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Liquidity Traps in a Monetary Union

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Abstract
The closed economy macro literature has shown that a liquidity trap can result from the self-fulfilling expectation that *future* inflation and output will be low. This paper investigates expectations-driven liquidity traps in a two-country New Keynesian model of a monetary union. In the model here, a rise in government purchases in an individual country has a weak effect on GDP in the rest of the union. The results here cast doubt on the view that, in the current era of ultra-low interest rates, a rise in fiscal spending by Euro Area (EA) core countries would significantly *boost* GDP in the EA periphery.

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1. Introduction

Since the global financial crisis (2008-9), short-term riskless nominal interest rates in the Euro Area have been close to zero, while Euro Area inflation too has remained low (below the ECB's target). Understanding this 'low rates' environment is one of the key challenges for economic analysis. This paper provides a novel perspective on the effect of a low rates environment on the transmission of aggregate supply and demand disturbances, in a *monetary union*. The analysis is based on a two-country New Keynesian business cycle model of a monetary union with a zero lower bound (ZLB) constraint for the nominal interest rate.

As is well-known from the *closed economy* macro literature (Benhabib et al., 2001a,b), the presence of the ZLB can give rise to multiple equilibria that exhibit self-fulfilling fluctuations of inflation and real activity, under the standard assumption that monetary policy follows an 'active' Taylor rule, i.e. a policy rule with a strong interest rate response when the inflation rate deviates from the central bank's inflation target. A liquidity trap, i.e. a situation in which the ZLB binds, can then be due to the self-fulfilling expectation that *future* inflation and output will be low. Mertens and Ravn (2014) and Aruoba et al. (2018) have shown that when a closed economy liquidity trap is caused by pessimistic expectations, then a rise in government purchases can have a deflationary effect, which mutes the rise in GDP triggered by the fiscal shock. The mechanism is that a rise in government purchases triggers a rise in the natural real interest rate. In a pessimism-driven liquidity trap, this induces a *fall* in the (current and expected future) inflation rate. This deflationary effect dampens the rise in output.

The contribution of the present paper is to study expectations-driven liquidity traps in a *monetary union*, in order to shed light on key policy issues facing the Euro Area. The model assumes fully integrated goods markets and financial markets. Each country produces a distinct set of goods, while consuming both domestic and imported goods. Due to a spending bias towards locally produced goods, consumer price index (CPI) inflation can differ across countries. The union's central bank follows a Taylor rule that targets *union-wide* inflation.

In this setting, I study expectations-driven sunspot equilibria that feature an occasionally binding ZLB. The model predicts that, in a liquidity trap, a rise in government purchases in country 'Home' raises local GDP, but the effect on 'Foreign' GDP is weak (and possibly negative). When government purchases shocks are persistent, the fiscal multiplier is smaller than unity. The finding of a weak *domestic* fiscal multiplier resonates with Mertens and Ravn's

(2014) prediction of a muted response of output to fiscal spending shocks, in a closed economy expectations-driven liquidity trap. The weak transmission (of a Home government purchases increase) to Foreign GDP reflects the muted rise in Home GDP, which only generates a weak demand spillover to Foreign output. The Home terms of trade improve, which tends to shift private-sector demand towards Foreign goods. However, price rigidity dampens this competitiveness effect, which contributes to the weak response of Foreign GDP. By contrast, a persistent country-specific rise in total factor productivity (TFP) raises both domestic and foreign GDP.

The effect of the expectations-driven liquidity traps considered here differs from that of liquidity traps triggered by large adverse exogenous aggregate demand shocks, such as an autonomous fall in households' subjective discount rates. 'Fundamentals-driven' liquidity traps of the latter type have been studied by an extensive literature; see, e.g., the closed economy models of Krugman (1998), Eggertsson and Woodford (2003), Holden (2016,2019) and Roeger (2015). That literature predicts that fiscal spending multipliers can be much larger at the ZLB than when the ZLB does not bind. A key assumption of that literature is that the liquidity trap ends permanently when the adverse aggregate demand shock subsides. The large predicted fiscal multipliers during a 'fundamentals-driven' liquidity trap are due to the fact that, with a forward looking Phillips curve, a small increase in the expected inflation rate, at the date at which the economy permanently emerges from the liquidity trap, has a large positive effect on inflation and output in *earlier* periods. Widely discussed studies by Erceg and Lindé (2010) and Blanchard et al. (2016) have presented models of a monetary union in which a liquidity trap is triggered by a large adverse demand shock. These authors find that, at the ZLB, a country-specific rise in government purchases can have a strong positive effect on domestic and *foreign* output, i.e. the predicted international fiscal spillover can be sizable. This theory provides a basis to the view that, in the aftermath of the global financial crisis, fiscal 'austerity' (and weak demand more generally) in Germany contributed to the slump in the rest of the Euro Area (e.g., Krugman, 2013). It also predicts that expansionary fiscal policy in Germany (and other Euro Area core countries) could significantly help the Euro Area 'periphery', in a low interest rate environment.

The present paper cautions against the idea of strong cross-border fiscal transmission in a monetary union, during a liquidity trap. If a liquidity trap is caused by self-fulfilling pessimism about future inflation, then cross-country fiscal spending spillovers can be much weaker than in a liquidity trap induced by an adverse aggregate demand shock.

2. Model of a monetary union

I consider a New Keynesian open economy model with a standard structure of goods, labour and financial markets (e.g., Kollmann, 2001a,b, 2002, 2004, 2005). There are two countries, referred to as Home (H) and Foreign (F), that belong to a monetary union. A common central bank sets the short-term nominal interest for both countries. In each country there are: (i) a government that makes exogenous purchases which are financed using lump-sum taxes; (ii) a representative infinitely-lived household; (iii) monopolistic firms that produce a continuum of differentiated tradable intermediate goods using domestic labour; (iv) competitive firms that bundle domestic and imported intermediates into composite non-tradable goods that are used for household and government consumption. Intermediate goods prices are sticky; all other prices are flexible. Each country's household owns the domestic firms, and it supplies labour to those firms (labour is immobile internationally). The labour market is competitive. For analytical tractability, the model abstracts from physical capital. The Foreign country is a mirror image of the Home country. The two countries are identical in steady state. The following description focuses on the Home country. Analogous conditions describe the Foreign country.

2.1. Home firms

The Home country's household consumes a composite final consumption good $C_{H,t}$ that is produced using the Cobb-Douglas technology $C_{H,t} \equiv (Y^H_{H,t}/\xi)^{\xi}(Y^F_{H,t}/(1-\xi))^{1-\xi}$ where $Y^H_{H,t}$ and $Y^F_{H,t}$ are, respectively, a composite of domestic intermediate goods and a composite of imported intermediates, used by country H. (The superscript on intermediate good quantities denotes the country of origin, while the subscript indicates the destination country.) There is a bias towards using local intermediates, in household consumption: $0.5<\xi<1$. Each country produces a distinct set of intermediates indexed by $s \in [0,1]$. (Intermediate good 's' produced by country H differs from intermediate 's' produced by country F.) The composite intermediate $Y^k_{H,t}$ is given by $Y^k_{H,t} \equiv \{\int_0^1 (y^k_{H,t}(s))^{(\nu-1)/\nu} ds\}^{\nu/(\nu-1)}$ with $\nu>1$, for k=H,F, where $y^k_{H,t}(s)$ is the quantity of the variety s input produced by country k that is sold to country H.

Home government consumption, denoted $G_{H,t}$, too is a composite of intermediary inputs, but government consumption only uses local intermediates (no imports): $G_{H,t} \equiv \{\int_0^1 (g^H_{H,t}(s))^{(\nu-1)/\nu} ds\}^{\nu/(\nu-1)}$, where $g^H_{H,t}(s)$ is the quantity of the Home produced variety s input that enters Home government consumption.

Let $p_{k,t}(s)$ be the price of intermediate good s produced by country k. Cost minimization in Home final good production implies: $y^k_{H,t}(s)=(p_{k,t}(s)/P_{H,t})^{-\nu} Y^k_{H,t}$ for k=H,F and $g^H_{H,t}(s)=(p_{H,t}(s)/P_{H,t})^{-\nu} G_{H,t}$, as well as $Y^H_{H,t}=\xi \cdot CPI_{H,t} \cdot C_{H,t}/P_{H,t}$, $Y^F_{H,t}=(1-\xi) \cdot CPI_{H,t} \cdot C_{H,t}/P_{F,t}$ where $P_{k,t} \equiv \{\int_{s=0}^1 p_{k,t}(s)^{1-\nu} ds\}^{1/(1-\nu)}$ and $CPI_{H,t} \equiv (P_{H,t})^{\xi} (P_{F,t})^{1-\xi}$. $P_{k,t}$ is a price index of intermediates produced by country k=H,F. Perfect competition implies that the country H final consumption good price index is $CPI_{H,t}$ (its marginal cost).

The technology of the firm producing intermediate good s in country H is: $y_{H,t}(s) = \theta_{H,t} L_{H,t}(s)$, where $y_{H,t}(s)$ and $L_{H,t}(s)$ are the firm's output and labour input at date t, while $\theta_{H,t} > 0$ is exogenous productivity in country H (all intermediate good producers located in a country have identical productivity). The firm's good is sold domestically and exported: $y_{H,t}(s)=y^H_{H,t}(s)+y^H_{F,t}(s)+g^H_{H,t}(s)$.

Intermediate good producers face quadratic price adjustment costs. The real profit, in units of Home consumption, of the firm that produces Home intermediate good s is:

$$\pi_{H,t}(s) \equiv (p_{H,t}(s) - W_{H,t}/\theta_{H,t}) y_{H,t}(s)/CPI_{H,t} - \tfrac{1}{2}\psi \cdot ([p_{H,t}(s) - \Pi \cdot p_{H,t-1}(s)]/P_{H,t-1})^2, \quad \psi>0,$$

where $W_{H,t}$ is the nominal wage rate in country H. The last term in this equation is the real price adjustment cost, where $\Pi > 1$ is the central bank's gross inflation target (see below). The firm sets $p^H_{H,t}(s)$ to maximises the present value of profits $E_t \sum_{\tau=0}^{\infty} \rho^H_{t,t+\tau} \pi_{H,t+\tau}(s)$, where $\rho^H_{t,t+\tau}$ is the Home household's intertemporal marginal rate of substitution in consumption between periods t

and $t+\tau$. All Home intermediate good firms face identical decision problems, and they produce identical quantities and set identical prices: $p_{H,t}(s)=P_{H,t} \quad \forall\ s \in [0,1]$.

