# Green Stakeholders in Two-Sided Markets*

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**Abstract**

Green stakeholders boycott firms with carbon emissions. We analyze their effects on competitive two-sided markets, such as bank lending, employee talent and suppliers. Matching with green stakeholders requires firms to address their carbon-emissions externality by spending on costly abatement. Green stakeholders match with less productive firms. They receive lower earnings than brown stakeholders — a greenium reflecting both sorting and abatement costs. Compared to the first-best carbon tax, there are distortions — aggregate output is lower and productive firms' profits are higher. Calibrating a green-stakeholders equilibrium for the US labor market, we find small output distortions but large distributional ones.

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# 1 Introduction

We analyze the welfare consequences of green stakeholders, who boycott firms with carbon emissions, on competitive two-sided markets. We have three prominent settings in mind. The first is bank lending. Nearly 41% of global banking assets, encompassing 138 banks across 44 countries, have committed to aligning their lending portfolios with net-zero emissions by 2050 (United Nations Net-Zero Banking Alliance (2022)). The second is employee talent. For instance, an IBM Business Institute Survey found that one-third of workers polled recently accepted a job at an environmentally conscious company for lower pay. The third is customer-supplier relationships. Consumer-facing firms such as Apple are putting pressure on their suppliers to meet emissions abatement guidelines, according to a recent Wall Street Journal report.

These green stakeholders are as important if not more than the portfolio restrictions of shareholders in public asset markets to hold green companies (United Nations Net-Zero Asset Managers Initiative (2022)). After all, only a subset of firms tap these public markets. In contrast, key stakeholders are vital for output. While there is extensive modeling of the effects of boycott by asset owners in liquid capital markets[1], there is relatively little work on the impact of green stakeholders on equilibrium and welfare in competitive two-sided markets.

To address this shortfall, we develop a model of a two-sided market where firm output is determined each period by complementarities between firm productivity and stakeholder productivity. Carbon emissions, which increase with firm output, accumulate over time, damage the economy and reduce social welfare as in integrated

[1] Heinkel, Kraus, and Zechner 2001, Hong and Kacperczyk 2009, Pástor, Stambaugh, and Taylor 2021, Pedersen, Fitzgibbons, and Pomorski 2021, Hong, Wang, and Yang 2023b, Broccardo, Hart, Zingales, et al. 2022 and Oehmke and Opp 2023 model how non-pecuniary tastes or mandates of green investors influence asset pricing, capital investments and welfare under a variety of assumptions regarding market structure, financial constraints and governance or voting policies. See Hong and Shore 2023 for a survey.

assessment models (Nordhaus 1992, Golosov et al. 2014, Jensen and Traeger 2014, Cai and Lontzek 2019, Barnett, Brock, and Hansen 2020, Hong, Wang, and Yang 2023a). Firms do no spend on costly abatement due to an externality. However, a fraction of stakeholders are green and adhere to a mandate that they will only work with firms that address the emissions externality by engaging in a given amount of costly abatement.

Firms and stakeholders otherwise match in a standard frictionless and competitive market over time. Output, wage and profit distributions are determined endogenously as in assignment models of competitive labor markets (Koopmans and Beckmann 1957, Tinbergen 1957, Sattinger 1979, Gabaix and Landier 2008, Tervio 2008). Risk-neutral firms consume profits and risk-neutral agents consume compensation or earnings each period, respectively.

Abatement in the green-stakeholder equilibrium only arises as a result of the efforts of firms to attract and match with stakeholders. In equilibrium, value-maximizing firms have to be indifferent between matching with a green stakeholder versus a brown stakeholder. Sorting can be summarized by an adjusted index of stakeholder productivity, whereby green stakeholder productivity is discounted by a fraction that is equal to firm abatement costs divided by its output. This discount reflects the abatement costs incurred in order to match with a green stakeholder. For example, if green stakeholders adhere to an abatement mandate of 10% of output, then a green stakeholder' productivity will be discounted by 10%.

Due to complementarities, there is positive matching or sorting of more productive firms with stakeholders with more discounted-productivity. In the decentralized sorting equilibrium, brown stakeholders with less productivity can be as attractive as more productive green stakeholders. The larger are the abatement costs as a fraction of output, the more is this discount on green stakeholder productivity. As a result, green stakeholders, all else equal, match with less productive firms.

Green stakeholders also receive lower earnings relative to brown stakeholders in the economy. We term this an earnings gap and define a greenium as this earnings gap divided by the earnings of brown stakeholders to reflect the willingness of green stakeholders to pay for abatement. The natural analog in the context of green shareholders in public markets is the willingness of green investors to forgo dividend yield to fund abatement (Heinkel, Kraus, and Zechner 2001, Broccardo, Hart, Zingales, et al. 2022, Hong, Wang, and Yang 2023b).

In contrast to shareholders in public markets, our greenium has two components. The first component is abatement costs incurred in order to work with green stakeholders — that is, compensating differentials for working for a green firm that abates emissions. The second component, unique to our setting, is bidding by productive firms for brown stakeholders since they benefit the most by hiring brown stakeholders, i.e a competitive sorting effect arising from avoidance of abatement costs.

The green-stakeholders equilibrium differs from the first-best solution in three important dimensions. In the first-best solution, the planner optimally chooses the sorting to maximize output and sets the emissions tax for a firm equal to the social cost of carbon. The most productive firms match with the most productive stakeholders. In contrast to the first best, there are potential misallocation or output distortions in the green-stakeholders' equilibrium since the most productive stakeholders need not pair with the most productive firms.

The second difference is distributional distortions. Brown productive firms do better in the green-stakeholder equilibrium than with a first-best carbon tax, i.e. their tax burden is lower under a mandate regime than an emission-tax regime. The size of these distortions in general depends on the distributions of firm and stakeholder productivity, the cost of abatement, the fraction of green stakeholders, and whether green stakeholders are likely to be productive. We show that these two differences are largest when there is no correlation of stakeholder productivity and stakeholder type.

The third difference is potential firm shutdown. If green stakeholders adhere to a net-negative emissions mandate (i.e. green firms must clean up more for others), then firms might then choose to shut down if they have to pay too high an abatement cost. When this happens, green stakeholders can again distort aggregate production.

In our quantitative analysis, we seek to quantify the size of these distortions. We apply our model to the setting of employee talent. Using US data on wages, profits, and abatement costs, we calibrate our model to target a number of moments, including the distributions of wage and profits per employee, and the emissions of firms. Based on the IBM Business Institute Survey, we assume that 33% of employees are green and are willing to take around a 28% lower pay to work for a green firm.[2] Moreover, we assume that the stakeholder type (green versus brown) is uncorrelated with stakeholder productivity. For our firms, the abatement cost as a fraction of output is around 21%.

We compare the green-stakeholders' equilibrium to a first-best carbon tax that corresponds to a social cost of carbon at around 350.[3] Compared to the first-best carbon tax, there is a small distortion in overall firm output even though the productivity of green stakeholders is discounted by 21%. There is also no distortions from firms shutting down since the calibrated green-stakeholders mandate is relatively modest — green firms emit around 15 tons of emissions per employee.

But there are big distortions when it comes to the distribution of firm profits. The flipside of productive firms doing better is that productive stakeholders benefit relatively more from being brown since productive firms are able to avoid green stakeholders because they bid for productive brown stakeholders. Indeed, the percent of the greenium due to abatement costs is around 85%. That is, 15% of the greenium is due

[2]Similarly, Krueger, Metzger, and Wu 2021 find using data from Sweden that green firms pay around 17% lower wages than brown firms.

[3]Our calibration for the first-best is more aggressive than the social cost of carbon in the literature at around \$200 as we assume more upfront costs for carbon removal.

to sorting effects.

We use comparative statics to show that our conclusions regarding output and distributional distortions are robust to key parameters in our model such as the fraction of stakeholders that are green. We also discuss why our conclusions are likely to extrapolate to other settings such as bank lending and customer-supplier relationships.

