Quantile Approach to Asset Pricing Models

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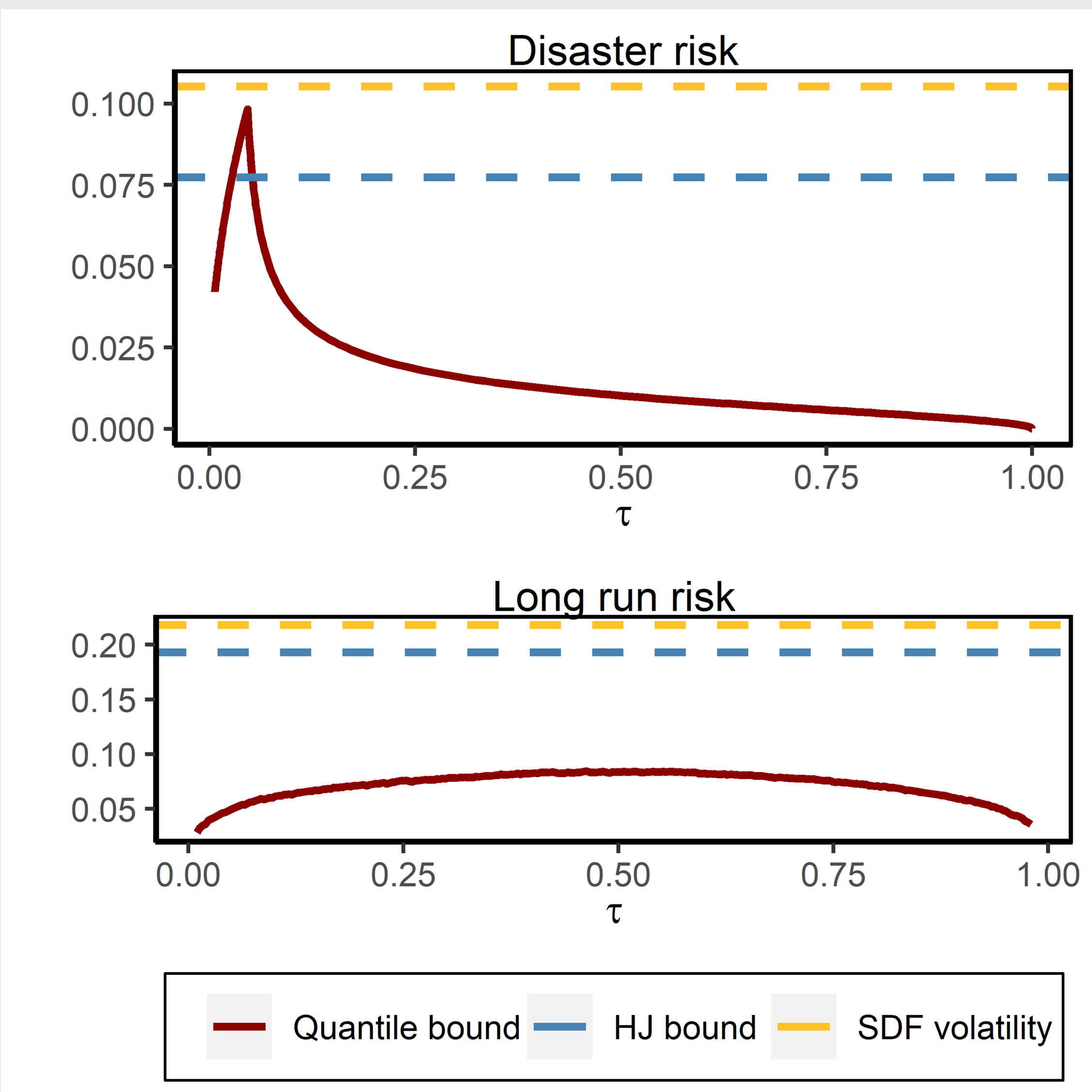
Motivation

- Misspecification of asset pricing models typically confined to mean-variance analysis.
- Example: Any proposed SDF (M) needs to overcome the Hansen-Jagannathan (HJ) bound

