Yield Curve Momentum

Markus Sihvonen*

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Abstract

I analyze time series momentum along the Treasury term structure. Past bond returns predict future returns both due to autocorrelation in bond risk premia and because unexpected bond return shocks increase the premium. Yield curve momentum is primarily due to autocorrelation in yield changes rather than autocorrelation in bond carry and can largely be captured using a single bond return or yield change factor. Because yield changes are partly induced by changes in the federal funds rate, yield curve momentum is related to post-FOMC announcement drift. The momentum factor is unspanned by the information in the term structure today and is hence inconsistent with standard term structure, macrofinance and behavioral models. I argue that the results are consistent with a model with unpriced longer term dependencies.

Keywords: Bond risk premia, time series momentum, term structure models, post-FOMC announcement drift.

JEL classification: G12, E43, E47

*Bank of Finland, Research Unit, Snellmaninaukio, P.O. Box 160. Email: markus.sihvonen@bof.fi. I thank Michael Bauer, Ryan Chahrour, Emanuel Mönch, George Pennacchi, Andrea Polo, Dimitri Vayanos and Michael Weber as well as participants at the Econometric Society European Winter Meeting and Dynamic Econometrics Conference for useful comments.

1 Introduction

Past returns can predict future returns (Fama, 1965). Moskowitz et al. (2012) find evidence of medium horizon return autocorrelation among a large set of asset classes. They dub this phenomenon ”time series momentum”.[1]

Possibly due to a focus on a large set of asset classes, the time series momentum literature has evolved largely separately from the vast literature on term-structure modelling and bond risk premia (see e.g. Ang and Piazzesi (2003), Fama and Bliss (1987) and Cochrane and Piazzesi (2005)). Because of this disconnect it is for example not clear whether time series momentum of government bonds is consistent with standard term-structure models.[2] This paper is an attempt to study the finer dynamics of time series momentum of government bonds, or yield curve momentum, and close the gap between the two literatures.

I argue that the findings of Moskowitz et al. (2012) are not necessarily inconsistent with standard models. However, I present new evidence related to yield curve momentum, which clearly is incompatible with such models.

First, I find that the term structure of momentum coefficients is downward sloping. Slope coefficients from regressing bond returns on the past return of the same maturity bond decline in bond maturity.

Second, I argue that yield curve momentum occurs both because of autocorrelation in bond yield changes and bond carry. However, because bond carry has small variation, most of the covariance between current and

[1] This is a growing literature, see e.g. Pitkäjärvi et al. (2020), Huang et al. (2020) and Goyal and Jegadeesh (2018).

[2] Durham (2013) analyzes the performance of a duration neutral cross-sectional momentum strategy with government bonds. He argues that some its profitability can be explained by a specific affine term structure model. However, he does not address time series momentum. Asness et al. (2013) study a cross-country momentum strategy with government bonds finding that such a strategy yields positive yet fairly small returns. Brooks and Moskowitz (2017) explain bond returns using value, momentum and carry factors. However, they do not study the sources of momentum or relate the findings to the term structure modelling literature. Osterrieder and Schotman (2017) connect bond return autocorrelations with model risk parameters but do not explicitly address momentum.

past returns is due to autocorrelation in bond yield changes.

I also decompose yield curve momentum into autocovariance in bond risk premia and covariance between bond risk premia and past unexpected news to bond returns. On average the first channel explains roughly one third of yield curve momentum while the second explains two thirds of it.

Third, I analyze the factor structure of yield curve momentum. I find that yield curve momentum can be largely captured by the change in the first principal component of yields or a single momentum factor defined as the average past return of different maturity bonds.

Fourth, I assess the relationship between monetary policy and yield curve momentum. Because changes in the Treasury yield curve are related to changes in the federal funds target rate, yield curve momentum is partly induced by monetary policy. That is yield curve momentum is in part driven by a drift pattern following a recent, expected or unexpected, rate change by the Fed. However, because especially long maturity yields display movements unrelated to target rate changes, yield curve momentum is not identical to post-FOMC announcement drift discussed in Brooks et al. (2019).

Fifth, I analyze whether yield curve momentum is consistent with standard term-structure models. The standard models imply that yields are affine in a set of factors. This form is also implied by standard macrofinance models, at least up to first order. These models can in principle generate covariance between current and past bond returns. However, this correlation should vanish after controlling for information in the current yield curve. The intuition is that, in this class models, the current factors determine the expected bond returns and after controlling for these factors no other variable should predict bond returns. On the other hand, these current factors are priced in the yield curve today. But then controlling for sufficiently many yields today is equivalent to controlling for the factors. I explain that this intuition carries to more complicated models after controlling for the generally non-linear relationship between bond returns and past yields.

I find that past bond returns predict future returns also conditional on the information in the yield curve today. Hence the spanning condition implied by standard models is violated in the data.

This point is similar to that made by Joslin et al. (2014), who study affine term structure models with macroeconomic factors. Emprically macro variables predict returns even after controlling for current yields. They therefore argue that the data can only be explained by a model with unspanned macro factors. However, they do not address momentum or consider the predictive power of past returns.

Can behavioral theories resolve my findings? Not necessarily. The reasons is that the current behavioral models still imply the same affine form for yields though the coefficients and factors might be different from rational models. Therefore these models still cannot generate yield curve momentum conditional on all the information in the yield curve today.

However, I propose a model that is consistent with the above empirical findings. In this model, factors exhibit longer term dependencies. However, these longer term relations are not priced in the term structure of interest rates today. Because past returns include information about such unpriced dependencies, they predict future returns also conditional on current yields.

I discuss two possible economic explanations for longer term dependencies to be unpriced. The first is a simple behavioral narrative: agents do not understand that factor dynamics have longer dependencies and bond prices reflect this misunderstanding. Second, I sketch a model with rational arbitraugeurs and simple rule-based traders. In this model, the demand from rule-based traders affects the duration risk that must be absorbed by arbitraugers. This effect can offset some of the effects of expected short rates on bond prices and imply a violation of the standard spanning condition.

2 Data and Definitions

I use the dataset on zero coupon US Treasury yields constructed by Liu and Wu (2020). These yields are built using a novel non-parametric method, which implies lower pricing errors compared to previous interpolation procedures. I apply a sample of end of month data between August 1971 and

December 2019 and focus on the yields and returns on 1 to 10 year bonds as well 1 month bills. In the appendix I show that the key results are robust to using the alternative dataset constructed by Gürkaynak et al. (2007), the data concerning the German yield curve available on the Bundesbank webpage and the Bloomberg US Treasury Index.

I obtain the federal funds target rate and the relevant target ranges from FRED. For monetary policy shock identification I utilize a series of the front month federal funds futures contract listed on the CME. Finally, I use the information on the Federal Reserve web page to create a series of the meeting dates of the Federal Open Market Committee.

I denote the monthly continuously compounded yield of maturity n by y_t^n. The logarithmic excess monthly return of maturity n bond is then given by

$$rx_{t+1}^n = -(n-1)y_{t+1}^{n-1} + ny_t^n - y_t^1 \tag{1}$$

and the return between periods t and any $t+h$, $rx_{h,t+h}^n$, is given by the sum over the one period excess returns.

3 Regression Evidence

I start by considering a simple regression of the form

$$rx_{t+1}^n = \alpha + \beta rx_{t-h,t}^n + \epsilon_{t+1} \tag{2}$$

that is I regress the excess monthly return of an n maturity bond on the excess return of an n maturity bond between periods $t-h$ and t. When calculating excess returns I hold maturity constant that is roll over the bond each month. I focus on lookback horizons (h) of 1,3,6 and 12 months. The results are given in table 1 and demonstrated further in figure 1.

The results are statistically significant for the return over the past month. However, the results for longer horizon past returns are not significant. Therefore, for the rest of this paper, I focus on the one month horizon. This is in contrast to Moskowitz et al. (2012) who focus on 1 year past returns.[3] I also ignore the volatility scaling applied by Moskowitz et al. (2012) as it can induce return predictability unrelated to raw momentum in returns as discussed in Kim et al. (2016) and Huang et al. (2020).

The regression betas decline in bond maturity. Hence the term structure of momentum coefficients is downward sloping. In the theoretical section I show that this is inconsistent with one factor interest rate models.

The results for the 1 month horizon have strong economic significance illustrated in table 2 and figures 2 and 3. These show the mean excess returns and annualized Sharpe ratios for different maturity bonds both for the full sample and in two subsamples with positive and negative past month excess returns for the same maturity bond. The mean returns and Sharpe ratios are substantially higher following positive rather than negative past month returns. The mean returns are increasing in bond maturity but Sharpe ratios decreasing in maturity. The Sharpe ratios of short maturity bonds are over 0.8 for months following positive excess returns in the previous month.

Figure 4 provides an alternative way to look at the above momentum patterns. It shows the share of total excess bond returns explained by excess returns in months with positive past month excess returns. For all maturities the bulk of returns comes from months with positive past month returns. For many maturities this share is more than 100 per cent because average returns in months with negative past month returns are negative. Because on average only 56 % of months show positive excess returns, these relationships are not mechanical. The appendix contains additional results concerning the invesment performance of a simple momentum strategy.

Factor momentum Yields and bond returns are often found to exhibit strong factor structures (see e.g. Cochrane and Piazzesi (2005)). Hence yield curve momentum might also be captured well using a simple factors. I next demonstrate that most of this momentum can indeed be catched using a

[3]Note that here the significance of 1 year past returns is somewhat better than for 3 and 6 month past returns.

	1 month lookback					3 month lookback				
Mat.	α	t-value	β	t-value	R^2	α	t-value	β	t-value	R^2
1	0.06	3.42	0.19	3.01	3.69	0.06	2.82	0.03	0.57	0.30
2	0.08	2.67	0.18	3.70	3.20	0.10	2.40	0.02	0.31	0.08
3	0.12	2.82	0.15	3.13	2.27	0.14	2.63	0.01	0.20	0.03
4	0.15	2.57	0.12	2.73	1.51	0.17	2.45	0.01	0.20	0.02
5	0.17	2.44	0.12	2.71	1.38	0.19	2.34	0.01	0.25	0.02
6	0.20	2.38	0.10	2.02	0.91	0.21	2.30	0.01	0.31	0.03
7	0.20	2.17	0.10	2.05	0.91	0.22	2.13	0.01	0.24	0.02
8	0.22	2.11	0.10	2.11	0.98	0.24	2.08	0.01	0.31	0.03
9	0.22	1.90	0.10	2.14	0.92	0.23	1.89	0.01	0.28	0.02
10	0.24	1.90	0.09	1.99	0.79	0.26	1.89	0.01	0.24	0.02
	6 month lookback					12 month lookback				
1	0.06	2.73	0.02	0.62	0.26	0.05	1.81	0.02	1.23	0.97
2	0.10	2.28	0.01	0.47	0.12	0.07	1.56	0.03	1.47	1.05
3	0.14	2.49	0.01	0.47	0.09	0.10	1.72	0.03	1.54	0.95
4	0.16	2.34	0.01	0.45	0.06	0.12	1.69	0.02	1.53	0.79
5	0.19	2.26	0.01	0.37	0.04	0.15	1.70	0.02	1.44	0.63
6	0.21	2.21	0.01	0.49	0.06	0.17	1.69	0.02	1.45	0.61
7	0.21	2.04	0.01	0.47	0.05	0.17	1.62	0.02	1.37	0.53
8	0.23	1.99	0.01	0.58	0.08	0.19	1.61	0.02	1.40	0.54
9	0.23	1.83	0.01	0.39	0.04	0.20	1.49	0.02	1.30	0.47
10	0.26	1.84	0.01	0.31	0.02	0.22	1.51	0.02	1.23	0.42

Table 1: shows the results from regressing the excess returns of different maturity bonds (years) on the past return for the same maturity bond for lookback horizons of 1,3,6 and 12 months. The t-values are based on Newey and West (1987) standard errors and the lag selection procedure of Newey and West (1994)

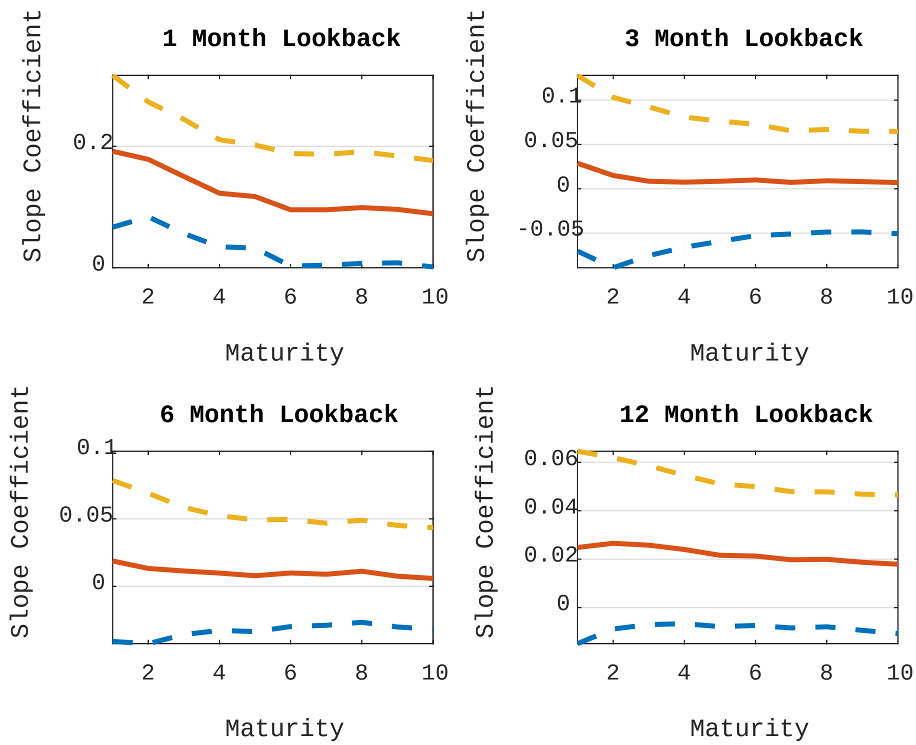

Figure 1: shows the slope coefficients and the relevant 95% confidence bounds from regressing the returns of different maturity bonds (years) on the past return for the same maturity bond for lookback horizons of 1,3,6 and 12 months.

