The Optimal Nominal Price of a Stock: A Tale of Two Discretenesses

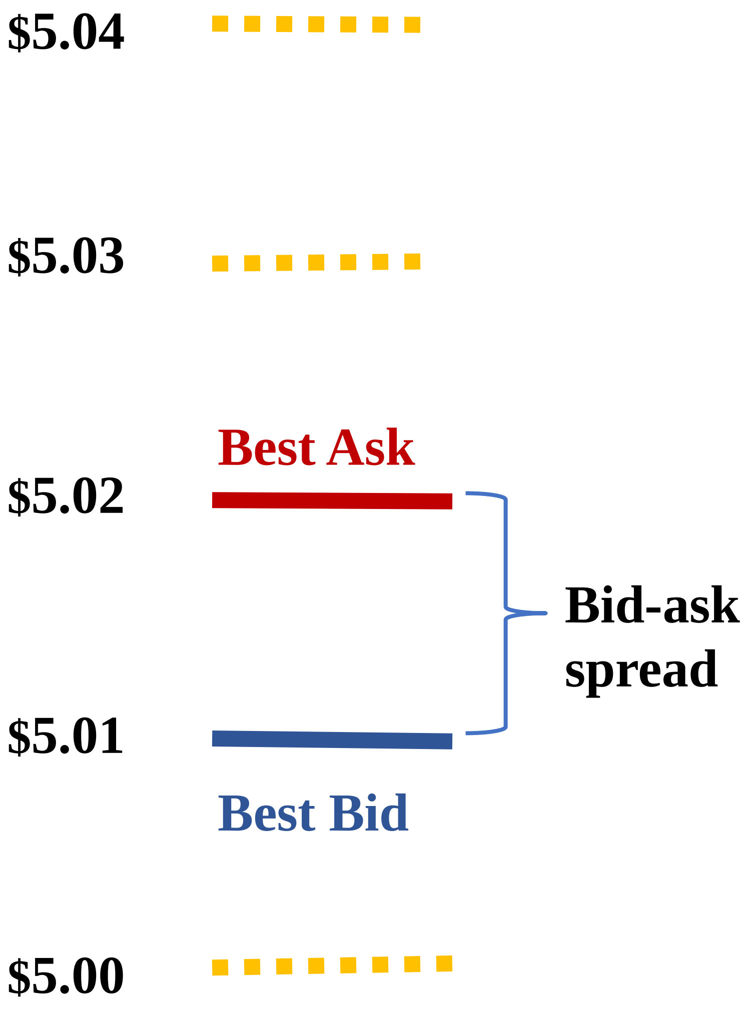
Sida Li

University of Illinois at Urbana-Champaign

Mao Ye

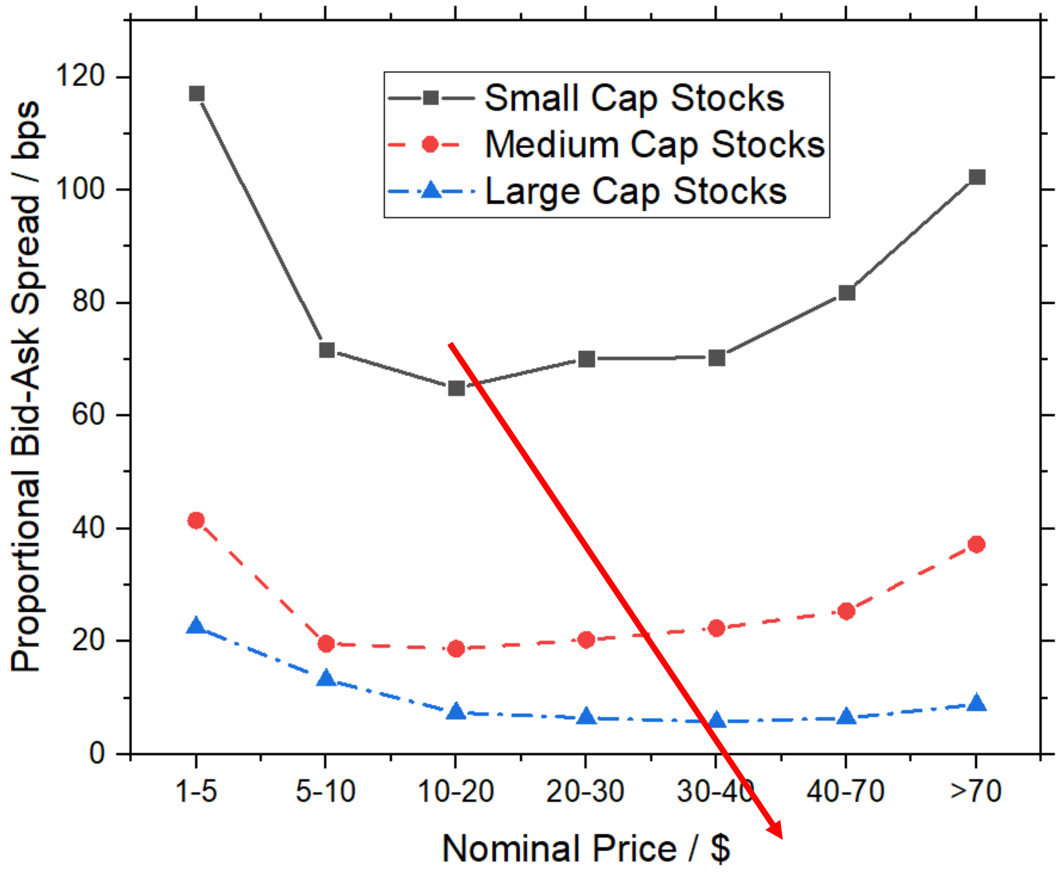
University of Illinois at Urbana-Champaign and NBER

Discrete Prices, Discrete Quantities and the Tradeoff



- An important and implicit assumption in most models
  - Both price and quantity are continuous variables
- A liquidity provider in U.S. stock exchanges
  - Minimum price variation (tick size) of 1 cent for bid and ask prices
  - Minimum lot size of 100 shares to display quotes
    - Trades can be in odd lots but quotes are in round lots
  - The National Best Bid and offer (NBBO) is defined over one-cent tick and 100-share lot
- Tradeoff for a U.S. listed firm
  - Choose a high price to make prices more continuous
  - Choose a low price to make quantities more continuous
  - What is the optimal price for liquidity?
- Regulators can change tick and lot sizes
  - What are the impacts of tick and lot sizes?
  - Should regulators choose one-size-fit-all tick and lot sizes?