**Related literature.** Corporations face pressure from a range of stakeholders to address the global warming externality. The literature has mainly focused on consumer (Besley and Ghatak 2007), investors in public markets (Heinkel, Kraus, and Zechner 2001, Hong, Wang, and Yang 2023b, Broccardo, Hart, Zingales, et al. 2022) and governance arrangements (Magill, Quinzii, and Rochet 2015)(see Hong and Shore 2022 for a review). In these studies, willingness to pay on the part of either consumers or investors as picked up by greeniums (holding fixed the proportion of green consumers or investors) is a sufficient statistic for mitigation of externalities.

There are few studies on the welfare impact of green stakeholders in two-sided markets. In our setting, competition for brown stakeholders means that the greenium often exceeds abatement costs. And if the proportion of brown stakeholders become scarce, the greenium becomes even larger. That is, the greenium is not a sufficient statistic for mitigation of externalities in our setting. Moreover, our quantitative conclusions regarding output and distributional distortions are new to the literature.

In addition, we contribute to the rich literature on assignment models (Koopmans and Beckmann 1957, Tinbergen 1957, Sattinger 1979, Gabaix and Landier 2008, Tervio 2008) by examining the welfare consequences of the non-pecuniary preferences of agents on one side of the market. There are few papers on this issue for labor markets despite its growing importance.

## 2 Model

**Production.** Time is continuous. There is a continuum of heterogeneous firms and stakeholders. We assume that firm and stakeholder characteristics affecting production can be summed up by one number, which we refer to as their productivity, denoted by $k$ and $s$, respectively. Let $A_t f(k, s)$ denote the flow production within the pair $(k, s)$, where $A_t$ represents the aggregate productivity.

**Assumption 1.** *The production function is continuous and multiplicatively separable* $f(k, s) = a(k)b(s)$, *where* $a'(k) > 0$ *and* $b'(s) > 0$.

For example, in the labor market assignment literature (Gabaix and Landier 2008, Tervio 2008), $k$ is the firm assets and $a(k) = k^\theta$ and $b(s) = s$. In the market for banks (Chang, Gomez, and Hong 2023, Schwert 2018), one can then interpret $a(k)$ as the firm's productivity and $b(s)$ as the loan size provided by bank $s$. In customer-supplier relationships, output is a function of productive capitals from the corporate customer and corporate supplier, respectively.

The distribution of firms is denoted by $G_f(k)$ with support $[k_L, k_H]$, and the distribution of stakeholders is denoted by $G_w(s)$ with support $[s_L, s_H]$. As we explain later in Section 5, our model, which features one-to-one matching, can be applied to environments where a firm hires multiple stakeholders or a bank lends to multiple firms by reinterpreting $G_w(s)$ and $G_f(k)$. All stakeholders and capital owners (firms) are risk-neutral and consume their profits each period, respectively.

Aggregate output each period is given by

$$Y_t = A_t F_t, \tag{1}$$

where $F_t$ is endogenous and depends on the matches between firms and stakeholders in the economy. The process that changes the aggregate productivity is Poisson with an arrival rate $\mu$. When there is a change, the new value of $A'$ is drawn from the fixed distribution $H(A)$, with support $[A_L, A_H]$ and a mean $\bar{A}$. For simplicity, we assume that there are no frictions regarding the matching decisions, and we normalize stakeholders' outside option to zero.

**Carbon emissions and abatement.** Following integrated assessment models (Nordhaus 1992, Jensen and Traeger 2014), we assume that production results in emissions and all firms have access to an abatement technology. Production between firm $k$ and stakeholder $s$ at time $t$ leads to firm emissions $\sigma A_t f(k,s)$ and aggregate emission of $\sigma \left(A_t F_t\right)$. In the meantime, all firms can remove $m_t$ of emissions at a linear cost $c$. Let $E_{t-}$ denote the accumulated stock of emissions *before* period $t$. The aggregate level of emissions at period $t$ is then given by $E_{t-} + \sigma A_t F_t - M_t$, where $M_t$ represents the abatement by all firms at period $t$.

We assume that the accumulated emissions decays at the rate $\delta$. Hence, the law of motion for carbon emissions is given by

$$dE_t = \left(\left(\sigma A_t F_t - M_t\right) - \delta E_{t-}\right) dt, \tag{2}$$

where the first two terms represent the newly added emissions net of abatement at period $t$.

**Damages from carbon emissions.** The damages of emissions to the economy at each point in time is strictly convex and increasing in the level of aggregate emission. It is modeled as a flow cost,

$$\frac{d}{1+\chi} \left(E_{t-} + \sigma A_t F_t - M_t\right)^{\chi+1}, \tag{3}$$

with $d > 0$ and $\chi > 0$.

We assume that the damage function is strictly convex ($\chi > 0$) to ensure interior solutions. Moreover, recent integrated assessment models emphasize that convex damage functions are more in line with climate science due to concerns about climate tipping points (Cai and Lontzek 2019, Bretschger and Pattakou 2019, Lemoine and Traeger 2014).

**Assumption 2.** *(Abatement is Socially Optimal)*

$$(\sigma A_L f(s_L, k_L))^{\chi} > c.$$

Assumption 2 means that the abatement cost $c$ is low enough so that zero abatement $m$ is never socially optimal. Specifically, this condition states that the abatement cost $c$ is lower than the damage of emissions for the least productive pair of firm and stakeholder (denoted by $k_L$ and $s_L$) even when the accumulated stock is zero $E_{t-} = 0$. Hence, for any other pair and/or for any $E_{t-} > 0$, it is socially optimal to have positive abatement.

**Emissions abatement mandate.** The mandate, denoted by $\zeta_t$, is modeled as a constraint on a firm's emissions minus its abatement $m$, which yields

$$\sigma A_t f(k, s) - m \leq \zeta_t. \tag{4}$$

That is, $\zeta_t$ represents the carbon emissions tolerance, where the lower the $\zeta_t$ means a lower tolerance or a tighter mandate. In the special case where $\zeta = 0$, the firm is abating just its own emissions. In general, it can be negative, which would require the firm to abate for others.

**Green stakeholders.** Some stakeholders are only willing to match with firms that satisfy the emissions abatement mandates. We refer to them as green stakeholders. For example, one can interpret these stakeholders as banks, workers or corporate customers that only lend to or work with firms that satisfy net-zero emissions. Equation (4) thus serves as an additional constraint for firms that match with green stakeholders. Such a constraint would not exist if were a firm to match with stakeholders who do not have an emissions abatement mandate.

Stakeholders' types are thus two-dimensional $(s, \theta) \subseteq [s_L, s_H] \times \{0, 1\}$, where $\theta = 1$ denotes green stakeholders and zero otherwise. Let $g_w(s, \theta)$ denote the joint probability density, and the measure of green stakeholders is given by $\lambda \equiv \int_S g_w(s, 1)ds$.

# 3 Decentralized Equilibrium with Green Mandate

**Firms optimization problem.** We now analyze the equilibrium outcome given any mandate. Given the mandate $\zeta_t$, the firm chooses the type of stakeholder $(s, \theta)$ to match with (if any) and the amount of removal $m$ to maximize the present value of firm profits discounted at the risk-free rate $r$.

Let $w_t(s, \theta)$ denote the fee for the stakeholder $(s, \theta)$, which can be interpreted as interest payments for banks, wages for workers, and fees for suppliers. The Hamilton-Jacobi-Bellman (HJB) equation of the firm can be expressed as

$$\begin{aligned} rJ_t(k, A_t, \zeta_t) = \max_{\{m_t \geq 0, (s,\theta)\}} & A_t f(k, s) - w_t(s, \theta) - cm_t \\ & + \mu \left( \int J_t(k, A_t, \zeta_t) dH(A') - J(k, A_t, \zeta_t) \right) + J_\zeta d\zeta_t \end{aligned} \tag{5}$$

subject to Equation (4) iff $\theta = 1$ (i.e. the firm matches with a green stakeholder).

The HJB Equation (5) has the following terms. The first three terms on the right-hand side are the flow revenues net of fees and the cost of abatement. The fourth term is the probability of a change in aggregate productivity times the expected change in the value function $J$ depending on the draw of the productivity distribution $A_t$. The fifth term is the change in the value function with changes in the mandate $\zeta_t$.