Average excess returns (%)

Maturity	1	2	3	4	5	6	7	8	9	10
All months	0.07	0.10	0.15	0.17	0.20	0.22	0.22	0.25	0.24	0.26
Positive past month ret.	0.09	0.17	0.26	0.30	0.37	0.40	0.38	0.41	0.49	0.57
Negative past month ret.	0.04	0.01	0.00	0.01	-0.02	-0.01	0.03	0.05	-0.05	-0.11
Sharpe ratios (annual)										
All months	0.57	0.43	0.44	0.40	0.38	0.36	0.33	0.33	0.29	0.29
Positive past month ret.	0.83	0.82	0.78	0.71	0.72	0.66	0.56	0.53	0.59	0.64
Negative past month ret.	0.26	0.04	0.00	0.02	-0.04	-0.01	0.04	0.06	-0.07	-0.12

Table 2: shows the mean excess returns and annualized Sharpe ratios for different maturity bonds in both the full sample and in two subsamples: following positive and negative past month excess returns.

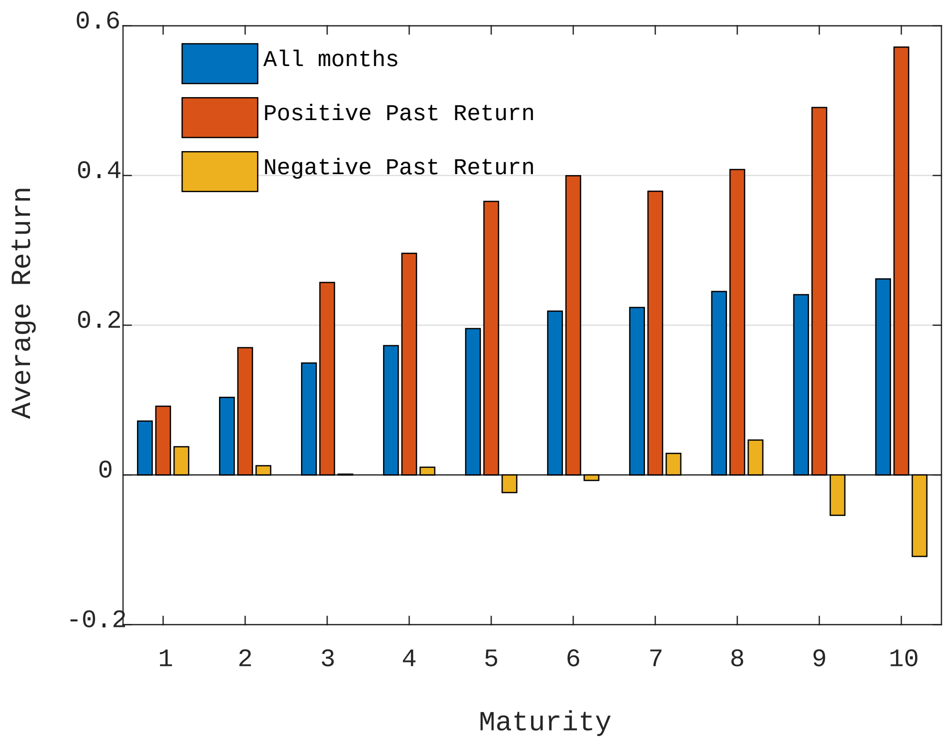

Figure 2: shows the mean returns for different maturity bonds both for the full sample and in subsamples following positive and negative past month returns.

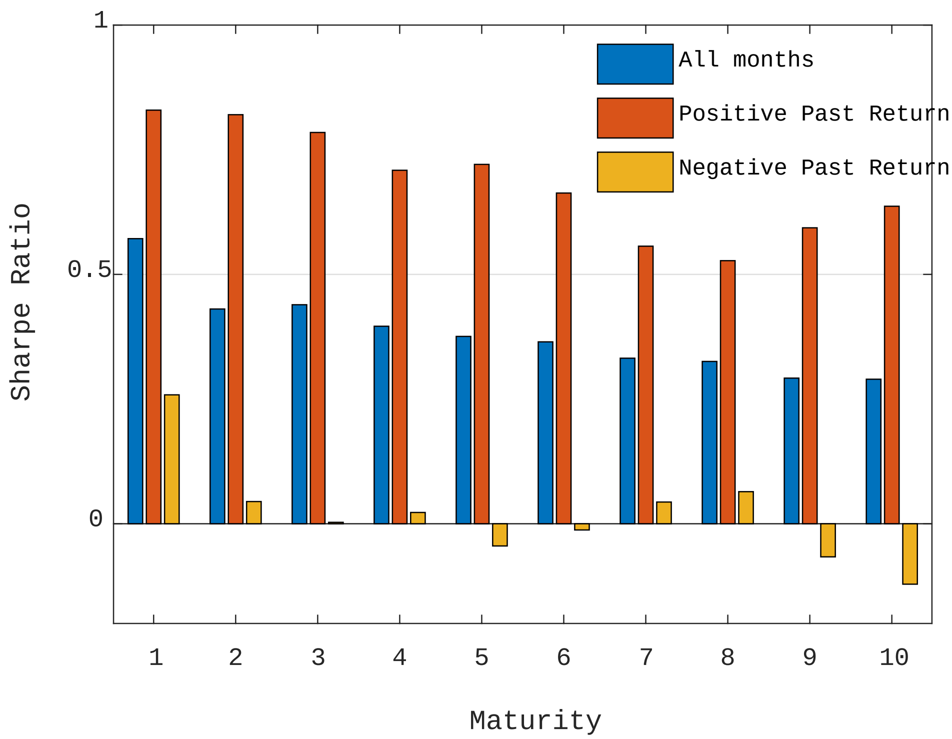

Figure 3: shows the annualized Sharpe ratios for different maturity bonds both for the full sample and in subsamples following positive and negative past month returns.

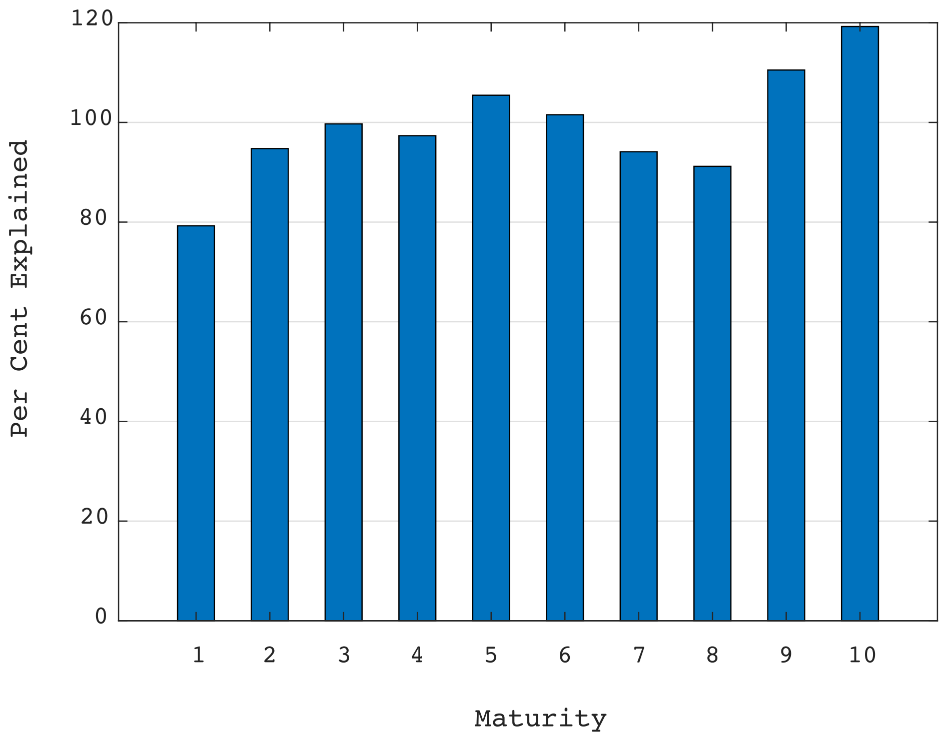

Figure 4: shows the share of total excess returns of different maturity bonds earned in months with positive past month excess returns.

Mat.	α	t-value	β	t-value	R^2
1	0.06	3.58	0.04	2.37	3.16
2	0.09	2.68	0.08	2.75	2.73
3	0.13	2.84	0.09	2.57	2.04
4	0.15	2.56	0.09	2.43	1.31
5	0.17	2.43	0.11	2.45	1.22
6	0.19	2.37	0.14	2.65	1.46
7	0.19	2.14	0.15	2.59	1.42
8	0.21	2.08	0.17	2.55	1.33
9	0.21	1.85	0.17	2.45	1.17
10	0.23	1.85	0.18	2.27	1.05

Table 3: shows the results from regressing the returns of different maturity bonds on the previous month average return of different maturity bonds. The t-values are based on Newey and West (1987) standard errors.

single factor.

Let us create a simple average of the different maturity bond returns as

$$\bar{rx}_{t+1} = \frac{1}{10} \sum_{n \in N} rx^n_{t+1}, \tag{3}$$

where $N = \{12, 24, 36, 48, 60, 72, 84, 96, 108, 120\}$. I then run a regression

$$rx^n_{t+1} = \alpha + \beta \bar{rx}_t + \epsilon_{t+1} \tag{4}$$

The results are given in table 3. Using the average of excess returns across different maturity bonds leads to only a minor loss in predictive power relative to using the past return of a bond with the corresponding maturity. For longest maturity bonds the R^2 actually increases but this improvement is small. I confirm this overall result in the next section by showing that the momentum is driven by a change in the first principal component of yields. Note that the loadings for the momentum factor are still different for returns based on different maturity bonds.

4 Decompositions

What is driving the results obtained in the previous section? I next analyze the sources of yield curve momentum using three decompositions. The first is based on decomposing bond returns into a carry and yield change component. The second decomposes returns into a risk premium and news component. The third divides returns to a part that is spanned by yields and to an unspanned residual component.

Carry-yield change decomposition To begin note that we can decompose the excess return on a bond as

$$rx_{t+1}^{n} = -(n-1)y_{t+1}^{n-1} + ny_t^n - y_t^1 = \tag{5}$$

$$\underbrace{-(n-1)y_t^{n-1} + ny_t^n - y_t^1}_{\text{excess carry}} - \underbrace{(n-1)(y_{t+1}^{n-1} - y_t^{n-1})}_{\text{yield change}} \equiv c_t^n - yc_{t+1}^n, \tag{6}$$

where (excess) carry and yield change are given by

$$c_t^n = -(n-1)y_t^{n-1} + ny_t^n - y_t^1$$

and

$$yc_{t+1}^n = (n-1)(y_{t+1}^{n-1} - y_t^{n-1})$$

Here carry describes the excess return on a bond assuming the yield curve would remain unchanged. This part of the return between t and $t+1$ is observable already at time t. On the other hand, yield change represents the effect of a change in the yield curve on the bond excess return. Therefore for the covariance between current returns and past returns we have

$$Cov(rx_{t+1}^n, rx_t^n) = Cov(rx_{t+1}^n, c_{t-1}^n - yc_t^n) = \tag{7}$$

$$Cov(c_t^n, c_{t-1}^n) + Cov(-yc_{t+1}^n, c_{t-1}^n) + Cov(c_t^n, -yc_t^n) + Cov(-yc_{t+1}^n, -yc_t^n) \tag{8}$$

Maturity	$Cov(c_t^n, c_{t-1}^n)$	$Cov(c_t^n, yc_t^n)$	$Cov(yc_{t+1}^n, c_{t-1}^n)$	$Cov(yc_{t+1}^n, yc_t^n)$
1	5.3	-0.9	5.6	90.0
2	7.2	1.4	2.7	88.7
3	5.7	2.4	3.0	88.9
4	5.4	2.2	4.8	87.5
5	4.7	4.1	2.3	88.9
6	4.9	5.3	4.9	85.0
7	4.1	1.1	3.4	91.5
8	3.7	4.8	6.1	85.3
9	3.3	3.7	6.3	86.7
10	3.6	0.2	2.8	93.4

Table 4: shows the share of covariance between bond return and past month bond return in per cent accounted by the four channels

This implies that past bond returns can predict future bond returns either because (i) past carry predicts current carry, (ii) past carry predicts future yield changes, (iii) past yield changes predict current carry or (iv) past yield change predicts future yield change.

Because current carry is observable one might argue that (i) and (iii) do not constitute an economically interesting form of predictability. Moreover, it is not clear that such covariance should be called ”momentum”. However, an investor would certainly benefit from being able to predict future yield changes. Moreover, especially covariance between future yield changes and past yield changes would aptly be called momentum. Such separations are not clear from standard treatments of time series momentum such as Moskowitz et al. (2012).

Table 4 gives the covariance decomposition above. One can see that covariance between future and past bond returns is mainly due to covariance between future and past yield changes. I also test these dependencies using the following regressions:

$$c_t^n = \alpha + \beta c_{t-1}^n + \epsilon_{t+1} \quad (9)$$

$$c_t^n = \alpha + \beta yc_t^{n-1} + \epsilon_{t+1} \quad (10)$$

$$yc_{t+1}^n = \alpha + \beta c_t^{n-1} + \epsilon_{t+1} \quad (11)$$

$$yc_{t+1}^n = \alpha + \beta yc_t^{n-1} + \epsilon_{t+1} \quad (12)$$

The results are given in table 5.The coefficient for the past carry in the carry prediction regression and the coefficient for past yield change in the yield change prediction regression are statistically significant. On the other hand, I do not find evidence of significant cross carry-yield change predictability. Note that even though there is a statistically robust relationship between past carry and future carry, because carry does that vary much its contribution to the covariance between future and past returns is small. Autocorrelation between yields appears to be strongest for shorter maturity bonds, which explains why the relationship between past and future returns is also strongest for these maturities.