U-Shaped Relationship Between Nominal Price and Liquidity



Large stocks bring higher optimal prices

- Choose a low price: tick constraints
  - 1 cent of $2 is 50 bps
  - Stock splits reduce liquidity when tick size is binding
    - Nominal spread: still ~1 cent
    - The percentage spread increases
- Choose a high price: lot constraints
  - 100 shares of a $3,000 stock is $300k
  - Larger adverse-selection risk for market-makers
  - Stock splits improve liquidity
    - The percentage spread decreases
- How can a firm achieve its optimal price?
- Which stock characteristics affect the optimal price?

Main Findings

Theory

- Two-Tick rule
  - Optimal price depends on volatility, dollar volume, and tick and lot size
  - All firms achieve their optimal prices when their nominal bid-ask spreads are two ticks
    - Contribution from tick size = Contribution from lot size
- Split ratio to achieve optimal price: $\sqrt{\frac{Current\ bid{-}ask\ spread}{Tick\ size}} - 1$

Empirics

- ➢ Explains 81% of cross-sectional variation in liquidity
- ➢ Explains 57% of cross-sectional variation in stock prices
- ➢ Stock splits increase liquidity if they move bid-ask spreads closer to two ticks
- ➢ The liquidity channel explains 1/3 of the split announcement returns

Policy

- ❖ Change tick and lot sizes
- ❖ Switch from uniform to proportional tick and lot sizes

Three-Stage Model, Five Types of Players

- Stage 1: A regulator chooses lot size L and tick size Δ
  - Can either be uniform or proportional

- Stage 2: A firm with value v chooses share price p
  - $H = \frac{v}{p}$ is shares outstanding
  - The firm aims to minimize expected trading costs for its traders

- Stage 3: A market-maker sets a competitive bid-ask spread s_t
  - Security value $v_{t\in(0,+\infty)}$ evolves as a compound Poisson process
    - Intensity of the jump event: λ_J
    - Size of the jump: σv_t or $-\sigma v_t$
  - The market-maker chooses s_t given p_t, L, Δ and strategies of informed and uninformed traders
    - Informed traders aim to adversely select the market-maker before the value jumps
    - Uninformed traders, each of whom has an inelastic need to buy/sell v_t, arrive at intensity λ_I
      - Choose how to divide their demand into child orders

Budish, Cramton, and Shim (BCS 2015) as a Special Case

- BCS do not model tick size, lot size, or share price choices
  - Alternatively, you can assume that the regulator and the firm in BCS make suboptimal choices
  - Stage 1: The regulator chooses $\Delta = 0$ and $L = 1$
  - Stage 2: The firm chooses a large price, $p = v$
    - Traders and the market-maker cannot further divide their demand into smaller units
    - The lot size is binding
  - Stage 3: Traders interact under the following parameters:
    - $\Delta = 0$
    - $L = 1$
    - $p = v$

- Adverse selection in BCS comes from HFTs' quick response to public information
  - BCS propose frequent batch auctions to solve the latency arbitrage problem
  - Adverse selection in our model can come from private information, following Baldauf and Mollner (2020)

Roadmap

- Continuous pricing ($\Delta = 0$) and discrete lot ($L = 1$): as in BCS
  - Difference: We allow the firm to choose its share price p

- Discrete pricing ($\Delta > 0$) and discrete lot ($L > 0$)
  - Reflect current U.S. regulation: Uniform tick and lot sizes
  - Theory
  - Empirical evidence
    - Cross-sectional evidence
    - Stock splits

- The regulator
  - Increases or decreases (uniform) tick and lot sizes
    - Lot and tick sizes are the same for all stocks
  - Switches to proportional tick and lot sizes
    - Lot and tick sizes are proportional to price