Profit-maximizing firms do not internalize the cost of emissions and thus the damage of emissions do not directly enter their objective function. The cost of emissions and the accumulated stock of emissions will affect the firm's problem only through the mandate $\zeta_t$. It is also clear from Equation 5 that firms will not have incentives to abate if the firm does not match with a green stakeholder.

## 3.1 Sorting with Green Stakeholders in Competitive Equilibrium

**Definition 1.** Given $(A_t, \zeta_t)$, a competitive equilibrium consists of a fee function $w_t(s, \theta)$, the assignment $\kappa_t(s, \theta)$, and abatement $m_t(k)$ at period $t$ such that (1) matching is stable, (2) $m_t(k)$ solves Equation (5) given the optimal match; and (3) the stakeholder market clears.

Observe that, taking the mandate $\zeta_t$ and equilibrium fee function $w_t(s, \theta)$ as given, firms' hiring and abatement decisions are effectively static. This occurs for two reasons that need to be highlighted. First, matching between firms and stakeholders is assumed to be frictionless. Hence, firms and stakeholders can change their matches at any point of time. Second, the mandate $\zeta_t$ imposes a constraint only on time $t$ emission but is not history dependent.

We thus solve the sorting and abatement problem for each time $t$ and for given any $(\zeta_t, A_t)$. The *flow* surplus between firm $k$ and stakeholder type $(s, \theta)$ can be expressed as

$$\Omega_t(k,(s,\theta)) = A_t f(k,s) - C^\theta \max\left\{0, (\sigma A_t f(k,s) - \zeta_t)\right\}, \tag{6}$$

where

$$C^\theta = \begin{cases} c & \text{if } \theta = 1 \\ 0 & \text{otherwise} \end{cases}. \tag{7}$$

To simplify our analysis, we consider the case where the mandate is binding in the sense that every firm needs to do *some* abatement in order to satisfy the mandate.[4]

**Assumption 3.** *(Binding Mandate)*

$$\sigma A_t f(k,s) > \zeta_t \ \forall k, s.$$

Under Assumption 3, firms that match with green stakeholders must then choose abatement (a positive value of $m$) so that the constraint Equation (4) is binding $\forall(k,s)$. Thus, the flow surplus in Equation 6 can be further reduced to

$$\Omega_t(k,(s,\theta)) = (1 - C^\theta\sigma)A_t f(k,s) + C^\theta \zeta_t. \tag{8}$$

**One-dimensional sorting by pseudo-index $z$.**

**Lemma 1.** *Under Assumption 3, the sorting outcome can be summarized by an one-dimensional index $z(s,\theta) = (1 - C^\theta\sigma)b(s)$, where a firm with higher $k$ is matched with a stakeholder with a higher index $z(s,\theta)$.*

To see this, observe that

$$\frac{\partial \Omega_t(k,(s,\theta))}{\partial k} = \left\{(1 - C^\theta\sigma)b(s)\right\} A_t a'(k)$$

[4] In the quantitative analysis, when we will focus on the environment where net-zero is socially optimal, this condition is naturally satisfied.

only depends on $(s, \theta)$ through the one-dimensional index $z(s, \theta)$. That is, the marginal gain of matching to a more productive firm is the same for two stakeholders that have the same index $z(s, \theta)$. Hence, these two stakeholders must have the sorting outcome. Moreover, since $a'(k) > 0$, this also means that $\frac{\partial \Omega_t(k, z(s,\theta))}{\partial k \partial z} > 0$. In other words, the sorting can thus be understood as more productive firms are matched with stakeholders with higher index $z$.[5]

Intuitively, since hiring green stakeholders requires costly abatement, the index summarizes the ranking of stakeholders' added value. All else equal, the green stakeholder now receives a discount of $(1 - c\sigma)$ relative to the stakeholder's latent productivity; and thus the green stakeholder's ranking is the same as a lower skilled non-green or brown stakeholder $b(s') < b(s)$ where $b(s') = (1 - c\sigma)\, b(s)$.

**Assumption 4.** *(Production is Socially Optimal)*

$$(1 - c\sigma) > 0$$

Assumption 4 means that, taking into account the emission and the abatement costs, the production is nevertheless socially optimal even for the least productive pair. As we explain below in our discussion of the planner's problem, Assumption 4 implies that the first-best solution is to maximize production and to abate accordingly without shutting down the firms.

**Very negative $\zeta_t$ and potential for firm shut down.** It is important to note that, despite that $1 - c\sigma > 0$, the surplus can be negative with a very negative $\zeta_t$, as

$$\Omega_t(k, (s, 1)) = (1 - c\sigma) A_t f(k, s) + c\zeta_t. \tag{9}$$

[5] According to Chiappori (2016), this is the case is referred as "pseudo-index" model.

Recall from our discussion of Equation 4 that $\zeta_t = 0$ represents the mandate that requires firms to abate their own emission. A very negative $\zeta_t$, however, can arise when green firms are required to clean up for other brown firms. As a result, when facing a very tight mandate (when $\zeta_t$ is very negative), firms may optimally choose to shut down. Moreover, according to Equation 9, given any $\zeta_t$, firms that produce less are more likely to shut down.[6]

**Distribution of pseudo-index $z$.** Given that the surplus can become negative for green stakeholders, let $\hat{s}_t$ denote the cutoff type of green stakeholder such that $\Omega_t(k, (\hat{s}_t, 1)) = 0$. Given any $\hat{s}_t$, the distribution of $z$ depends on the joint distribution $g_w(s, \theta)$, which is given by

$$G_w(z|\hat{s}_t) \equiv \int_{\hat{s}_t}^{b^{-1}\left(\frac{z}{1-c\sigma}\right)} g_w(\tilde{s}, 1)d\tilde{s} + \int_{s_L}^{b^{-1}(z)} g_w(\tilde{s}, 0)d\tilde{s}. \tag{10}$$

From Equation 10, the first term is the mass of green stakeholders such that $(1 - c\sigma)b(s) \leq z$ conditional on being employed $s \geq \hat{s}_t$, and the second term is the mass of brown stakeholders such that $b(s) \leq z$. Note that, because of some green stakeholder may not be in the market, the measure of total stakeholders that are matched is given by $G_w(s_H|\hat{s}_t) = 1 - \int_{s_L}^{\hat{s}_t} g_w(\tilde{s}, 1)d\tilde{s} \in [(1 - \lambda), 1]$. For simplicity, we focus on the case where the lowest value of $z$ in the market is the least skill green stakeholder $(1 - c\sigma)\,\hat{s}_t$, as we show that it is also the relevant case under the optimal mandate.[7]

[6] A positive $\zeta_t$, on the other hand, means that firms can produce positive net emission, which automatically guaranteed positive surplus under Assumption 4.

[7] More generally, the lowest value of $z$ can also be the least skilled brown stakeholder, which happens when there are lots of green stakeholders that are out of market and thus $(1 - c\sigma)\hat{s}_t \geq s_L$.

## 3.2 Equilibrium Assignment, Compensation, and Green vs. Brown Firms

**Proposition 1.** *Given any* $\zeta_t$*, the equilibrium is characterized by the cutoff type* $(\hat{s}_t, \hat{k}_t)$*, where stakeholders (firms) are out of the market if and only if* $s \leq \hat{s}_t$ $(k < \hat{k}_t)$*. For any stakeholder* $(s, \theta)$*, the assignment function is given by* $\kappa(s, \theta) = \kappa^*(z(s, \theta))$*, where* $\kappa^*(z(s, \theta))$ *solves (1)* $G_w(s_H|\hat{s}_t) - G_w(z|\hat{s}_t) = 1 - G_f(\kappa_t^*(\tilde{z}))$ *for any* $z \geq z_{L,t}$ *with* $\hat{k}_t = \kappa^*(z_{L,t})$*,and (2) for any* $\hat{s}_t \in (s_L, s_H)$*,* $\Omega_t\left(\kappa^*\left((1 - c\sigma)\hat{s}_t\right), \hat{s}_t\right) = 0$*. If the firm* $k$ *is matched with a green stakeholder with skill* $s$*,*

$$m^*(k) = \sigma A_t f(k, s) - \zeta_t, \tag{11}$$

*and zero otherwise. The compensation for a green stakeholder is given by*

$$w_t(s, \theta) = W_t(z(s, \theta)) + C^\theta \zeta_t \tag{12}$$

*where*

$$W_t(z) = \int_{z_{L,t}}^{z} A_t a\left(\kappa^*(\tilde{z})\right) d\tilde{z} + W_t\left(z_{L,t}\right) \tag{13}$$

*and* $W_t(z_{L,t}) = -c\zeta_t$*.*

Since firms only abate carbon emissions when they end up matching with a green stakeholder, the sorting outcome thus determines firms' abatement. That is, if a firm works with a green stakeholder in equilibrium, it chooses the abatement so that the mandate is satisfied, which gives Equation 11 and zero otherwise. We thus refer the firms that (do not) hire green stakeholders and thus engage emissions abatement as the green (brown) firms.