Given these findings I now revisit the question about whether yield curve momentum can be captured using a single factor. In particular I explore this further using principal component analysis. I extract the first three principal components using all the 120 maturities between 1 month and 10 years. I then consider the following regressions:

$$rx_{t+1}^n = \alpha + \beta(pc_t^1 - pc_{t-1}^1) + \epsilon_{t+1} \quad (13)$$

$$rx_{t+1}^n = \alpha + \beta(pc_t^2 - pc_{t-1}^2) + \epsilon_{t+1} \quad (14)$$

$$rx_{t+1}^n = \alpha + \beta(pc_t^3 - pc_{t-1}^3) + \epsilon_{t+1} \quad (15)$$

	c_t^n on c_{t-1}^n					c_t^n on yc_t^n				
Mat.	α	t-value	β	t-value	R^2	α	t-value	β	t-value	R^2
1	0.01	3.76	0.85	28.79	72.59	0.07	10.38	-0.00	-0.14	0.01
2	0.01	2.59	0.92	42.82	83.68	0.09	9.77	0.00	0.33	0.04
3	0.01	2.43	0.90	27.50	81.06	0.13	12.01	0.00	0.61	0.13
4	0.01	2.57	0.93	44.80	85.66	0.14	11.89	0.00	0.58	0.11
5	0.01	2.42	0.93	48.49	87.18	0.16	12.05	0.01	1.21	0.41
6	0.01	2.48	0.94	52.78	88.38	0.18	12.52	0.01	1.45	0.53
7	0.01	2.57	0.93	48.16	87.18	0.17	11.92	0.00	0.34	0.03
8	0.01	2.40	0.94	55.93	89.14	0.19	12.02	0.00	1.44	0.59
9	0.01	2.37	0.94	51.06	88.21	0.18	11.19	0.00	1.38	0.39
10	0.01	2.28	0.95	57.19	89.75	0.19	10.97	0.00	0.08	0.00
	yc_{t+1}^n on c_{t-1}^n					yc_{t+1}^n on yc_t^n				
1	0.01	0.23	-0.07	-0.03	0.00	0.00	0.23	0.16	2.38	2.72
2	-0.02	-0.25	0.55	0.32	0.15	0.01	0.30	0.16	3.13	2.64
3	-0.00	-0.04	0.23	0.20	0.04	0.02	0.35	0.14	2.70	1.84
4	-0.06	-0.45	0.80	0.59	0.25	0.02	0.40	0.11	2.28	1.19
5	-0.10	-0.69	1.15	0.82	0.40	0.03	0.44	0.11	2.38	1.12
6	-0.16	-0.97	1.49	1.13	0.64	0.04	0.47	0.08	1.71	0.68
7	-0.05	-0.30	0.74	0.59	0.15	0.04	0.47	0.09	1.84	0.78
8	-0.22	-1.15	1.73	1.44	0.82	0.05	0.48	0.09	1.82	0.74
9	-0.26	-1.32	2.00	1.78	1.09	0.05	0.49	0.08	1.85	0.72
10	-0.06	-0.25	0.74	0.66	0.15	0.06	0.48	0.08	1.82	0.70

Table 5: shows the results of regressing carry c_t^n and yield change yc_{t+1}^n on their past values. The t-values are based on Newey and West (1987) standard errors.

$$rx^n_{t+1} = \alpha + \beta_1(pc^1_t - pc^1_{t-1}) + \beta_2(pc^2_t - pc^2_{t-1}) + \beta_3(pc^3_t - pc^3_{t-1}) + \epsilon_{t+1} \quad (16)$$

That is I explain returns using the change in the first three principal components of yields, first individually and then including them all in one regression. The principal components appear standard. The first principal component explains roughly 98.5% of the variation in yields. While this component is often called a level factor, the yield loadings decline slightly in bond maturity. That is they drop from around 0.098 for 1 month yields to 0.082 for 10 year bonds.

This is important because a pure level shift in the yield curve does not create variation in excess bond returns. The average contemporaneous correlation between the change in this factor and excess bond returns is -0.95. That is an increase in this factor is related to an upward shift in the yield curve but also to negative excess returns on long-term bonds.

The second component has positive loadings on short term yields and negative loadings on long term yields and could be called a slope factor. The third component has positive loadings on short and long maturity yields and negative loadings on mid-maturity yields. This component represents curvature. The first three components together account for 99.97% of the variation in yields.

The results for individual regressions are given in table 6. Here only the change in the first principal component of yields is clearly significant, though in some regressions changes in the curvature factor are significant at a 10%-confidence level. The results for the regressions with all three included at the same time are given in table 7. Again only the first principal component is significant. This suggests that yield curve momentum is driven by changes in a single factor.[4]

[4]These findings are related to those in Hoogteijling et al. (2021), who in contemporaneous work find evidence that yield changes can predict bond returns. Using an annual rather than monthly horizon, they also find evidence that changes in the slope and curvature factors can forecast returns.

	pc^1					pc^2				
Mat.	α	t-value	β	t-stat	R^2	α	t-value	β	t-value	R^2
1	0.07	4.00	-0.02	-2.47	3.41	0.07	3.63	-0.02	-0.85	0.27
2	0.10	3.05	-0.03	-2.86	2.59	0.10	2.75	-0.01	-0.23	0.02
3	0.14	3.14	-0.04	-2.57	1.85	0.15	2.85	-0.01	-0.11	0.00
4	0.17	2.79	-0.04	-2.29	1.07	0.17	2.60	0.01	0.10	0.00
5	0.19	2.65	-0.05	-2.22	0.94	0.19	2.48	0.02	0.31	0.02
6	0.21	2.60	-0.06	-2.39	1.17	0.22	2.41	0.02	0.27	0.02
7	0.22	2.37	-0.07	-2.37	1.22	0.22	2.21	-0.01	-0.15	0.01
8	0.24	2.30	-0.07	-2.36	1.21	0.24	2.15	-0.04	-0.40	0.04
9	0.23	2.06	-0.07	-2.23	1.02	0.24	1.94	-0.02	-0.21	0.01
10	0.25	2.05	-0.08	-2.09	0.92	0.26	1.93	-0.03	-0.23	0.01
	pc^3									
1	0.07	3.55	0.06	0.99	0.74					
2	0.10	2.76	0.12	1.17	0.71					
3	0.15	2.88	0.16	1.32	0.70					
4	0.17	2.63	0.17	1.21	0.49					
5	0.19	2.51	0.23	1.34	0.59					
6	0.22	2.45	0.37	1.85	1.20					
7	0.22	2.23	0.38	1.71	0.98					
8	0.24	2.17	0.40	1.61	0.87					
9	0.24	1.96	0.45	1.68	0.93					
10	0.26	1.95	0.45	1.55	0.76					

Table 6: shows the results of predicting returns of different maturity bonds on the change in the first three principal components separately. The t-values are based on Newey and West (1987) standard errors.

Mat.	α	t-value	β_1 (pc^1)	t-value	β_2 (pc^2)	t-value	β_3 (pc^3)	t-value	R^2
1	0.07	3.99	-0.02	-2.25	-0.01	-0.60	0.03	0.53	3.65
2	0.10	3.02	-0.03	-2.39	0.00	0.02	0.06	0.59	2.76
3	0.14	3.10	-0.04	-2.04	0.00	0.06	0.10	0.73	2.08
4	0.17	2.77	-0.04	-1.70	0.01	0.20	0.10	0.62	1.27
5	0.19	2.62	-0.04	-1.61	0.03	0.35	0.15	0.75	1.25
6	0.21	2.58	-0.05	-1.60	0.02	0.21	0.28	1.22	1.87
7	0.22	2.35	-0.05	-1.63	-0.01	-0.15	0.29	1.15	1.73
8	0.24	2.28	-0.06	-1.58	-0.04	-0.39	0.31	1.08	1.68
9	0.23	2.04	-0.06	-1.46	-0.03	-0.24	0.36	1.16	1.54
10	0.26	2.02	-0.06	-1.38	-0.03	-0.25	0.35	1.06	1.33

Table 7: shows the results of predicting returns of different maturity bonds on the change in the first three principal components together. The t-values are based on Newey and West (1987) standard errors.

Risk premium-news decomposition I next study the anatomy of yield curve momentum using a variant of a well-known decomposition of returns into a risk premium and news component. In particular we have

$$rx^n_{t+1} = \underbrace{\mathbb{E}_t[rx^n_{t+1}]}_{\text{Expectation}} + \underbrace{rx^n_{t+1} - \mathbb{E}_t[rx^n_{t+1}]}_{\text{News}} \tag{17}$$

Therefore also

$$Cov(rx^n_{t+1}, rx^n_t) = Cov(\mathbb{E}_t[rx^n_{t+1}], \mathbb{E}_{t-1}[rx^n_t]) + Cov(\mathbb{E}_t[rx^n_{t+1}], rx^n_t - \mathbb{E}_{t-1}[rx^n_t]) \tag{18}$$

Here I used the fact that the news component should be uncorrelated with information known when forming the expectation. The decomposition implies that bond returns are correlated with past bond returns either due to autocorrelation in bond risk premia or because the bond risk premium is correlated with past unexpected shocks to the premium.

One benefit of the carry-yield change decomposition is that both components can be easily measured in the data. However, risk premia are not directly observable and must be approximated using a model. Here I consider a simple linear predictive regression:

$$rx_{t+1}^{n} = A'X_t + \epsilon_{t+1} \tag{19}$$

Note that such a form is implied by standard term structure models, though the exact number of principal components depends on the number of factors. I focus on yield curve factors as predictors.[5] In particular I include the first five principal components as well as their lagged values. Now we have

$$Cov(rx_{t+1}^{n}, rx_t^{n}) = Cov(A'X_t, A'X_{t-1}) + Cov(A'X_t, \epsilon_t) + \tag{20}$$

$$Cov(\epsilon_{t+1}, A'X_{t-1}) + Cov(\epsilon_{t+1}, \epsilon_t) \tag{21}$$

The last two terms can be seen as the effect of approximation error of the model, which arises if the model is not exactly correct or if principal components are measured with error.

The results are given in table 8. We can see that for short maturity bonds, momentum is mainly because unexpected past bond returns increase the next period bond risk premium. However, for longer maturity bonds, the two channels are roughly equally important. The approximation error components are fairly small, with perhaps the exception of 5 and 6 year bonds. This suggests that the model provides a reasonable approximation to bond risk premia.

Spanning decomposition Past bond returns can predict future bond returns either because i) past bond returns contain information about current yield

[5] We could also include macroeconomic variables as predictors. However, yield factors seem better at forecasting returns.

Maturity	Risk premia	News	Error
1	21.4 %	78.6 %	0.0 %
2	22.0 %	78.4 %	-0.4 %
3	25.5 %	80.3 %	-5.9 %
4	29.2 %	71.1 %	-0.2 %
5	27.2 %	66.1 %	6.7 %
6	30.3 %	77.9 %	-8.3 %
7	34.7 %	68.1 %	-2.7 %
8	40.8 %	60.2 %	-1.1 %
9	44.3 %	55.5 %	0.2 %
10	51.6 %	49.3 %	-0.9 %

Table 8: shows the decomposition of covariance between the return of different maturity bonds and their past value into the autocovariance of risk premia, covariance between risk premia and past unexpected bond returns and covariance between past returns and an approximation error component.

curve factors that predict future bond returns or ii) past returns contain additional information relevant for future returns. Formally the first explanation implies that past returns are *spanned* by current yields whereas the second implies that they are not. As explained later standard term structure models imply that the spanning condition holds so that yield curve momentum should be explained by the first channel.

To test the relevant importance of the two channels consider a linear projection of returns on the principal components of yields

$$rx_{t+1}^{n} = A'PC_t + \epsilon_{t+1} \tag{22}$$

The autocovariance in bond returns can then be decomposed to spanned and unspanned parts.

Maturity	Spanned	Unspanned
1	32.3 %	67.7 %
2	27.1 %	73.0 %
3	35.2 %	64.8 %
4	47.2 %	52.8 %
5	51.3 %	48.7 %
6	65.5 %	34.6 %
7	66.6 %	33.4 %
8	69.2 %	30.8 %
9	69.5 %	30.5 %
10	77.6 %	22.4 %

Table 9: shows the decomposition of covariance between the return of different maturity bonds and their past value into a part spanned by yields and an unspanned part.

$$Cov(rx_{t+1}^n, rx_t^n) = \underbrace{Cov(rx_{t+1}^n, A'PC_t)}_{\text{Spanned}} + \underbrace{Cov(rx_{t+1}^n, rx_t^n - A'PC_t)}_{\text{Unspanned}} \tag{23}$$

I apply seven principal components of yields as including further components has minor effects on the results. The results are given in table 9 and suggest that both the spanned and unspanned components of returns are important to explaining yield curve momentum. For short maturity bonds the unspanned components account for most of momentum but for longer maturities the spanned component appears more important.

Testing Spanning Results from the spanning decomposition above suggest that unspanned variation in returns is important to explaining yield curve momentum. This appears true especially for short maturity bonds. I now test this result more formally by including the first three principal components into the predictive regression shown in table 1. The results are given by in

Mat.	β_1 (rx_{t-1})	t-stat	β_2 (pc^1)	t-stat	β_3 (pc^2)	t-stat	β_4 (pc^3)	t-stat	R^2
1	0.19	2.61	0.0012	1.63	0.00	-0.39	-0.03	-1.28	5.12
2	0.18	3.27	0.0014	1.15	-0.01	-0.82	-0.08	-2.22	5.02
3	0.15	2.86	0.0017	1.05	-0.02	-1.29	-0.11	-2.46	4.43
4	0.13	2.50	0.0016	0.77	-0.03	-1.64	-0.15	-2.58	3.93
5	0.12	2.51	0.0016	0.65	-0.04	-1.81	-0.17	-2.32	3.70
6	0.09	1.81	0.0013	0.47	-0.06	-2.12	-0.15	-1.80	3.00
7	0.09	1.81	0.0010	0.32	-0.07	-2.44	-0.15	-1.52	3.06
8	0.09	1.87	0.0013	0.34	-0.08	-2.67	-0.15	-1.29	3.24
9	0.09	1.90	0.0004	0.09	-0.09	-2.64	-0.13	-1.01	2.92
10	0.08	1.77	0.00	0.00	-0.10	-2.71	-0.13	-0.87	2.74

Table 10: shows the results of predicting returns of different maturity bonds on the past return of the bond and the first three principal components of yields. The t-values are based on Newey and West (1987) standard errors.