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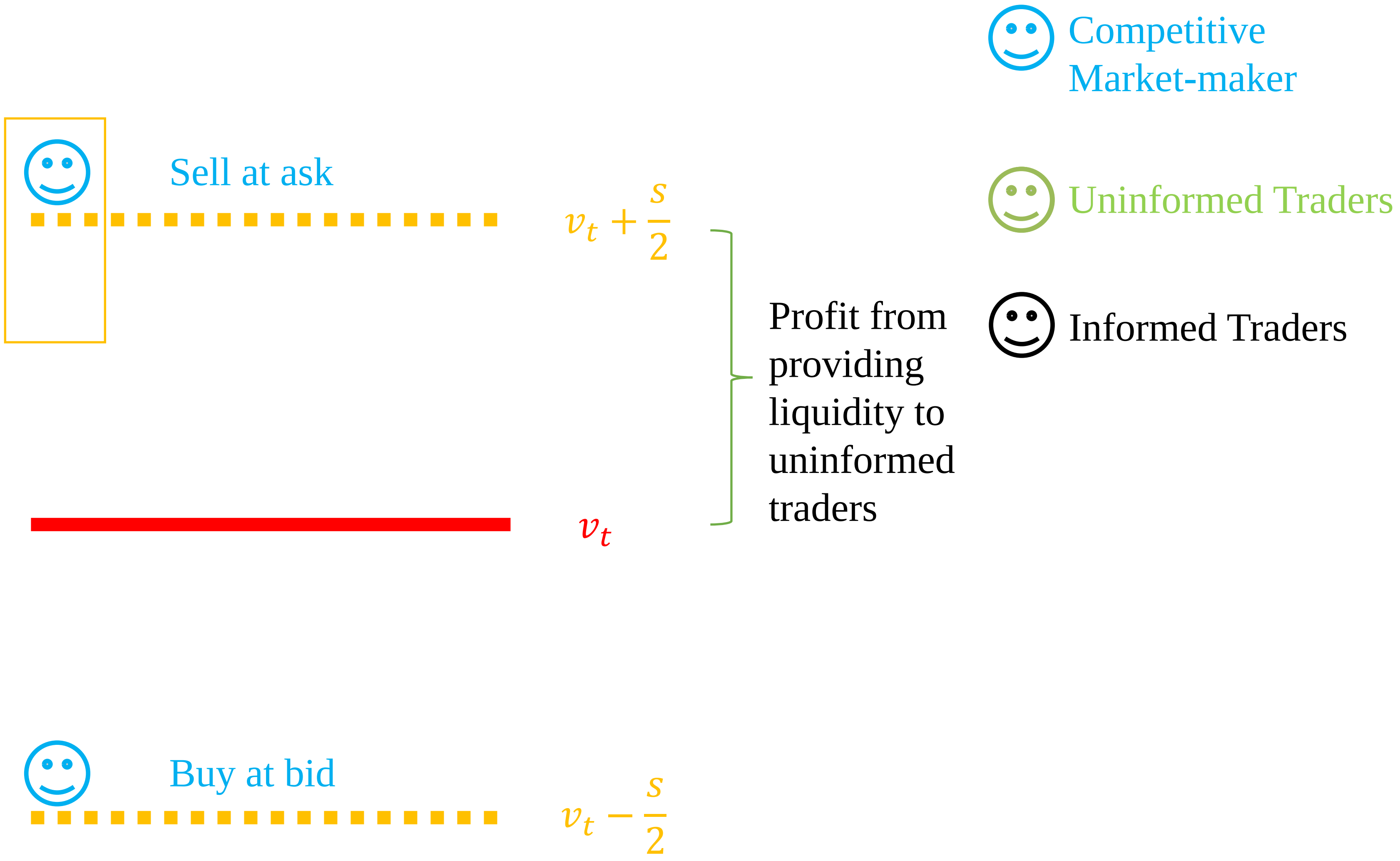
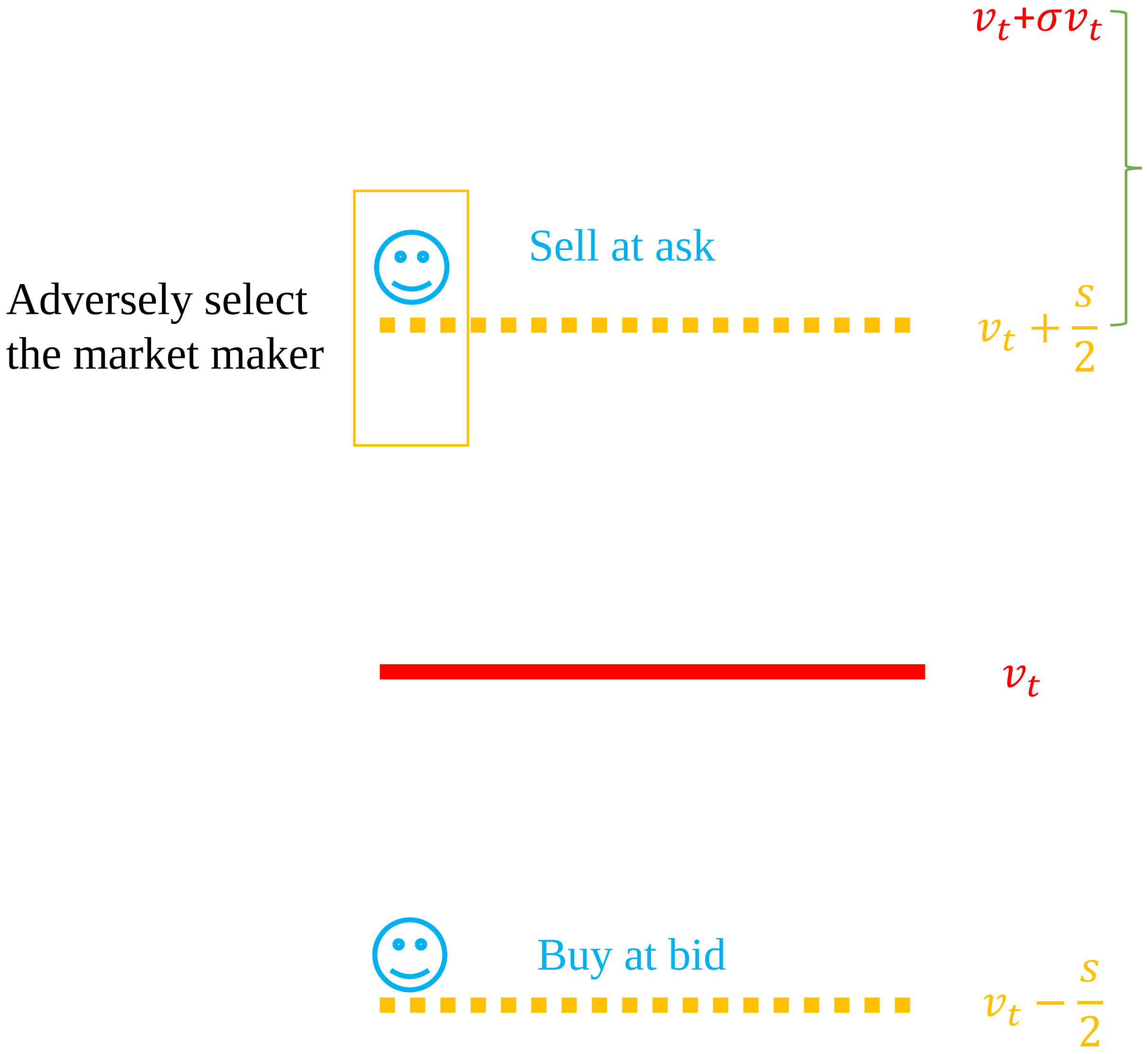
Revenue for Supplying Liquidity to an Uninformed Trader



Costs of Being Adversely Selected



Equilibrium bid-ask spread in BCS equalizes the profit and losses

- $s_t^{BCS} = \frac{2\sigma v_t \lambda_J}{\lambda_I + \lambda_J}$

λ_I: Investor arrival rate
λ_J: Value jump arrival rate
σ: Volatility
s: Bid-ask spread ($)

Strategy Spaces for the Firm

- The firm chooses a nominal price level at time 0 through H-for-1 splits
- Nominal price $p \equiv \frac{v}{H}$
- "One share" in BCS becomes H shares
  - Holding demand fixed, uninformed traders now need to trade H shares
- The firm minimizes expected trading cost per unit of time for its traders
  - Equivalently, the firm maximizes its liquidity

Strategy Spaces for Traders

- Uninformed traders minimize transaction costs by dividing the H-share into child orders
  - $f(q)$: the frequency of child order of q lots
  - s.t. $\Sigma\, qLf(q) = H$
  - Example of two extreme choices
    - Submits H shares all at once: $f(q) = 1$ for $q = \frac{H}{L}$
    - Slice orders into the minimum lot: $f(q) = \frac{H}{L}$ for $q = 1$
  - The arrival rate for order of size q becomes $\lambda_I f(q)$

- The competitive market-maker chooses B_t^q and A_t^q after knowing $f(q)$
  - Bid and ask prices for the q^{th} lot
  - In equilibrium, $A_t^1 \le A_t^2 \le \ldots \le A_t^{\max(q)} \le p_t(1+\sigma)$ and $B_t^1 \ge B_t^2 \ge \cdots \ge B_t^{\max(q)} \ge p_t(1-\sigma)$.
    - Larger child orders tend to walk up the book and execute at worse prices
  - Define bid-ask spread $s_t = A_t^1 - B_t^1$ and proportional spread $\mathcal{S}_t \equiv \frac{s_t}{p_t}$

