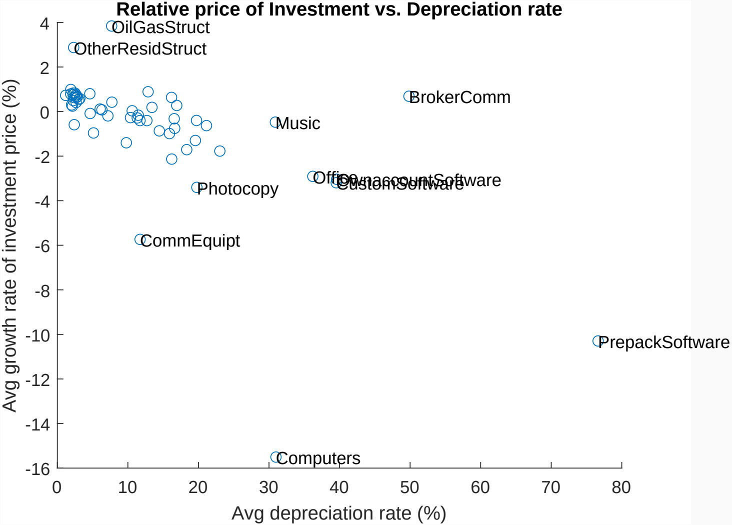
# I-w vs. K-w prices



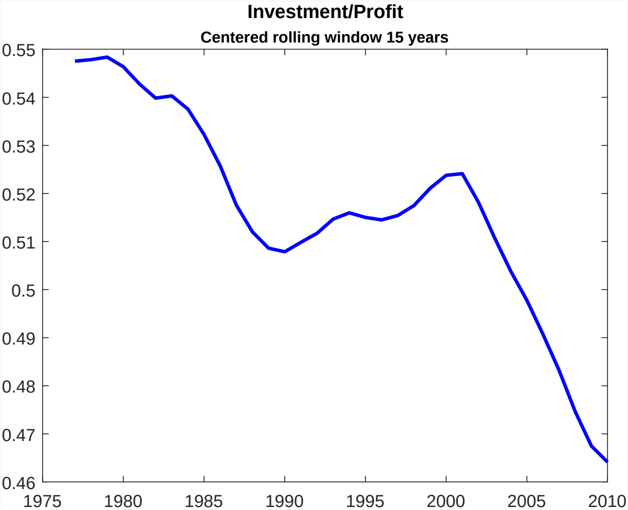
# I-w vs. K-w prices



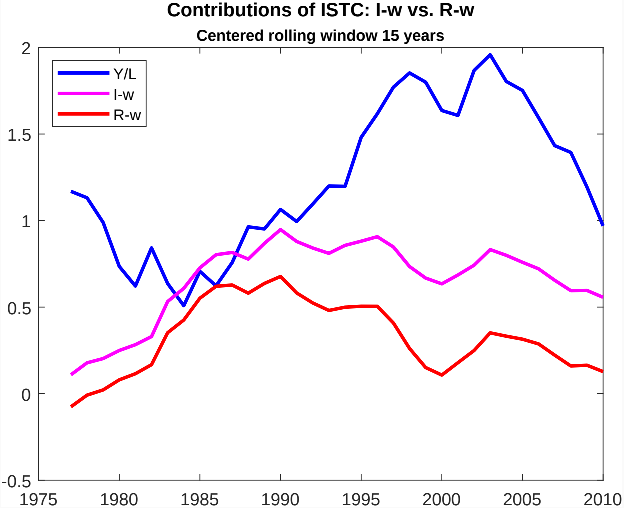
# Depreciation and Price Trend



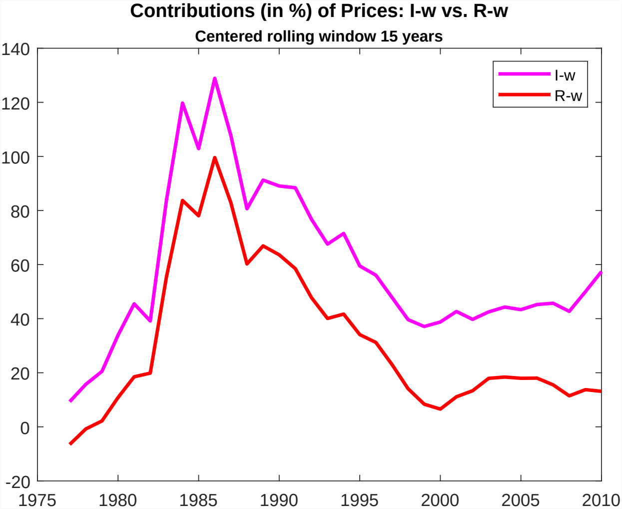
# Investment-Profit Ratio



# Comparison of contribution of ISTC



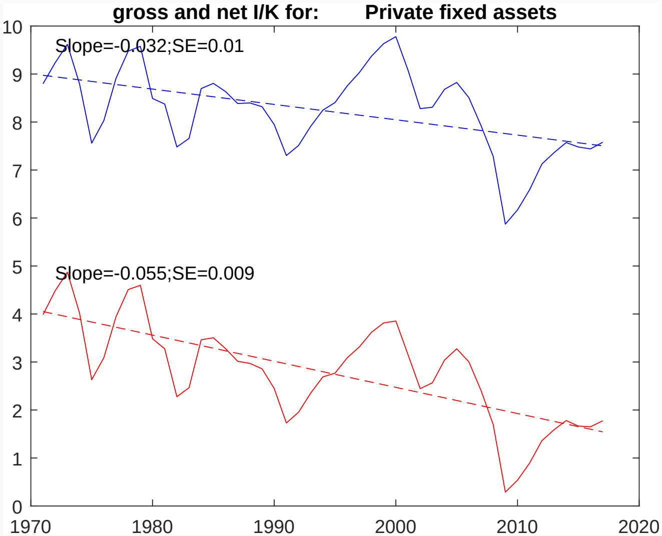
# Comparison of contribution of ISTC: Percentages



# Macroeconomic Puzzles

- Decline of investment
- High profitability
- Decline of labor share
- Decline of $r^*$ (TBA)

# Decline in net I/K



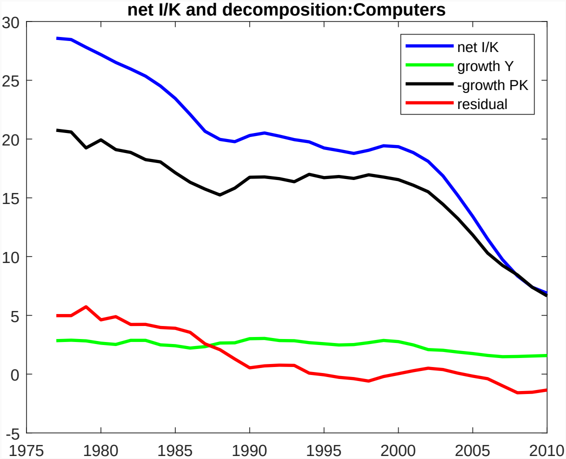
# Decline of I/K

Write BGP condition, adding an error term:

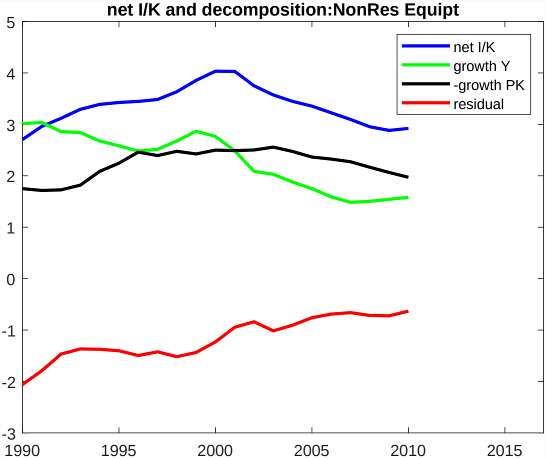