The Home terms of trade and the real exchange rate (CPI-based) are $q_t \equiv P_{H,t}/P_{F,t}$ and $RER_t \equiv CPI_{H,t}/CPI_{F,t}$, respectively. Note that $RER_t=(q_t)^{2\xi-1}$. Due to consumption home bias $(2\xi-1>0)$, the real exchange rate is an increasing function of the terms of trade. The real price of the domestic intermediate good, in units of final consumption, too is an increasing function of the terms of trade:

$$P_{H,t}/CPI_{H,t}=(q_t)^{1-\xi}. \tag{1}$$

2.2. Household preferences and labour supply

The intertemporal preferences of the representative Home household are described by $E_0 \sum_{t=0}^{\infty} \beta^t \Psi_{H,t} \{\ln(C_{H,t}) - \frac{1}{1+1/\eta}(L_{H,t})^{1+1/\eta}\}$ where $C_{H,t}$ and $L_{H,t}$ are final consumption and aggregate hours worked. $0<\beta<1$ is the household's steady state subjective discount factor and $\eta>0$ is the Frisch labour supply elasticity. $\Psi_{H,t}>0$ is a stationary exogenous preference shock that alters the household's rate of time preference. The household equates the marginal rate of substitution between leisure and consumption to the real wage rate, which implies

$$(1/C_{H,t})(W_{H,t}/CPI_{H,t}) = (L_{H,t})^{1/\eta}. \tag{2}$$

2.3. Financial markets

The model assumes complete international financial markets, and so consumption risk is efficiently shared across countries. In equilibrium, the ratio of Home to Foreign households' marginal utilities of consumption is, thus, proportional to the Home real exchange rate (Kollmann, 1991,1995; Backus and Smith, 1993). This implies:

$$C_{H,t}/C_{F,t}=\Lambda \cdot (\Psi_{H,t}/\Psi_{F,t})/RER_t, \tag{3}$$

where Λ is a date- and state-invariant term that reflects countries' (relative) initial wealth. I assume that countries have identical initial wealth, so that $\Lambda=1$.

There is also a market for a one-period riskless nominal bond. The *nominal* interest rate on that bond is r_t between periods t and $t+1$. The gross nominal rate is denoted $R_t \equiv 1+r_t$.

The Home household's Euler equation for this bond is:

$$R_t \cdot E_t\beta(\Psi_{H,t+1}/\Psi_{H,t})(C_{H,t}/C_{H,t+1})/\Pi^{CPI}_{H,t+1} = 1, \qquad (4)$$

where $\Pi^{CPI}_{H,t+1} \equiv CPI_{H,t+1}/CPI_{H,t}$ is the Home gross CPI inflation rate between t and $t+1$.

2.4. Monetary policy

The union-wide (gross) CPI inflation rate is $\Pi_t \equiv \frac{1}{2}\Pi^{CPI}_{H,t} + \frac{1}{2}\Pi^{CPI}_{F,t}$.[1] The monetary union's central bank sets the interest rate r_t according to a feedback rule that targets union-wide CPI inflation, subject to the zero lower bound (ZLB) constraint $r_t \geq 0$, i.e. $R_t \geq 1$. The monetary policy rule is

$$1 + r_t = Max\{1, \Pi/\beta + (\gamma_\pi/\beta)\cdot(\Pi_t - \Pi)\},\ \ \gamma_\pi > 1, \qquad (5)$$

where $\Pi > 1$ is the central bank's gross inflation target. The parameter γ_π captures the central bank's policy response to inflation. The 'Taylor principle' $(\gamma_\pi > 1)$ is assumed to hold: when the ZLB constraint is slack, a rise in inflation by 1 percentage point (ppt) triggers a rise of the policy rate by more than 1 ppt.

2.5. Market clearing

Real GDP in country k=H,F $(Y_{k,t})$ equals aggregate intermediate good output, $Y_{k,t} = \theta_{k,t} L_{k,t}$. Intermediate good firms meet all demand at posted prices. This implies $Y_{k,t} = Y^k_{H,t} + Y^k_{F,t} + G_{k,t}$ i.e. GDP equals the sum of domestic and foreign intermediate good demand. Using the demand functions described above, this can be expressed as $Y_{H,t} = \xi CPI_{H,t} C_{H,t}/P_{H,t} + (1-\xi) CPI_{F,t} C_{F,t}/P_{H,t} + G_{H,t}$, $Y_{F,t} = (1-\xi) CPI_{H,t} C_{H,t}/P_{F,t} + \xi CPI_{F,t} C_{F,t}/P_{F,t} + G_{F,t}$.

2.6. Solving the model

Following much of the previous literature on macro models with a ZLB constraint (see Holden, 2016, 2019, for detailed references), I linearise all equations, **with the exception of the interest rate rule (5)**. This allows to capture the macroeconomic effects of the occasionally binding ZLB constraint, while keeping analytical tractability.

[1] The Euro-Area price level measure (HICP) used by ECB policy-makers is a Laspeyres-type index. The expression for union-wide inflation used in this model corresponds to an index that was linearised around the symmetric steady state.

I take a linear approximation around a steady state in which (in both countries) the gross inflation rate equals the inflation target Π; the corresponding steady state gross interest rate is $1+r=\Pi/\beta$. Let $\hat{x}_t \equiv (x_t - x)/x$ denote the relative deviation of a variable x_t from its steady state value $x \neq 0$ (variables without time subscript denote steady state values). To simplify the analysis, I assume that government purchases are zero, in steady state.[2] I define $\widehat{G}_{k,t} \equiv G_{k,t}/Y_k$ as the ratio of government purchases to steady state GDP. Linearization of the risk-sharing condition (3) and of goods market clearing conditions (using (1)) gives:

$$\widehat{C}_{H,t} - \widehat{C}_{F,t} = -(2\xi - 1)\hat{q}_t + \widehat{\Psi}_{H,t} - \widehat{\Psi}_{F,t}, \tag{6}$$

$$\widehat{Y}_{H,t} = \xi \widehat{C}_{H,t} + (1-\xi)\widehat{C}_{F,t} - 2\xi(1-\xi)\hat{q}_t + \widehat{G}_{H,t},\ \widehat{Y}_{F,t} = (1-\xi)\widehat{C}_{H,t} + \xi \widehat{C}_{F,t} + 2\xi(1-\xi)\hat{q}_t + \widehat{G}_{F,t}. \tag{7}$$

The linearised Euler equation (4) of country $k=\mathrm{H,F}$ is:

$$\widehat{R}_t = E_t\{\widehat{\Pi}^{CPI}_{k,t+1} + \widehat{C}_{k,t+1} - \widehat{C}_{k,t} + \widehat{\Psi}_{kt} - \widehat{\Psi}_{k,t+1}\}. \tag{8}$$

Linearizing the first-order condition of the intermediate good firms' decision problem in country $k=\mathrm{H,F}$ gives a standard 'forward-looking' Phillips equation:

$$\widehat{\Pi}_{k,t} = \kappa_w \cdot \widehat{mc}_{k,t} + \beta E_t \widehat{\Pi}_{k,t+1}, \tag{9}$$

where $\Pi_{k,t} \equiv P_{k,t}/P_{k,t-1}$ is the gross inflation rate of the country k *producer price index (PPI)*, while $mc_{k,t} = (W_{k,t}/\theta_{k,t})/P_{k,t}$ is real marginal cost (deflated by the producer price) in k's intermediate good sector (e.g., Kollmann, 2002). $\kappa_w > 0$ is a coefficient that is a decreasing function of the price adjustment-cost parameter ψ. Using the nominal wage implied by the Home household's labour supply equation (2) (and the analogous Foreign equation) allows to express real marginal costs as:

$$\widehat{mc}_{H,t} = \widehat{C}_{H,t} + \tfrac{1}{\eta}\widehat{Y}_{H,t} - (1+\tfrac{1}{\eta})\hat{\theta}_{H,t} - (1-\xi)\hat{q}_t \text{ and } \widehat{mc}_{F,t} = \widehat{C}_{F,t} + \tfrac{1}{\eta}\widehat{Y}_{F,t} - (1+\tfrac{1}{\eta})\hat{\theta}_{F,t} + (1-\xi)\hat{q}_t. \tag{10}$$

Expressing the interest rate rule (5) using 'hatted' variables gives

$$\widehat{R}_t = Max\{-(\Pi - \beta)/\Pi, \gamma_\pi \cdot \widehat{\Pi}_t\}. \tag{11}$$

The ZLB constraint binds when $\gamma_\pi \widehat{\Pi}_t \leq -(\Pi - \beta)/\Pi$.

All exogenous variables follow univariate AR(1) processes with autocorrelation $0<\rho<1$:

[2] An interpretation of negative government purchases is that government occasionally has an autonomous supply of resources that it distributes to the private sector.

$\hat{\theta}_{k,t+1}=\rho\hat{\theta}_{k,t}+\varepsilon^{\theta}_{k,t+1}$, $\hat{G}_{k,t+1}=\rho\hat{G}_{k,t}+\varepsilon^{G}_{k,t+1}$, $\hat{\Psi}_{k,t+1}=\rho\hat{\Psi}_{k,t}+\varepsilon^{\Psi}_{k,t+1}$ for k=H,F where $\varepsilon^{\theta}_{k,t+1},\varepsilon^{G}_{k,t+1},\varepsilon^{\Psi}_{k,t+1}$ are mean-zero innovations.

2.6.1. Union-wide inflation and output

The dynamic equations that govern union-wide variables are isomorphic to a closed economy model. In what follows, variables without country (H,F) subscripts denote union-wide averages. Union-wide GDP, consumption, productivity, government purchases and preference shocks are $\hat{Y}_t\equiv\frac{1}{2}\hat{Y}_{H,t}+\frac{1}{2}\hat{Y}_{F,t}$, $\hat{C}_t\equiv\frac{1}{2}\hat{C}_{H,t}+\frac{1}{2}\hat{C}_{F,t}$, $\hat{\theta}_t=\frac{1}{2}\hat{\theta}_{H,t}+\frac{1}{2}\hat{\theta}_{F,t}$, $\hat{G}_t=\frac{1}{2}\hat{G}_{H,t}+\frac{1}{2}\hat{G}_{F,t}$ and $\hat{\Psi}_t\equiv\frac{1}{2}\hat{\Psi}_{H,t}+\frac{1}{2}\hat{\Psi}_{F,t}$, respectively.[3] Note that $\hat{Y}_t=\hat{C}_t+\hat{G}_t$ (from (7)). Union-wide CPI inflation equals union-wide PPI inflation: $\hat{\Pi}_t\equiv\hat{\Pi}^{CPI}_t=\frac{1}{2}\hat{\Pi}_{H,t}+\frac{1}{2}\hat{\Pi}_{F,t}$. Averaging the Home and Foreign Phillips equations (9) (using (10)) gives a union-wide Phillips curve:

$$\hat{\Pi}_t=\kappa\cdot(\hat{C}_t-\hat{\theta}_t+\tfrac{1}{1+\eta}\hat{G}_t)+\beta E_t\hat{\Pi}_{t+1},\ \text{with}\ \kappa\equiv\kappa_w(1+\eta)/\eta>0. \tag{12}$$

Averaging Home and Foreign Euler equations (8) gives:

$$\hat{R}_t=E_t\{\hat{\Pi}_{t+1}+\hat{C}_{t+1}-\hat{C}_t-(\hat{\Psi}_{t+1}-\hat{\Psi}_t)\}. \tag{13}$$

Combining (13) and the interest rate rule (11), and substituting out union-wide consumption using the union-wide Phillips curve (12) gives:

$$Max\{-(\Pi-\beta)/\Pi,\gamma_\pi\cdot\hat{\Pi}_t\}=-\tfrac{1}{\kappa}\hat{\Pi}_t+(1+\tfrac{1+\beta}{\kappa})E_t\hat{\Pi}_{t+1}-\tfrac{\beta}{\kappa}E_t\hat{\Pi}_{t+2}+\hat{R}^{nat}_t, \tag{14}$$

$$\text{with}\ \hat{R}^{nat}_t\equiv E_t\{(\hat{\theta}_{t+1}-\hat{\theta}_t)-\tfrac{1}{1+\eta}(\hat{G}_{t+1}-\hat{G}_t)-(\hat{\Psi}_{t+1}-\hat{\Psi}_t)\}=(1-\rho)\{-\hat{\theta}_t+\tfrac{1}{1+\eta}\hat{G}_t+\hat{\Psi}_t\}.$$

I call (14) the union-wide 'Euler-Phillips' equation and I refer to $\hat{R}^{nat}_t$ as the (union-wide) natural real interest rate. If prices were flexible $(\kappa=\infty)$, the union-wide risk-free gross real interest rate between dates t and $t+1$ (expressed as a deviation from the steady state rate) would be $\hat{R}^{nat}_t$. Note that $\hat{R}^{nat}_t$ reflects expected one-period-ahead changes of exogenous variables.

