Sequential Learning, Asset Allocation, and Bitcoin Returns

with James Yae

AFA 2022 Annual Meeting

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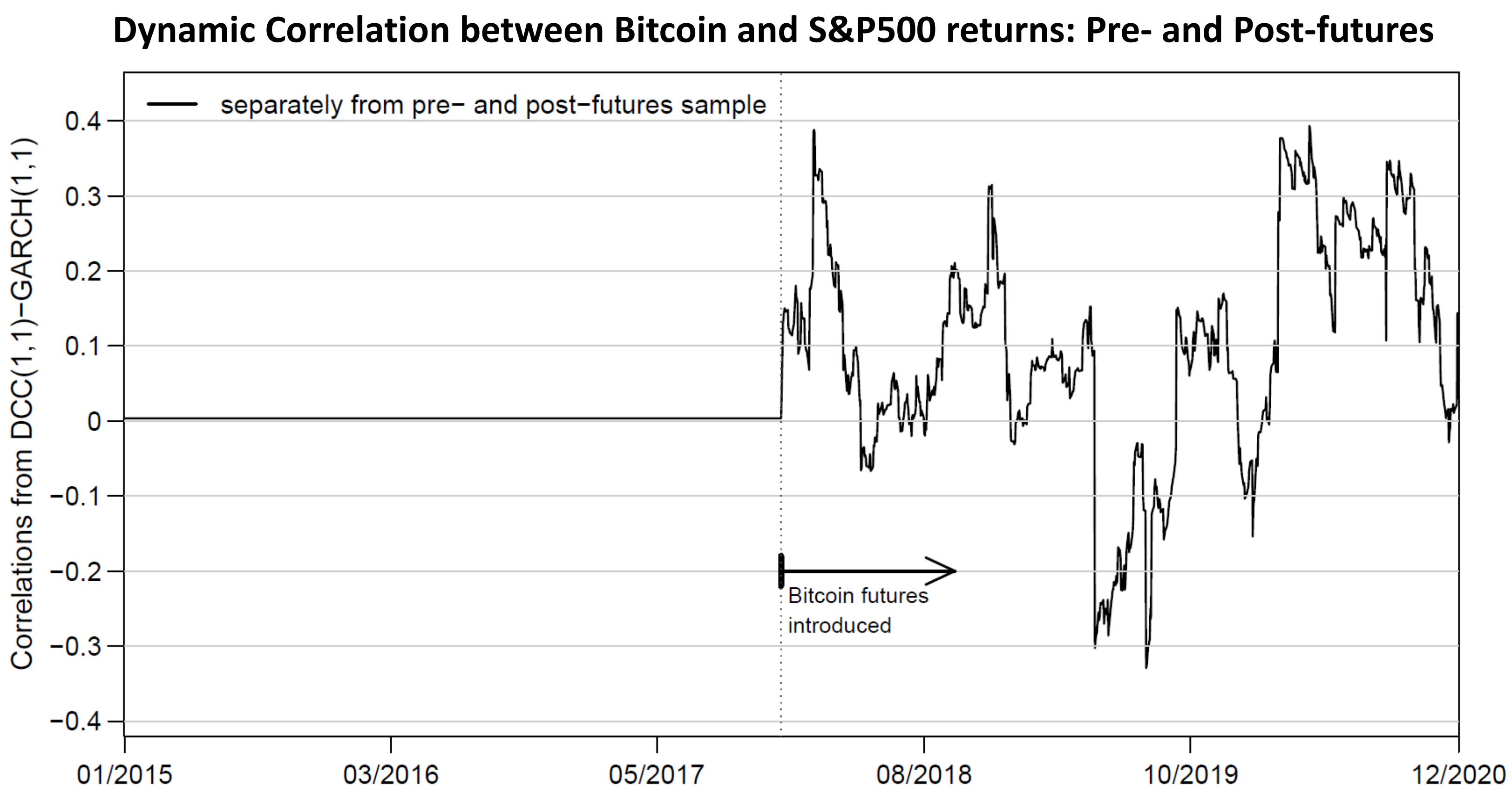
University of Houston

