

# MULTI-PRODUCT FIRMS AND MISALLOCATION

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## Abstract

Shocks and distortions could affect the aggregate economy via the multi-product channel (Bernard, Redding, and Schott, 2010, BRS henceforth; Miniti and Turino, 2013; Jaef, 2018). This paper finds that (i) compared to the U.S., multi-product firms are fewer and smaller in China; (ii) firms with higher Hsieh and Klenow(2009, HK henceforth) taxes are less likely to be multi-product producers, controlling for the size effect. While the empirics confirm the existence of the product channel, our quantitative analysis using a discrete multi-product choice model with endogenous firm entry and exit suggests that this channel generates a small misallocation. This is because marginal firms that drop products due to distortions are of medium productivities. The output loss from the product extensive margin is small once the granularity of the firm size distribution is matched in the model. Most welfare loss come from the firm extensive margin and the product intensive margin.

## Empirical Analysis

We use the National Bureau of Statistics firm-level data, 1998-2007 and the Economic Census 2004 data in China. Products are defined as 5-digit SIC levels, similar to BRS (2010). In the data, the number of products is topcoded at 3. We hence focus on the dimension of single-product versus multi-product firms.

	CN: AIES 98-07 Pooled			CN: Census		U.S.
	All	Non-SOE	Non-Exporter	2004	BRS(2010)	
Number share	0.27	0.26	0.26	0.17	0.39	
Output share	0.44	0.41	0.36	0.42	0.87	
Relative Size	2.08	2.03	1.58	3.50	10.47	

Fig. 1: Number, Output Share and Relative Size of Multi-Product Firms, China vs U.S.

Figure 1 shows that multi-product firms are fewer and smaller in China. The fewer result is robust to industry composition differences and possible measurement error in product definitions in the Chinese data.

We run the following regression to test if firms with a higher tax distortion is less likely to be a multi-product firm, where distortions are the TFPR measure in HK:

$$P(\text{Multi}_{ist}) = \Phi(\beta_0 + \beta_1 \text{Distortion}_{ist} + \beta_2 \text{SOE}_{ist} + \beta_3 \text{NonExporter}_{ist} + \beta_4 \text{LogAsset}_{ist} + \delta_s + \delta_t + \epsilon_{ist})$$

Figure 2 shows the negative effect are statistically significant in the baseline regression (column 3). The results are robust after controlling for markups (De Loecker and Warzynski, 2012) or contemporaneous distortions are replaced by lagged distortions.

	(1)	(2)	(3)	(4) 04-07 sub-sample	(5)	(6)	(7)
tau1	-0.168*** (0.004)						
tau2		-0.002*** (0.001)					
tau			-0.088*** (0.003)	-0.074*** (0.005)			
markup				-0.048*** (0.006)			
L.tau					-0.041*** (0.006)		
L.multi					2.894*** (0.005)		
L2.tau						-0.040*** (0.006)	
L2.multi						2.037*** (0.005)	
L3.tau							-0.041*** (0.006)
L3.multi							1.444*** (0.005)
age	0.013*** (0.000)	0.013*** (0.000)	0.013*** (0.000)	0.008*** (0.000)	0.004*** (0.000)	0.004*** (0.000)	0.004*** (0.000)
soe	0.047*** (0.003)	0.047*** (0.003)	0.046*** (0.003)	0.078*** (0.007)	-0.013** (0.007)	-0.006 (0.007)	-0.009 (0.007)
nonexporter	-0.070*** (0.003)	-0.089*** (0.003)	-0.086*** (0.003)	-0.076*** (0.004)	-0.039** (0.005)	-0.054*** (0.005)	-0.058*** (0.006)
logasset	0.134*** (0.001)	0.128*** (0.001)	0.125*** (0.001)	0.106*** (0.001)	0.062*** (0.002)	0.077*** (0.002)	0.089*** (0.002)
constant	-1.896*** (0.011)	-1.967*** (0.011)	-1.905*** (0.011)	-1.606*** (0.017)	-2.363*** (0.022)	-2.143*** (0.021)	-1.990*** (0.023)
Industry FE	Yes						
Year FE	Yes						
N	1,432,808	1,432,808	1,432,808	640,428	831,655	570,379	376,464