10. The table suggests that the past return is still significant. However, for higher maturity bonds this significance is obtained only at the 10% level.

Higher principal components of yields can contain information useful for predicting bond returns. Therefore in the appendix I extend this regression by controlling for more information in the yield curve as well as potential non-linearities. Here the past return is significant for shorter but not for longer maturities. These results further confirm that, at least for short maturities, the unspanned components of returns are important for explaining yield curve momentum.

5 Momentum and Post-FOMC Announcement Drift

How are these findings related to monetary policy? Because especially the short end of the yield curve tends to be tightly controlled by the Fed, yield curve momentum might be induced by policy rate changes. This is also due to recent findings related to post-FOMC announcement drift. Brooks et al.

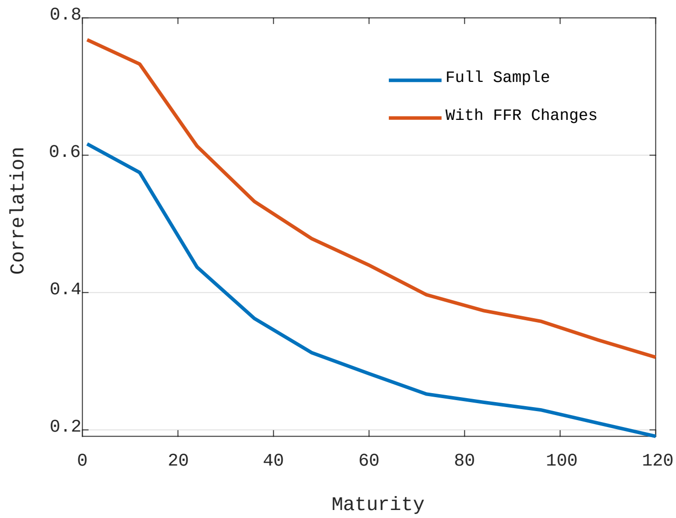

Figure 5: shows the correlation between the change in the Federal funds target rate (FFTR) and the change in the yield of different maturity (in months) bonds in two subsamples: full and months with non-zero FFTR changes.

(2019) find that longer term bond yields respond sluggishly to changes in the federal funds target rate.[6]

I now study this relationship using data on the federal funds target rate. I also utilize data on federal funds futures and the FOMC announcement dates to construct a series of surprise changes in the federal funds rate as in Kuttner (2001). The data period for the federal funds target rate begins in October 1982 and the data for monetary policy surprises on October 1988.

Figure 5 shows the correlation between changes in yields and changes in the federal funds target rate. It does so in two samples: the full sample starting in 82 and a subsample of months with a non-zero change in this policy rate. Excluding months with no rate changes this correlation is close to 0.8 at the short end of the yield curve but only around 0.3 at the long end. The decline in correlation for longer maturity bonds is natural since the

[6]There is a similar drift pattern in equity markets after rate changes, see Neuhierl and Weber (2018).

federal funds rate is an overnight rate. All of these correlations are somewhat smaller in the full sample; overall roughly 30% of months included changes in the policy rate.

I now consider the following regressions

$$rx_{t+1}^{n} = \alpha + \beta \Delta FFTR_t + \epsilon_{t+1}. \tag{24}$$

$$rx_{t+1}^{n} = \alpha + \beta \Delta UEFFTR_t + \epsilon_{t+1}. \tag{25}$$

That is I explain the returns of different maturity bonds on the raw change of the past month federal funds target rate as well the unexpected change in this rate. These regressions are related to those considered by Cook and Hahn (1989) and Kuttner (2001) except that I consider the past rather than the contemporaneous change in the policy rate.[7]

The results are given in table 11. Here I also show the results from regressing bond returns on the change in the previous month change in the corresponding yield for the same period when the target rate is available. Results when using the federal funds target rate and bond yield are similar for shorter maturities, which is perhaps not surprising since these yields are highly correlated with the target rate. However, for longer maturities the target rate change is not significant while the yield change is. Therefore it seems that yield curve momentum is closely related but still separate from post-FOMC announcement drift.

Table 11 also shows the results when the independent variable is the past surprise change in the federal funds rate. Interestingly the results are not significant for 1 and 2 year bonds but become significant for longer maturities. Therefore long maturity bonds seem to have a stronger drift pattern after surprise changes in the federal funds rate. The sample period for these regressions is somewhat shorter though.

[7] Cook and Hahn (1989) and Kuttner (2001) also look at yield changes rather than excess returns.

	FFTR change					yc^n				
Mat.	α	t-value	β	t-value	R^2	α	t-value	β	t-value	R^2
1	0.08	6.23	-0.21	-3.08	3.80	0.08	6.33	-0.17	-3.02	5.38
2	0.13	4.58	-0.33	-2.47	1.87	0.08	6.25	-0.16	-3.36	4.99
3	0.19	4.31	-0.39	-1.92	1.02	0.08	6.07	-0.14	-3.24	3.94
4	0.23	3.97	-0.42	-1.57	0.64	0.08	5.96	-0.12	-3.16	3.15
5	0.27	3.71	-0.43	-1.28	0.43	0.08	5.90	-0.11	-3.02	2.45
6	0.31	3.60	-0.32	-0.82	0.17	0.08	5.80	-0.10	-2.61	1.98
7	0.34	3.39	-0.12	-0.27	0.02	0.08	5.75	-0.09	-2.50	1.58
8	0.38	3.34	-0.15	-0.28	0.02	0.08	5.71	-0.09	-2.30	1.37
9	0.39	3.12	-0.11	-0.19	0.01	0.08	5.69	-0.09	-2.28	1.21
10	0.43	3.09	-0.11	-0.17	0.01	0.08	5.69	-0.09	-2.21	1.07
	Unexpected FFTR change									
1	0.09	4.27	-0.26	-1.32	2.04					
2	0.14	2.49	-0.49	-1.54	1.08					
3	0.19	2.23	-0.96	-1.87	1.73					
4	0.22	1.99	-1.45	-2.08	2.20					
5	0.25	1.86	-1.84	-2.11	2.41					
6	0.28	1.83	-2.28	-2.16	2.76					
7	0.31	1.81	-2.52	-2.02	2.60					
8	0.34	1.78	-2.76	-1.95	2.54					
9	0.35	1.72	-3.02	-1.95	2.55					
10	0.39	1.78	-3.43	-2.05	2.85					

Table 11: shows the results from regressing the returns of different maturity bonds (years) on the previous change in federal funds target rate, change in the previous yield for the same maturity bond and the previous month unexpected change in the federal funds target rate (Kuttner, 2001). The t-values are based on Newey and West (1987) standard errors.

Monetary policy is related to the news part of the return decomposition analyzed in the previous section. The average correlation between target rate changes and the news component of returns is -0.3 with higher absolute values for short maturity bonds. Bond return shocks are related but not fully driven by changes in the policy rate.

We can also analyze the contribution of target rate changes to yield curve momentum using a decomposition. I project bond returns on contemporaneous changes in the federal funds rate as follows:

$$rx_t^n = \alpha + \beta \Delta FFTR_t + \epsilon_t. \tag{26}$$

Using this projection, I can then decompose bond return autocovariance into an effect caused by changes in the federal funds target rate and a residual component:

$$Cov(rx_{t+1}^n, rx_t^n) = \underbrace{Cov(rx_{t+1}^n, \beta \Delta FFTR_t)}_{\text{FFR effect}} + \underbrace{Cov(rx_{t+1}^n, \epsilon_t)}_{\text{Other}} \tag{27}$$

The results are given in table 12. This simple decomposition suggests that target rate changes are an important contributor to momentum for shorter maturities but less so for longer maturities.

Overall, yield curve momentum is therefore connected with but not identical to post-FOMC announcement drift. Past month yield hikes predict low returns in the following month. These yield changes can be partly but not fully explained with same month movements in the policy rate. For example the momentum coefficients are still significant in the subsample of months with no policy rate changes. The appendix contains additional discussion concerning the post-FOMC announcement drift.

Finally, note that the above discussion is unlikely to fully capture the broad relationship between monetary policy and yield curve momentum. Yields tend to fluctuate also in periods without any formal monetary policy decisions. However, this does not imply that such changes are unrelated to

Maturity	FFTR effect	Other
1	47.3 %	52.7 %
2	31.0 %	69.0 %
3	20.3 %	79.8 %
4	21.3 %	78.8 %
5	17.1 %	82.9 %
6	13.6 %	86.4 %
7	5.3 %	94.7 %
8	7.4 %	92.6 %
9	4.8 %	95.2 %
10	4.8 %	95.2 %

Table 12: shows the decomposition of covariance between the return of different maturity bonds and their past value into a part explained by change in the federal funds target rate and a residual component.

monetary policy. These fluctuations might for example still reflect changes in the market participants' views about future monetary policy actions.

6 Momentum and Affine Term Structure Models

How to account for the above empirical findings in a term structure model? I start by introducing a baseline affine term structure model and [8] discussing minimal requirements implied by the data. It is seen that especially the violation of the spanning condition implies strong restrictions for such a model.

Assume that bond prices are a function of an $m \times 1$ dimensional factor X_t. This factor follows:

$$X_t = \mu + \phi X_{t-1} + v_t, \tag{28}$$

[8] See e.g. Ang and Piazzesi (2003) and Cochrane and Piazzesi (2009)

where v_t is multivariate Gaussian $v_t \sim N(0, V)$. The log nominal discount factor is a linear function of the factors

$$M_{t+1} = \exp\left(-\delta_0 - \delta_1' X_t - \frac{1}{2}\lambda_t' V \lambda_t - \lambda_t' v_{t+1}\right)$$

$$\lambda_t = \lambda_0 + \lambda_1 X_t$$

We can then solve bond prices recursively using

$$p_t^1 = \log \mathbb{E}_t(M_{t+1}) \tag{29}$$

$$p_t^n = \log \mathbb{E}_t(M_{t+1} \exp(p_{t+1}^{n-1})) \tag{30}$$

In this model prices and yields take an affine form.

$$p_t^n = A_n + B_n' X_t \tag{31}$$

here

$$A_0 = 0, \ B_0 = 0$$

$$B_{n+1} = -\delta_1 + B_n' \phi^*$$

$$A_{n+1} = -\delta_0 + A_n + B_n' \mu^* + \frac{1}{2} B_n' V B_n$$

Here the risk neutral parameters are given by

$$\phi^* = \phi - V\lambda_1 \tag{32}$$

$$\mu^* = \mu - V\lambda_0 \tag{33}$$

The model implies an analytical expression for the momentum slope coefficient given in the following proposition:

Proposition 1. *The slope coefficient in the momentum regression is given by*

$$\beta^{n,m} = \frac{Cov(rx^n_{t+1}, rx^n_t)}{Var(rx^n_t)}$$

Here

$$\begin{gathered} Cov(rx^n_{t+1}, rx^n_t) = \\ (n-1)^2 B'_{n-1}\phi V B_{n-1} - (n-1)B'_{n-1}\phi^2\Sigma(nB_n - B_1) \\ -(n-1)(nB'_n - B'_1)VB_{n-1} + (nB'_n - B'_1)\phi V(nB_n - B_1) \end{gathered}$$

and

$$\begin{gathered} Var(rx^n_t) = \\ (n-1)^2 B(n-1)' VB(n-1) - 2(n-1)B(n-1)'\phi V(nB(n) - B(1)) + \\ (nB(n)' - B(1)')V(nB(n) - B(1)) \end{gathered}$$

Proof: see appendix.

What type of affine term structure model can generate momentum? I first discuss the general restrictions imposed by the empirical findings. To begin note that in order to generate yield curve momentum, one needs a model with time-varying bond risk premium:

Remark 1. *The momentum slope coefficient is zero in a model with a constant (but possibly maturity specific) risk premium* $\lambda_1 = \mathbf{0}$.

The proof of the remark follows from the decomposition of bond returns into a risk premium and news component. The news component cannot be forecasted with past returns by definition. Now also the best forecast of the risk premium is a constant so the slope coefficient in the momentum regression would be zero.

Table 1 shows that the regression slope coefficient is decreasing in bond maturity. This effectively rules out models in which the coefficient is constant across maturities. In particular we have the following remark:

Remark 2. *The momentum slope coefficient is constant across maturities in a one factor model.*

Proof: see appendix.

This result is related to fact that in one factor interest rate models all bond yields are perfectly correlated (see e.g. Vasicek (1977)).

In the empirical part I established that yield curve momentum is primarily driven by the change in the first principal component of yields. But does this imply that one could capture most of momentum using a single factor term structure model? This reasoning is incorrect as this finding rather suggests that the model should include information about both the first principal component and its past value rather suggesting a minimum of two factors.

Our empirical results suggest that momentum should be explained by a model in which past returns are not spanned by information in current yields. As discussed in the next section this observation poses difficulties for standard models. These models tend to imply that the same model factors that forecast bond returns also drive variation in yields. Therefore controlling for sufficiently many yields is equivalent to controlling for the factors and no other variable should contain additional information for forecasting bond returns. However, similarly to Joslin et al. (2014), we can generate a violation of this spanning condition by parametrizing the model to a knife-edge case for which an invertibility condition condition holds.

Remark 3. *Past bond returns can predict future returns conditional on the information in the term structure today only if the following condition holds:* $[B_{n(1)}, B_{n(2)}, ..., B_{n(m)}]$ *is not invertible for* $n(i) \in \mathbb{Z}_{++}$.

Proof: see appendix.

The intuition for this result is that if an invertibility condition fails, controlling for the yields is not generally equivalent to controlling for the factors. Now some factors can predict returns and yield changes but not be priced in the current term structure of yields.

To conclude remarks 1-3 put constraints on the model that can explain the key empirical findings. In particular they imply that we need a multifactor, unspanned term structure model with a time-varying risk premium.