Structural Break



Bitcoin Return Predictability Puzzle

$$r_{b,t+1} = a_0 + a_1 x_t + \varepsilon_{t+1}$$

	Predictor x_t	coefficient a_1	t-statistic (NW)	R^2 (%)
✗	ρ_t	0.001	0.049	0.000
✗	β_t	-0.002	-0.735	0.063
✗	cov_t	2.682	0.664	0.063
✓	$\Delta\rho_t$	-0.153	-3.682	1.055
	$\Delta\rho_t$ (pre-COVID)	-0.153	-2.888	1.079
	$\Delta\rho_t$ (LAD)	-0.129	-3.816	1.017
	$\Delta\rho_t$ (Rank)	-0.142	-4.085	1.039
	$\Delta\rho_t$ (INT)	-0.134	-3.598	1.787
	$\Delta\rho_t$ (Trimmed)	-0.245	-2.824	1.255

In this analysis, we use post-futures data at daily frequency.

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- **No Bitcoin return predictability from ρ_t, β_t, or cov_t**

we would expect investors directly learn ρ_t

- **Increase in correlation ρ_t predicts lower Bitcoin returns**

high ρ_t → *small benefits from diversification* → *expect high RP to compensate*

→ *but negative coefficient of* $\Delta\rho_t$ *suggests lower returns after an increase in correlation*

Bitcoin Demand

the optimal weight on Bitcoin

$$w_{b,t} = \frac{\mu_t^* - \rho_t \sigma_t^*}{(\mu_t^* - \rho_t \sigma_t^*) + (\sigma_t^* - \rho_t \mu_t^*)\sigma_t^*}$$ Max Sharpe Ratio!

Conditional risk premium ratio $\mu_t^* = \mu_{b,t} / \mu_{m,t}$

Conditional volatility ratio $\sigma_t^* = \sigma_{b,t} / \sigma_{m,t}$

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Conditional volatility ratio $\sigma_t^* = \sigma_{b,t} / \sigma_{m,t}$

⇨ $w_{b,t}(\mu_t^*, \sigma_t^*, \rho_t)$ as a proxy for conditional Bitcoin demand!

Investors learn about $(\mu_t^*, \sigma_t^*, \rho_t)$ and adjust their portfolios

Bitcoin Demand

1. First layer Bitcoin demand decomposition

$$w_{b,t} - \overline{w}_b \approx \underbrace{[w_{b,t}^{(c+v)} - \overline{w}_b]}_{\text{Non-speculative}} + \underbrace{[w_{b,t}^{(mean)} - \overline{w}_b]}_{\text{Speculative}}$$

$$\overline{w_b} = w_b(\overline{\mu^*}, \overline{\sigma^*}, \bar{\rho})$$

$$w_{b,t}^{(mean)} \triangleq w_b(\mu_t^*, \overline{\sigma^*}, \bar{\rho})$$ ⇒ investors only learn μ_t^*

$$w_{b,t}^{(c+v)} \triangleq w_b(\overline{\mu^*}, \sigma_t^*, \rho_t)$$ ⇒ investors only learn (σ_t^*, ρ_t)

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$$w_{b,t}^{(c+v)} \triangleq w_b(\overline{\mu^*}, \sigma_t^*, \rho_t)$$ ⇨ investors only learn (σ_t^*, ρ_t)

2. Second layer Bitcoin demand decomposition

$$w_{b,t}^{(c+v)} - \overline{w}_b \approx \underbrace{[w_{b,t}^{(cor)} - \overline{w}_b]}_{\text{from correlation}} + \underbrace{[w_{b,t}^{(vol)} - \overline{w}_b]}_{\text{from volatility}}$$