The natural real interest rate is a decreasing function of the date t level of (union-wide) productivity and an increasing function of government purchases and of the preference shock. As

[3] $\hat{Y}_t,\hat{C}_t,\hat{G}_t$ correspond to linearised quantity indices (Laspeyres/Paasche) of union-wide output, consumption and government purchases.

these forcing variables follow AR(1) processes with autocorrelation ρ, the natural rate too is an AR(1) process with autocorrelation ρ. Because of the assumed mean reversion of productivity, a positive productivity shock reduces the expected future growth rate of productivity; in a flex-prices economy, a positive productivity shock increases consumption on impact; future consumption rises less than current consumption, i.e. the expected growth rate of consumption falls, and hence the real natural interest rate drops. Similar logic explains why positive fiscal spending and preference shocks raise the natural real interest rate.

2.6.2. Country-level variables

Country-level inflation, consumption and output depend on the terms of trade. Note that

$$\widehat{\Pi}_{i,t}=\widehat{\Pi}_t+\tfrac{1}{2}(\widehat{\Pi}_{i,t}-\widehat{\Pi}_{j,t}),\ \widehat{C}_{i,t}=\widehat{C}_t+\tfrac{1}{2}(\widehat{C}_{i,t}-\widehat{C}_{j,t}),\ \widehat{Y}_{i,t}=\widehat{Y}_t+\tfrac{1}{2}(\widehat{Y}_{i,t}-\widehat{Y}_{j,t})\ \text{ for } i,j=H,F;\ j\neq i. \tag{15}$$

Relative (Home vs. Foreign) PPI inflation equals the change of the Home terms of trade: $\widehat{\Pi}_{H,t}-\widehat{\Pi}_{F,t}=\hat{q}_t-\hat{q}_{t-1}$. Relative output is a function of the terms of trade, of the relative preference shock and of relative government purchases (from (6),(7)):

$$\widehat{Y}_{H,t}-\widehat{Y}_{F,t}=-\hat{q}_t+(2\xi-1)(\widehat{\Psi}_{H,t}-\widehat{\Psi}_{F,t})+(\widehat{G}_{H,t}-\widehat{G}_{F,t}). \tag{16}$$

Subtracting the Foreign Phillips curve from the Home Phillips curve (see (9)) gives $\widehat{\Pi}_{H,t}-\widehat{\Pi}_{F,t}=\kappa_w\cdot(\widehat{mc}_{H,t}-\widehat{mc}_{F,t})+\beta E_t\{\widehat{\Pi}_{H,t+1}-\widehat{\Pi}_{F,t+1}\}$. Thus, $\hat{q}_t-\hat{q}_{t-1}=\kappa_w\cdot(\widehat{mc}_{H,t}-\widehat{mc}_{F,t})+\beta E_t\{\hat{q}_{t+1}-\hat{q}_t\}$. Relative real marginal cost can be written as (using (6),(7),(10)):

$$\widehat{mc}_{H,t}-\widehat{mc}_{F,t}=-\tfrac{1+\eta}{\eta}\hat{q}_t-\tfrac{1+\eta}{\eta}(\hat{\theta}_{H,t}-\hat{\theta}_{F,t})+\tfrac{1}{\eta}(\widehat{G}_{H,t}-\widehat{G}_{F,t})+\tfrac{\eta+2\xi-1}{\eta}(\widehat{\Psi}_{H,t}-\widehat{\Psi}_{F,t}).$$

These expressions imply that the terms of trade are governed by the difference equation

$$E_t\hat{q}_{t+1}=\tfrac{1+\kappa+\beta}{\beta}\hat{q}_t-\tfrac{1}{\beta}\hat{q}_{t-1}+\tfrac{\kappa}{\beta}(\hat{\theta}_{H,t}-\hat{\theta}_{F,t})-\tfrac{1}{1+\eta}\tfrac{\kappa}{\beta}(\widehat{G}_{H,t}-\widehat{G}_{F,t})-\tfrac{\kappa}{\beta}\tfrac{\eta+2\xi-1}{1+\eta}(\widehat{\Psi}_{H,t}-\widehat{\Psi}_{F,t}). \tag{17}$$

$0<\beta<1$ and $\kappa>0$ ensure that (17) has a unique non-explosive solution

$$\hat{q}_t=\Xi\cdot\hat{q}_{t-1}-a_\theta(\hat{\theta}_{H,t}-\hat{\theta}_{F,t})+a_G(\widehat{G}_{H,t}-\widehat{G}_{F,t})+a_\Psi(\widehat{\Psi}_{H,t}-\widehat{\Psi}_{F,t}), \tag{18}$$

with $0<\Xi<1$ and $a_\theta,a_G,a_\Psi>0$. Ξ is a root of the polynomial

$$\Gamma(z)\equiv-z^2+\tfrac{1+\kappa+\beta}{\beta}z-\tfrac{1}{\beta}=0.\ ^{4} \tag{19}$$

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[4] $\Xi=a/2-((a/2)^2-1/\beta)^{0.5}$ with $a\equiv(1+\kappa+\beta)/\beta$; $a_\theta\equiv-(\kappa/\beta)/\zeta$, $a_G\equiv-(1/(1+\eta))(\kappa/\beta)/\zeta$, $a_\Psi\equiv-((\eta+2\xi-1)/(1+\eta))(\kappa/\beta)/\zeta$, where $\zeta\equiv\Xi+\rho-a<0$.

(18) shows that the terms of trade are a function of current and lagged relative (Home vs. Foreign) fundamental shocks. A rise in relative Home productivity worsens the Home terms of trade, while a rise in Home government purchases and the Home preference shifter Ψ_H improve the Home terms of trade. When prices are more sticky (lower Phillips-curve slope κ), the terms of trade respond more sluggishly to exogenous shocks.

Relative variables depend on (current and lagged) relative forcing variables. Given the terms of trade process (18), relative output and consumption are uniquely pinned down by (6) and (16). Given a process for union-wide inflation and output, the country-level variables can then be determined using (15). Importantly, monetary policy affects union-wide output and inflation, but it does *not* affect the terms of trade and relative variables. While union-wide inflation and output are indeterminate (see below), relative variables are *determinate*.

2.7. Flex-prices model

A flex-prices (Real Business Cycle, RBC) model provides a useful benchmark for understanding the dynamics of real variables in the sticky-prices economy. The flex-prices model is a special case of the above model in which price adjustment costs are zero, $\psi=0$, and the slope of the Phillips curve is infinite, $\kappa=\infty$.[5] The flex-prices equilibrium allocation is described by:

$$\widehat{Y}_{i,t}=\widehat{\theta}_{i,t}+\tfrac{\eta}{1+\eta}\widehat{G}_{i,t}-\tfrac{(1-\xi)\eta}{1+\eta}\cdot(\widehat{\Psi}_{i,t}-\widehat{\Psi}_{j,t}),\quad \text{for } i\in\{H,F\};$$

$$\widehat{C}_{i,t}=\xi\widehat{\theta}_{i,t}+(1-\xi)\widehat{\theta}_{j,t}-\tfrac{\xi}{1+\eta}\widehat{G}_{i,t}-\tfrac{1-\xi}{1+\eta}\widehat{G}_{j,t}+\tfrac{(1-\xi)(2\xi+\eta)}{1+\eta}\cdot(\widehat{\Psi}_{i,t}-\widehat{\Psi}_{j,t}),\quad \text{for } i,j\in\{H,F\},\ i\neq j;$$

$$\hat{q}_t=\tfrac{2\xi-1+\eta}{1+\eta}\cdot(\widehat{\Psi}_{H,t}-\widehat{\Psi}_{F,t})-(\hat{\theta}_{H,t}-\hat{\theta}_{F,t})+\tfrac{1}{1+\eta}\cdot(\widehat{G}_{H,t}-\widehat{G}_{F,t}).$$

The Home terms of trade are a decreasing function of relative (Home vs. Foreign) productivity and an increasing function of relative government purchases and of the relative preference shock, under flexible prices. In a flex-prices world, output in country *i* depends (positively) on domestic productivity and government purchases, but is independent of *foreign* productivity and government purchases. The zero international output spillover of productivity and government purchases reflects the household preferences of the Cole and Obstfeld (1991)-type assumed here, i.e. the combination of a unitary intertemporal consumption substitution elasticity and a unitary

[5] Under flexible prices, real variables are uniquely pinned down (and independent of inflation), but inflation remains indeterminate.

intratemporal substitution elasticity between domestic and imported intermediates.[6] By contrast, consumption in a given country depends positively on both domestic and foreign productivity, and negatively on domestic and foreign government purchases, in the RBC world. A positive country *i* preference shock raises *i*'s consumption, and it lowers *i*'s output (as the rise in consumption triggers a fall in labour supply).

3. Expectations-driven liquidity traps

3.1. Constant productivity, government purchases and preferences

The model has multiple bounded solutions. To see this in the simplest possible way, consider first an economy in which productivity, government purchases and the preference parameter Ψ are constant, so that the natural real interest rate is likewise constant: $\widehat{R}_t^{nat}=0 \ \ \forall t.$ As the Taylor principle holds $(\gamma_\pi>1),$ the union-wide Euler-Phillips equation (14) is then solved by two steady state inflation rates: $\widehat{\Pi}=0$ and $\widehat{\Pi}=-(\Pi-\beta)/\Pi.$ The ZLB binds in the latter steady state. This finding is in line with Benhabib et al. (2001a,b) who show (in a simpler model) that the combination of the ZLB and an 'active' Taylor rule produces two steady states, and that the ZLB binds in one of these steady states.