The assignment function $\kappa^*(z)$ determines the firm type for the stakeholder with index $z$. A firm with size $k$ can thus either be matched with a brown stakeholder or a green stakeholder but with a relatively high skill, as long as both have the same index $z$. The compensation equation further implies that firm $k$ is indeed indifferent between these two options. This is because that, for any $b(s) = (1 - c\sigma)b(s') = z$, they result in the same flow payoff to the firm, which yields

$$A_t f(k, s) - w_t(s, 0) = (1 - c\sigma)A_t f(k, s') + c\zeta_t - w_t(s', 1),$$

where we use the fact that $w_t(s', 1) = W_t(z) + c\zeta_t$ according to Equation 12. Note that $W_t(z)$ given in Equation (13) is simply the equilibrium utility for a stakeholder with index one-dimensional index $z$, as in Tervio (2008).

The second term in Equation (12) is an adjustment for the abatement cost. Only for the special case with $\zeta_t = 0$, two stakeholders with the same index $z$ generates the same matching surplus value. More generally, the level of the mandate affects firms' abatement costs and hence the surplus and thus compensations differ for green and brown stakeholders conditional on index $z$.

Note that, since we normalize stakeholder's outside option to be zero, the utility for the lowest type $W_t(z_{L,t})$ is pinned down so that his utility is zero. Specifically, since the lowest type is a green stakeholder, then $w(\hat{s}_t, 1) = W_t(z_{L,t}) + c\zeta_t = 0$.

**The effect of mandate on aggregate abatement.** Given $\zeta_t$, the aggregate level of abatement is essentially the abatement by all firms that hire green stakeholders, which can be expressed as

$$M_t(\zeta_t) = \int_{\hat{s}_t(\zeta_t)}^{s_H} \{\sigma A_t f(\kappa^* ((1 - c\sigma)s), s) - \zeta_t\} g_w(s, 1) ds. \tag{14}$$

This highlights two effects of the mandate. First of all, it directly affects the level of abatement of green firms. Second, it affects the surplus function and may result in a firm shutting down, affecting the cutoff type $\hat{s}_t(\zeta_t)$. Recall that the sorting outcome only depends on the underlying distribution $G_w(z|\hat{s}_t)$. Hence, if the mandate does not affect the cutoff type $\hat{s}_t(\zeta_t)$, a lower $\zeta_t$ only increases the aggregate abatement but does not affect the sorting outcome.

**Earnings premium for brown stakeholders.** Since green stakeholders require firms to clean up, they would thus receive a lower compensation relative to brown stakeholders, all else equal, in equilibrium. Formally, the earnings premium for brown stakeholders with skill $s$ yields

$$EP_t(s) \equiv w_t(s,0) - w_t(s,1) = W_t\left(z(s,0)\right) - W_t\left(z(s,1)\right) - c\zeta_t. \tag{15}$$

Similarly, the premium is affected by the mandate $\zeta_t$ through two channels. Fixing $\hat{s}_t$, a lower $\zeta_t$ leads to more abatement and thus higher premium. When the mandate also affects the cutoff type, it results in different $G_w(z|\hat{s}_t)$ and thus $W_t(z)$.

Equation 15 highlights that the earnings premium is determined by the difference in compensation of two different z-indices, which can be expressed as

$$\begin{aligned} w(s,0) - w(s,1) &= A_t \int_{(1-c\sigma)b(s)}^{b(s)} a\left(\kappa^*(z)\right) d\tilde{z} - c\zeta_t \qquad (16) \\ &= A_t \underbrace{\left\{ \int_{(1-c\sigma)b(s)}^{b(s)} \left\{ a\left(\kappa^*(z)\right) - a\left(\kappa^*\left((1-c\sigma)b(s)\right)\right) \right\} d\tilde{z} \right\}}_{sorting\ effect} \\ &+ \underbrace{c\left(\sigma A_t f\left(\kappa^*\left((1-c\sigma)b(s)\right), s\right) - \zeta_t\right)}_{abatement\ cost}. \end{aligned}$$

The earning premium in our model can thus be decomposed into two terms. The first term captures the fact that type-$s$ green stakeholder is now hired by the smaller firm, relative to type-$s$ brown stakeholder, which is the loss of having a lower ranking. This term is zero if and only if firms are homogeneous, and increases in firm dispersion. The second term represents the abatement cost for the firms that match with type-$s$ green stakeholder, who needs to abate $\{\sigma A_t f(\kappa^*((1-c\sigma)s, s) - \zeta_t\}$ in order to satisfy the mandate. This expression further highlights that, without misallocation (i.e., when firms are all homogeneous), the wage gap collapses to the standard compensating differential, i.e. the abatement cost.

**Lemma 2.** *Given $\zeta_t$, $EP_t(s)$ increases with $s$ and $\lambda$ $\forall t$.*

Lemma 2 highlights that the premium is larger for more productive stakeholders. This is because more productive stakeholders are employed at more productive firms, which thus predicts a higher $\int_{(1-c\sigma)b(s)}^{b(s)} \kappa^*(\tilde{z})d\tilde{z}$ in Equation 16. This result holds for any correlation between skill and stakeholder type. In other words, this result does not rely on the fact that higher skilled stakeholders are more likely to be green.

The effect of $\lambda$, the total measure of green stakeholders is also related to the sorting effect. Intuitively, when there are more green stakeholders (higher $\lambda$), brown stakeholders become more scare and attractive, and thus enjoy higher rents. Formally, one can show that $\kappa^*(z)$ is weakly higher for a higher $\lambda$, and thus increases the premium.

# 4 Comparison to First-Best Solution

In this section, we compare the green-stakeholder equilibrium outcomes to those of the first-best carbon emissions tax.

## 4.1 First-Best Emissions Tax

**The planner's problem** The planner chooses (1) the assignment function $\kappa_t(s)$, which determines the total production in this economy, denoted by $F\{\kappa_t(s)\}$, and (2) the aggregate emission removal, denoted by $M_t$, to maximize social welfare:

$$\max_{\{\kappa_t(s),M_t\}} \mathbb{E}\int_0^\infty e^{-rt}\left\{A_tF\{\kappa_t(s)\} - \frac{d}{1+\chi}\left(E_{t-} + \sigma A_tF\{\kappa_t(s)\} - M_t\right)^{\chi+1} - cM_t\right\}dt. \tag{17}$$

Since the planner can change the sorting at each point of time (i.e., no reallocation or search frictions), the assignment problem is again effectively static and it only affects the production $F\{\kappa_t(s)\}$, at period $t$. However, unlike the firms in competitive markets that take the policy at period $t$ as given, the planner's abatement problem is dynamic as the continuation value is affected by the accumulated emissions. The HJB equation thus yields

$$\begin{aligned} rV^{FB}(A_t, E_{t-}) = \max_{\{\kappa_t(s),M_t\}} & A_tF\{\kappa_t(s)\} - \frac{1}{1+\chi}\left(E_t + \sigma A_tF\{\kappa_t(s)\} - M_t\right)^{\chi+1} - cM_t \\ & + \mu\left\{\int V^{FB}(A', E_{t-})dG(A') - V(A_t, E_{t-})\right\} + \frac{\partial V^{FB}(A_t, E_{t-})}{\partial E}dE_t \end{aligned} \tag{18}$$

The first three terms on the right-hand size are the total production, net of total damages and cost of abatement. The fourth term is the probability of a change in aggregate productivity times the expected change in the value function depending on the draw of the productivity distribution $A_t$. The fifth term is the change in the value function with changes in the accumulated emission stock $E_t$.