Bitcoin Demand

3. Bitcoin demand change decomposition

$$\Delta w_{b,(t-1):t} \triangleq w_{b,t} - w_{b,t-1} = \Delta w^{(cor)}_{b,(t-1):t} + \Delta w^{(vol)}_{b,(t-1):t} + \Delta w^{(mean)}_{b,(t-1):t} + e_t$$

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non-speculative speculative

Bitcoin Demand

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non-speculative   speculative

estimation

DCC(1,1)-GARCH(1,1) Engle (2002)

- obtain (σ_t^*, ρ_t) estimates
- set $\overline{\mu^*} = 1$ for simplicity
- use median of (σ_t^*, ρ_t) for $(\overline{\sigma^*}, \bar{\rho})$

Equilibrium Model: Intuition

BTC Investors

Speculative Investors	Non-speculative Investors
$(\mu_t^*, \overline{\sigma^*}, \bar{\rho})$	$(\overline{\mu^*}, \sigma_t^*, \rho_t)$

conditional correlations affect subsequent Bitcoin prices through
asynchronous portfolio rebalancing

Equilibrium Model: Intuition

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Speculative Investors
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conditional correlations affect subsequent Bitcoin prices through
asynchronous portfolio rebalancing

t ——— t+1

SI trade based on μ_t^* (or $P_{b,t}$) → trade based on μ_{t+1}^* (or $P_{b,t+1}$)

NI buys BTC from SI

Equilibrium

NI estimate ρ_t (then set $w_{b,t}^{(c+v)}$) → rebalance portfolio at t+1

delay in order executions

asynchronous

Data Description

1. Data Source

- Cryptocurrency price levels from coinmarketcap.com
- Bitcoin attributes data from Blockchain.com
- Equity market and forex data from investing.com
- Daily treasury yield from US Department of Treasury

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2. Specs

- Daily closing price; different closing hours
- Holidays and weekends
- International equity markets
- Bitcoin future contracts introduced in December 2017
  - ✓ whole period 01/01/2015 – 12/31/2020
  - ✓ pre-futures period 01/01/2015 – 12/17/2017
  - ✓ post-futures period 12/18/2017 – 12/31/2020

Bitcoin Return Predictability

Predictive Regression

$$r_{b,t+1} = b_0 + b_1 \Delta w^{(cor)}_{b,(t-1):t} + b_2 \Delta w^{(vol)}_{b,(t-1):t} + Z_t \gamma + \varepsilon_{t+1}$$

b_1: effects of change in demand due to the learning of correlation on subsequent BTC returns

Z_t: Control variables

- $r_{b,t}, \beta_t, Volume_t,$ and EPU_t
- Market attributes (M)
- Blockchain attributes (B)
- Extra lagged returns (L)

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effects of change in demand due to the learning of correlation on subsequent BTC returns

Control variables

- $r_{b,t}, \beta_t, Volume_t, and$ EPU_t
- Market attributes (M)
- Blockchain attributes (B)
- Extra lagged returns (L)

- Stambaugh bias?
- Sample size sufficient?
- Look ahead bias?

Bitcoin Return Predictability

$$r_{b,t+1} = b_0 + b_1 \Delta w^{(cor)}_{b,(t-1):t} + b_2 \Delta w^{(vol)}_{b,(t-1):t} + Z_t \gamma + \varepsilon_{t+1}$$