There also exist stochastic equilibria with an *occasionally* binding ZLB. Those equilibria seem especially relevant empirically. Arifovic et al. (2018), Aruoba et al. (2018) and Nakata and Schmidt (2020) construct equilibria in which an exogenous random sunspot variable determines whether the ZLB constraint is slack or whether it binds. Let union-wide inflation follow a Markov chain with two possible values: $\widehat{\Pi}_t \in \{\widehat{\Pi}^S, \widehat{\Pi}^B\}$ such that $\widehat{\Pi}^B < \widehat{\Pi}^S$ and

$$\gamma_\pi \widehat{\Pi}^B \leq -(\Pi-\beta)/\Pi < \gamma_\pi \widehat{\Pi}^S. \tag{20}$$

(20) implies that the ZLB constraint binds (is slack) when low (high) inflation $\widehat{\Pi}^B$ $(\widehat{\Pi}^S)$ obtains, as can be seen from the linearised Taylor rule (11). Denote the transition probabilities between $\widehat{\Pi}^B$ and $\widehat{\Pi}^S$ by $p_{ij} \equiv \text{Prob}(\widehat{\Pi}_{t+1}=\widehat{\Pi}^j | \widehat{\Pi}_t=\widehat{\Pi}^i)$ for $i,j \in \{S;B\},$ with $0 \leq p_{ij} \leq 1,$ $p_{iS}+p_{iB}=1.$

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[6] A positive shock to Home government purchases raises the relative price of Home output. As the shock lowers Home and Foreign consumption, it has opposing income and substitution effects on household demand for Foreign output. Under Cole-Obstfeld preferences, these opposite effects cancel out, and Foreign output does not change. A Home productivity shock too induces countervailing income and substitution effects, such that Foreign output is unaffected.

An equilibrium with an occasionally binding ZLB is defined by inflation rates $\widehat{\Pi}^S, \widehat{\Pi}^B$ and probabilities p_{SS}, p_{BB} such that the inequalities in (20) and the Euler-Phillips equation (14) hold (see Not-for-Publication Appendix).

Model calibration

One period in the model represents one quarter in calendar time. I set $\beta=0.9975$, which implies a 1% per annum (p.a.) steady state riskless real interest rate. The Frisch labour supply elasticity is set at unity, $\eta=1$, a conventional value in macro models. The local content of private consumption spending is set at $\xi=0.8$. [7] The Central Bank's quarterly gross inflation target is set at $\Pi=1.005$, in line with the 2% annual ECB inflation target. The inflation coefficient of the interest rate rule (5) is set at the conventional value $\gamma_\pi=1.5$. The slope coefficient κ_w of the Phillips curve (9) is set at a value such that the observationally equivalent Phillips curve under Calvo (1983) staggered price setting entails an average 4-quarter duration between price changes, consistent with empirical evidence for the Euro Area (Kollmann, 2001a; Giovannini et al., 2019).[8] *The preceding parameters are used in all simulations below.*

The existence of an equilibrium with an occasionally binding ZLB constraint requires persistent ZLB regimes, i.e. the probabilities p_{SS}, p_{BB} must to be close to unity.[9] Assume, e.g., $p_{SS}=p_{BB}=0.95$; thus, if the economy is in a liquidity trap at date t, then there is a 95% probability that the liquidity trap will still be in effect at $t+1$, and the average duration of a liquidity trap is 20 quarters. Then $\widehat{\Pi}^B=-0.0080$ and $\widehat{\Pi}^S=-0.0011$. This corresponds to annualised union-wide inflation rates in the states with binding and slack ZLB constraints of

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[7] This value matches the fact that the ratio of within-Euro Area trade to EA GDP is about 20%.

[8] Under Calvo price setting, the slope of the Phillips curve is $\kappa_w=(1-D)(1-\beta D)/D$, where 1-$D$ is the probability that an individual firm can change its price in a given period. I set $D=0.75$, which implies $\kappa_w=0.08395$.

[9] When p_{SS}, p_{BB} are *not* sufficiently close to unity, then the values of $\widehat{\Pi}^S, \widehat{\Pi}^B$ pinned down by the Euler-Phillips equation (14) violate inequalities (20). Intuitively, if agents believe that a liquidity trap is transient, then inflation is too high during a liquidity trap (as agents expect a rapid escape from the trap), i.e. a liquidity trap is impossible. An equilibrium with expectations-driven liquidity traps exists when $p_{SS}+p_{BB}>1.67$. This condition is met under the plausible assumption that the expected duration of each ZLB regime exceeds 6 quarters. The ECB policy rate has been at (or close to zero) since late 2013, i.e. for more than 6 years.

-1.22% and 1.55%, respectively.[10] Union-wide GDP $\widehat{Y}$ in these two states is -0.22% and 0.20%, respectively. The nominal interest rate is 2.33% p.a. in the non-ZLB state.

3.2. Time-varying productivity, government purchases and preference shocks

I next consider an economy with time-varying productivity, government purchases and preference shocks. Interest centres on how the transmission of fundamental shocks depends on the ZLB regime. I continue to assume that the ZLB regime follows a Markov chain. That Markov chain is independent of shocks to productivity, government purchases and preferences. The model variants with expectations-driven ZLB regimes assume that the ZLB regime is *solely* driven by agents' self-fulfilling beliefs about future ZLB regimes. In those model variants, it is postulated that fundamental shocks are sufficiently small, so that those shocks do not trigger a change in the expectations-driven ZLB regime.

I construct equilibria in which union-wide inflation follows state-dependent linear policy functions of the natural real interest rate, $\widehat{R}_t^{nat}$:

$$\widehat{\Pi}_t^B = \mu^B + \lambda^B \widehat{R}_t^{nat} \text{ if the ZLB constraint binds at } t; \tag{21}$$

$$\widehat{\Pi}_t^S = \mu^S + \lambda^S \widehat{R}_t^{nat} \text{ if the ZLB constraint is slack at } t, \tag{22}$$

$$\text{with} \quad \gamma_\pi \widehat{\Pi}_t^B \leq -(\Pi - \beta)/\Pi < \gamma_\pi \widehat{\Pi}_t^S. \tag{23}$$

The coefficients $\mu^B, \lambda^B, \mu^S, \lambda^S$ can be solved for using the method of undetermined coefficients (after substituting (21)-(22) into Euler-Phillips equation (14)).

3.2.1. Permanent expectations-driven liquidity trap

To build intuition, I first consider an economy that is in a permanent expectations-driven liquidity trap $(p_{BB}=1)$, so that $\widehat{\Pi}_t = \widehat{\Pi}_t^B \leq -\{(\Pi-\beta)/\Pi\}/\gamma_\pi \;\; \forall t.$ In a permanent liquidity trap, the Euler-Phillips equation (14) becomes:

[10] As pointed out by a referee, the model-predicted inflation rate during a liquidity trap (-1.22%) is lower than recent Euro Area inflation (about +1%). The Not-for-Publication Appendix presents a model variant in which safe bonds generate a positive 'convenience yield' for investors (Caballero and Farhi, 2017; Pfeiffer et al., 2020). That variant generates a liquidity trap with realistic (positive) inflation. Importantly, responses to fundamental shocks are unchanged in that model variant. Thus, the key results below, regarding shock transmission under the two ZLB regimes, continue to hold.

$$-(\Pi-\beta)/\Pi = -\tfrac{1}{\kappa}\widehat{\Pi}_t^B + (1+\tfrac{1+\beta}{\kappa})E_t\widehat{\Pi}_{t+1}^B - \tfrac{\beta}{\kappa}E_t\widehat{\Pi}_{t+2}^B + \widehat{R}_t^{nat}. \tag{24}$$

Substitution of the policy function (21) into (24) gives:

$$-(\Pi-\beta)/\Pi = -\tfrac{1}{\kappa}\{\mu^B+\lambda^B\widehat{R}_t^{nat}\} + (1+\tfrac{1+\beta}{\kappa})\{\mu^B+\lambda^B\rho\widehat{R}_t^{nat}\} - \tfrac{\beta}{\kappa}\{\mu^B+\lambda^B\rho^2\widehat{R}_t^{nat}\} + \widehat{R}_t^{nat}, \tag{25}$$

as $\mathrm{E}_t\widehat{\Pi}_{t+s}^B = \mu^B+\lambda^B\rho^s\widehat{R}_t^{nat}$ for s>0.

(25) holds for arbitrary values of $\widehat{R}_t^{nat}$ iff $\mu^B = -(\Pi-\beta)/\Pi$ and $\{-\tfrac{1}{\kappa}+(1+\tfrac{1+\beta}{\kappa})\rho-\tfrac{\beta}{\kappa}\rho^2\}\lambda^B+1=0$. Thus, the slope of the policy function in a permanent liquidity trap is: $\lambda^B = -1/\{-\tfrac{1}{\kappa}+(1+\tfrac{1+\beta}{\kappa})\rho-\tfrac{\beta}{\kappa}\rho^2\} = -(\kappa/\beta)/\Gamma(\rho)$, where $\Gamma(\rho)$ is defined in (19). Note that $\Gamma(0) = -\tfrac{1}{\beta} < 0$ and $\Gamma(1) = \tfrac{\kappa}{\beta} > 0$; furthermore $\Gamma'(\rho) > 0$ for $0 \le \rho \le 1$. Therefore, $\Gamma(\rho) > 0$ holds for $0 < \Xi < \rho \le 1$ where Ξ (defined by $\Gamma(\Xi) = 0$) is the autoregressive coefficient of the law of motion of the terms of trade (18). For the calibrated values of β, κ (see above), we have $\Xi = 0.67$.

Empirical estimates of the quarterly autocorrelation of productivity and government purchases (and other macroeconomic shocks) are typically in the range between 0.95 and 1, and thus clearly larger than Ξ.[11] Thus, $\Gamma(\rho) > 0$ and $\lambda^B < 0$ hold for a plausible autocorrelation ρ, i.e. a rise in the natural interest rate lowers the union-wide inflation rate, in a permanent liquidity trap. As the natural real interest rate is a decreasing function of productivity, and an increasing function of government purchases and of the household preference shifter Ψ, a persistent productivity shock raises inflation, while persistent positive aggregate demand (government purchases and preference shift) shocks lower inflation, in a permanent expectations-driven liquidity trap.

Intuitively, a persistent rise in the union-wide natural real interest rate triggers a rise in the expected real interest rate. In a permanent liquidity trap, the nominal interest rate is stuck at zero, and the rise in the real interest rate is brought about by a fall in the union-wide inflation rate. This can be seen most easily when ρ is very close to (but below) unity. Then $\widehat{\Pi}_t^B \approx E_t\widehat{\Pi}_{t+1}^B \approx E_t\widehat{\Pi}_{t+2}^B$, and (24) gives $\widehat{\Pi}_t^B \approx -(\Pi-\beta)/\Pi - \widehat{R}_t^{nat}$, so that a positive shock to the natural

[11] Backus et al. (1992)'s classic estimates of laws of motion of TFP for a European aggregate and for the US (1970-86) imply TFP first-order autocorrelations of 0.99. The autocorrelation of the Euro Area quarterly government purchases/GDP ratio (1995-2017) is 0.97 (data source: ECB-AWM database 2018).

real rate triggers (approximately) a one-to-one negative response of the current and expected future union-wide inflation rate.