**Lemma 3.** *Under Assumption 4,* $\kappa^{FB}(s) = \arg\max_{\kappa(s)} \int f(\kappa(s), s) dG_w(s)$.

$$V^{FB}(A_t, E_{t-}) = \left( \frac{(1 - c\sigma) F\{\kappa^{FB}(s)\}}{r} \right) \left( \frac{rA_t + \mu \bar{A}}{r + \mu} \right) + \gamma_E E_{t-} + v_0,$$

*where* $\gamma_E = -\left(\frac{c}{1+r+\delta}\right)$. *The first-best allocation and welfare can be achieved with an emissions tax in competitive markets, where*

$$T_t(e) = \left\{ \frac{d}{1+\chi} (e + E_t)^{\chi+1} - \gamma_E e \right\} + \tau(E_t),$$

*where* $E_t = E_{t-} + \hat{e}_t$ *is the current stock of emission, and* $\tau(E_t) = \left(\frac{\chi}{1+\chi}\right) (\gamma_E + c)^{\frac{\chi+1}{\chi}} - (c + \gamma_E) E_t$.

Recall that Assumption 4, $(1 - c\sigma) > 0$, implies that it is optimal to maximize the production, taking into account the abatement costs. Hence, the optimal sorting must simply maximize total production at each point of time, obtaining the first-best production, which yields $F^{FB} \equiv \max_{\kappa(s)} \int f(\kappa(s), s) dG_w(s)$.

Given the production $F^{FB}$, the optimal mandate is then simply chosen to generate the optimal abatement $M_t$, so that the marginal benefit of decreasing emission stock must equal to the cost of abatement. That is, the FOC is satisfied

$$d \left(E_{t-} + \sigma A_t F^{FB} - M_t\right)^{\chi} - \frac{\partial V^{FB}(A_t, E_{t-})}{\partial E} = c, \tag{19}$$

where RHS is the marginal benefit of decreasing emission stock that includes the flow costs as well as the cost of increasing the stock next period.

To implement this, the tax function must internalize the social cost of emissions. The first term in the tax is the flow damage cost and the second term represents the cost of increasing future emission stock, using the fact that $\frac{\partial V^{FB}(A_t, E_{t-})}{\partial E} = \gamma_E$.

The flow payoff to the firm, denoted by $J_t^{Tax}(k)$, under the emissions tax can be rewritten as

$$J_t^{Tax}(k) = \max_s A_t f(k,s) - C_t(f(k,s)) - w_t(s),$$

where $C_t(f) \equiv \min_m T_t(\sigma f - m) + cm$ represents the the effective cost of production, taking into account the abatement costs and tax payment.

**Redistribution of tax revenues.** Note that the constant term $\tau(E_t)$, which represents the redistribution of tax revenue and possible subsidies, affects the level of profits for all firms equally and thus does not have distributional effects. For simplicity, we chose the constant term $\tau(E_t)$ so that firms' problem under the taxation schedule can be expressed as $C_t(f) = c\sigma A f$. That is, the effective production cost is normalized to zero for firms without any production.

**Firm profits under first-best carbon tax.** The flow payoff to the firm can thus be simplified to

$$J_t^{Tax}(k) = \max_s (1 - c\sigma) A_t f(k,s) - w(s).$$

In other words, taxation effectively generates a discount factor $(1-c\sigma)$ in productivity for all pairs. While such a discount will change firm's profits and agent's earnings, it will *not* affect the sorting in decentralized market; hence, the first-best allocation is thus guaranteed.

**Comment on intensive margin.** Since we focus on heterogeneous agents in our framework, we take the productivity distribution as given (the capital stock and skill) as given and shut down the intensive margin for simplicity. If one adds back such a choice, then the taxation will induce distortion on the intensive margin (such as the capital and labor investment), consistent with standard model with homogeneous agents with tax distortion (Hong, Wang, and Yang 2023b).

## 4.2 Distortion under Green-Stakeholder Equilibrium

Relative to the first-best benchmark, the green-stakeholder equilibrium has three sets of differences. First, there is misallocation in the decentralized equilibrium. Second, there are distributional consequences. To our earlier point regarding brown agents' earning premium, we show that more productive firms do relatively better under the green-stakeholder equilibrium than under a carbon-emissions tax. Third, the potential for shutdown when there is a very negative abatement mandate $\zeta_t$ limits how much green firms can abate for brown firms.

### 4.2.1 Misallocation

Let $F(\zeta_t)$ denote the total production in decentralized markets given any mandate $\zeta_t$. Proposition 4 below highlights the skill misallocation as low skilled brown stakeholders become relatively valuable, and thus $F(\zeta_t) \leq F^{FB}$.

**Lemma 4.** *Compared to the first-best allocation, green stakeholders (brown stakeholders) work for smaller (larger) firms $\kappa_t(s,1) \leq \kappa^{FB}(s)$ $\left(\kappa_t(s,0) \geq \kappa^{FB}(s)\right)$ and $F(\zeta_t) \leq F^{FB}$. The output distortion in the green-stakeholder equilibrium, denoted by $\frac{F(\zeta_t)}{F^{FB}}$, increases with $c$, $\sigma$, and heterogeneity of stakeholders and firms, and is non-monotonic in the correlation between stakeholder productivity and type (green or brown).*

Since the sorting can be summarized by the discounted-productivity index, the higher are $c$ and $\sigma$, the lower is the ranking of green stakeholders, thereby implying a higher deviation from the first-best allocation.

**Correlation between stakeholders' productivity and preference.** To understand the effects of the correlation between stakeholder productivity and type, let $\lambda(s) \equiv \frac{g_w(s,1)}{g_w(s,1)+g_w(s,0)}$ represent the measure of green stakeholders conditional on productivity $s$. In the special case that there is no correlation between productivity and type, we thus have $\lambda(s) = \lambda \ \forall s$.

To illustrate the effects, it is useful to consider three special cases: no correlation $\lambda^0(s) = \lambda \ \forall s$, extreme positive correlation, and extreme negative correlation, which are denoted by $\lambda^+(s)$ and $\lambda^-(s)$, respectively.

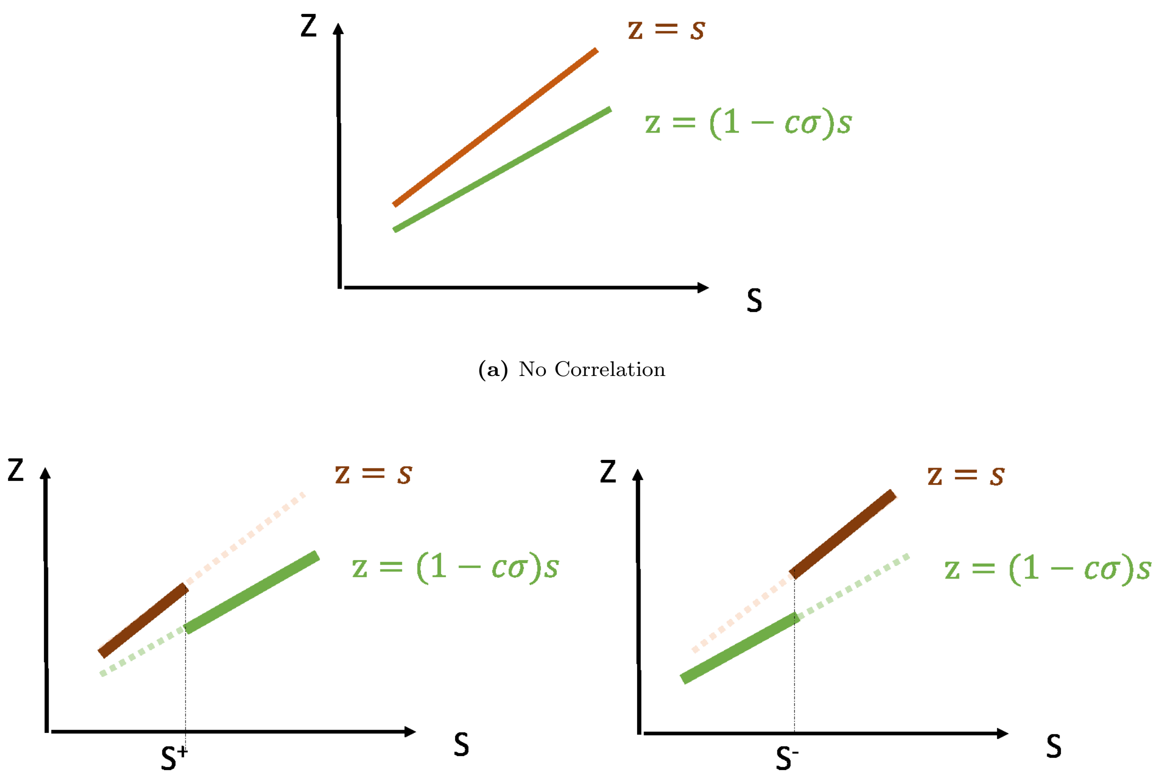
$$\lambda^+(s) = \begin{cases} 1 & \text{if } s \geq s^+ \equiv G_w^{-1}(1-\lambda) \\ 0 & \text{otherwise} \end{cases} . \ \lambda^{-1}(s) = \begin{cases} 0 & \text{if } s \geq s^- \equiv G_w^{-1}(\lambda) \\ 1 & \text{otherwise} \end{cases} .$$