6.1 Spanning Puzzle and Problem with Standard Models

The finding that past returns can predict future returns controlling for information in the yield curve today poses difficulties for standard models. These models imply that bond returns and yields are both described by the same small set of factors. The models do not naturally generate a violation of the invertibility condition described in remark 3. I next discuss some of these models:

Macrofinance Models I first consider the three main macrofinance models used to explain asset returns: the long run risk model, the habit model and the disarters model. In the long-run risk model (see e.g. Bansal and Shaliastovich (2012)) bond yields take an affine form in the economic state variables. Therefore this model is of the form discussed in the previous section and for standard parametrizations cannot generate momentum conditional on information in the term structure today.

In the habit model, bond yields are a generally non-linear function of habit (see e.g. Wachter (2006)). Therefore the argument of the previous section is strictly valid only up to a first order approximation of the underlying model. However, as discussed in the appendix one can generalize Remark 3 to any well-defined function $y_t = g(X_t)$ so that there is no conditional momentum after controlling for the generally non-linear relationship between past yields and returns. The results obtained in the appendix also suggest that controlling for non-linearities also does not alter the key conclusions.

Also the disasters model of Gabaix (2012) implies that yields are of the form $y_t = g(X_t)$ for state variables X_t. This is also true for any Markovian model such as standard DSGE models. For example Rudebusch and Swanson (2012) offer a macroeconomic interpretation of term premia using a DSGE model with Epstein-Zin preferences. Therefore the general results apply to this model subject to excluding knife-edge cases in which an invertibility condition fails.

Models with Financial Frictions Vayanos and Woolley (2013) posit that momentum might be explained by frictions in delegated asset management. Because the equilibrium is linear in state variables, the model can only generate unconditional momentum. Similarly the preferred habitat term structure model of Vayanos and Vila (2020) takes a standard affine form and hence is unable to generate conditional momentum.

Behavioral Models I now turn to behavioral models and models with heterogenous beliefs. Cieslak (2017) argues that short rate forecast errors can explain bond return predictability. To explain the findings she estimates an affine model using survey data under the assumption of zero subjective risk premia . Because survey forecasts generally differ from rational predictions, the coefficients of the model are generally different than under rational beliefs. However, because the model is still of the standard affine form, it cannot create momentum conditional on the information in the term structure today.

Granziera and Sihvonen (2020) assume that agents have sticky rather than perfectly rational expectations concerning short rates. This slow updating creates a drift pattern in bond returns following short rates changes.[9] Hence the model naturally generates unconditional momentum. In this model biased beliefs enter as new state variables but again the model takes a standard affine form, which is inconsistent with conditional momentum.

In Xiong and Yan (2010) yields are a generally non-linear function of the beliefs of different types of investors. Again this model cannot generate conditional momentum controlling for non-linear dependencies between returns and past yields.

[9] Brooks et al. (2019) also argue that a similar model can explain the post FOMC announcement drift.

The classic momentum model of Hong and Stein (1999) features only one asset. The authors solve for an linear equilibrium. It is not obvious how to extend the model to multiple assets but assuming such an extended model were still linear the problems discussed above apply.

6.2 Accounting for Momentum in a Term Structure Model

I next discuss how to account for momentum in a term structure model. For intuition I start with a simplified example and then move to a more realistic estimated term structure model.

Simple Example

Consider a one factor model as in for example Vasicek (1977). However, make the following twist. First, instead of the standard AR(1) dynamics assume the factor follows an AR(2)-process. In such a model bond prices generally depend on both the current value of the state variable x_t and its lag x_{t-1}. However, assume the second lag is not priced that is under the risk neutral pricing measure the factor follows an AR(1) process. This implies that bond prices depend only on the current value of the factor x_t.

I estimate the true factor dynamics from 1 month rates and find significant persistence parameters of 1.077 for the first lag and −0.088 for the second. For comparison fitting an AR(1) process would result in a persistence parameter of 0.98. The predictability results hinge on a single parameter, the risk neutral persistence of the factor. I calibrate this to match the relative volatility between 10 year and 1 month rates. The corresponding market price of risk parameters could be solved from equations 33 and 32 but are not relevant for the exercise.

Now consider regressing the past excess return of a 5 year bond on the previous month return of a 5 year bond. Using simulations I obtain a slope coefficient of 0.08, that is the model is able to generate momentum, though this coefficient is slightly smaller than in the data (0.12). Moreover, because this is effectively a one factor model, this coefficient is actually constant across maturities, whereas in the data it is decreasing.

But then I repeat this exercise but now explain the return using the past month return and the beginning of period yield of the bond. The coeffient on the past return is still positive at roughly 0.07. That is the model is able to generate momentum conditional on the information in the term structure today.

Why is this model able to generate conditional momentum? In the data, yields effectively follow an AR(2)-process. However, agents price bonds as if the process is AR(1). The higher lag is not priced. Still this second lag is useful for predicting future yields and returns. Because past returns incorporate information about this second lag, including them into the regression increases the model's predictive power. Note that if the second lag were priced, one could effectively back it out from the current yield curve for example using principal component analysis.

Numerical Model

I next consider a more realistic estimated term structure model. I consider a five factor model with the state variable $[pc_t^1, pc_t^2, pc_t^3, pc_t^4, pc_{t-1}^1]$. That is the state variables consists of the first four principal components of yields and the lag of the first component of yields. The demeaned factor dynamics are given by VAR(1) model in companion form with a coefficient matrix

$$\phi = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} \\ \phi_{21} & \phi_{22} & \phi_{23} & \phi_{24} & 0 \\ \phi_{31} & \phi_{32} & \phi_{33} & \phi_{34} & 0 \\ \phi_{41} & \phi_{42} & \phi_{43} & \phi_{44} & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{34}$$

These dynamics are otherwise standard but the first principal component depends also on its second lag. I estimate the coefficient matrix directly from the data using least squares. I estimate the risk neutral factor dynamics and the short rate sensitivity paratemeters δ_1 to minimize the following loss criterion:

$$\Theta \frac{1}{N}\frac{1}{T}\sum_{n=1}^{N}\sum_{t=1}^{T}(y_t^{n,m} - y_t^n)^2 + (\beta^m - \beta)'(\beta^m - \beta) + (\beta^{m,c} - \beta^c)'(\beta^m - \beta^c)$$

The first term is essentially identical to the penalty function in Ang et al. (2006) and Cochrane and Piazzesi (2009), the sum of squared deviations between model implied and actually observed yields.[10] The second term is new: the sum of squared deviations between model implied and observed momentum betas. The third term is also new: the squared deviations between model implied and observed momentum betas conditional on information in the yield curve. Finally Θ is a scaling parameter between the first and the two other moment conditions. Overall, we can view this as a GMM-type estimator with a weighting matrix

$$\begin{bmatrix} \frac{\Theta}{N} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \tag{35}$$

To generate a violation of the spanning condition, I assume that the lag of the first principal component is not priced that is under the risk neutral measure the factor follows a standard VAR(1) model. Hence the yield factor coefficients are of the form $[B_1(n), B_2(n), B_3(n), B_4(n), 0]$. This implies that as in Remark 3, this lag cannot be inferred from the current yield curve, which results in non-zero values for the conditional momentum betas.
Figure 6 shows the resulting population momentum betas and conditional momentum betas along with the values measured from the data. Overall one can see that the model is able to replicate yield curve momentum in the data quite accurately. The root mean squared error between model implied and actual yields is 0.3%. Fit could be further improved by including additional factors to the model.

[10]Here for the model implied yield we have $y_t^{n,m} = -\frac{A_n + B_n X_t}{n}$.

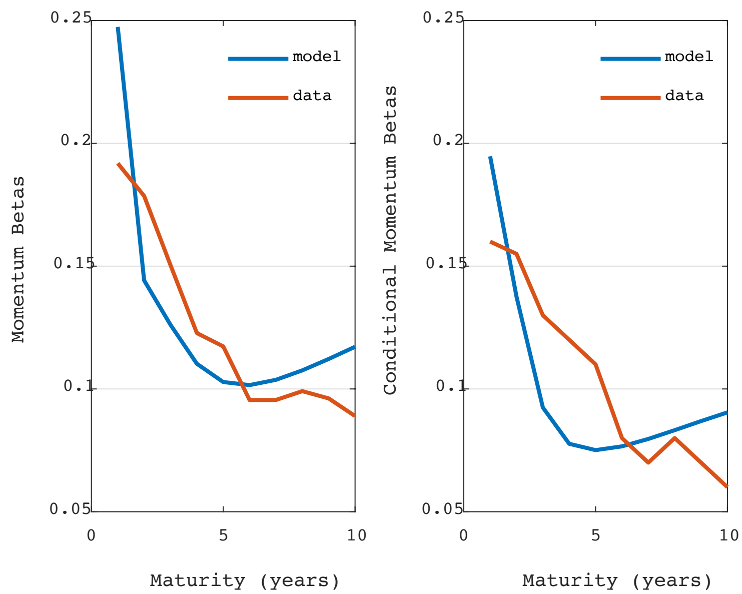

Figure 6: shows the plain and conditional momentum coefficients observed in the data and those implied by the estimated term structure model.

6.3 Spanning Puzzle and Measurement Error

Duffee (2011) and Joslin et al. (2014) find evidence that some macroeconomic variables can forecast bond returns conditional on the information in the term structure today. They argue that the data should be explained by a term structure model in which these macroeconomic variables are not spanned by current yields. Similarly I argue that the empirical observations documented in this paper must be explained by a model in which past bond returns are not spanned by current yields.[11]

However, Bauer and Rudebusch (2017) argue that the empirical evidence presented by Joslin et al. (2014) is rather due to measurement error in yields. They estimate standard term structure models and show that introducing a measurement error can explain why some macroeconomic variables appear to forecast bond returns even after controlling for yields.

To study whether measurement error can account for my findings, I estimate a spanned version of the 4 factor term structure model discussed in the previous section to match average yield errors and momentum betas. Here I impose a VAR(1) structure on the yields and assume all factors are generally priced. I then simulated the regression slope coefficients controlling for all yield curve information. Similarly to Bauer and Rudebusch (2017), I introduce a normally distributed noise term to yields with the standard deviation based on the yield measurement error found by Liu and Wu (2020).

The simulated 5 per cent critical values for the momentum betas when controlling for all yield curve information are between 0.07 - 0.1.[12] Because for short maturities the empirically observed values are above these thresholds, measurement error does not appear to explain the violations of the spanning hypothesis documented in this paper.

Of course I cannot fully rule that there exists some spanned term structure model that is consistent with my empirical results after accounting for the

[11] See also Feunou and Fontaine (2014).

[12] The critical values in previous version of the paper were higher due to a typo in the code.

effects of measurement error. However, I have not found much support for the measurement error explanation. Note that in addition to Joslin et al. (2014), for example Moench and Siavash (2021) argue that unspanned variables are important to explaining yield curve dynamics.

6.4 Economic Interpretations

I have argued that the empirical results of this paper are problematic for standard theories that do not naturally generate a violation of the spanning condition. But what is the economic reason that the spanning condition is not satisfied? Why are past returns important for predicting future returns but not be priced in the term structure of interest rates today?

Answering this question is challenging because unspanned models still lack a full structural interpretation. Moreover, as in Duffee (2011) and Joslin et al. (2014), these models require knife-edge restrictions on model parameters. However, I now discuss two possible explanations.

A Simple Behavioral Explanation

The first possible explanation is behavioral. The unspanned model presented above is consistent with a situation where the true expected value of the first principal component of yields depends also on its second lag yet agents price bonds as if it does not. That is agents ignore longer term dependencies in the state variable process. Note that assuming AR(1) dynamics is also fairly common in the term structure literature. The results of this paper show that relaxing this assumption has important implications for bond return dynamics.

Rational Arbitraugers and Rule-Based Traders

Another possible explanation is that demand from rule-based traders offsets some of the effects of short rates on bond prices. I next sketch such a model that is loosely motivated by the term structure model of Vayanos and Vila (2020) and its modification in Hamilton and Wu (2012). This exercise also gives an economic interpretation to the simple unspanned one factor model discussed before.

Assume there are two types of investors: rational arbitraugers and rule-based traders. Arbitraugers maximize a mean variance objective over the return of their portfolio $r_{t,t+1}$:

$$\mathbb{E}_t[r_{t,t+1}] - \frac{1}{2}\gamma \mathbb{V}ar_t[r_{t,t+1}]$$

Here the portfolio return is given by

$$r_{t+1} = \sum_{n=1}^{N} z_n r_{n,t,t+1}$$

Here z_n is the number of n maturity bonds held by the arbitrauger and $r_{n,t,t+1}$ is the return of the corresponding bond. Somewhat similarly to Hong and Stein (1999), the model also features rule based traders. Assume their demand for each bond ($n \geq 2$) is given by χr_{t-1}, where χ is a constant.[13] By market clearing

$$z_n = -\chi r_{t-1}$$

Finally assume, as in the simple one factor example, that short rate dynamics are given by:

$$r_{t+1} = c + \rho_1 r_t + \rho_2 r_{t-1} + \epsilon_{t+1}$$

We obtain the following result:

Proposition 2. *There is a $\phi > 0$ such that $p_t^n = A_n + B_n r_t$. Here $B_n = B_{n-1}\rho_1 - 1$.*

Proof: see appendix.

The interest rate sensitivity parameters B_n are identical to our previous one factor model that was able to generate both unconditional and conditional

[13] We could also add a maturity specific constant to this demand.

momentum. However, here we do not have a free risk parameter to calibrate the persistence separately from its objective counterpart. Using the already estimated AR(2)-process for short rates, we can now simulate a plain momentum slope coefficient of 0.26 and a conditional coefficient of 0.15. While this model generates a sligthly stronger momentum pattern than in the data, given its simplicity it does surprisingly well.

Given our estimated process, the model implies that high interest rates are associated with high bond returns. An interpretation of the rule χr_{t-1} is then that these traders demand more bonds during times of high interest rates because they associate this with high returns. We could justify the use of past month rather than current month interest rate, if they have some sluggishness in executing trades and must decide their period t holdings already at $t-1$. Interest rate levels could also be related to financial institutions hedging demands.

Why do the arbitraugers price bonds as if the process is AR(1)? Conditional on the current month rate r_t, a high past month rate r_{t-1} predicts a lower value for next month rate r_{t+1}. This force pushes bond prices up already this month. However, a high past month rate r_{t-1} implies high demand from rule based traders. This implies that the arbitraugers must absorb more duration risk. Because these two forces cancel out bond prices do not depend on r_{t-1}.