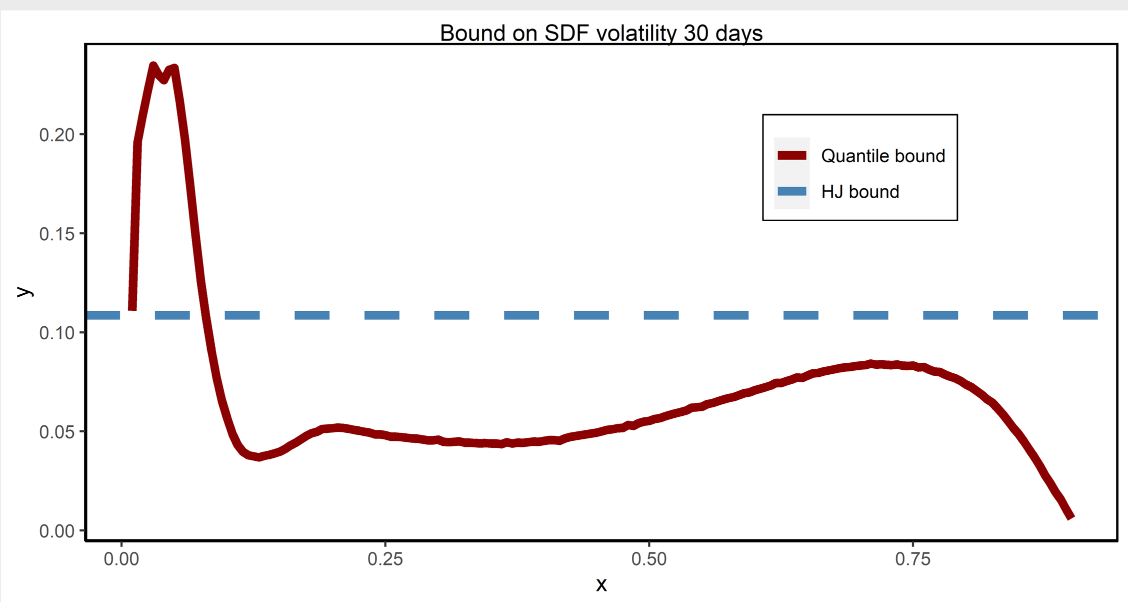
$$\sigma(M) \geq \frac{\mathbb{E}[R] - R_f}{R_f \cdot \sigma(R)}$$

- Consider the consumption based SDF $M = \beta g_c^{-\gamma}$, where g_c and γ denote consumption growth and risk-aversion respectively.
- This model is misspecified since $\sigma(g_c)$ is low in the data. Thus we need counterfactually high levels of γ to overcome the HJ bound.
- Question: Can we use other statistics than mean and variance that gives more insight into determinants of misspecification?

Bound comparison for different asset pricing models



Bounds for S&P500 data



- Shape of quantile bound roughly similar to disaster risk model.
- More formal testing shows that quantile bound is significantly stronger than HJ bound.
- The LRR model can only reconcile this for very high levels of risk-aversion ($\gamma \geq 90$).
- Intuition: disaster risk induces a peak in quantile bound for small τ. For conditional lognormal models, the quantile bound is essentially symmetric.

A new bound

- Let $F(x) = \mathbb{P}(R \leq x)$ denote the CDF of the return DGP.
- Similarly, $\widetilde{F}(x) = \widetilde{\mathbb{P}}(R \leq x)$, is the CDF of the risk-neutral distribution. Define $\widetilde{Q}_\tau$ as the risk-neutral quantile function $\widetilde{F}(\widetilde{Q}_\tau) = \tau,\ \forall \tau \in (0,1)$.
- Consider the ordinal dominance curve $\phi(\tau) := F(\widetilde{Q}_\tau)$.
- We then obtain a new bound on the SDF volatility for all $\tau \in (0,1)$:

Theorem 0.1 (Quantile bound)**.**

$$\sigma(M) \geq \frac{\tau - \phi(\tau)}{R_f\sqrt{\phi(\tau)\cdot(1-\phi(\tau))}}. \tag{1}$$