That is, $\lambda^+(s)$ ( $\lambda^-(s)$) means that stakeholders with relatively high productivity $s \geq s^+$ ($s \geq s^-$) are all green (non-green) stakeholders, where $s^+$ and $s^-$ are pinned down so that the total measure of green stakeholders is $\lambda$.

Figure 1 shows that when productivity and type are uncorrelated, the modified-productivity $z$ distribution of brown stakeholders (red line) lies above that of the green stakeholders. Hence, brown stakeholders are more attractive to productive firms and will be matched with and offered higher compensation by productive firms. Less productive firms match with green stakeholders and do the abatement for society.

Figure 2 below illustrates the effect of distribution on $z$ when productivity and type are correlated. We use the width of the green and brown line represent the size of green stakeholders and brown stakeholders given the productivity $s$.

Recall that $\kappa^*(z)$ is increasing in $z$. Hence, in the negative correlation case, the most productive firm must work with brown stakeholders. On the other hand, in the case with positive correlation, stakeholders with highest $z$ also have higher skills and thus are matched with the most productive firms.




**(a)** No Correlation



**(b)** Perfect Positive Correlation

**(c)** Perfect Negative Correlation

**Figure 1:** The top subfigure represents the case of no correlation between productivity and green status. The bottom left subfigure represents the extreme positive correlation case and the bottom right subfigure represents extreme negative correlation case. The width of the green and brown line represent the size of green stakeholders and brown stakeholders given the skill $s$.

This example also illustrates why the size of distortion is generally non-monotonic in the correlation between productivity and type. Note that the distortion arises whenever the index $z$ and $s$ are not perfectly correlated. In the case with the positive correlation, the most productive stakeholders are always stakeholders with the largest discounted-productivity index. However, moving from the positive correlation to zero correlation, some of these high productivity green stakeholders now have lower ranking $z$ and thus will be out-competed by brown stakeholders with lower productivity, thereby increasing the distortion. Similarly, the distortion also increases when moving from the negative correlation case to the zero correlation case. In fact, the negative correlation case is the special case where $z$ and $s$ is perfectly correlated (i.e., higher $s$ must have higher $z$), which thus means no distortion.

#### 4.2.2 Distributional Effects

We now analyze how firm profits compare in the green-stakeholders equilibrium with the first best. To be precise, we compare firm profits under the green-stakeholder equilibrium with an emission tax, which are denoted by $J_t^M(k)$ and $J_t^{Tax}(k)$, respectively.

To allow for arbitrary correlation between productivity and type, we assume that $\lambda(s) = \lambda + \chi(s) \in [0, 1]$, where $\int \chi(s) dG_w(s) = 0$ and thus $\int \lambda(s) dG_w(s) = \lambda$.

**Lemma 5.** *$J_t^M(k)$ decreases with $\lambda$ and $J_t^M(k) - J_t^{Tax}(k) \geq 0$, where equality holds when $\lambda(s) = 1\ \forall s$. Moreover, $J_t^M(k) - J_t^{Tax}(k)$ increases with firm size $k$.*

The idea behind the proof uses the fact the outcome of taxation is equivalent to the special case where all stakeholders are green ($\lambda = 1$), where all firms effectively face the same discount factor $(1 - c\sigma)$ in all pairs. For any $\lambda < 1$, it is as if there is a better distribution of types $z$, as some stakeholders do not care about the abatement mandate. As a result of that, firms now can work with stakeholders with a weakly higher index $z$ and thus earn higher profits.

Moreover, since more productive or large firms benefit most for the improvement of skill or talent distribution, they thus benefit relatively more as well. In other words, compared to taxation, the effective costs for larger firms are now lower. A lower $\lambda$ means more brown stakeholders, which again improves the effective stakeholders' skills distribution further. Hence, for the same reason, it increases firm profits.

In the taxation benchmark, firms have lower marginal profits and stakeholders thus also have lower earnings. On the other hand, in the green-stakeholders equilibrium, brown stakeholders enjoy higher earnings as they are more valuable to firms. Hence, a higher measure of green stakeholders means brown stakeholders become more scarce, which thus increase the fees of brown stakeholders $w_t(s, 0)$ or the earnings premium $EP$.

**Lemma 6.** *$w_t(s, 0)$ increases with $\lambda$, and $w_t(s, 0) - w_t^{Tax}(s) \geq 0$ where equality holds when $\lambda = 1$.*

#### 4.2.3 Potential Firm Shut Down

A final key difference of the green-stakeholder equilibrium to the first best is that, unlike the carbon tax, only firms that are matched with green stakeholders are abating emissions. As a result, the green firms might have to abate for others. Firms may optimally choose to shut down the production as a result. To capture this formally, we refer the abatement mandate that is subject to a No-Shut-Down constraint if and only if the mandate $\zeta_t$ ensures positive surplus for all pairs.

According to Equation 9, since the least productive pairs are more likely to shut down given by $\zeta_t$, the no-shut down constraint can thus be further reduced to having non-negative surplus for the least productive pair, which yields

$$\Omega_t(\kappa(s_L, 1), (s_L, 1)) = A_t z(s_L, 1) a(\kappa(s_L, 1)) + c\zeta_t \geq 0. \tag{20}$$

That is, the no-shut-down constraint implies that there is a limit to how much green firms are willing to abate for brown firms. One can see that the no-shut-down constraint is likely to bind when the emission stock is high ($E_{t-}$), the measure of green agents is low, and brown firms produce more emissions.

Importantly, which firms become brown or green depends on the equilibrium sorting. Intuitively, if green firms happen to be larger firms, then the constraint is unlikely to bind as they only need to abate for firms that have lower emissions. Through this channel, the correlation between productivity and type is thus crucial for aggregate abatement.

**Lemma 7.** *The no-shut-down constraint is more likely to bind with lower $\lambda$, higher emission stocks $E_{t-}$, lower correlation between between productivity and type.*

# 5 Quantitative Analysis

We now apply our model to the setting of employee talent.

## 5.1 Data and Moments

Our data for firm wages and profits for US firms come from COMPUSTAT. For 2019, there are a total of 932 firms that report staff salary, number of employees and EBITDA. Among the variables, wage is the most likely to be unpopulated in COMPUSTAT. The firms that report all the variables of interest tend to be in the top 1000 firms by assets.