7 Conclusion

I analyze time series momentum along the Treasury term structure. I find that past returns predict future bond returns largely because of autocorrelation in yield changes. This autocorrelation is further due to both autocorrelation in bond risk premia and correlation between bond risk premia and past unexpected bond returns. Because Treasury yields are correlated with the federal funds rate, yield curve momentum is partly driven by post-FOMC announcement drift. Finally, past returns are not spanned by information in the current term structure of interest rates.

The last finding is particularly problematic for standard theory models, which predict that yield curve momentum should vanish after controlling for sufficiently many yields. However, I show that the results are consistent with a term structure model with unpriced longer term dependencies.

8 Appendix

8.1 Controlling for More Yield Curve Information

The main text shows the result from predicting bond returns using past bond returns and the first three principal components of yields. I now extend these results using the following regression:

$$rx_{t+1}^{n} = \alpha + \beta_1 rx_t^n + \sum_{i \in S} \beta_s y_t^i + \epsilon_{t+1}, \tag{36}$$

where the selected yields are the 1 month and 1 to 10 year rates. Note that this is equivalent to controlling for the 1 month rate and the corresponding 10 forward rates and spans the tent-shaped factor discussed by Cochrane and Piazzesi (2005). The results are shown in table 13. The coefficient on the past return is statistically significant for shorter maturities though less so for longer maturities. This suggests that at least for shorter maturities yield curve momentum exists after controlling for the information in the yield curve today.

In some models, for example in the habit model of Wachter (2006), yields affect future returns non-linearly. We now test this possibility by considering the more general partially linear regression

$$rx_{t+1}^{n} = \beta_1 rx_t^n + f(\mathbf{y}_t) + \epsilon_{t+1}. \tag{37}$$

As explained later, assuming an invertibility condition, any Markovian model of yields implies that

$$rx_{t+1}^{n} = f(\mathbf{y}_t) + \epsilon_{t+1}. \tag{38}$$

Mat. (y)	1	2	3	4	5	6	7	8	9	10
α	-0.02	-0.01	-0.05	-0.10	-0.22	-0.27	-0.39	-0.59	-0.77	-0.99
t-value	-0.42	-0.10	-0.32	-0.49	-0.86	-0.89	-1.15	-1.49	-1.70	-2.01
β_1 (rx_t^1)	0.16	0.16	0.13	0.12	0.11	0.08	0.07	0.08	0.07	0.06
t-value	2.41	2.97	2.64	2.30	2.24	1.54	1.56	1.60	1.52	1.42
β_2 (y_t^1)	-0.23	-0.41	-0.63	-0.92	-1.11	-1.19	-1.38	-1.61	-1.72	-1.91
t-value	-4.42	-3.94	-4.27	-4.52	-4.66	-4.51	-4.70	-4.70	-4.62	-4.70
β_3 (y_t^{12})	0.51	0.64	1.32	1.95	2.33	2.53	2.88	3.35	3.60	3.95
t-value	2.68	1.74	2.59	2.85	2.96	2.84	3.06	3.22	3.30	3.37
β_4 (y_t^{24})	-0.44	-0.18	-1.40	-1.33	-1.25	-1.46	-1.76	-2.14	-2.56	-3.08
t-value	-0.92	-0.22	-1.24	-0.94	-0.75	-0.77	-0.86	-0.99	-1.16	-1.33
β_5 (y_t^{36})	0.46	0.01	0.53	-1.74	-1.67	-0.95	-0.57	-0.50	0.38	1.15
t-value	0.51	0.01	0.28	-0.83	-0.69	-0.34	-0.19	-0.16	0.11	0.32
β_6 (y_t^{48})	-0.22	0.49	1.51	3.90	1.56	1.10	1.30	1.79	0.31	-0.00
t-value	-0.18	0.23	0.57	1.28	0.43	0.28	0.30	0.36	0.05	-0.00
β_7 (y_t^{60})	-1.19	-2.39	-3.71	-4.14	-2.37	-5.23	-6.47	-7.15	-5.49	-5.67
t-value	-1.64	-1.95	-2.17	-1.86	-0.89	-1.43	-1.45	-1.39	-0.94	-0.91
β_8 (y_t^{72})	1.39	2.84	4.04	4.59	5.71	9.15	9.01	9.79	9.93	11.31
t-value	1.83	2.00	2.05	1.80	1.88	2.31	1.93	1.89	1.78	1.94
β_9 (y_t^{84})	-0.05	-0.83	-1.57	-2.23	-2.34	-2.88	-1.71	-3.35	-4.43	-6.18
t-value	-0.11	-1.02	-1.29	-1.28	-1.06	-1.06	-0.46	-0.78	-0.95	-1.23
β_{10} (y_t^{96})	0.01	0.22	0.41	0.33	-0.80	-0.04	0.02	0.89	-0.97	-1.36
t-value	0.03	0.26	0.32	0.18	-0.36	-0.01	0.00	0.20	-0.20	-0.26
β_{11} (y_t^{108})	-0.28	-0.81	-1.26	-1.32	-0.89	-2.21	-2.52	-2.56	-1.04	-1.80
t-value	-0.52	-0.89	-0.96	-0.79	-0.43	-0.76	-0.66	-0.57	-0.21	-0.35
β_{12} (y_t^{120})	0.02	0.41	0.76	0.87	0.81	1.15	1.17	1.47	1.97	3.60
t-value	0.08	0.83	1.11	1.02	0.77	0.86	0.65	0.69	0.85	1.44
R^2	0.11	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09

Table 13: shows the results of predicting returns of different maturity bonds on the past return of the bond and the yields of 1 month bill and 1 to 10 year bonds. The t-values are based on Newey and West (1987) standard errors.

Therefore these models imply that $\beta_1 = 0$. However, the challenge is that f is generally unknown. I tackle this using two approaches. The first approach is to estimate the model using the semiparametric approach described by Wood (2011). Here the standard errors are calculated using quasi-maximum likelihood.[14] The second approach is to simply add the squared yields, on top of the yields, to the regression. The results are given in table 14, which shows the results for the β_1 parameter. For the first approach β_1 is always significant. However, the model produces a high in sample fit and might achieve low standard errors by overfitting. For the second approach, the slope coefficient is significant for shorter but not for longer maturity bonds. These exercises suggest that accounting for non-linearities does not strongly alter the main conclusions of this paper.

8.2 Predicting Bonds Returns with Carry and Yield Change

The results of the main section suggest that including information about both past carry and yield change might be beneficial to predicting bond returns. I now test this prediction by including both variables separately into the predictability regression.

$$rx_{t+1}^n = \alpha + \beta_1 c_t^n + \beta_2 yc_t^n + \epsilon_{t+1} \tag{39}$$

Note that because period t carry is observable we include this rather than the previous period carry into the regression. The results are given in table 15. For most maturities both carry and past yield change are significant. There is a small increase in R^2 relative to a regression with past return.

8.3 Post Announcement Drift: Further Analysis

This section provides some further results related to the post-FOMC announcement drift. Figure 7 shows the changes in different maturity yields per one basis

[14] To avoid problems with overfitting I only include yields of every second year.

	Semipar.		Squares	
Mat	β_1	t-value	β_1	t-value
1	0.22	4.51	0.20	2.23
2	0.25	5.09	0.22	2.62
3	0.23	4.63	0.18	2.40
4	0.20	4.12	0.16	2.02
5	0.19	3.98	0.14	1.90
6	0.16	3.27	0.10	1.42
7	0.14	2.87	0.09	1.42
8	0.12	2.65	0.08	1.25
9	0.11	2.42	0.06	1.02
10	0.11	2.40	0.06	0.93

Table 14: shows the slope coefficient on past return when explaining excess bond returns on past excess bond returns on an arbitrary non-linear function of yields, estimated using a semiparametric method, as well as a linear regression with yields and squared yields. The t-values for the first regression are obtained using quasi-maximum likelihood (Wood, 2011). The t-values for the second regression are based on Newey and West (1987) standard errors.

Maturity	α	t-value	β_1 (c_t^n)	t-value	β_2 (yc_t^n)	t-value	R^2
1	0.01	0.19	0.97	2.50	0.16	2.39	5.17
2	-0.02	-0.29	1.29	2.40	0.16	3.06	5.02
3	-0.00	-0.03	1.15	1.40	0.14	2.63	3.18
4	-0.06	-0.46	1.54	2.19	0.11	2.19	2.95
5	-0.09	-0.66	1.74	2.51	0.10	2.23	2.98
6	-0.15	-0.95	2.06	2.75	0.08	1.56	2.84
7	-0.06	-0.34	1.58	2.05	0.09	1.81	1.83
8	-0.21	-1.15	2.35	2.94	0.08	1.66	2.97
9	-0.25	-1.33	2.71	3.13	0.08	1.69	3.23
10	-0.07	-0.30	1.66	1.97	0.08	1.80	1.63

Table 15: shows the results of predicting returns of different maturity bonds on carry and past yield change. The t-values are based on Newey and West (1987) standard errors.

point change in the federal funds target rate. Shorter maturity yields show a clear drift pattern after target rate changes.

In this particular sample long maturity yields do not exhibit similar drifts. However, as explained by Brooks et al. (2019) results for long maturities are stronger when considering unexpected target rate changes. This can explain why are regression results are stronger for long maturity bonds when using unexpected rather than plain changes in the target rate.

Figure 8 plots the historical development of different maturity yields along with that for the target rate. One can see that all the yields share the same broad developments. However, the contemporaneous correlation between yield changes and changes in the federal funds target rate is far from perfect. Post-FOMC announcement drift seems to contribute to this correlation being fairly low. However, this is likely not the only reason. For example theoretically longer maturity yields should reflect expectations about the long run path of future short rates and also anticipate target rate

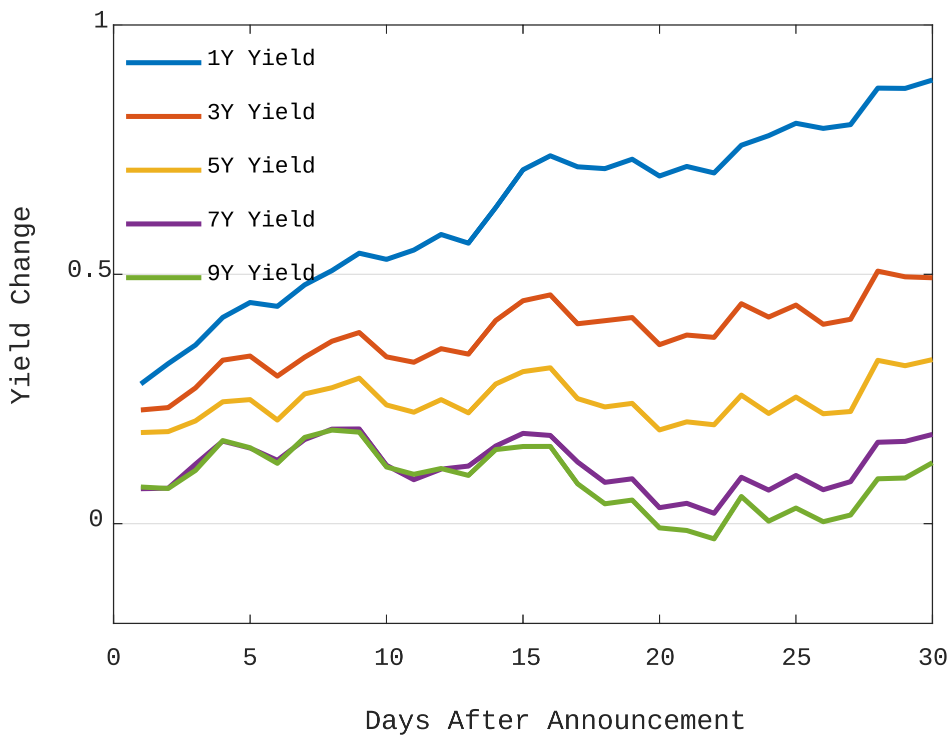

Figure 7: shows the change in different maturity yields after a change in the federal funds target rate (FFRT). Changes are measured per one basis point change in the FFRT. Days after announcement are measured using trading days.

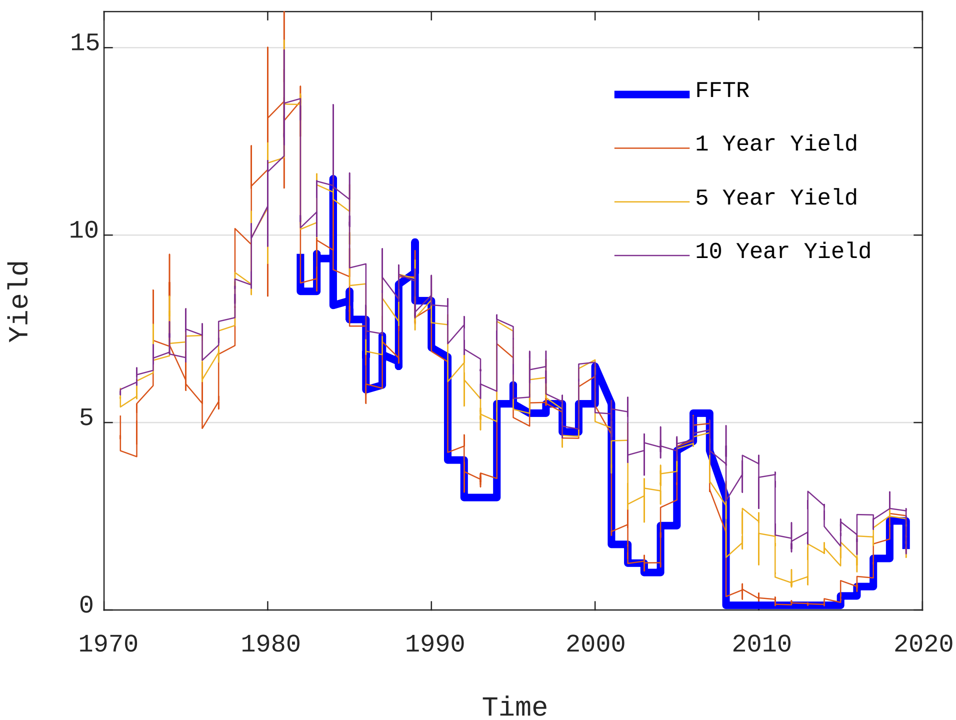

Figure 8: shows the historical development of 1,5 and 10 year yields along with the federal funds target rate.

changes.[15]

8.4 Robustness with Respect to Gürkaynak et al. (2007) data

Liu and Wu (2020) construct the yield curve using a novel procedure that results in lower pricing errors compared to standard procedures such as the Svensson (1994) method applied by Gürkaynak et al. (2007). How does this affect the key results of this paper?