$$\frac{I_{it}}{K_{it}} = \delta_i + g_Y - \frac{\dot{p}_{it}}{p_{it}} + \varepsilon_{it}.$$

True at any level of aggregation (w. stock-weighted indices)

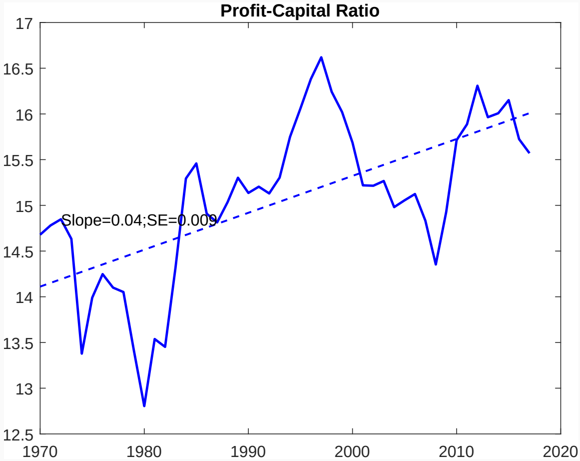
# Evolution of net I/K: computers



# Evolution of net I/K: non-res equipment



# Stability of Profit/K



# Data

| | DlogY/H | Inv/Prof | Price IW | Price KW | Price RW |
|---|---|---|---|---|---|
| **1970-2017** | 1.19 | 0.51 | -1.02 | 0.23 | -0.41 |
| **1970-1984** | 1.17 | 0.55 | -0.23 | 0.65 | 0.16 |
| **1985-2005** | 1.49 | 0.52 | -1.49 | 0.09 | -0.73 |
| **2006-2017** | 0.68 | 0.45 | -1.12 | -0.01 | -0.51 |

Avg. growth of Y/L, I/Profits, and I-w, K-w, R-w prices