- Informed traders adversely select the market-maker with intensity λ_J

Equilibrium: Slice-and-Dice to the Minimum Lot

Uninformed traders slice demand into one-lot child orders

- One-lot orders never walk up the book
- One-lot orders increase arrival intensity of uninformed orders
- Trading intensity: $\lambda_I v_t \equiv \lambda_I p_t H$

The market-maker maintains one lot at the best bid and ask

- One-lot depth minimizes the adverse-selection loss
- Quickly refills 1 lot once the previous lot is consumed
- The informed trader can pick off only one lot
    - Smaller dollar lot size reduces the market-maker's loss

Informed traders pick off the one lot when value jumps

- Trading intensity: $\lambda_J p_t L$
- Market maker's loss: $\lambda_J p_t L \cdot (\sigma - \frac{s_t}{2})$

Empirical support

- We find that 87.5% of depths at the best bid and ask price are exactly one lot if the bid-ask spread is not constrained by one tick

The Percentage Bid-Ask Spread and the Firm's Choice

- The equilibrium percentage spread equalizes revenues and losses

$$\lambda_I p_t H \cdot \frac{\mathcal{S}_t}{2} = \lambda_J p_t L \cdot \left(\sigma - \frac{\mathcal{S}_t}{2}\right)$$

- $\mathcal{S}_t = \frac{2\sigma\lambda_J L}{\lambda_I H + \lambda_J L} < \mathcal{S}_t^{BCS} = \frac{2\sigma\lambda_J}{\lambda_I + \lambda_J}$
  - BCS assume that $H = 1$ and $L = 1$
  - They propose narrowing the spread using frequent batch auctions

λ_I: Investor arrival rate
λ_J: Value jump arrival rate
σ: Volatility (jump size)
$\mathcal{S}$: Bid-ask spread (bps)
H: Shares outstanding

- We find two alternative ways to narrow the BCS bid-ask spread:
  - Regulator can reduce lot size L
  - The firm can aggressively split the stock (increasing H)
    - Intuition: when lot size is the only friction, the firm aggressively splits to minimize lot-size constraints

The Square Rule for Bid-ask Spread

- $s_t = \mathcal{S}_t p_t = \frac{2\sigma\lambda_J L}{\lambda_I H + \lambda_J L} p_t = \frac{2\sigma\lambda_J p_t^{2} L}{(\lambda_I H + \lambda_J L) p_t} = \frac{2\sigma\lambda_J p_t^{2} L}{DVol}$
  - Dollar trading volume $DVol = (\lambda_I H + \lambda_J L) p_t$
  - The bid-ask spread increases quadratically with price
    - An p_t-time increase comes mechanically from share price
    - The other p_t-time comes from increased adverse-selection risk

λ_I: Investor arrival rate
λ_J: Value jump arrival rate
σ: Volatility (jump size)
$\mathcal{S}$: Bid-ask spread (bps)
s: Bid-ask spread ($)
H: Shares outstanding
$DVol$: Trading volume ($)

- The bid-ask spread under continuous pricing
  - Proportional to the square of p_t
  - Proportional to the volatility $\sigma\lambda_J$
  - Inversely proportional to the dollar trading volume

- Next step: Adding discrete price and generating the Modified Square Rule

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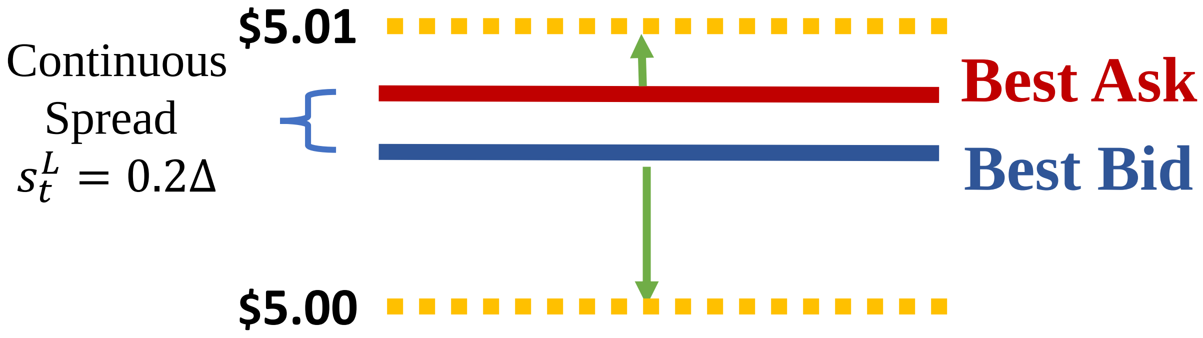
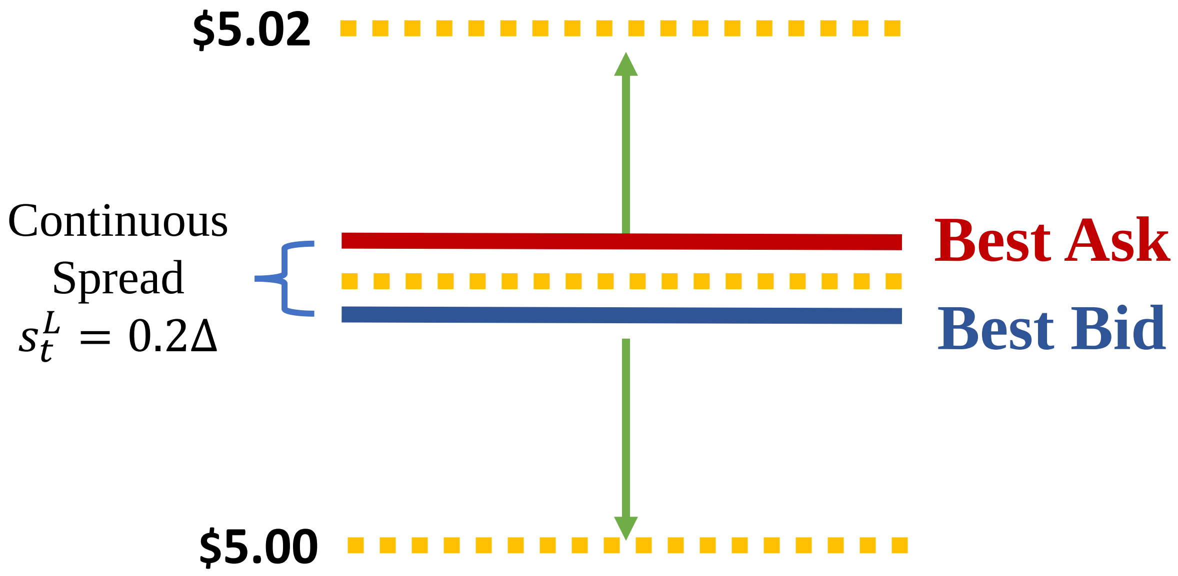
The Bid-Ask Spread under Discrete Pricing