By contrast, when the natural real interest rate is less persistent, $\rho<\Xi$, then a positive shock to the union-wide natural real interest rate *raises* the union-wide inflation rate, in a permanent expectations-driven liquidity trap (e.g., when $\rho=0$ we have $\lambda^B=\kappa>0$). Hence a transitory positive productivity shock *lowers* union-wide inflation, while a transitory positive shock to government purchases *raises* union-wide inflation. [12]

Unless stated otherwise, the following simulations assume $\rho=0.95$, so that $\lambda^B<0$. Shock autocorrelations equal, or close to, 0.95 are widely assumed in macroeconomic models.

For $\rho=0.95$, the policy function for union-wide inflation in a permanent liquidity trap is

$$\widehat{\Pi}_t^B = -0.0074 - 1.070\,\widehat{R}_t^{nat} = -0.0074 + 0.05\,\widehat{\theta}_t - 0.03\widehat{G}_t - 0.05\,\widehat{\Psi}_t\,. \tag{26}$$

Thus, a 1% increase in union-wide productivity raises union-wide (gross) inflation by 0.05% (this corresponds to a rise of the annualised inflation rate by 0.2 ppt); while a 1% increase in union-wide government purchases lowers gross inflation by 0.03%. [13]

In a permanent liquidity trap, the policy rule for union-wide output is

$$\widehat{Y}_t^B = -0.0001 + 1.02\,\widehat{\theta}_t + 0.49\,\widehat{G}_t - 0.02\,\widehat{\Psi}_t\,, \tag{27}$$

i.e. the union-wide government purchases multiplier is 0.49. Although a rise in government purchases lowers union-wide inflation (see above), the government purchases multiplier is positive, because a rise in government purchases lowers union-wide consumption, which raises union-wide labour supply.

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[12] This can be seen most easily when $\rho=0$. Then, a one-time government purchases increase at date t has not effect on future natural real interest rate; thus, the shock has zero effect on union-wide inflation, output and consumption at $t+1$. The union-wide aggregate Euler equation (13) shows that, hence, union-wide consumption at date t does not respond to the shock, in a liquidity trap. Union-wide inflation at t rises to trigger a one-time rise in union-wide output to meet the higher output demand caused by higher government purchases. When the shock serial correlation ρ is strictly positive but smaller than Ξ, then it remains true that a positive government purchases shock raises union-wide inflation and output, in an expectations-driven liquidity trap.

[13] The existence of an equilibrium with a permanent liquidity trap requires that the forcing variables are bounded, to ensure that $\gamma_\pi \widehat{\Pi}_t^B \leq -(\Pi-\beta)/\Pi$ holds $\forall t$ (see (23)). For example, if productivity and the preference shifter take steady state values, then $\widehat{G}_t \geq -9\%$ has to hold: when government purchases fall below this lower bound, then the inflation rate defined by (26) rises to a level such that the Taylor rule prescribes a strictly positive nominal interest rate, which violates the first inequality shown in (23). The subsequent analysis assumes that all exogenous forcing variables remain sufficiently close to steady state values, so that the inequalities in (23) are satisfied.

3.2.2. Permanently slack ZLB constraint

I next consider an equilibrium in which the economy stays forever away from the ZLB, so that $\widehat{\Pi}_t = \widehat{\Pi}_t^S > -\{(\Pi - \beta)/\Pi\}/\gamma_\pi \ \ \forall t$ (see (25)). The Euler-Phillips equation (14) then implies

$$\gamma_\pi \widehat{\Pi}_t^S = -\tfrac{1}{\kappa}\widehat{\Pi}_t^S + (1+\tfrac{1+\beta}{\kappa})E_t\widehat{\Pi}_{t+1}^S - \tfrac{\beta}{\kappa}E_t\widehat{\Pi}_{t+2}^S + \widehat{R}_t^{nat}. \tag{28}$$

Substitution of policy rule (22) into (28) shows that the coefficients of the policy rule are

$$\mu^S = 0 \ \text{ and } \ \lambda^S = -(\kappa/\beta)/\{\Gamma(\rho) - \gamma_\pi \cdot (\kappa/\beta)\}. \tag{29}$$

$\gamma_\pi > 1$ (Taylor principle) implies that $\Gamma(\rho) - \gamma_\pi \cdot (\kappa/\beta) < 0 \ \ \forall \ 0 \le \rho \le 1$, and so $\lambda^S > 0$: when the ZLB is always slack, then a rise in the natural real interest rate triggers a rise in the inflation rate, and thus an increase in the nominal interest rate.

When the ZLB never binds, then a rise in productivity (which reduces the natural interest rate) lowers thus union-wide inflation, while positive aggregate demand shocks raise inflation. For $\rho = 0.95$, the policy rule for union-wide inflation, under a permanently slack ZLB constraint is

$$\widehat{\Pi}_t^S = 1.77\,\widehat{R}_t^{nat} = -0.09\,\hat{\theta}_t + 0.04\,\widehat{G}_t + 0.09\,\widehat{\Psi}_t, \tag{30}$$

while the policy rule for union-wide output is:

$$\widehat{Y}_t^S = 0.97\,\hat{\theta}_t + 0.51\,\widehat{G}_t + 0.03\,\widehat{\Psi}_t. \tag{31}$$

Quantitatively, the union-wide output responses to persistent shocks $(\rho = 0.95)$ are, thus, similar across the regimes with permanently binding/slack ZLB constraints. With a permanently slack ZLB constraint, union-wide output is slightly less responsive to productivity shocks, but slightly more responsive to government purchase shocks than in the permanent liquidity trap (see (27),(31)). Persistent shocks have a muted effect on the natural real interest rate, and hence the response of the inflation rate to these shocks is likewise muted. This helps to understand why, although the response of union-wide inflation to shocks differs *qualitatively* across the two regimes, the response of output is so similar across ZLB regimes. (As shown in Sect. 3.3, responses of consumption and the terms of trade too are very similar across ZLB regimes.)

3.2.3. Occasionally binding ZLB constraint

I next consider an equilibrium with random switches in/out of the liquidity trap. Assume as in Sect. 3.1. that the persistence of each ZLB regime is $p_{SS}=p_{BB}=0.95$, and that fundamental shocks are persistent ($\rho=0.95$). Then, the policy rules for union-wide inflation and output with a binding ZLB constraint ('B') are

$$\widehat{\Pi}_t^B = -0.0080 - 1.36\,\widehat{R}_t^{nat} = -0.0080 + 0.07\,\hat{\theta}_t - 0.03\widehat{G}_t - 0.07\,\widehat{\Psi}_t\ ,$$

$$\widehat{Y}_t^B = -0.0022 + 1.06\,\hat{\theta}_t + 0.47\,\widehat{G}_t - 0.06\,\widehat{\Psi}_t\ .$$

The policy rules with a slack ZLB ('S') are

$$\widehat{\Pi}_t^S = -0.0011 + 1.28\,\widehat{R}_t^{nat} = -0.0011 - 0.06\,\hat{\theta}_t + 0.03\,\widehat{G}_t + 0.06\,\widehat{\Psi}_t\ ,$$

$$\widehat{Y}_t^S = 0.0020 + 0.94\,\hat{\theta}_t + 0.53\,\widehat{G}_t + 0.06\,\widehat{\Psi}_t\ .$$

Given that the ZLB regimes are persistent, it is not surprising that these policy functions are similar to the ones that obtain when each regime is permanent ((26),(27),(30),(31)). It remains true that, in a liquidity trap, a rise in the natural interest rate lowers union-wide inflation, so that a positive productivity shock raises union-wide inflation, while a positive government purchases shock lowers union-wide inflation.[14] Again, the responses of union-wide output to productivity and government purchases shocks are similar across ZLB regimes. The union-wide government purchases multiplier is close to 0.5 in both regimes.

3.3. Dynamic shock responses

Table 1 reports shock responses for the baseline New Keynesian model with an occasionally binding ZLB constraint ($p_{SS}=p_{BB}=0.95$). The autocorrelation of the forcing variables is $\rho=0.95$. 1% innovations to Home productivity and to the Home preference shifter are considered, as well as an innovation that raises Home government purchases by 1% of Home steady state output. Responses after 0, 4 and 10 periods are reported; see Column labelled 'Horizon'. Panel (a) of Table 1 shows responses that obtain in a liquidity trap, while Panel (b) shows responses under a slack ZLB constraint. For comparison purposes, Table 2 shows

[14] $\lambda^B<0, \lambda^S>0$ holds for ZLB regime persistence parameters such that $p_{SS}+p_{BB}>1.71$, i.e. for plausible p_{SS}, p_{BB}.

dynamic responses for a New Keynesian model in which the ZLB constraint is always slack (p_{SS}=1). Table 3 shows dynamic response in an RBC (flex-prices) world.

In the **RBC model**, a positive shock to Home productivity raises Home output while Foreign output is unaffected. Home and Foreign consumption both rise. The rise in Home consumption exceeds the increase in Foreign consumption. The Home terms of trade deteriorate. In the RBC model, a positive shock to Home government purchases raises Home output, while Foreign output is unaffected; private consumption falls in both countries; the Home terms of trade improve. [15]

As pointed out above, the dynamic equations that govern union-wide variables are isomorphic to a closed economy model. Tables 1 and 2 show that, in the sticky-prices world, the responses of union-wide output and consumption are close to the responses in the RBC economy. This is consistent with the well-known fact that, in a *closed* economy with sticky prices, a monetary policy rule that stabilises the inflation rate can imply a dynamics of aggregate output and consumption that mimics a flex-prices economy, and that in the face of a both aggregate demand and aggregate supply shocks (e.g., Kollmann, 2008). For example, a rise in Home government purchases by 1% of Home output (i.e. 0.5% of union-wide output) raises union-wide output by 0.25% on impact, in the RBC model, compared to a rise of 0.26% in the sticky-prices model without ZLB constraint (Table 2); in the sticky-prices model with an occasionally binding ZLB constraint, the corresponding increases of union-wide output are 0.24% in a liquidity trap, and 0.26% when the ZLB constraint is slack (see Table 1, Panels (a.2),(b.2)).

However, the adjustment of relative (Home vs. Foreign) variables to country-specific shocks is distorted significantly, in the short term, by price stickiness, and thus it differs from the response in an RBC world. This is so because price stickiness slows the adjustment of the terms of trade to country-specific shocks. Monetary policy in a currency union cannot undo this distortion. However, importantly, the adjustment of country-level variables to persistent shocks does not depend much on whether the ZLB binds or not.

[15] A rise in Home government purchases worth 1% of Home steady state output reduces Home and Foreign consumption by 0.40% and 0.10%, on impact, so that world consumption falls by 0.25%. Thus, there is sizable consumption crowding out, which contributes to the modest fiscal spending multiplier (Home output rises by 0.50%, while Foreign output is unaffected, in the RBC model).