Table 16 replicates the results in table 1 for the 1 month lookup using the Gürkaynak et al. (2007) data updated on the Federal reserve webpage.

[15] Also yield levels reflect the cumulative effect of yield changes and hence tend to be more correlated.

Mat	α	t-value	β	t-value	R^2
1	0.06	3.31	0.19	3.12	3.77
2	0.09	2.86	0.17	3.39	2.83
3	0.12	2.70	0.14	3.12	2.02
4	0.15	2.59	0.12	2.78	1.46
5	0.18	2.49	0.10	2.47	1.08
6	0.20	2.38	0.09	2.19	0.83
7	0.22	2.28	0.08	1.94	0.65
8	0.23	2.17	0.07	1.69	0.51
9	0.25	2.08	0.06	1.46	0.39
10	0.26	1.99	0.05	1.23	0.29

Table 16: shows the results from regressing the excess returns of different maturity bonds on their past returns using the atlernative data from Gürkaynak et al. (2007). The t-values are based on Newey and West (1987) standard errors.

The sample period is as before. While this alternative data yields somewhat smaller coefficients for long maturity bonds, overall the results are fairly similar across the two datasets.

8.5 Robustness with Respect to German Data

Are the results robust to data from other developed countries? Next I study this using data on the German government yield curve available on the Bundesbank webpage. These curves are constructed using the interpolation procedure of Svensson (1994). Because standard interpolation procedures often have large pricing errors for short maturity yields (Liu and Wu, 2020), I focus on actual rather than excess returns that do not require specifying a 1 month risk-free rate.

I replicate the exercise of explaining the return of different maturity bonds on their return in the prior month. The results are given in table 17 and are fairly similar for both countries. The R^2 is quite high for short maturity bonds in both countries as their returns are strongly related to

	Germany					USA				
Mat.	α	t-value	β	t-value	R^2	α	t-value	β	t-value	R^2
1	0.23	5.10	0.40	3.86	15.65	0.26	7.55	0.42	7.31	17.28
2	0.28	7.76	0.35	5.47	12.59	0.37	9.53	0.24	4.46	5.82
3	0.35	8.22	0.27	4.56	7.68	0.43	9.12	0.18	3.27	3.16
4	0.39	8.22	0.25	4.66	6.17	0.48	8.18	0.13	2.67	1.77
5	0.43	7.88	0.22	4.52	5.01	0.50	7.32	0.12	2.59	1.46
6	0.47	7.44	0.20	4.13	3.92	0.54	6.58	0.09	1.91	0.90
7	0.50	7.01	0.17	3.60	2.95	0.55	6.01	0.09	1.88	0.85
8	0.54	6.61	0.15	3.02	2.14	0.56	5.52	0.10	1.95	0.91
9	0.57	6.26	0.12	2.45	1.45	0.56	5.04	0.09	1.94	0.81
10	0.61	5.93	0.09	1.86	0.87	0.59	4.80	0.08	1.78	0.68

Table 17: shows the results from regressing returns of different maturity bonds on their past returns in both Germany and US. The t-values are based on Newey and West (1987) standard errors.

short-term yields that are highly autocorrelated. This suggests that the results are robust to the German yield curve though this curve might be measured with larger pricing errors.

8.6 Time Series vs Cross-Sectional Momentum

The literature on equity momentum (e.g. Chan et al. (1996)) has focused on a cross sectional strategy that goes long stocks with relatively high past returns and short stocks with relatively low past returns. Could a similar strategy be applied with different maturity government bonds?

The finding that time series momentum is largely associated with a single factor suggests that such a strategy is unlikely to provide high returns. I now demostrate this further by considering a simple cross-sectional momentum strategy. I consider the returns of bonds with maturities from 1 to 10 years. As in Lewellen (2002) assume the weight of each bond is given by $w_i = (r_{i,t} - r_{p,t})/10$, where $r_{i,t}$ is the return of the bond and $r_{p,t}$ is the return of

$\mathbb{E}[r_{s,t}]$	$\frac{1}{10}\sum_{i=1}^{10}\rho_i$	$\frac{1}{10}\sum_{i=1}^{10}\mathbb{E}[r_{i,t}]^2$	$-\rho_m$	$-\mu_m^2$
0.0003	0.0044	0.0033	-0.0041	-0.0032

Table 18: shows a decomposition of the mean return from a cross sectional momentum strategy (%)

an equal weighted portfolio of all the ten bonds. The mean return of this strategy can be decomposed as follows:

$$\mathbb{E}[r_{s,t}] = \frac{1}{10}\sum_{i=1}^{10}(\rho_i + \mathbb{E}[r_{i,t}]^2) - (\rho_m + \mu_m^2).$$

Here ρ_i and ρ_p are the autovariances of the individual bonds and the equally weighted portfolio of bonds respectively. Moreover, μ_i and μ_p are the bonds' unconditional mean returns.

The results from this decomposition are given in table 18. The strategy yields a 0.0003 per cent monthly return with a modest annualized Sharpe ratio of 0.087. This is largely because the mean autocovariance of the bonds is close to the autocovariance of an equally weighted portfolio of the bonds. This zero net investment strategy cannot benefit from time series momentum related to shifts in a single factor that manifests itself somewhat similarly for all the different maturity bonds.

8.7 Investment Performance

The results of this paper suggest that an investor could gain using momentum strategies in Treasury bonds. But how big are these gains? Answering this question is complicated because such momentum strategies can be implemented in multiple ways. While more sophisticated strategies might provide higher returns, for transparency I focus on a particularly simple strategy. In particular assume an investor buys a bond assuming its past month excess return was positive. On the other hand, if this past return was negative, assume the investor instead chooses to hold short term bills earning her zero excess returns. Note that this simple strategy naturally

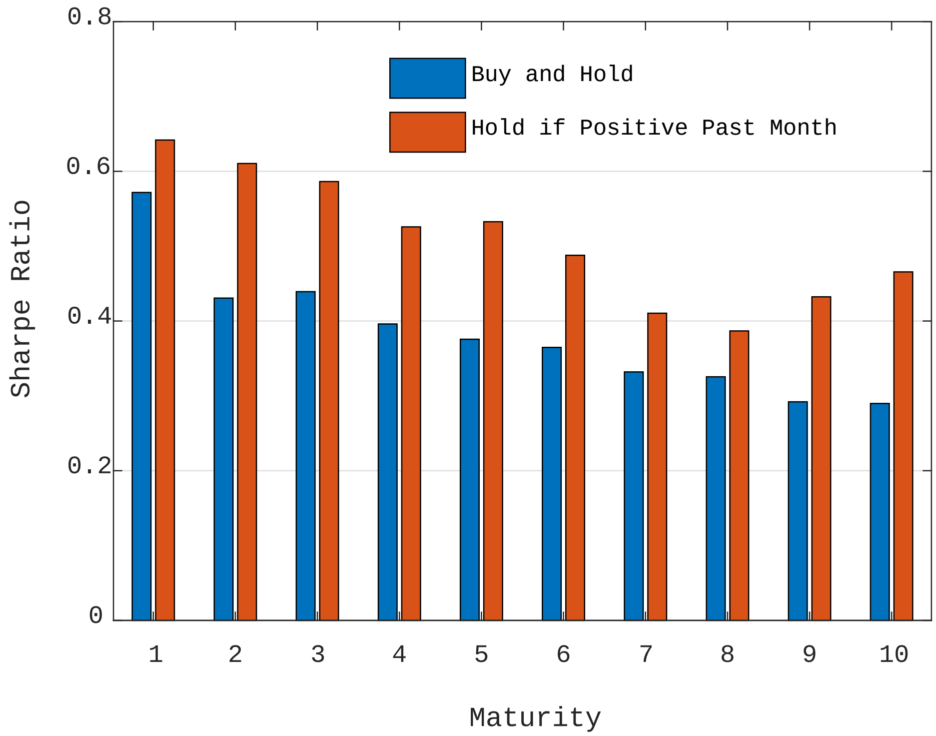

Figure 9: shows the annualized Sharpe ratios for different maturity bonds for a simple momentum strategy and a buy and hold strategy.

also constitutes an ”out of sample” evaluation for the relavant trading performance.

Figure 9 shows the Sharpe ratios from this simple momentum strategy along with those for a buy and hold strategy that passively holds given maturity bonds. One can see that the momentum strategy earns higher Sharpe ratios for all maturities. The average Sharpe ratio of the momentum strategy is 0.51 compared to 0.38 for the buy and hold strategy. This momentum stategy also enjoys a positive average skewness of 1.27 compared to 0.22 for the buy and hold strategy. The Sharpe ratios for an equally weighted portfolio of simple momentum strategies would be 0.50 compared to 0.36 for an equally weighted buy and hold strategy. Here the improvement in Sharpe ratio is therefore 39%.

Figure 3 conveys an interesting additional point. The mean excess returns are fairly close to zero following months with negative past month returns. Hence it is not clear that an investor could benefit from twisting our momentum strategy by also going short bonds after such months. This long short strategy would improve mean returns for some maturity bonds but not all. Moreover, because this improvement in mean returns is fairly small but such a strategy involves higher volatility, the Sharpe ratios for this long-short strategy are lower for all maturities.[16]

Finally note that a more comprehensive analysis of the investment performance of yield curve momentum strategies should take into account the broader constitution of the investor's portfolio and other signals used. For example Hurst et al. (2017) notes that trend followers can clearly improve Sharpe ratios by diversifying exposures to momentum strategies for different asset classes. They also show that momentum returns tend to survive after controlling for reasonable estimates of transaction costs.

8.8 Results for a Bond Index

The key results of this paper are based on a yield curve constructed using a numerical approximation scheme. A possible concern is that these errors contribute to the key findings regarding yield curve momentum. I next demonstrate that these errors are unlikely to invalidate the main regression results of this paper.

In particular, I use the excess returns on the Bloomberg Aggregate Treasury bond index, available from 1973, that is a few years before the start of our main data. This index is calculated directly using Treasury bonds and hence represents tradable returns. It serves as perhaps the most widely followed benchmark index for Treasuries. However, the results obtained with this index are not fully comparable with our main results because of

[16]This point is somewhat nuanced though. If the unconditional bond risk premium represents rational compensation for risk, going short following months with negative returns might hedge macroeconomic risk and is not necessarily suboptimal.

two reasons. First, this index is based on coupon paying bonds, while our main results are for zero coupon bonds. Second, this index represents a broad portfolio of different maturity Treasury bonds.

I replicate the key regression of this paper by explaining one month excess return on this index by its past value. The slope coefficient is 0.11, which is close to the slope coefficients for longer maturity bonds in table 1. The corresponding t-value is 2.67 and hence the results are strongly significant.

I also replicated the investment strategy that holds bonds only in months following positive past month excess returns. The Sharpe ratio for this strategy is 0.55 compared to 0.44 for a buy and hold strategy. Note that because the strategy is effectively implemented for a portfolio of bonds, it cannot benefit from any individual time series predictability for different maturity bonds.

8.9 Stability Analysis

Is yield curve momentum stronger during some periods than others? I now analyze potential structural breaks in the relationship between current and past returns. I consider a simple 10 year rolling regression. Figure 10 plots the results when one month return is explained with the one month return in the past month. One can see that the slope coefficients are fairly stable overall but clearly fall after the financial crisis.

Explaining this break is beyond the scope of the paper. However, the period is characterized by extraordinarily low interest rates and unconventional monetary policies. For example an effective lower bound on yields can alter the relationship between current and past bond returns. The Fed policies during the period pushed yields down and led to high bond returns. However, if yields are close to an effective floor, these high bond returns do not predict similar elevated returns going forward.

As discussed in the section on investment performance, bond excess returns tend to be close to zero following months with negative returns.

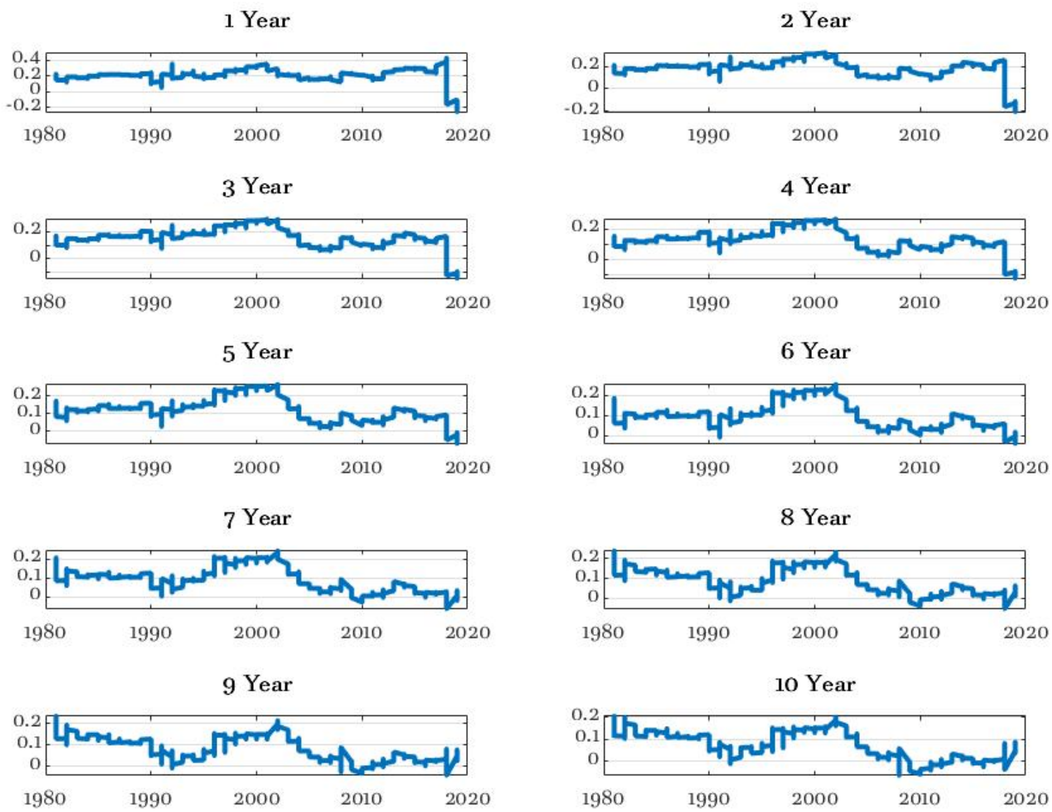

Figure 10: Momentum slope coefficient in a rolling 10 year sample for different maturity bonds

Effectively the negative momentum effect is offset by a substantial unconditional bond risk premium. On the other hand, following positive months the positive momentum effect increases expected bond returns on top of the unconditional risk premium. Because high bond returns are associated with increasing interest rates, momentum strategy returns tend to be higher during subperiods with declining rather than increasing interest rates.