Our data on carbon emissions for 2019 comes from MSCI. To synchronize with our wage and employee data, we also focus on the top 1000 firms by assets. We focus on the 2019 cross-section since these cross-sectional distributions are fairly stable across years and 2019 is the most recent year that we can simultaneously obtain wages, profits and emissions data.

| | Mean | STDEV | 25th pct | Median | 75th |
|---|---|---|---|---|---|
| Wage/employee ($) | 141,688 | 300,262 | 61,308 | 90,625 | 132,714 |
| EBITDA/employee ($) | 156,241 | 1,073,012 | 22,296 | 99,141 | 176,868 |
| Emissions/revenue (tons/mil $) | 549.5 | 1255.9 | 36.4 | 104.3 | 432.9 |

**Table 1:** Summary statistics. US firm wage and EBITDA per employee and tons of carbon emissions per million dollars of revenue.

**Wage and EBITDA per employee distributions.** In Table 1, we report the summary statistics of three key distributions that inform our calibration. The first is the distribution of wage per employee. The mean is $141,688 with a standard deviation of $300,262. The 25th percentile is $61,308. The median is $90,625. And the 75th percentile is $132,714. This distribution of wages that we obtain for our subset of large firms matches well the distributions of wages and profits for S&P 500 firms where wage data comes from Glassdoor (Branikas et al. 2022).

The profits of the firm, which we measure using EBITDA, is also scaled by the number of employees. The mean of this distribution is $156,241 and the standard deviation is $1,073,012. Even though the means of wage per employee and EBITDA per employee are comparable ($141,688 versus $156,241), the standard deviation of EBITDBA per employee is three times larger. This larger dispersion partially reflects accounting treatments as some of the firms at the bottom end of the distribution can have negative EBITDA. For the purposes of our calculation, we will focus on the interquartile range of EBITDA per employee. The 25th percentile is $22,295 and the 75th percentile is $176,868.

**Emissions and abatement costs.** The third row of Table 1 reports the distribution of tons of emissions per million dollars of revenue. The mean is 549.5 tons and the standard deviation is 1255.9 tons. The 25th percentile is 36.4 tons while the 75th percentile is 432.9 tons. The distribution is highly right skewed as firms from the energy sectors generate far more tons of emissions per million of revenue.

The abatement cost per million dollars of revenue can be obtained once we have an estimate of the cost of removing a ton of carbon emissions. Cost estimates range widely for carbon capture and storage technologies (see a special report from the IPCC Metz et al. 2005 for an overview of these technologies). Since these technologies require energy, the avoidance of carbon emissions depends on the cost of commodity prices. Moreover, the operating cost of abatement for certain heavy sectors are much higher than for other sectors. Importantly, many estimates of cost in the literature ignore the upfront fixed costs of establishing these projects. Operating costs only constitute 10-15% of the total costs.

To estimate these total costs, we use data from the CRU Group which has a database of over 300 carbon capture and storage projects. Figure 2 shows the cost of carbon capture and storage across a range of projects in real 2022 dollars. Notice that for heavy sectors, the costs are quite high, around $1000 per ton of carbon emissions. For others, they are around a few hundred dollars per ton of emissions.

**Fraction of employees that are green.** As we highlighted at the outset, an IBM Business Institute Survey polled 14,000 people across 10 countries including the US. The survey found that one-third of workers polled recently accepted a job at an environmentally conscious company for lower pay. Their estimate of willingness-to-pay was substantial at 28%. We assume that employee skill and whether they are green is uncorrelated.

## 5.2 Functional Forms and Underlying Distributions

We assume that $b(s) = s$ and $a(k) = k^{\theta}$, where $\theta$ represents the impact of firm size on output. While our baseline model is about one-to-one matching, it can be reinterpreted

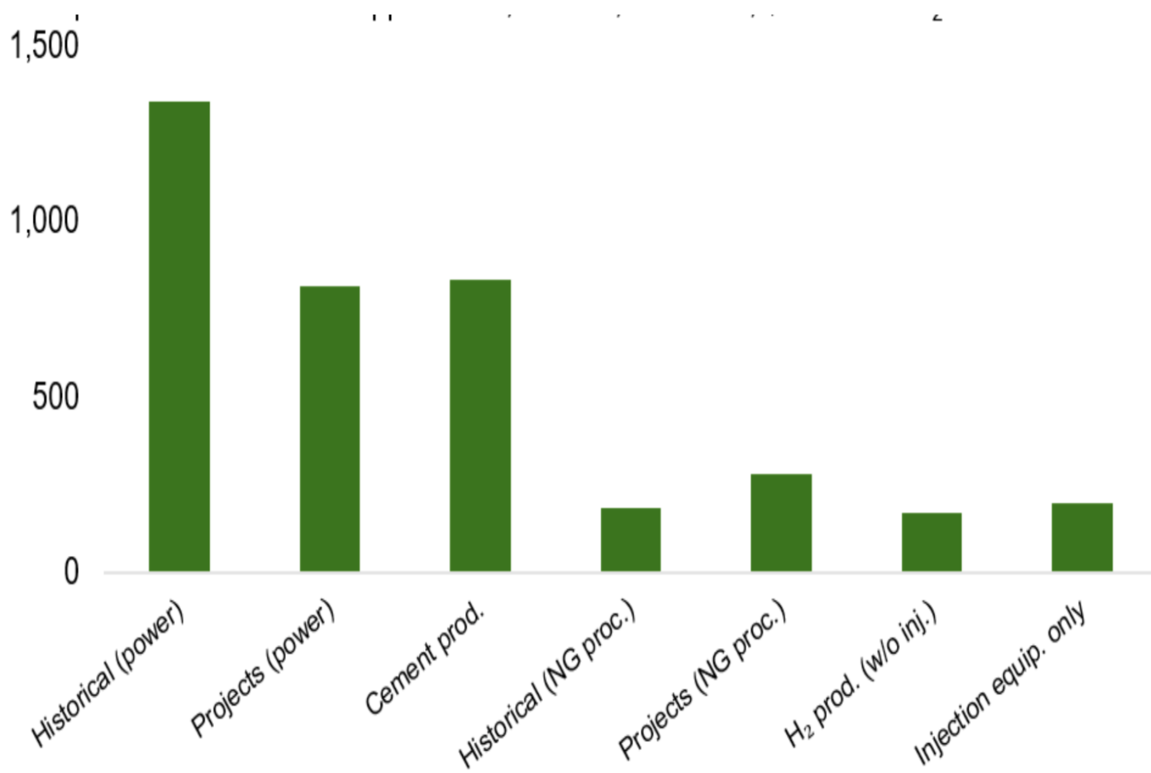



**Figure 2:** Total cost of abatement across projects. Unit capex. for different CCS applications, median, real 2022, $/annual tCO2

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as firm $k$ having multiple positions and the matching is between positions and workers, under the assumption that the production function is additively separable across types and within types.

The supports of firm productivity distribution are denoted by $[k_L, k_H]$. Assuming that the distribution of firms $k$ is Pareto with index $\alpha$ with density $g_f(k)$ and the number of position for firm $k$ is given by $\ell(k) = \ell_0 \left(\frac{k}{k_L}\right)^m$, the measure of positions provided by firms that is smaller than $k$ is then given by $G(k) = \int \ell(k) g_f(k) dk$, which is a Pareto distribution with index $\gamma = \alpha - m$.

The distribution of employee talent is specified as $G_w(s) = 1 - \left(\frac{s_H - s_L}{s_H - s}\right)^{-\frac{1}{\beta}}$, where $s_L$ and $s_H$ represents the lowest and highest worker skill, respectively. Since the production function is assumed to be $Ak^\theta s$, the level effect of $A$ and $s$ are observational equivalent, we thus normalize $A = 1$.