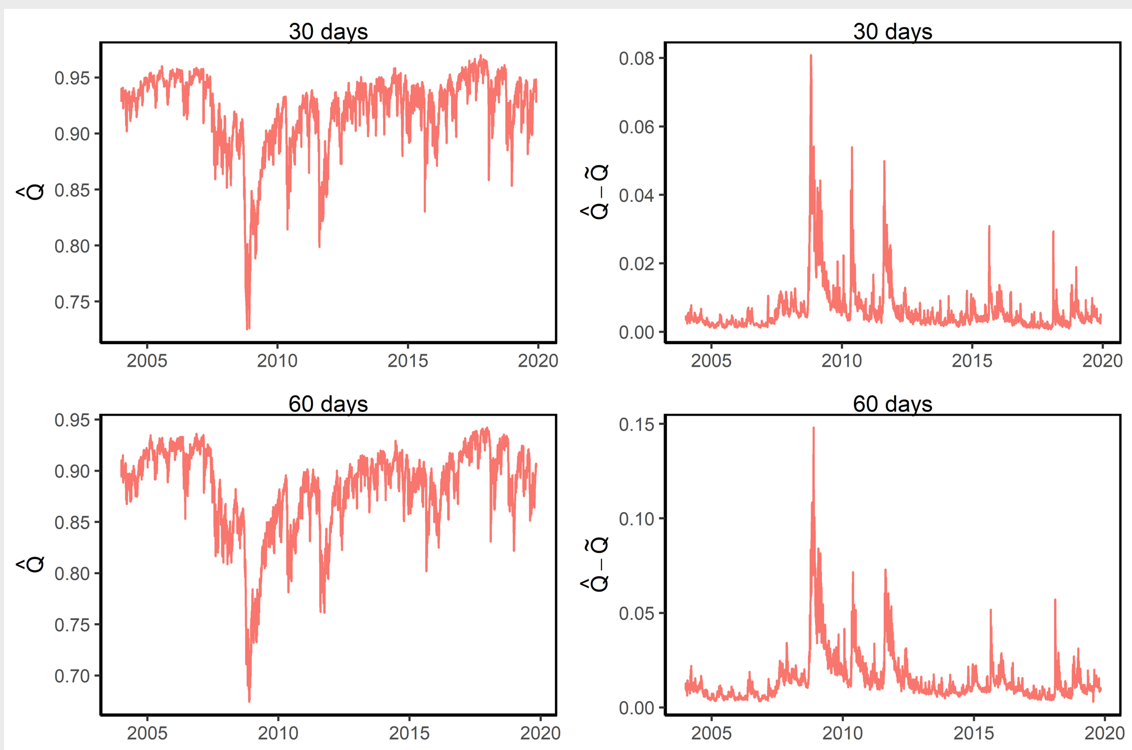
Conditional quantile premium

- We also consider the conditional difference $Q_{t,\tau} - \underbrace{\widetilde{Q}_{t,\tau}}_{\text{observed}}$.
- Von-Mises approximation yields $Q_{t,\tau} \approx \widetilde{Q}_{t,\tau} + \frac{\tau - F_t(\widetilde{Q}_{t,\tau})}{\widetilde{f}_t(Q_{t,\tau})}$
- Building on Chabi-Yo and Loudis (2020), we bound $\tau - F_t(\widetilde{Q}_{t,\tau}) \geq LRB_t(\tau)$, where $LRB_t(\tau)$ is inferred at time t from option data.
- In spirit of Martin (2017), we test tightness of the bound, using quantile regression $Q_{t,\tau} = \beta_0(\tau) + \beta_1(\tau)\cdot\left(\widetilde{Q}_{t,\tau} + \frac{LRB_t(\tau)}{\widetilde{f}_t(\widetilde{Q}_{t,\tau})}\right)$

Maturity:	30 days				
	$\widehat{\beta}_0(\tau)$	$\widehat{\beta}_1(\tau)$	Wald test	$R^1(\tau)[\%]$	$R^1_{oos}(\tau)[\%]$
$\tau = 0.01$	0.06 (0.3132)	0.97 (0.3506)	0.97	21.08	17.26
$\tau = 0.05$	0.20 (0.2944)	0.80 (0.3130)	0.41	9.27	9.21
$\tau = 0.1$	0.17 (0.2661)	0.83 (0.2766)	0.46	5.67	6.14
$\tau = 0.2$	0.21 (0.3808)	0.79 (0.3881)	0.54	1.7	3.78

- Conclusion: $\widehat{Q}_{t,\tau} := \widetilde{Q}_{t,\tau} + \frac{LRB_t(\tau)}{\widetilde{f}_t(\widetilde{Q}_{t,\tau})}$ is a good approximation of latent $Q_{t,\tau}$.