Consider the effects of a 1% Home productivity increase. In the sticky-prices model, the Home terms of trade deteriorate much less (-0.30%), on impact, than in the RBC world (-1%). This means that, under sticky prices, *relative* demand for Home output rises less (see (16)), while the risk sharing condition (6) implies a more muted rise in Home relative consumption. This explains why, under sticky prices, Home output and consumption rise less, while Foreign output and consumption rise more than in the flex-prices world. (In the flex-prices world, the 1% Home productivity increase raises Home output by 1%, on impact, while Foreign output is unchanged; in the sticky-prices model, Home and Foreign output rise by 0.68% and 0.38% on impact, respectively, when the ZLB binds; Home and Foreign output rise by 0.62% and 0.32%, respectively when the ZLB is slack.)

Price rigidity weakens the appreciation of the Home terms of trade triggered by a positive shock to Home government purchases. This implies that, under nominal rigidities, the shock triggers a stronger rise in Home output (than in the RBC world), and a *fall* in Foreign output. The international transmission of a (persistent) country-specific government purchases shock is, thus, negative in the sticky-prices model. This is the case both at the ZLB and away from the ZLB (see Table 1, Panels (a.2),(b.2)). In an expectations-driven liquidity trap, a rise in Home government purchases worth 1% of Home output raises Home GDP by 0.66%, on impact, while Foreign GDP drops by 0.19%. Home and Foreign consumption drop by 0.31% and 0.22%, on impact; Home consumption is crowded out less than in the flex-price model, but Foreign consumption falls more; this contributes to the greater rise in Home output and to the fall in Foreign output that occur in the liquidity trap. When the ZLB does not bind, then Home and Foreign output respond by 0.69% and -0.16%, respectively, on impact. As the fiscal shock triggers a fall in union-wide inflation at the ZLB, but a rise in union-wide inflation away from the ZLB, the union-wide fiscal spending multiplier is slightly smaller at the ZLB than away from the ZLB triggers.[16] This explains why the fall in Foreign output is slightly greater at the ZLB than away from the ZLB. [17]

[16] At the ZLB, Home inflation rises by 0.24 ppt p.a., while Foreign inflation falls by 0.37 ppt p.a.; the corresponding inflation changes away from the ZLB are +0.37 ppt and -0.24 ppt, respectively.

[17] Away from the ZLB, the fiscal spillover to Foreign output becomes weakly positive after 7 periods, but the cumulative effect (over an infinite horizon) of the Home fiscal shock on Foreign output is negative and amounts to -0.16% of quarterly steady state output, while the cumulative effect on Home output is 10.74%. At the ZLB, the corresponding cumulative effects on Foreign and Home output are -0.74% and 10.14%, respectively.

A similar mechanism also operates in response to a Home household demand (preference) shock. Price stickiness dampens the appreciation of the Home terms of trade, after a positive Home preference shock. This helps to understand why, under nominal rigidities, a positive Home preference shock *raises* Home output and lowers Foreign output, on impact. This is the case both when the ZLB constraint binds and when it is slack. (By contrast, in the RBC world, Home output falls, while Foreign output rises.)

4. Fundamentals-driven liquidity traps

This paper focuses on expectations-driven liquidity traps. As discussed in the Introduction, an extensive literature assumes that liquidity traps are triggered by large adverse demand shocks, such as an autonomous fall in households' subjective discount rates. Mertens and Ravn (2014) refer to liquidity traps of this type as 'fundamentals-driven' liquidity traps. A key assumption in fundamentals-driven liquidity trap models is that the liquidity trap ends permanently when the adverse aggregate demand shock subsides. See Holden (2016, 2019) for a comprehensive analysis of deterministic equilibria that feature a permanent exit from a liquidity trap. The literature shows that, in a fundamentals-driven liquidity trap, fiscal spending multipliers can be markedly higher than when the ZLB does not bind; also, a positive technology shock can trigger an output *contraction* (Eggertsson and Woodford, 2003).

Widely discussed studies by Erceg and Lindé (2010) and Blanchard et al. (2016) have presented quantitative models of a two-country monetary union that experiences a fundamentals-driven liquidity trap triggered by a large adverse household demand (preference) shock; these authors show that, in such a liquidity trap, a country-specific rise in government purchases can have a strong positive effect on domestic and *foreign* output, i.e. the predicted international fiscal spillover is sizable. [18]

For comparison purposes with the expectations-driven liquidity traps analysed in this paper, I now discuss a fundamentals-driven liquidity trap, in the stylised two-country model of a monetary union used above. Following Blanchard et al. (2016), I consider a scenario in which a liquidity trap is brought about by an unanticipated one-time shock at some initial date $t=0$ that depresses the union-wide natural real interest rate below its steady level, so that $\hat{R}_0^{nat}<0$. Except

[18] Open economy models of fundamentals-driven liquidity traps are also studied by, e.g., Cook and Devereux (2013), Fujiwara and Ueda (2013), Farhi and Werning (2016) and Acharya and Bengui (2018).

for shocks at date $t=0$, there are no random disturbances. Thus the economy evolves deterministically (perfect foresight), after $t=0$. There exists a unique deterministic equilibrium in which the economy permanently leaves the liquidity trap after a finite time interval.[19]

As there are no exogenous innovations after date $t=0$, the natural real interest rate at $t\geq 0$ is: $\widehat{R}_t^{nat} = \rho^t \cdot \widehat{R}_0^{nat} < 0$, where $0<\rho<1$ is the autocorrelation of the exogenous forcing processes and of the natural rate (see (14)).

In a deterministic equilibrium *without* ZLB constraint, the union-wide inflation rate and the nominal interest rate at dates $t\geq 0$ would be

$$\widehat{\Pi}_t^* = \lambda^S \rho^t \widehat{R}_0^{nat} \text{ and } \widehat{R}_t^* = \gamma_\pi \lambda^S \rho^t \widehat{R}_0^{nat},$$

respectively, where $\lambda^S > 0$ is the policy rule coefficient for a regime with a permanently slack ZLB constraint defined in (29). A fundamentals-driven liquidity trap occurs when the unconstrained nominal interest rate is negative at date $t=0$, i.e. when (expressing the interest rate in deviation from steady state):

$$\widehat{R}_0^* < -(\Pi - \beta)/\Pi. \tag{32}$$

This inequality holds when the real natural rate at date $t=0$ is sufficiently low. Assume that (32) applies, and let T^* be the smallest value of $t\geq 1$ for which $\widehat{R}_t^* \geq -(\Pi-\beta)/\Pi$. Thus,

$$\widehat{R}_{T^*-1}^* < -(\Pi-\beta)/\Pi \text{ and } \widehat{R}_{T^*}^* \geq -(\Pi-\beta)/\Pi.$$

The fundamentals-driven liquidity trap equilibrium studied by Blanchard et al. (2016) has the property that the ZLB constraint binds until period T^*-1, and that the ZLB does not bind in $t \geq T^*$. Thus, $\widehat{\Pi_t} = \widehat{\Pi_t^*}$ and $\widehat{R}_t = \widehat{R}_t^*$ hold for $t \geq T^*$. In periods $t < T^*$, the nominal interest rate is zero, i.e. $\widehat{R}_t = -(\Pi-\beta)/\Pi$. From Euler-Phillips equation (14), we see that union-wide inflation obeys the condition

$$\widehat{\Pi}_t = \kappa(\Pi-\beta)/\Pi + (1+\beta+\kappa)\widehat{\Pi}_{t+1} - \beta\widehat{\Pi}_{t+2} + \kappa\widehat{R}_t^{nat} \text{ for } 0 \leq t < T^*. \tag{33}$$

[19] The Phillips-Euler equation for union-wide inflation (14) does not include lagged endogenous state variables. Holden (2016, 2019) shows that this ensures that an equilibrium featuring eventual exit from the liquidity trap is unique.

Iterating backward in time using (33) allows to compute union-wide inflation in periods $0 \le t < T^*$. Importantly, the largest root of the "backward" inflation equation (33) exceeds unity. Thus, the backward inflation loop is explosive. This implies that the impact $(t=0)$ inflation response to a shock that triggers a fundamentals-driven liquidity trap can be large, as confirmed by the simulations below. Also, shocks to the natural real interest rate that induce small changes in the inflation rate in periods T^*, i.e. when the economy emerges from the liquidity trap, may have a big effect on inflation, and thus on output, at the start of the liquidity trap. This explains why fiscal multipliers in a fundamentals-driven liquidity trap can be (very) large.

Table 4 reports dynamic shock responses, in a fundamentals-driven liquidity trap. All preference and technology parameters are set at the values used previously. The forcing processes again have autocorrelation $\rho=0.95$.

Following Blanchard et al. (2016), I consider a baseline fundamentals-driven liquidity trap scenario that starts in period $t=0$, and that lasts 12 quarters. This baseline scenario is brought about by an unanticipated one-time -9.90% innovation to Home and Foreign preference shifters (Ψ) at $t=0$ that depresses the union-wide natural real interest rate by 49.50 basis points, on impact.

Panel (a) of Table 4 reports the adjustment process, under the baseline scenario. Panels (b.1)-(b.3) report dynamics under alternative scenarios that obtain when positive 1% date $t=0$ innovations to Home productivity, Home government purchases and the Home preference parameter Ψ_H are *added* to the baseline scenario. Effects of those Home shocks are shown as deviations from the baseline scenario.

The baseline liquidity trap scenario (Panel (a)) exhibits a sharp, but short-lived, contraction in union-wide inflation and output. Union-wide inflation drops by 28.56 ppt, 19.42 ppt, 13.36 ppt, 9.36 ppt and 6.71 ppt p.a. below steady state, in quarters 0 to 4 after the baseline shock, while union-wide output is 13.68%, 9.07%, 6.00%, 3.96% and 2.61% below steady state in the same periods.

During the liquidity trap, exogenous Home productivity, government purchases and preference shocks have a strong effect on output and inflation, in both countries, as can be seen in Panel (b) of Table 4. A 1% positive innovation (at $t=0$) to Home productivity lowers union-wide inflation and output by 13.14 ppt and 6.10%, respectively, on impact. Given the muted short-run response of the terms of trade, due to price stickiness, Home and Foreign output both drop sharply on impact, by 5.95% and 6.26%, respectively (Panel (b.1)). An innovation to Home government purchases worth 1% of Home steady state output raises union-wide inflation and output by 4.64 ppt and 2.59%, on impact (Panel (b.2)). This very strong rise in union-wide output and the muted terms of trade response imply that Home *and* Foreign output both rise strongly, in the short run. On impact, Home and Foreign output go up by 3.01% and 2.16%, respectively. Thus, the cross-country spillover of government purchases is positive and sizable.

5. Sensitivity analysis: less persistent shocks

The simulations presented so far have assumed persistent fundamental shocks $(\rho=0.95)$, in accordance with empirical autocorrelations and with autocorrelations typically assumed in macro models. However, it seems interesting to understand the effect of less persistent shocks, in a liquidity trap. Table 5 reports *impact* effects of Home productivity and government purchases shocks, in the sticky-prices model, assuming a $\rho=0.5$ shock autocorrelation; all other model parameters remain unchanged.[20] Panel (I) considers the model variant with expectations-driven ZLB regimes, while Panel (II) assumes a fundamentals-driven liquidity trap (12 quarters).