Fig. 2: Higher Distortions, Less Likely Multi-Product Firms

## Model

We build a general equilibrium model with firm entry and exit. Firms can choose 1 (single-product) or 2 (multi-product) products. The modelling of discrete product number choice connects tightly to the data and helps the following quantitative work, compared to models with a continuum of products (BRS, 2010; Jaef, 2018). The household's utility function is

$$Q = \left[ \int_{\omega \in \Omega} (\lambda(\omega)q(\omega))^\rho d\omega \right]^{\frac{1}{\rho}}, \quad 0 < \rho < 1 \quad (1)$$

where  $q(\omega)$  and  $\lambda(\omega)$  are the quantity and product appeal of the product variety  $\omega$ . Households supply unskilled labor  $L$  and human capital  $H$  inelastically.

Each firm pays entry cost  $f_e$  in units of unskilled labor. Upon entry, they draw a permanent productivity  $\phi \sim g(\psi)$ , and two taste shocks  $a_1, a_2$  independently from a Binomial distribution, with  $P(a_i = a_h) = p, P(a_i = a_l) = 1 - p, a_l < a_h, i = 1, 2$ . Potential distortions are size-dependent (Gunner, Ventura, and Xu, 2008): if  $\psi > \psi_{tax}$ , firms are taxed by  $\tau_t > 0$ ; otherwise, they are subsidized by  $\tau_s < 0$ .

Conditional on producing, firm chooses to set price  $p$  and hire labor and human capital  $\{l, h\}$ , given the states of  $\{\psi, a_1, a_2, \tau; P, Q\}$ , a headquarter cost  $f_0$ , and a per product line fixed cost  $f_i$ :

$$\max_{p, l, h} \{ (1 - \tau)p_1q_1 - w_Ll_1 - w_Hh_1 - w_Lf_0 - w_Lf_1 + \mathbb{I}((1 - \tau)p_2q_2 - w_Ll_2 - w_Hh_2 - w_Lf_2) \}$$

subject to

$$\text{Inverse demand: } q_i = \frac{Q}{\lambda_i} \left[ \frac{p_i}{\lambda_i P} \right]^{-\sigma}, \quad i = 1, 2$$

$$\text{Linear production: } q_i = l_i \phi, \quad i = 1, 2$$

$$\text{Appeal production: } \lambda_1 = a_1 h_1^\alpha$$

$$\text{Appeal production: } \lambda_2 = \xi a_2 h_2^\alpha$$

$\xi \in [0, 1]$  span of control parameter: limited management capacity for the second product line

The decisions on entry and the number of products are cutoff rules as in Melitz (2003). Figure 3 colors the region of firms producing 2 products in yellow, and the region of firms producing 1 product in blue. Taxes increases the cutoffs of multi-product and single-product firms, while subsidies invite low productivity firms to produce and even produce 2 products.

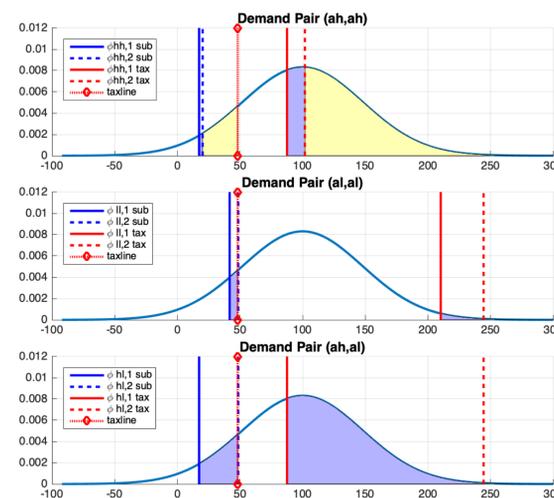


Fig. 3: How Distortions Change the Cutoffs

We solve the model in the stationary equilibrium. As in Melitz(2003), the general equilibrium are determined by the free entry condition of entrants and the markets clear conditions. Contact email: wenyaw111@gmail.com ; yang.ei@mail.shufe.edu.cn.