Predictor	Post-futures (12/18/2017 to 12/31/2020)					Post-futures before COVID-19 (12/18/2017 to 02/29/2020)				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\Delta w^{(cor)}_{b,(t-1):t}$	0.51		0.47	0.47	0.47	0.50		0.48	0.47	0.48
	(4.15)		(2.89)	(2.91)	(2.80)	(3.16)		(2.78)	(2.63)	(2.37)
$\Delta w^{(vol)}_{b,(t-1):t}$		0.06	0.03	0.04	0.03		-0.19	-0.23	-0.22	-0.20
		(0.42)	(0.28)	(0.30)	(0.20)		(-1.27)	(-1.66)	(-1.67)	(-1.56)
$r_{b,t}$			-0.42	-0.42	-0.42			-0.22	-0.23	-0.22
			(-1.79)	(-1.76)	(-1.74)			(-0.83)	(-0.83)	(-0.82)
β_t			0.04	0.06	0.07			0.06	0.07	0.09
			(0.22)	(0.31)	(0.31)			(0.28)	(0.32)	(0.40)
$Volume_{b,t}$			-0.58	-0.55	-0.55			-0.74	-0.91	-0.87
			(-2.59)	(-1.35)	(-1.39)			(-3.52)	(-1.93)	(-1.78)
$EPU_{b,t}$			0.63	0.64	0.63			0.21	0.20	0.21
			(1.71)	(1.68)	(1.76)			(0.98)	(0.93)	(0.96)
Controls			M	MB	MBL			M	MB	MBL
R^2 (%)	1.15	0.01	3.94	3.99	4.00	1.17	0.16	4.04	4.14	4.38
$Adj.R^2$	1.02	-0.12	2.92	2.71	2.47	0.99	-0.02	2.62	2.36	2.24

Newey-West t statistics are reported in parenthesis.

Bitcoin Return Predictability

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Uncertainty in Portfolio Weights

Why $\Delta w^{(cor)}_{b,(t-1):t}$ not $\Delta w^{(vol)}_{b,(t-1):t}$?

Consider an AR(1) $z_t = a_0 + a_1 z_{t-1} + e_t$

$$z_t = (\sigma^*_t, \rho_t) \text{ or } (w^{(vol)}_{b,t}, w^{(cor)}_{b,t})$$

z_t	DCC-GARCH est.		Portfolio weights	
	σ^*_t	ρ_t	$w^{(vol)}_{b,t}$	$w^{(cor)}_{b,t}$
a_1	0.92 (0.01)	0.98 (0.01)	0.92 (0.01)	0.98 (0.01)
$var(e_t)$	0.16	0.05	11.13	0.37
$\frac{var(e_t^{(\sigma)})}{var(e_t^{(\rho)})}$		3.48		30.12

Estimation is obtained by the Post-futures sample. Standard errors are reported in parenthesis.

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$\frac{var(e^{(\sigma)}_t)}{var(e^{(\rho)}_t)}$		3.48		30.12

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Investors disregard the noisy volatility-ratio signals!

Global Equity and Crypto Markets

$$r_{c,t+1} = b_0 + b_1 \Delta w^{(cor)}_{b,(t-1):t} + \varepsilon_{t+1} \quad \text{for} \quad c \in \{BTC, ETH, XRP\}$$

	Coefficient estimates			Newey-West t-stat			R^2 (%)		
	BTC	ETH	XRP	BTC	ETH	XRP	BTC	ETH	XRP
S&P500	0.51	0.81	0.68	4.15	4.10	2.75	1.15	1.75	0.83
NYSE	0.48	0.75	0.57	3.45	3.73	1.91	1.02	1.51	0.57
NASDAQ	0.50	0.86	0.61	4.20	4.32	2.61	1.07	1.99	0.66
China	0.55	0.80	0.78	3.29	3.23	2.97	1.30	1.64	1.08
Japan	0.59	0.68	0.86	2.83	2.37	2.80	1.42	1.13	1.22
India	0.96	1.15	0.86	2.05	1.83	1.97	4.00	3.47	1.32
UK	0.46	0.62	0.56	3.20	3.16	3.29	0.93	1.04	0.56
Germany	0.45	0.67	0.74	2.37	3.26	3.03	0.92	1.23	1.04
France	0.40	0.60	0.65	2.57	2.79	2.84	0.73	0.95	0.77
Italy	0.21	0.32	0.43	1.21	1.28	1.89	0.19	0.29	0.36
Equal-weighted	0.53	0.79	0.75	3.55	3.96	3.25	1.25	1.65	1.01
GDP-weighted	0.62	0.94	0.86	4.58	4.74	3.68	1.71	2.38	1.34
Volume-weighted	0.61	0.92	0.80	4.91	4.85	3.50	1.62	2.24	1.16
Combination forecast	0.60	0.86	0.85	3.97	4.39	3.64	1.56	1.98	1.28