Transitory shocks have a bigger effect on the natural real interest rate than persistent shocks. In the model variant with expectations-driven ZLB regimes, union-wide and Home inflation are hence impacted more strongly by transitory Home shocks than by persistent shocks. When the ZLB constraint is slack, transitory shocks thus induce greater responses of the policy interest rate (see Panel (Ib)). This explains why responses to transitory shocks are less similar across the two (expectations-driven) ZLB regimes (than the responses to persistent shocks reported in Table 1). In an expectations-driven liquidity trap, the *qualitative* features of shock responses depend on shock persistence: e.g., a transitory rise in government purchases $(\rho=0.5)$ *increases* union-wide inflation (while a persistent shock *lowers* union-wide inflation; see Sect. 3.2.1.). A transitory innovation to Home government purchases worth 1% of steady state Home output boosts union-wide inflation by 0.46%; that strong inflation increase leads to a stronger increase in union-wide output (compared to the $\rho=0.95$ case) that is accompanied by a *rise* in

[20] With $\rho=0.5$, the dynamic effects of shocks die out very fast, and thus only impact effects are reported.

consumption. Home and Foreign output *both* increase, by 1.06% and 0.14%, respectively. The Home terms of trade improve less (+0.08%) than in response to a persistent government purchases increase, which explains why the spillover to Foreign output remains weak, although it is positive now. Note also that, in the expectations-driven liquidity trap, a transitory rise in Home productivity lowers Home and Foreign inflation and output.

In fact, for ρ=0.5, shock responses in the expectations-driven liquidity trap are qualitatively and quantitatively similar to responses in a fundamentals-driven liquidity trap (see Panel (II), Table 5). In the fundamentals-driven liquidity trap, the qualitative shock transmission features are not affected by shock persistence. With less persistent shocks, it remains true that a rise in Home government purchases increases Home and Foreign inflation and output, and that a rise in Home productivity depresses Home and Foreign inflation and output; however, inflation and output responses are much weaker when shocks are transitory. For ρ=0.5, the Home government purchases shock raises Home and Foreign output by 1.08% and 0.16%, respectively. In the fundamentals-driven liquidity trap, the cross-country spillover of a transitory fiscal spending shock is, hence, much more muted than the spillover of a persistent fiscal shocks (see Table 4). Thus, irrespective of the nature of the liquidity trap, a transitory fiscal shocks fails to generate strong cross-country output spillovers.

6. Conclusion

The paper investigates expectations-driven liquidity traps in a two-country New Keynesian model of a monetary union. In the model here, a rise in government purchases in an individual country has a weak effect on GDP in the rest of the union. The results caution against the idea of strong cross-border fiscal transmission in a monetary union.

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Table 1. New Keynesian model with expectations-driven ZLB regimes: dynamic responses to persistent exogenous shocks

Horizon	Y	C	Π	R	Y_H	Y_F	C_H	C_F	Π_H	Π_F	q	*Exogenous variables*
(a) Binding ZLB constraint												
(a.1) Home productivity increase												θ_H
0	0.53	0.53	0.14	0.00	0.68	0.38	0.62	0.44	-0.47	0.75	-0.30	1.00
4	0.43	0.43	0.11	0.00	0.78	0.09	0.64	0.23	0.06	0.16	-0.69	0.81
10	0.32	0.32	0.08	0.00	0.62	0.02	0.50	0.14	0.13	0.03	-0.60	0.60
(a.2) Home government purchases increase												G_H
0	0.24	-0.27	-0.07	0.00	0.66	-0.19	-0.31	-0.22	0.24	-0.37	0.15	1.00
4	0.19	-0.22	-0.06	0.00	0.43	-0.04	-0.32	-0.11	-0.03	-0.08	0.34	0.81
10	0.14	-0.16	-0.04	0.00	0.29	-0.01	-0.25	-0.07	-0.07	-0.02	0.30	0.60
(a.3) Home private demand increase (preference shock)												Ψ_H
0	-0.03	-0.03	-0.14	0.00	0.15	-0.21	0.40	-0.46	0.35	-0.62	0.24	1.00
4	-0.02	-0.02	-0.11	0.00	-0.06	0.01	0.22	-0.27	-0.07	-0.16	0.55	0.81
10	-0.02	-0.02	-0.08	0.00	-0.08	0.04	0.14	-0.17	-0.12	-0.04	0.48	0.60
(b) Slack ZLB constraint												
(b.1) Home productivity increase												θ_H
0	0.47	0.47	-0.13	-0.19	0.62	0.32	0.56	0.38	-0.74	0.48	-0.30	1.00
4	0.38	0.38	-0.11	-0.16	0.73	0.04	0.59	0.18	-0.16	-0.05	-0.69	0.81
10	0.28	0.28	-0.08	-0.12	0.58	-0.02	0.46	0.10	-0.03	-0.13	-0.60	0.60
(b.2) Home government purchases increase												G_H
0	0.26	-0.24	0.06	0.10	0.69	-0.16	-0.28	-0.19	0.37	-0.24	0.15	1.00
4	0.22	-0.19	0.05	0.08	0.45	-0.02	-0.30	-0.09	0.08	0.03	0.34	0.81
10	0.16	-0.14	0.04	0.06	0.31	0.01	-0.23	-0.05	0.01	0.07	0.30	0.60
(b.3) Home private demand increase (preference shock)												Ψ_H
0	0.03	0.03	0.13	0.19	0.21	-0.15	0.46	-0.40	0.62	-0.36	0.24	1.00
4	0.02	0.02	0.11	0.16	-0.01	0.05	0.27	-0.22	0.15	0.06	0.55	0.81
10	0.02	0.02	0.08	0.12	-0.04	0.08	0.17	-0.14	0.04	0.12	0.48	0.60

Notes: Dynamic responses to 1% innovations to Home forcing variables are shown (innovation to government purchases represents 1% of steady state Home output). Persistence of ZLB regimes: $p_{SS}=p_{BB}=0.95$. Autocorrelation of fundamental shocks: $\rho=0.95$. Responses 0, 4 and 10 periods after the shock are shown (see Column labelled 'Horizon'). Panel (a) assumes that the ZLB constraint binds (liquidity trap); Panel (b) assumes that the ZLB constraint is slack. Variables: union-wide output, consumption, inflation and nominal interest rate (Y,C,Π,R); Home (H) and Foreign (F) output (Y_H,Y_F), consumption (C_H,C_F), inflation (Π_H, Π_F) and Home terms of trade (q). Output, consumption and the terms of trade are reported as % deviations from steady state; interest rate and inflation are reported as percentage point (ppt) per annum differences from steady state.
Source: Author's calculations.

Table 2. New Keynesian model in which the ZLB never binds: dynamic responses to persistent exogenous shocks

Horizon	*Y*	*C*	*Π*	*R*	Y_H	Y_F	C_H	C_F	$Π_H$	$Π_F$	*q*
(1) Home productivity increase											
0	0.49	0.49	-0.18	-0.27	0.64	0.33	0.58	0.40	-0.79	0.43	-0.30
4	0.40	0.40	-0.15	-0.22	0.74	0.05	0.60	0.19	-0.20	-0.09	-0.69
10	0.29	0.29	-0.11	-0.16	0.59	-0.01	0.47	0.11	-0.05	-0.16	-0.60
(2) Home government purchases increase											
0	0.26	-0.24	0.09	0.13	0.68	-0.17	-0.29	-0.20	0.39	-0.22	0.15
4	0.21	-0.20	0.07	0.11	0.45	-0.03	-0.30	-0.10	0.09	0.04	0.34
10	0.15	-0.15	0.05	0.08	0.30	0.00	-0.24	-0.06	0.03	0.08	0.30
(3) Home private demand increase (preference shock)											
0	0.01	0.01	0.18	0.27	0.19	-0.17	0.44	-0.41	0.67	-0.31	0.24
4	0.01	0.01	0.15	0.22	-0.02	0.04	0.25	-0.23	0.19	0.10	0.55
10	0.01	0.01	0.11	0.16	-0.05	0.07	0.16	-0.15	0.07	0.15	0.48

Notes: Responses to 1% innovations to Home forcing variables are reported. See Table 1 for further information. Source: Author's calculations.

Table 3. RBC model: dynamic responses to persistent exogenous shocks

Horizon	*Y*	*C*	*Π*	*R*	Y_H	Y_F	C_H	C_F	$Π_H$	$Π_F$	*q*
(1) Home productivity increase											
0	0.50	0.50	---	---	1.00	0.00	0.80	0.20	---	---	-1.00
4	0.41	0.41	--	---	0.81	0.00	0.65	0.16	---	---	-0.81
10	0.30	0.30	---	---	0.60	0.00	0.48	0.12	---	---	-0.60
(2) Home government purchases increase											
0	0.25	-0.25	---	---	0.50	0.00	-0.40	-0.10	---	---	0.50
4	0.20	-0.20	---	---	0.41	0.00	-0.33	-0.08	---	---	0.41
10	0.15	-0.15	---	---	0.30	0.00	-0.24	-0.06	---	---	0.30
(3) Home private demand increase (preference shock)											
0	0.00	0.00	---	---	-0.10	0.10	0.26	-0.26	---	---	0.80
4	0.00	0.00	---	---	-0.08	0.08	0.21	-0.21	---	---	0.65
10	0.00	0.00	---	---	-0.06	0.06	0.16	-0.16	---	---	0.48

Notes: Responses to 1% innovations to Home forcing variables are reported. See Table 1 for further information. Source: Author's calculations.