- Discrete price widens the quote to the next available tick
$$s_t^{tot} = s_t^L + s_t^\Delta$$
- The widening effect s_t^Δ depends on the relative location of $p_t \pm \frac{s_t^L}{2}$ within the tick grid
- Our model shows that the expected widening effect $\mathbb{E}\left[s_t^\Delta\right]$ is one tick Δ



Widens less than Δ



Widens more than Δ

The Average Bid-Ask Spread Under Discrete Pricing

- Decompose average bid-ask spread s^{tot} into two components

$$\mathbb{E}[s_t^{tot}] = \underbrace{\mathbb{E}\left[\frac{2\sigma\lambda_J p_t^2 L}{DVol}\right]}_{Lot\ Driven} + \underbrace{\Delta}_{Tick\ Driven}$$

- s^L: Lot-driven spread
  - The bid-ask spread under continuous pricing
  - Splits reduce the lot-driven spreads following the Square Rule

- s^Δ: Tick-driven spread is one tick
  - No matter how firm splits their stocks, the nominal bid-ask spread cannot be lower than one tick
  - Splits do not change the tick-driven nominal spread, thereby increasing the percentage spread

Proposition 4: Two-Tick Rule and the Optimal Price

Homogeneous Optimal Spread

- $\min_{p} E\left[\frac{s_t^{tot}}{2p_t} \cdot DVol\right]$

$= \min_{p} \sigma\lambda_J pL + \frac{\Delta}{p}\frac{\lambda_I v}{2} + \text{constant}$

- $\sigma\lambda_J pL + \frac{\Delta}{p}\frac{\lambda_I v}{2} \geq 2\sqrt{\sigma\lambda_J pL \cdot \frac{\Delta}{p}\frac{\lambda_I v}{2}}$
  - Equality only when $\sigma\lambda_J pL = \frac{\Delta}{p}\frac{\lambda_I v}{2}$
  - Tick-driven spread = Lot-driven spread

- All firms reach maximum liquidity when their bid-ask spreads are two ticks

Heterogeneous Optimal Price

$$p^* = \sqrt{\frac{\lambda_I v\Delta}{2\sigma\lambda_J L}}$$

- Volatile stocks should choose low prices
  - σ is the jump size and λ_J is the jump frequency
  - High break-even percentage spread
  - The main constraint comes from lot size

- Active stocks should choose high prices
  - v is market cap and λ_I is the turnover rate
  - $\lambda_I v$ is the dollar volume
  - Low break-even percentage spread
  - The main constraint comes from tick size

- Heterogeneous p^* leads to homogeneous optimal spread

Proposition 5: The Modified Square Rule

- $p^* = \sqrt{\frac{\lambda_I v \Delta}{2\sigma\lambda_J L}}$

- Fortunately, the firm does not need to calibrate σ, v, λ_I, or λ_J
  - Its current bid-ask spread provides a sufficient statistic for the optimal split ratio

- We can decompose the firm's current spread: $\mathbb{E}[s_t^{tot}] = \mathbb{E}[s_t^L] + \Delta$
  - An H-for-1 split reduces the lot component from $\mathbb{E}[s_t^L] = \mathbb{E}[s_t^{tot}] - \Delta$ to $\frac{\mathbb{E}[s_t^{tot}]-\Delta}{H^2}$
  - The total spread after the split = $\frac{\mathbb{E}[s_t^{tot}]-\Delta}{H^2} + \Delta$

- The optimal split ratio $H^* = \sqrt{\frac{\mathbb{E}[s_t^{tot}]-\Delta}{\Delta}}$

An Example for the Modified Square Rule

- The total spread after the split = $\frac{\mathbb{E}[s_t^{tot}]-\Delta}{H^2} + \Delta$

- Example: Why does Amazon have a sixfold wider spread than Microsoft?
  - Amazon: \$3,305 stock price; \$1.53 average bid-ask spread (4.7 bps)
  - Microsoft: \$255 stock price; \$0.0195 average bid-ask spread (0.77 bps)

- If Amazon splits 13-for-1 to make its price similar to Microsoft's
  - Amazon's bid-ask spread would change to $\frac{153-1}{13^2} + 1 = 1.90$ cents
  - The split leads to a spread similar to Microsoft's and the percentage spread narrows

- The optimal split ratio $H^* = \sqrt{\frac{\mathbb{E}[s_t^{tot}]-\Delta}{\Delta}}$
  - For Amazon, it is $\sqrt{\frac{1.53-0.01}{0.01}} = 12.3$, and we predict that such a split can lead to a two-tick spread

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Cross-Sectional Tests

- Three-factor model of liquidity (81% R-squared)
  - $s_t^L = s_t^{tot} - s_t^{\Delta} = \frac{2\sigma\lambda_J p_t^2 L}{\lambda_I v_t}$
  - $log\left(s_t^{tot} - s_t^{\Delta}\right) = 2\,\underbrace{log(p_t)}_{Price} - \underbrace{log(\lambda_I v_t)}_{Dollar\ Volume} + \underbrace{log\left(\sigma\lambda_J\right)}_{Volatility} + Const$
    - $E(s_t^{\Delta}) = \Delta$
  - $log\left(\overline{Spread} - \Delta\right)_i = 2 \cdot log(Price)_i + log(Volume)_i + log(Volatility)_i + \varepsilon_i$

- Two-factor model of stock price (57% R-squared with 2 factors)
  - $p_t^* = \sqrt{\frac{\lambda_I \Delta v_t}{2\sigma\lambda_J L}}$
  - $log(p_t^*) = \frac{1}{2}\underbrace{log(\lambda_I v_t)}_{Dollar\ Volume} - \frac{1}{2}\underbrace{log\left(\sigma\lambda_J\right)}_{Volatility}$
  - $log(Price)_i = \frac{1}{2} log(Dollar\ Volume)_i - \frac{1}{2} log(Volatility)_i + \varepsilon_i.$

Direct Test of the Modified Square Rule

$$log\left(\overline{Spread} - \Delta\right)_i = 2 \cdot log(Price)_i + log(Volume)_i + log(Volatility)_i + \varepsilon_i$$