Out-of-Sample Predictability

Evaluation period	1/2/2019 to 12/31/2020				1/2/2019 to 2/29/2020			
	OLS	WLS	LAD	Rank	OLS	WLS	LAD	Rank
$\Delta w^{(cor)}_{b,(t-1):t}$, from S&P500								
R^2_{IS} (%)	1.29	1.17	0.92	0.97	1.47	1.36	1.07	1.07
R^2_{OS} (%)	1.07	1.54	2.32	2.33	1.49	2.07	3.49	3.52
R^{2*}_{OS} (%)	-0.08	0.40	1.19	1.20	-0.55	0.04	1.49	1.52
P-value of DM test	0.54	0.31	0.01	0.01	0.56	0.50	0.02	0.02
$\Delta w^{(cor)}_{b,(t-1):t}$, globally aggregated by GDP								
R^2_{IS} (%)	1.49	1.49	1.26	1.16	1.01	0.69	0.38	0.50
R^2_{OS} (%)	1.37	1.95	2.86	2.84	1.56	2.30	3.61	3.67
R^{2*}_{OS} (%)	0.23	0.81	1.74	1.71	-0.48	0.27	1.61	1.68
P-value of DM test	0.47	0.26	0.04	0.04	0.63	0.47	0.06	0.06
$\Delta\rho_{(t-1):t}$, from S&P500								
R^2_{IS} (%)	1.11	0.92	0.78	0.78	1.53	1.42	1.11	1.04
R^2_{OS} (%)	0.92	1.41	2.13	2.17	1.36	1.97	3.29	3.36
R^{2*}_{OS} (%)	-0.22	0.27	1.00	1.04	-0.68	-0.07	1.29	1.35
P-value of DM test	0.63	0.37	0.01	0.01	0.60	0.54	0.02	0.02
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R^2_{IS} (%)	1.29	1.27	1.01	0.99	0.95	0.56	0.26	0.40
R^2_{OS} (%)	1.22	1.75	2.46	2.55	1.49	2.23	3.47	3.54
R^{2*}_{OS} (%)	0.08	0.61	1.34	1.42	-0.56	0.20	1.46	1.54
P-value of DM test	0.61	0.47	0.04	0.04	0.53	0.27	0.01	0.01

Technical details

- Data before 2019 for training. Re-estimating the predictor and b1 everyday with an expanding window.
- R^2_{IS}: in-sample R^2
- R^2_{OS}: out-of-sample R^2 against historical averages
- R^{2*}_{OS}: out-of-sample R^2 against zeros
- DM test: whether a prediction is better than a naïve forecast of zero
- Models: OLS, Weighted Least Square, Least Absolute Deviation, and a Rank-based regression

Main takeaways

❖ **Strong OOS predictability in general**

❖ **Change in weight predictor beats change in correlation predictor**

❖ **Predictability from using global equities is better than that from using S&P500 alone**

Conclusions

1) Dynamic correlation:
   Bitcoin's changing narrative characterized by dynamic correlation with stock markets since the introduction of Bitcoin futures.

2) Bitcoin return predictability "puzzle":
   Increase in daily Bitcoin demand change due to dynamic correlation predicts higher subsequent Bitcoin returns (puzzling predictor & sign)

3) Rational asset allocation:
   The empirical pattern is consistent with investors' learning on time-varying correlations and practice on rational portfolio optimization

4) Asynchronous portfolio rebalancing:
   We use an equilibrium model to explain how Bitcoin return predictability can be generated through asynchronous portfolio rebalancing

Thank You!