Table 4. New Keynesian model with fundamentals-driven liquidity trap: baseline liquidity trap scenario and dynamic responses to persistent exogenous shocks

Horizon	Y	C	Π	R	Y_H	Y_F	C_H	C_F	Π_H	Π_F	q	*Exogenous variables*
(a) Baseline scenario (triggered by adverse shocks to Home and Foreign private demand)												Ψ_H & Ψ_F
0	-13.68	-13.68	-28.56	0.00	-13.68	-13.68	-13.68	-13.68	-28.56	-28.56	0.00	-9.90
4	-2.61	-2.61	-6.71	0.00	-2.61	-2.61	-2.61	-2.61	-6.71	-6.71	0.00	-7.66
10	-0.20	-0.20	-2.12	0.00	-0.20	-0.20	-0.20	-0.20	-2.12	-2.12	0.00	-5.93
12	-0.15	-0.15	-1.89	0.15	-0.15	-0.15	-0.15	-0.15	-1.89	-1.89	0.00	-5.35
(b) Dynamic responses to shocks (shown as difference relative to baseline scenario)												
(b.1) Home productivity increase												θ_H
0	-6.10	-6.10	-13.14	0.00	-5.95	-6.26	-6.01	-6.19	-13.74	-12.53	-0.30	1.00
4	-0.88	-0.88	-2.53	0.00	-0.54	-1.23	-0.68	-1.09	-2.58	-2.47	-0.69	0.81
10	0.21	0.21	-0.19	0.00	0.51	-0.09	0.39	0.03	-0.14	-0.24	-0.60	0.60
12	0.26	0.26	-0.10	-0.14	0.54	-0.01	0.43	0.10	-0.04	-0.15	-0.54	0.54
(b.2) Home government purchases increase												G_H
0	2.59	2.09	4.64	0.00	3.01	2.16	2.04	2.13	4.94	4.33	0.15	1.00
4	0.66	0.25	0.88	0.00	0.90	0.43	0.15	0.36	0.91	0.86	0.34	0.81
10	0.17	-0.13	0.07	0.00	0.32	0.02	-0.21	-0.04	0.04	0.09	0.30	0.60
12	0.14	-0.13	0.05	0.07	0.27	0.01	-0.21	-0.05	0.02	0.07	0.27	0.54
(b.3) Home private demand increase (preference shock)												Ψ_H
0	4.65	4.65	9.23	0.00	4.83	4.47	5.08	4.23	9.71	8.74	0.24	1.00
4	0.91	0.91	1.76	0.00	0.88	0.94	1.15	0.67	1.80	1.71	0.55	0.81
10	0.05	0.05	0.13	0.00	-0.01	0.11	0.21	-0.11	0.09	0.18	0.48	0.60
12	0.01	0.01	0.10	0.14	-0.05	0.06	0.15	-0.13	0.05	0.14	0.44	0.54

Notes: Panel (a) reports baseline scenario of a fundamentals-driven liquidity trap lasting 12 quarters. Panel (b) reports responses to 1% innovations to Home forcing variables (the innovation to government purchases represents 1% of steady state Home output). Responses 0, 4, 10 and 12 periods after the shock are shown. Responses in Panel (b) are shown as differences compared to the baseline scenario. Output, consumption and the terms of trade are reported as % deviations from steady state; the interest rate and inflation are reported as ppt p.a. differences from steady state. Source: Author's calculations.

Table 5. New Keynesian model versions: impact responses to less persistent shocks

	Y	C	Π	R	Y_H	Y_F	C_H	C_F	Π_H	Π_F	q
(I) Model version with expectations-driven ZLB regimes											
(Ia) Binding ZLB constraint											
Home productivity increase	-0.20	-0.20	-0.92	0.00	-0.12	-0.28	-0.15	-0.25	-1.26	-0.58	-0.17
Home government purchases increase	0.60	0.10	0.46	0.00	1.06	0.14	0.08	0.12	0.63	0.29	0.08
(Ib) Slack ZLB constraint											
Home productivity increase	0.19	0.19	-0.44	-0.66	0.27	0.11	0.24	0.14	-0.78	-0.10	-0.17
Home government purchases increase	0.40	-0.10	0.22	0.33	0.86	-0.06	-0.12	-0.08	0.39	0.05	0.08
(II) Model version with fundamentals-driven liquidity trap											
Home productivity increase	-0.24	-0.24	-0.98	0.00	-0.16	-0.32	-0.19	-0.29	-1.32	-0.65	-0.17
Home government purchases increase	0.62	0.12	0.49	0.00	1.08	0.16	0.09	0.14	0.66	0.32	0.08

Notes: Impact responses to a 1% innovation to Home productivity and government purchases are shown. (the innovation to government purchases represents 1% of steady state Home output). Autocorrelation of productivity and government purchases: 0.5 (all other parameters unchanged).
Panel (I): model variant with expectations-driven ZLB regimes (same set-up as in Table 1).
Panel (II): model variant with a fundamentals-driven liquidity trap (12 quarters); same set-up as in Table 4. The fundamentals-driven liquidity trap is caused by a persistent one-time preference shock of autocorrelation 0.95 (baseline liquidity trap scenario is, thus, identical to scenario in Panel (a) of Table 4). See Tables 1 and 4 for further information. Source: Author's calculations.

Not-for-Publication Appendix

I) Expectations-driven liquidity traps: technical aspects

This Appendix provides technical explanations regarding the computation of equilibria with expectations-driven liquidity traps, in the economy with constant productivity, government purchases and preferences (Sect. 3.1). As explained in Sect. 3.1, the model with a ZLB constraint has equilibria in which an exogenous random sunspot variable determines whether the ZLB constraint is slack or whether it binds. Assume that union-wide inflation follows a Markov chain with two possible values: $\widehat{\Pi}_t \in \{\widehat{\Pi}^S, \widehat{\Pi}^B\}$ such that $\widehat{\Pi}^B < \widehat{\Pi}^S$ and

$$\gamma_\pi \widehat{\Pi}^B \leq -(\Pi-\beta)/\Pi < \gamma_\pi \widehat{\Pi}^S. \tag{20}$$

As can be seen from the linearised Taylor rule (11), equation (20) implies that the ZLB constraint binds when low inflation $\widehat{\Pi}^B$ is realised, and that the ZLB constraint is slack when high inflation $\widehat{\Pi}^S$ obtains. Denote the transition probabilities between union-wide inflation at dates t and at t+1 by $p_{ij} \equiv \text{Prob}(\widehat{\Pi}_{t+1}=\widehat{\Pi}^j | \widehat{\Pi}_t=\widehat{\Pi}^i)$ for $i,j \in \{\text{S;B}\}$, with $0 \leq p_{ij} \leq 1$ and $p_{iS}+p_{iB}=1$, and let $\Phi \equiv \begin{bmatrix} p_{SS} & p_{SB} \\ p_{BS} & p_{BB} \end{bmatrix}$ be the matrix of transition probabilities. Let $\vec{\widehat{\Pi}} \equiv [\widehat{\Pi}^S, \widehat{\Pi}^B]^{\mathrm{T}}$ (column vector). Expected future inflation is then $E(\widehat{\Pi}_{t+1} | \widehat{\Pi}_t=\widehat{\Pi}^S) = \Phi_{1,\bullet}\, \vec{\widehat{\Pi}}$, $E(\widehat{\Pi}_{t+1} | \widehat{\Pi}_t=\widehat{\Pi}^B) = \Phi_{2,\bullet}\, \vec{\widehat{\Pi}}$, $E(\widehat{\Pi}_{t+2} | \widehat{\Pi}_t=\widehat{\Pi}^S) = \Phi^2_{1,\bullet}\, \vec{\widehat{\Pi}}$, $E(\widehat{\Pi}_{t+2} | \widehat{\Pi}_t=\widehat{\Pi}^B) = \Phi^2_{2,\bullet}\, \vec{\widehat{\Pi}}$, where $\Phi^2 \equiv \Phi \times \Phi$. $\Phi_{i,\bullet}$ and $\Phi^2_{1,\bullet}$ (for i=1,2) denote row i of matrices Φ and Φ^2, respectively.

An equilibrium with an occasionally binding ZLB is defined by inflation rates $\widehat{\Pi}^S, \widehat{\Pi}^B$ and transition probabilities p_{SS}, p_{BB} such that the inequalities in (20) and the Euler-Phillips equation (14) hold:

$$\gamma_\pi \widehat{\Pi}^S = -\tfrac{1}{\kappa}\widehat{\Pi}^S + (1+\tfrac{1+\beta}{\kappa})\Phi_{1,\bullet}\vec{\widehat{\Pi}} - \tfrac{\beta}{\kappa}\Phi^2_{1,\bullet}\vec{\widehat{\Pi}},$$

$$-(\Pi-\beta)/\Pi = -\tfrac{1}{\kappa}\widehat{\Pi}^B + (1+\tfrac{1+\beta}{\kappa})\Phi_{2,\bullet}\vec{\widehat{\Pi}} - \tfrac{\beta}{\kappa}\Phi^2_{2,\bullet}\vec{\widehat{\Pi}}.$$

These equations are, respectively, the Euler-Phillips equation if the ZLB constraint is slack and if it binds.

II) Model variant with a convenience yield on safe bonds

As pointed out by a referee, the model-predicted inflation rate during a permanent expectations-driven liquidity trap (-1.22% per annum; see Sect. 3.1) is lower than recent Euro Area (EA) inflation. The ECB's main policy rate (MRO) fell to 0.25% p.a. in Nov. 2013, and has been zero since 2016. Mean EA inflation in 2014–2020 was 0.76%. The *real* ECB policy rate has been negative since the end of the global financial crisis, GFC (mean real policy rate 2010-20: -1.10%). Real rates on short-term government debt regarded as safe (i.e. sovereign debt issued by Germany and other core EA countries) too has been negative in the aftermath of the GFC. Real equity returns have been much higher during that period (mean MSCI Euro stock return 2014-2020: 5.72%).

It has been argued that ultra-low post-GFC rates on safe bonds (relative to returns on risky assets) have been driven by an elevated 'convenience yield' on those bonds, due to heightened risk aversion and a global shortage of safe assets (e.g., Caballero and Farhi, 2017). As shown in this Appendix, a safe-bond convenience yield can also explain why inflation in the recent EA liquidity trap has been positive.

To capture a convenience yield on safe bonds, assume that households discount safe bond returns using a subjective discount factor 'b' that is larger than the discount factor β used in the valuation of risky payment streams, and in firm price setting decisions, i.e. $b>\beta$ (see, e.g., Pfeiffer et al., 2020). Then, the bond Euler equation becomes $R_t E_t b(\Psi_{H,t+1}/\Psi_{H,t})(C_{H,t}/C_{H,t+1})/\Pi^{CPI}_{H,t+1}=1$ (instead of (4)), but the discount factor in the Phillips equation continues to be given by β (see (9)). To target inflation Π, the central bank now uses the interest rate rule $R_t=Max\{1,\Pi/b+(\gamma_\pi/b)\cdot(\Pi_t-\Pi)\}$ (instead of (5)). Provided $b<\Pi$, there then exists a steady state liquidity trap equilibrium, in which the gross inflation rate is b, while the gross real policy rate is $1/b$. Thus, a model with $\Pi>b>1>\beta$ has a liquidity trap equilibrium with *positive* inflation and a negative real policy rate.

E.g., setting b=1.0025 produces a -1% p.a real policy rate, and a 1% p.a. inflation rate in a permanent liquidity trap. Keeping all other parameters unchanged at the baseline values discussed in the main text (including β=0.9975), the model variant with occasionally binding ZLB constraints ($p_{BB}=p_{SS}$=0.95; see Sect. 3.2.3) now predicts that inflation rates under binding and slack ZLB constraints are 0.92% and 1.85% p.a., respectively.

When $b>\beta$ is assumed, the only change to the linearised model equations is that the term $Max\{-(\Pi-\beta)/\Pi,\gamma_\pi\cdot\widehat{\Pi}_t\}$ is replaced by $Max\{-(\Pi-b)/\Pi,\gamma_\pi\cdot\widehat{\Pi}_t\}$ in the linearised interest rate rule (11) and in the Euler-Phillips equation (14). Importantly, impulse responses to fundamental shocks are unchanged when $b>\beta$ holds (compared to the baseline model with $b=\beta$): shock responses depend on β, but not on b. Thus, the key results discussed in the main text (Sect. 3.2 and 3.3), regarding the transmission of fundamental shocks, under the two ZLB regimes, continue to go through.