- Sample: U.S. Common Stocks with 1 cent tick size and 100-share lot size

Panel A: Three-Factor Model of Liquidity

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent Variable	$Log(s_t^L) = Log(s_t^{tot} - \Delta)$					
Sample Period	2020	2019	2018	2017	2016	2015
Log(Price$_t$)	**2.09*** **	**2.08*** **	**2.12*** **	**2.06*** **	**2.08*** **	**2.06*** **
	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Log(Volatility$_t$)	**0.96*** **	**1.19*** **	**1.16*** **	**1.07*** **	**1.17*** **	**1.14*** **
	(0.05)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Log(Volume$_t$)	**-0.84*** **	**-0.82*** **	**-0.81*** **	**-0.79*** **	**-0.83*** **	**-0.81*** **
	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Obs.	3745	3652	3736	3711	3713	3850
R^2	0.8063	0.8389	0.8003	0.7704	0.8095	0.8298
Adj. R^2	0.8061	0.8387	0.8001	0.7702	0.8093	0.8296

- Data fit model not only qualitatively but also quantitatively
  - Coefficients are close to predictions
  - High R^2
- One possibility
  - Machines that make liquidity may use models like our parsimonious theoretical model

Horse Races with Canonical Models

	Our 3-factor	Madhavan (2000)	Stoll (2000)
Dependent Variable	*Spread−Δ*	Percentage Spread	Percentage Spread
Log(Price$_t$)	**2.09*** **		**6.20*** **
	(0.03)		(1.56)
Log(Volatility$_t$)	**0.96*** **		
	(0.05)		
Log(Volume$_t$)	**-0.84*** **	**-30.46*** **	**-19.17*** **
	(0.02)	(1.49)	(2.67)
Log(MKTCAP$_t$)		**19.08*** **	**9.69*** **
		(1.71)	(1.21)
Log(#Trades$_t$)			**-7.12*** **
			(2.72)
*Volatility$_t$ * 10^2*		**5.40*** **	
		(0.53)	
*Variance$_t$ * 10^4*			**0.19*** **
			(0.02)
1/(Price$_t$)		**-39.91*** **	
		(5.17)	
Obs.	3745	3745	3745
R^2	0.81	0.62	0.65
Adj. R^2	0.81	0.62	0.65

We explain bid-ask spreads with higher R^2 and fewer variables

- Our model: theory-driven three factors, $R^2 = 0.81$
- Madhavan (2000): four factors, $R^2 = 0.62$
- Stoll (2000 AFA presidential address): five factors, $R^2 = 0.65$

Improvement 1: Subtract the mechanical tick-driven Δ from the spread

- Canonical models assume monotonic relationship between spread and nominal price
- Our model captures the U-shaped relationship
- Using our dependent variable would improve the R^2 of traditional models to 0.81 and 0.82, respectively

Market cap does not add much value after controlling for dollar volume

- A small stock is as liquid as a large stock if they have similar dollar volumes and volatility
- Adding the market cap to our three-factor model of liquidity marginally improves R^2 from 0.81 to 0.82
- Keep market cap, delete volume, R^2 drops to 0.75

Cross-Sectional Variations in Stock Prices

- $p^* = \sqrt{\dfrac{\lambda_I \Delta v}{2\sigma\lambda_J L}}$
  - $log(p^*) = \frac{1}{2} log(\lambda_I v) - \frac{1}{2} log(\sigma\lambda_J) + const.$

- $log(Price)_i = \frac{1}{2} log(Dollar\ Volume)_i - \frac{1}{2} log(Volatility)_i + \varepsilon_i$

- Our two-factor model explains 57% of cross-sectional variations in nominal prices

- Rationalizes a number of puzzles in the behavioral finance literature
  - The volatility puzzle (Baker, Greenwood, and Wurgler 2009; Shue and Townsend 2021)
  - The social norm puzzle (Weld, Michaely, Thaler, and Benartzi 2009)

The Volatility Puzzle

	(1)
Dependent Variable	$Log(Price)$
Log(Volatility)	**-0.99*** **
	(0.04)
Log(Volume)	**0.29*** **
	(0.01)
Industry FE	N
Obs.	3745
Adj. R^2	0.57

- Baker, Greenwood, and Wurgler (2009)
  - Catering hypothesis: an increase in volatility should reduce the incentive to manage prices downward because there is a *"greater chance of reaching a low price anyway."*
  - *"A somewhat unexpected result is that volatile firms have a greater, not lesser, propensity to manage prices downward."*
- Interpretation by Shue and Townsend (2021)
  - Investors think in part about stock-price changes in dollars rather than percentage units
  - Low-priced stocks should experience more extreme return responses to news
- Our interpretations: lot or tick size
  - Volatility ↑ ⇒ Adverse-selection risk ↑
    - Firms should reduce prices to decrease lot-size constraints
  - Volatility ↑ ⇒ Percentage spread ↑
    - Firms should reduce prices because of weaker tick-size constraints

The Social Norm Puzzle

	Our Model	Our Model + Ind. FE	Our Model - Volatility + Ind. FE
	(1)	(2)	(3)
Dependent Variable	$Log(Price)$	$Log(Price)$	$Log(Price)$
Log(Volatility)	**-0.99*** **	**-0.93*** **	
	(0.04)	(0.04)	
Log(Volume)	**0.29*** **	**0.30*** **	**0.34*** **
	(0.01)	(0.01)	(0.03)
Industry FE	N	Y	Y
Obs.	3745	3745	3745
Adj. R^2	0.57	0.58	0.48

- Weld, Michaely, Thaler, and Benartzi (2009)
  - Firms choose prices like their industry and size peers
  - Evidence of social norm
- Our interpretation
  - Industry clustering
    - Explanatory power of industry FE diminishes after controlling for volatilities
    - Firms in the same industry subject to similar volatilities
  - Size clustering
    - Large firms have higher dollar volumes
    - Choose higher prices to relieve tick size constraints
    - Tick and lot channel explains not only why large firms choose similar prices but also why they cluster at *higher* prices

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The Sample of Splits

- Predicted change in the percentage spread $R = \frac{(s^{tot} - \Delta)/H^2 + \Delta}{p/H} - \frac{s^{tot}}{p}$
  - R is implied by the Modified Square Rule
  - p: the stock price before split announcements
  - s^{tot}: the average bid-ask spread before split announcements
  - H: split ratio

- Sample: U.S. common stock splits announced from June 2003 to December 2020
  - Exclude split ratios lower than 1.25-to-1 (Grinblatt, Masulis, and Titman 1984)
  - 1,196 stock splits