## Quantitative Analysis

We set some parameters exogenously. For example, the human capital share  $\alpha = 0.2$ , elasticity of substitution  $\sigma = 4.5$  and the productivity distribution to be normal  $\varphi \sim N(\mu_\varphi, \sigma_\varphi^2)$ . We set some parameters to match the U.S. data as a distortion free economy. Importantly, we set the scale of control parameter  $\xi$  and distortion related parameter to match the multi-product firm output share and the top 5% output share in Chinese data (Figure 4). The data displays features of fat tail in output share.

Parameter	Value	Moment	Data	Model
$\xi$	0.865	Number Share (Multi)	0.27	0.2972
$\tau_t$	0.615	Output Share (Multi)	0.44	0.4249
$P(\varphi > \varphi_{tax})$	0.855	Top 5% Output Share	0.61	0.6819
$\tau_s$	-0.225	Subsidized Share	0.16	0.1590

Fig. 4: Calibration with the Chinese NBS Firm-Level Data

We look into the role of distortion by removing them in the benchmark model. Tax distortions depress welfare hugely, dropping to 29.82%. Subsidy improves welfare to 32.57% by increasing the variety of products and the measure of firms.

Moments	Data	Benchmark		$\xi = 0.865$		$\xi = 1(U.S.)$	
		$\tau_{tax} \neq 0$ $\tau_{sub} \neq 0$	$\tau_{tax} = 0$ $\tau_{sub} = 0$	$\tau_{tax} \neq 0$ $\tau_{sub} = 0$	$\tau_{tax} = 0$ $\tau_{sub} \neq 0$	$\tau_{tax} = 0$ $\tau_{sub} = 0$	$\tau_{tax} = 0$ $\tau_{sub} = 0$
Number Share (Multi)	0.27	0.2972	0.3095	0.3118	0.3095	0.3829	
Output Share (Multi)	0.44	0.4249	0.6208	0.6046	0.6208	0.8591	
Top 5% Output Share	0.61	0.6819	0.8055	0.7942	0.8055	0.8120	
Subsidized Firm Share	0.16	0.1590	0	0	0	0	
Top 10% Output Share	0.71	0.8491	0.9033	0.8844	0.9033	0.9065	
Drop Rate of Entries	0.31	0.4148	0.4546	0.3755	0.4546	0.4575	
Subsidized MP Firm Share	0.05	0.0566	0	0	0	0.088	
Measure of Firms	-	0.197	0.270	0.112	0.270	0.269	
Aggregate Price	-	0.00054	0.00018	0.00052	0.00018	0.00018	
Welfare	-	32.57%	100%	29.82%	100%	102.8%	

Fig. 5: Counterfactual Economies for China

We decompose the welfare loss into three margins in Figure 6.

	Benchmark	PF-FF	PF	No Dist.
Measure of Firms	0.197	0.197	0.234	0.270
Aggregate Price	0.000537	0.000190	0.000187	0.000178
Welfare	32.57%	61.68%	97.77%	100%

Fig. 6: Decomposition of Misallocations into Three Margins

PF-FF: All distortions are removed while both the product cutoffs and the firm measure are fixed as in the Benchmark. PF: Distortions are removed, the firm measure adjusts while the product cutoffs are still fixed as in the Benchmark. Labor and human capital are reallocated in both cases.

The overall welfare loss is 67.43% compared to the distortion-free economy. The product extensive margin is least important, 2.23%, and the product intensive margin and firm extensive margin account for 29.1% and 36.09% respectively.

## Conclusions

We find that multi-product firms are fewer and smaller in China, and the product extensive margin of distortion is statistically significant in Probit regressions. To quantify the effects of distortions, we build a discrete multi-product choice model with endogenous firm entry and exit and calibrate it to match the China data. The model theoretically confirms the product extensive margin of distortions, but quantitatively the welfare loss is much smaller through this margin, compared to the product intensive margin and firm extensive margin.