8.10 Predicting Yield Changes: the Longer Run

In this section I study the longer run effects of a shock to bond yields. I consider a regression of the form

$$\Delta y^n_{t+h} = \alpha + \beta \Delta y^n_t + \epsilon_{t+h} \tag{40}$$

for different horizons h. That is I predict yield changes between $t+h$ and $t+h-1$ by the change in the same maturity bond yield between t and $t-1$. As in the local projection method of Jordà (2005), the slope coefficients can be interpreted as a type of impulse response function.

The resulting slope coefficients along with the 95% confidence intervals are shown in figure 11. The coefficients are high for the horizon of one month and then again high for the 11 month horizon. Many of the coefficients in between are negative though not statistically different from zero. These results can explain why the 1 month horizon works best in the regressions reported in table 1.

The slope coefficients for different horizons sum to numbers sligthly smaller than the coefficient for the first year. Therefore the total effect to yields after a year is positive but fairly small. Put alternatively, assume there is an increase in bond yields at period t. Because of short horizon autocorrelation in yields this predicts a further increase in yields in the next month. The longer horizon autocorrelations large offset each other so that on average yields after a year remain sligthly below but close to the level after a month following the yield change ($t+1$).

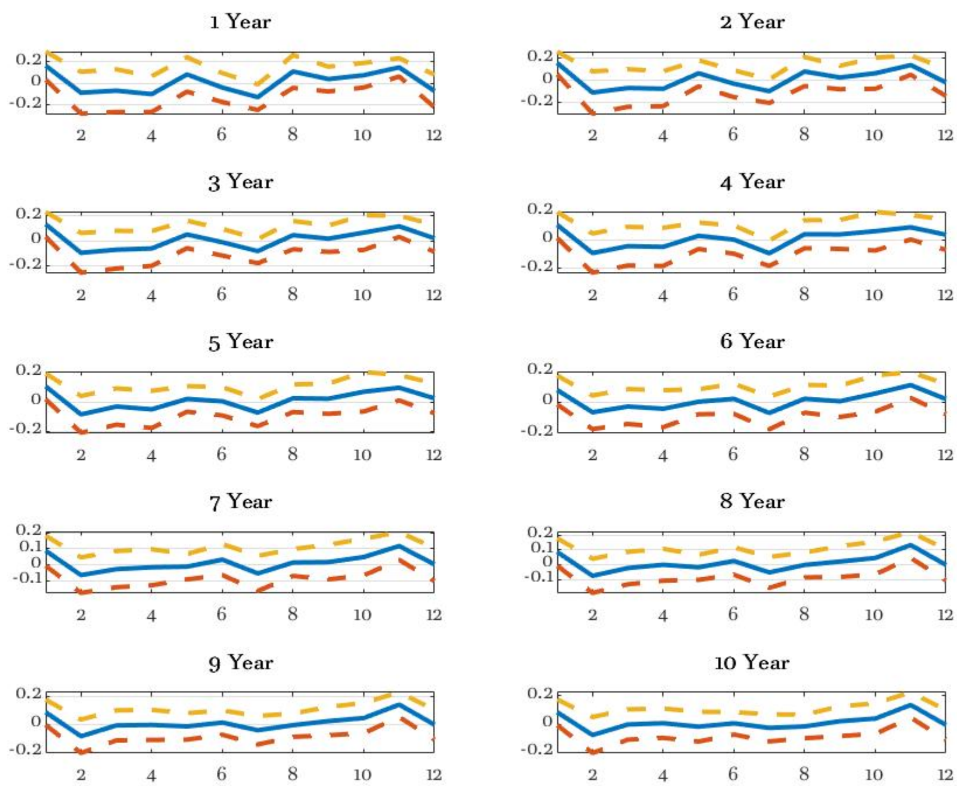

Figure 11: shows the slope coefficients on a regression of bond yield change on future bond yield changes

8.11 Proof of Proposition 1

We have

$$
\begin{gathered}
rx_{t+1}^{n} = -(n-1)y_{t+1}^{n-1} + ny_t^n - y_t^1 = \\
-(n-1)(A(n-1) + B(n-1)'X_{t+1}) + n(A(n) + B(n)'X_t) - (A(1) + B(1)'X_t) = \\
-(n-1)B(n-1)'X_{t+1} + (nB(n)' - B(1)')X_t - (n-1)A(n-1) + nA(n) - A(1)
\end{gathered}
$$

$$
\begin{gathered}
Cov(rx_{t+1}^{n}, rx_{t-1}^{n}) = \\
Cov(-(n-1)B_{n-1}'X_{t+1}, -(n-1)B_{n-1}'X_t) + Cov(-(n-1)B_{n-1}'X_{t+1}, (nB_n' - B_1')X_{t-1}) + \\
Cov((nB_n' - B_1')X_t, -(n-1)B_{n-1}'X_t) + Cov(nB_n' - B_1')X_t, (nB_n' - B_1')X_{t-1}) = \\
(n-1)^2 B_{n-1}'\phi V B_{n-1} - (n-1)B_{n-1}'\phi^2 V(nB_{n-1} - B_1) - (n-1)(nB_n' - B_1')VB_{n-1} + \\
(nB_n' - B_1')\phi V(nB_n - B_1)
\end{gathered}
$$

$$
\begin{gathered}
Var(rx_t^n) = \\
(n-1)^2 B_{n-1}' V B_{n-1} - 2(n-1)B_{n-1}'\phi V(nB_n - B_1) + \\
(nB_n' - B_1')V(nB_n - B_1)
\end{gathered}
$$

The regression slope coefficient is given by the ratio of the covariance and variance terms.

8.12 Proof of Remark 2

Due to normality, the standard pricing formula applies:

$$
p_t^n = -y_t^1 + \mathbb{E}_t[p_{t+1}^{n-1}] + \frac{1}{2}Var_t(p_{t+1}^{n-1}) + Cov_t(m_{t+1}, p_{t+1}^{n-1})
$$

Hence

$$rx_{t+1}^{n} = p_{t+1}^{n-1} - p_t^n - y_t^1 = p_{t+1}^{n-1} - \mathbb{E}_t[p_{t+1}^{n-1}] - Cov_t(m_{t+1}, p_{t+1}^{n-1}) - \frac{1}{2}Var_t(p_{t+1}^{n-1})$$

$$rx_{t+1}^{n} = B_{n-1}v_{t+1} + B_{n-1}V\lambda_t - \frac{1}{2}B_{n-1}'VB_{n-1}$$

Therefore

$$Var(rx_{t+1}^{n}) = B_{n-1}^2 Var(v_{t+1} + V\lambda_t)$$

and

$$Cov(rx_{t+1}^{n}, rx_t^n) = B_{n-1}^2 Cov(v_{t+1} + V\lambda_t, v_t + V\lambda_{t-1})$$

and the slope coefficient in the momentum regression (this is given $n \geq 2$, if $n = 1$, excess returns are always zero and the coefficient undefined) is

$$Var(rx_{t+1}^{n}) = B_{n-1}^2 Var(v_{t+1} + V\lambda_t)$$

and

$$\frac{Cov(rx_{t+1}^{n}, rx_t^n)}{Var(rx_{t+1}^{n})} = \frac{Cov(v_{t+1} + V\lambda_t, v_t + V\lambda_{t-1})}{Var(v_{t+1} + V\lambda_t)}$$

which is independent of bond maturity.

8.13 Proof of Remark 3

Excess return of an n maturity bond is given by

$$rx_{t,t+1}^{n} = -(n-1)y_{t+1}^{n-1} + ny_t^n - y_t^1 = \tag{41}$$

$$-(n-1)(A_{n-1} + B_{n-1}'X_{t+1}) + n(A_n + B_n'X_t) - (A_1 + B_1'X_t). \tag{42}$$

This implies the expected excess return is of the form

$$\mathbb{E}_t[rx^n_{t,t+1}] = \tilde{A}_n + \tilde{B}_n X_t, \tag{43}$$

where

$$\tilde{A}_n = -(n-1)A_{n-1} + nA_n - A_1$$

and

$$\tilde{B}_n = -(n-1)B_{n-1}\phi + nB_n - B_1.$$

Now consider an m dimensional collection of yields $\hat{y}_t$. Note that we have

$$\hat{y}_t = \hat{A} + \hat{B}X_t,$$

where $\hat{A}$ are $\hat{B}$ simply collect the relevant A_n and B_n for the corresponding maturities. If $\hat{B}$ is invertible:

$$X_t = \hat{B}^{-1}(\hat{y}_t - \hat{A})\hat{y}_t.$$

Therefore we have

$$\mathbb{E}_t[rx^n_{t,t+1}] = \tilde{A}_n + \tilde{B}_n\hat{B}^{-1}(\hat{y}_t - \hat{A})\hat{y}_t, \tag{44}$$

now we can write the conditional expectation for the excess return as a linear (affine) function of the yields $\hat{y}_t$. Therefore we can write the excess returns as

$$rx^n_{t+1} = \tilde{A}_n + \tilde{B}_n\hat{B}^{-1}(\hat{y}_t - \hat{A})\hat{y}_t + \varepsilon_{t+1}, \tag{45}$$

where ε_{t+1} is independent white noise. Now conditional on the yields $\hat{y}_t$, no other variable like past returns or previous period returns should forecast excess returns.

However, the argument fails if $\hat{B}$ is not invertible. Then controlling for current yields is not generally equivalent to controlling for the factors. Then past bond returns can also predict future returns conditional on the information in the yield curve today.

Remark 3: The Effect of Nonlinearities Remark 3 assumes that yields are an affine function of state variables. However, it can be generalized to arbitrary functions. Now assume excess returns are of the form

$$y_t^n = g_n(X_t).$$

and that

$$X_{t+1} = \xi(X_t) + \epsilon_{t+1}$$

for some g_n and ξ. We can view this as a generalized Markovian model. Now pick any m yields stacked into a vector $\tilde{y}_t$. Moreover, define $\tilde{g}$ as

$$\tilde{y} = \tilde{g}(X_t),$$

where this function simply collects the relevant elements using g_n. Assuming the inverse exists, we can solve

$$X_t = \tilde{g}^{-1}(\tilde{y}).$$

Now note that we have

$$rx_{t,t+1}^n = -(n-1)y_{t+1}^{n-1} + ny_t^n - y_t^1 = \tag{46}$$

$$-(n-1)g_{n-1}(f(X_{t+1})) + ng_n(X_t) - g_1(X_t) \tag{47}$$

Excess returns are of the form

$$rx_{t,t+1}^n = f_n(X_t) = f_n(\tilde{g}^{-1}(\tilde{y})).$$

Now no other variable should predict excess returns controlling for $f_n(\tilde{g}^{-1}(\tilde{y}))$.

8.14 Proof of Proposition 2

Conjecture $p_t^n = A_n + B_n r_t$. Similarly to Hamilton and Wu (2012) then approximate:

$$\mathbb{E}_t[r_{t+1}] \approx -z_{1t} r_t + \tag{48}$$

$$\sum_{n=2}^{N} z_{tn} \left[A_{n-1} + B_{n-1}(c + \rho_1 r_t + \rho_2 r_{t-1}) - A_n - B_n r_t - r_t + \frac{1}{2} B(n-1)^2 \sigma_\epsilon^2 \right] \tag{49}$$

and

$$\mathbb{V}ar_t[r_{t+1}] \approx \left(\sum_{n=2}^{N} z_{tn} B_{n-1} \right)^2 \sigma_\epsilon^2$$

Maximizing the arbitrauger's objective for maturity n bond gives:

$$A_{n-1} - A_n + B_{n-1}[c + \rho_1 r_t + \rho_2 r_{t-1}] - B_n r_t - r_t = \gamma B_{n-1} \sigma_\epsilon^2 \left(\sum_{n=2}^{N} z_{tn} B_{n-1} \right) \tag{50}$$

Plugging in $z_{tn} = -\chi r_{t-1}$

$$A_{n-1} - A_n + B_{n-1}[c + \rho_1 r_t + \rho_2 r_{t-1}] - B_n r_t - r_t = -\gamma B_{n-1} \sigma_\epsilon^2 \left(\sum_{n=2}^{N} (\chi r_{t-1}) B_{n-1} \right) \tag{51}$$

Set

$$\chi = -\frac{\rho_2}{\gamma \sigma_\epsilon^2 \sum_{n=2}^{N} B_{n-1}}$$

Then we obtain:

$$-A_{n-1} + A_n + B_{n-1}[c + \rho_1 r_t] - B_n r_t - r_t = 0$$

We can solve:

$$B_n = B_{n-1}\rho_1 - 1$$

$$A_n = A_{n-1} - B_{n-1}c$$

One can see that $B_n < 0$ and hence $\chi > 0$ assuming $\rho_2 < 0$ as in the data. The time-varying part of expected bond returns is

$$B_{n-1}(\rho_1 r_t + \rho_2 r_{t-1}) - B_n r_t = r_t + \rho_2 B_{n-1} r_t$$

This is increasing in r_t assuming $\rho_2 < 0$.

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