Predicted Spread Change Matches Realized Spread Change

$$\Delta\mathcal{S}_i = \beta \cdot R_i + Controls_{i,t} + Industry_i \times Year\ FE_t + \varepsilon_i$$

Dependent Variable	Realized $\Delta\mathcal{S}_i$ (bps)
R_i *(bps)*	**1.02*** **
	(0.20)
Log(H_i)	0.17*
	(0.10)
Controls	Y
Industry-Year FE	Y
Obs.	1196
Adj. R^2	0.336

Control variables: log(market cap), price, log(volume), and turnover rates (Weld et al. 2009)

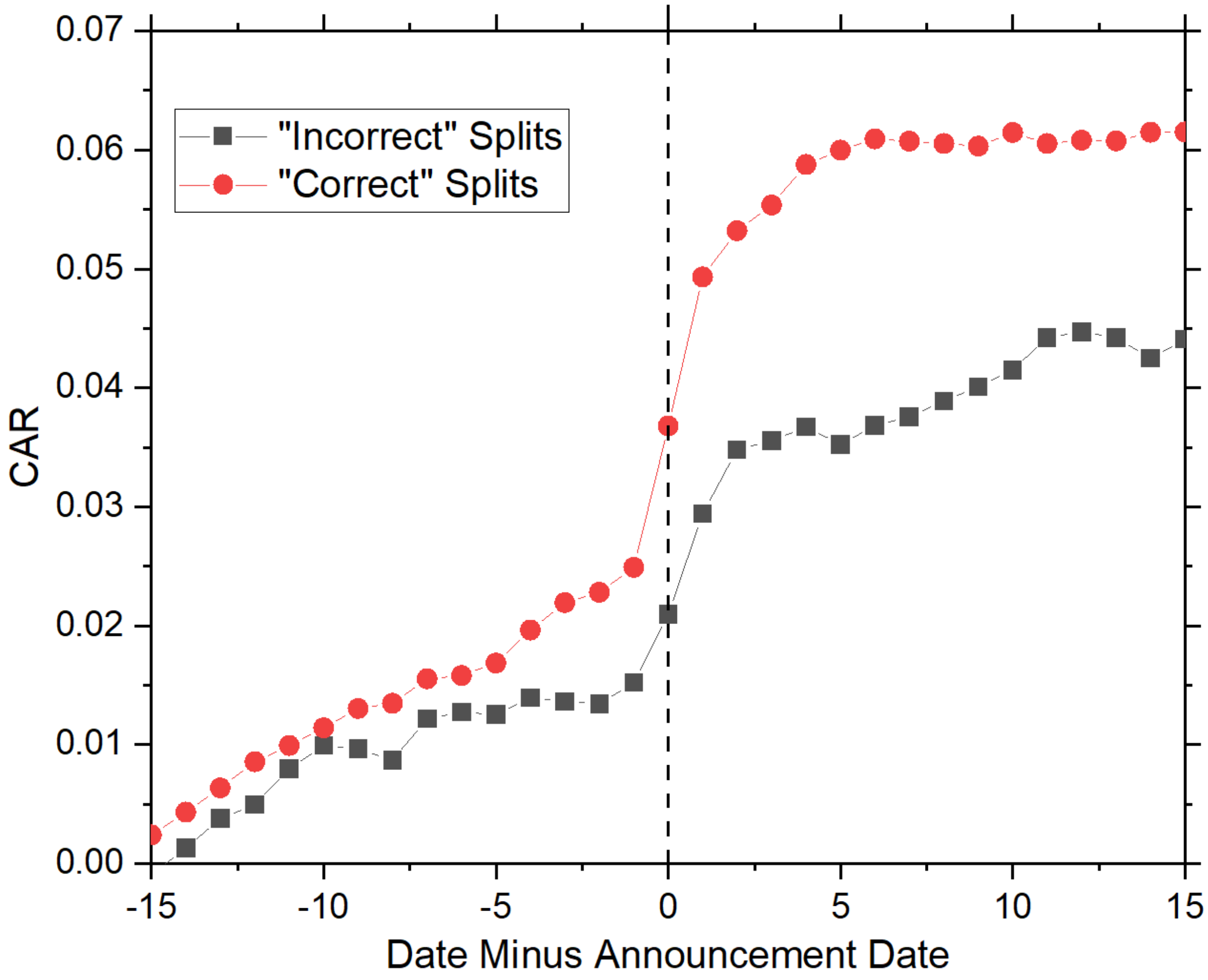
- Magnitude: 1 bps increase in predicted spread leads to 1.02 bps of realized spread change
- Average predicted spread change R_i is -15.22 bps
  - Most stock splits (1,089/1,196) are "correct," i.e., they reduce percentage spreads
  - Among 107 incorrect splits, 74 should have split but they split too aggressively

Predicting the Split Announcement Returns

- An increase in liquidity increases share prices, so we find that asset prices adjust to reflect the expected liquidity changes

- Announcement returns for "correct" splits: +2.87%
  - Predicted percentage spread change, $R_i < 0$

- Announcement returns for "incorrect" splits: +1.36%
  - Predicted percentage spread change, $R_i > 0$



Predicting Split-Announcement Returns

- $CAR_{i,[T-1,T+1]} = \delta \cdot R_i + Controls + Industry \times Year\ FE + \varepsilon_i$

- 1 bps predicted spread increase

  → -6.18 bps announcement return

  - Rationale: The present value of the excess transaction costs that will be paid in the future before the next split

- Correct split ratios explain -15.22×-6.18 = 94 bps of the overall split-announcement return of 273 bps

- Optimal pricing would reduce the median spreads from 42.85 bps to 25.66 bps
  - Improved liquidity would increase the value of the median U.S. stock by (42.85-25.66) × 6.18 = 106 bps

Dependent Variable	$CAR_{i,[T-1, T+1]}$ (bps)
R (bps)	**-6.18*** **
	(2.40)
Split Ratio	1.46
	(1.15)
Log(Market Cap)	**4.81** **
	(1.80)
Log(Price)	**-6.29*** **
	(1.76)
Turnover	**5.63*** **
	(1.81)
Log(Volume)	**-4.96*** **
	(1.72)
Industry-Year FE	Y
Obs.	1196
Adj. R^2	0.164