$\widehat{Q}_{t,\tau}$ and $\widehat{Q}_{t,\tau} - \widetilde{Q}_{t,\tau}$ for $\tau = 0.05$



References

Backus, D., Chernov, M., and Martin, I. (2011). Disasters implied by equity index options. *The Journal of Finance*, 66(6):1969–2012.

Bansal, R., Kiku, D., and Yaron, A. (2012). An empirical evaluation of the long-run risks model for asset prices. *Critical Finance Review*, (1):183–221.

Chabi-Yo, F. and Loudis, J. (2020). The conditional expected market return. *Journal of Financial Economics*.

Martin, I. (2017). What is the expected return on the market? *The Quarterly Journal of Economics*, 132(1):367–433.

Ross, S. A. (2015). The recovery theorem. *The Journal of Finance*, 70(2):615–648.

Disaster and long-run risk model

- The quantile bound (1) is quite different depending on the asset pricing model. We consider two models:
  (i) Disaster risk model (Backus et al., 2011):
  $$M = \beta g_c^{-\gamma}, \text{ where } \log g_c = \varepsilon + \eta$$
  and $\varepsilon \sim N(\mu, \sigma^2), \eta|(J = j) \sim N(j\theta, j\nu^2), J \sim$ **Poisson**(κ).
  (ii) Long-run risk (LRR) model (Bansal et al., 2012):
  $$\log M_{t+1} = \textbf{Constant} - \frac{\theta}{\psi}\log g_{c,t+1} + (\theta - 1)\log R_{c,t+1}$$
  Here, $R_{c,t+1}$ is the return on the consumption asset. Both $g_{c,t+1}, R_{c,t+1}$ are conditionally lognormal.
- We compare the quantile bound and HJ bound using the model calibration from Backus et al. (2011) and Bansal et al. (2012).
- The figure shows that the quantile bound can be stronger than the HJ bound in disaster risk model, but not in LRR model.

$\widehat{Q}_{t,\tau}$ and $\widehat{Q}_{t,\tau} - \widetilde{Q}_{t,\tau}$

- Left panels show $\widehat{Q}_{t,\tau}$ over time. Evidence for time varying disaster risk.
- Right panels show $\widehat{Q}_{t,\tau} - \widetilde{Q}_{t,\tau}$. Spikes occur amidst height of financial crisis.
- Since $\widehat{Q}_{t,\tau}$ goes down during crisis (left panels), but $\widehat{Q}_{t,\tau} - \widetilde{Q}_{t,\tau}$ goes up (right panels), we conclude that $\widetilde{Q}_{t,\tau}$ changes more than $\widehat{Q}_{t,\tau}$.
- $\widetilde{Q}_{t,\tau}$ captures insurance effect, whereas $\widehat{Q}_{t,\tau}$ captures forward looking loss of a crash.
- Data show that insurance effect is more dominant.
- As byproduct, we have shown how to recover part of the left tail distribution. Using quantile regression, we also verify that we can recover the right tail of the distribution.
- Using quantile regression, we estimate the equation $Q_{t,\tau} = \beta_0(\tau) + \beta_1(\tau)\cdot\widetilde{Q}_{t,\tau}$, for $\tau \geq 0.5$ and find $\beta_0(\tau) \approx 0, \beta_1(\tau) \approx 1$.
- This means that almost all risk-adjustment comes from the left tail. This complements the theoretical recovery theorem from Ross (2015).

$Q_{t,\tau} = \beta_0(\tau) + \beta_1(\tau)\widetilde{Q}_{t,\tau}$

Horizon	30 days		
	$\widehat{\beta}_0(\tau)$	$\widehat{\beta}_1(\tau)$	Wald test
$\tau = 0.05$	0.31 (0.2510)	0.69 (0.2680)	0.06
$\tau = 0.1$	0.32 (0.2273)	0.67 (0.2372)	0.02
$\tau = 0.2$	0.38 (0.3211)	0.62 (0.3278)	0.07
$\tau = 0.5$	0.06 (0.2273)	0.94 (0.2258)	0.88
$\tau = 0.8$	−0.04 (0.1842)	1.04 (0.1788)	0.92
$\tau = 0.9$	0.04 (0.1547)	0.96 (0.1486)	0.88
$\tau = 0.95$	0.00 (0.1518)	1.00 (0.1445)	1