New Perspectives for Two Questions in Corporate Finance

- **Why do firms split their stocks?**
  - Signaling: firms use splits to signal good news
    - The cost of the signal comes from reduced liquidity (Brennan and Copeland, 1988)
      - Yet, we find that most splits increase liquidity in our sample
    - Fama et al. (1969) and Lakonishok and Lev (1987) find that profits and stock prices increase significantly before splits but not after. Easley, O'Hara and Saar (1998) find that splits do not reduce information asymmetry
      - Inconsistent with signaling but consistent with tick and lot channels
      - A previous increase in stock prices increases lot constraints, and stock splits reduce lot constraints.
  - Trading range hypothesis: (Lamoureux and Poon, 1987; Maloney and Mulherin, 1992)
    - Firms use stock splits to attract retail traders
    - Yet, we find that institutional holdings increase after splits in our sample
  - Pay-to-play hypothesis (Angel 1997)
    - Splits increase bid-ask spread and market-making revenue for brokers/dealers
    - Yet, we find that that bid-ask spread narrows in our sample post-splits

- **What explains positive returns following split announcements?**
  - Our tick-and-lot channel explains 1/3 of post-split announcement returns

Back-of-Envelope Calculations

- Stock that should split
  - Amazon’ s ($3,305) percentage spread is 4.62 bps
    - Its shareholders paid $684 million transaction costs per year
  - Microsoft’s ($255) percentage spread is only 0.77 bps
    - Amazon’s shareholder would pay only $111 million per year if Amazon splits to the same price as Microsoft
  - Further reduce to $110 million per year with optimal pricing
- Stock that should reverse split
  - Ford ($7) percentage spread is 14 bps, which is four times greater than GM (3.3 bps)
  - Ford investors would save $66 million per year if the company chose a price that is similar to GM’s
- Optimal pricing would reduce the median spreads from 42.85 bps to 25.66 bps
  - Improved liquidity would increase the value of the median U.S. stock by (42.85-25.66) × 6.18 = 106 bps
- The total benefit of adopting optimal pricing would be $93.7 billion
  - Top winners: Google ($3.96 billion), Amazon ($3.89 billion), and UnitedHealth Group ($546 million)

Roadmap

- Continuous pricing ($\Delta = 0$) and discrete lot ($L = 1$): as in BCS
  - Difference: We allow the firm to choose its share price p

- Discrete pricing ($\Delta > 0$) and discrete lot ($L > 0$)
  - Reflect current U.S. regulation: Uniform tick and lot sizes
  - Theory
  - Empirical evidence
    - Cross-sectional evidence
    - Stock splits

- The regulator
  - Increases or decreases (uniform) tick and lot sizes
    - Lot and tick sizes are the same for all stocks
  - Switches to proportional tick and lot sizes
    - Lot and tick sizes are proportional to price

Uniform Tick and Lot Sizes: The Square Root Rule

- $p^* = \sqrt{\frac{\Delta\lambda_I v}{2\sigma\lambda_J L}}$

- Partially explain a puzzle: why decimalization did not lead to 6.25-for-1 split waves?
  - Stock prices happened to drop after the dot.com bubble, but not by 6.25 times

- If Δ decreases by 6.25 times, the firm should respond only by a $\sqrt{6.25}$-for-1 split
  - Contribution from tick size = Contribution from lot size = $\sqrt{6.25}$ = 2.5
  - Best nominal spread is still two ticks, except that it narrows by a factor of 6.25
  - Liquidity improve by a factor of $\sqrt{6.25}$ = 2.5

- Interestingly, a 6.25-for-1 split leads to the same outcome as doing nothing at all
  - Doing nothing: relative tick size decrease 6.25-fold and the dollar lot size remains the same
  - 6.25-for-1 split: relative tick size remains the same and the dollar lot size decreases 6.25-fold

- If L also decreases by 6.25 times; firms keep the original price
  - Algorithmic trading can effectively reduce lot sizes by slicing orders into smaller units

Proportional vs. Uniform Tick and Lot Sizes

Tick Size \ Lot Size	Fixed L	Proportional $\mathbb{L}(p) = k^L/p$
Fixed Δ	$p^* = \sqrt{\frac{\lambda_I \Delta v}{2\sigma\lambda_J L}}$	$p^* \to \infty$
Proportional $\Delta(p) = k^\Delta p$	$p^* \to 0$	Expected transaction cost does not depend on p

- If lot size is uniform but tick size is proportional to price, the firm should split
- If tick size is uniform but lot size is proportional to price, the firm should reverse split
- If tick and lot sizes are both proportional, the firm's decision is inconsequential
  - We find that proportional tick and lot sizes reduce liquidity, if the regulator chooses any existing stock as the benchmark

Proportional Tick and Lot Sizes May Harm Liquidity

- Uniform tick and lot sizes seem like "one-size-fits-all"
  - Mandate the same tick and lot sizes for stocks with varying *prices*
  - However, uninform tick and lot sizes offer one degree of freedom
    - Allow a firm to choose the optimal price to balance discrete prices and quantities

- Proportional tick and lot sizes are truly "one-size-fits-all"
  - Impose uniform discreteness for stocks with heterogeneous *fundamental characteristics*

- Example: Choose a $10 stock as a benchmark
  - 1 cent tick size ⟺ 10 bps; 100-share lot size ⟺ $1,000 lot size

- A $100 stock then has a 10-cent tick size and a 10-share lot size
  - Original spread = 2 cents: 1 cent from tick constraint and 1 cent from lot constraint
  - Proportional tick and lot ⟹10.1 cent bid-ask spread

Conclusion

- Market microstructure
  - All firms maximize their liquidity at the two-tick spread
    - Contribution from tick size = Contribution from lot size
  - The three-factor model of liquidity explains 81% of variations in bid-ask spreads

- We explain a number of puzzles raised in the behavioral finance literature
  - Volatile firms choose low prices to relieve lot constraints
  - Large firms cluster at higher prices to relieve tick constraints
  - Industrial peers choose similar prices because their volatilities are similar
  - Our two-factor model of price explains 57% of variations in stock prices

- Corporate decisions
  - Stock splits in general move prices closer to the two-tick optimal
  - Correct split ratios contribute 94 bps to the 273 bps of split-announcement returns
  - A $\sqrt{\frac{s_t^{tot}-\Delta}{\Delta}}$ split ratio to achieve the optimal two-tick spread
  - A median U.S. stock value would increase by 106 bps if it moved to its optimal price

Policy Implications

- We encourage policymakers to reduce tick and lot sizes
  - Especially for tick- and lot-constrained stocks
  - Reducing lot size can mitigate the adverse-selection problem in BCS

- Proportional tick and lot sizes may reduce liquidity
  - Continuous tick and lot sizes offer two degrees of freedom (Kyle and Lee 2017)
  - Uniform tick and lot sizes offer one degree of freedom
    - Firms can use prices to balance discrete prices and discrete quantities
  - Proportional tick and lot sizes have no degrees of freedom
    - Impose the same level of discreteness on prices and quantities for all firms