## 5.3 Calibration and Parameter Choices

We now calibrate our model. Table 2 below summarizes the parameter values. From the summary statistics in Table 1, the mean tons of emissions per million dollar of revenue is 549.5. Hence, we set $\sigma$ to be $5.94 \cdot 10^{-4}$. For the purposes of our calculations below, we will take \$350 per ton of emissions as our estimate of $c$, which is a rough average across all the projects from Figure 2. This implies \$219,600 of abatement costs per million dollars of revenue, around 21% of revenue, i.e. $c\sigma = 21\%$. Moreover, we set $\lambda(s) = \lambda = 33\%$. That is, we are assuming that employee type and employee talent are uncorrelated and that 33% of employees at any given productivity level is green. The parameters for the support of the firm asset distribution, $k_L$ and $k_H$, are from the minimum and maximum value of assets in our sample, respectively. For our calibration, we allow for a non-zero outside option for stakeholders. We use the minimum wage in

| Parameter | Definition | Value |
|---|---|---|
| $\zeta$ | abatement mandate (tons) | 15 |
| $\sigma$ | emissions from revenue (tons/million \$) | $5.94 \cdot 10^{-4}$ |
| $c$ | abatement cost per ton | 350 |
| $\gamma$ | Pareto index for firm positions | 25 |
| $\theta$ | Impact of firm assets on output | 0.2 |
| $\beta$ | Employee skills | $-0.02$ |
| $k_L, k_H$ | Support for firm assets | $146, 2.5 \cdot 10^6$ (million \$) |
| $s_L, s_H$ | Support for employee skills | $700, 56000$ |
| $\lambda$ | Fraction of green employees | 0.33 |
| $r$ | Discount rate | 0.05 |
| $\delta$ | Carbon decay rate | 0.003 |
| $d$ | Damage scaling | 2 |
| $\chi$ | Damage convexity | 0.085 |

**Table 2:** Parameter Values

our sample as a workers' outside option, which is around 10,000. [8]

We then simultaneously choose the remaining six parameters: the parameters governing production $\theta$ and firm positions $\gamma$, the parameters that govern the employee productivity distribution $s_L, s_H$, and $\beta$, and the abatement mandate $\zeta$ so that we target the following series of seven moments: (a) a wage per employee that of a mean of \$140,000; (b) standard deviation of wages; (c) mean EBITDA per employee of \$160,00 (hence the share of profits to wages is around 53% to 47%), (d) the 25th, median and 75th percentiles of EBITDA per employee, and (e) a mean greenium equal to 28%.

In Table 3, we report the targets which are a subset of moments drawn from Table 1 along with the mean greenium of 28% from survey data. We also report the model generated output corresponding to each of these targeted moments. Generally, we can match well the moments from the data.

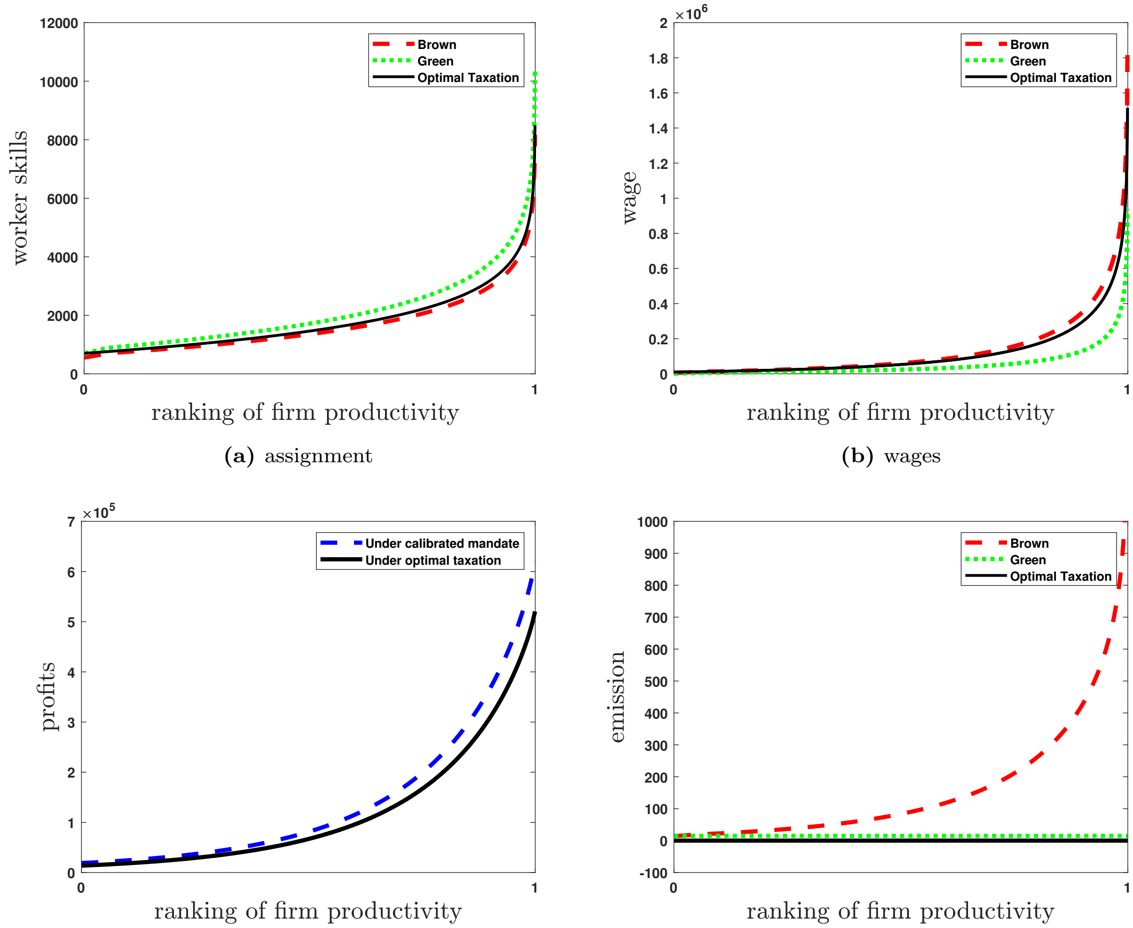
[8] Our baseline model is derived under zero outside option for simplicity. More generally, our model can accommodate non-zero outside option $w_0 > 0$, which affects the initial value of workers' wages as well as the shut-down constraint by the constant term $w_0$.

| | Data | Model |
|---|---|---|
| Mean Wage | $1.4 \cdot 10^5$ | $1.4 \cdot 10^5$ |
| Std Wage | $3 \cdot 10^5$ | $2 \cdot 10^5$ |
| Mean EMITDA/employee | $1.6 \cdot 10^5$ | $1.4 \cdot 10^5$ |
| 25th EBITDA/employee | $2.2 \cdot 10^4$ | $3.3 \cdot 10^4$ |
| 50th EBITDA/employee | $9.9 \cdot 10^4$ | $7.8 \cdot 10^4$ |
| 75th EBITDA/employee | $1.8 \cdot 10^5$ | $1.9 \cdot 10^5$ |
| Mean greenium | 28% | 31% |

**Table 3:** Target and Model Moments: Key moments we target from Table 1 and moments from models with parameters from Table 2.

**First-best carbon tax.** To evaluate our distortions, we also need to calculate a first-best optimal tax. This requires that we calibrate the discount rate ($r$), the parameter related to the dynamics of carbon stock ($\delta$) and the parameters governing damage from the stock of emissions for the economy ($d$ and $\chi$). This exercise is similar to the integrated assessment literature (Nordhaus 1992, Jensen and Traeger 2014, Cai and Lontzek 2019). There is a wide range of discount rates that are used in the literature. Following the literature, we choose choose $r$ to be 5% and the carbon decay rate to be 0.003.

We then choose $d$ and $\chi$ so that we hit the following moments which is in line with moments from calibrated integrated assessment models. We think of our data as a representative sample of the industrial sector, which is 11% of aggregate GDP (23 trillion) or around 2.54 trillion, which produces around 1.5 billion tons of new flow of emissions given $\sigma = 0.0006$. The current stock of the industrial sector emission is around 150 billion tons. Given these, we then choose $d$ and $\chi$ so that the ratio of abatement costs to revenue in the sector is 8% and the damage over aggregate GDP is 10%, which implies that the optimal emission after cleaning is around 0.9 billion tons. Using the fact that the workforce in this sector is 40% of the aggregate workforce (i.e., 67 millions workers), the calibrated first-best emission per worker is 13.66 tons.




**(a)** assignment

**(b)** wages

**(c)** profits

**(d)** emissions

**Figure 3:** Equilibrium outcomes for calibrated mandate and first-best carbon tax: assignment, wages-per-employee, profits-per-employee and emissions-per-employee. Parameters are described in Table 2.

## 5.4 Equilibrium Outcomes and Welfare Relative to First Best

We finally describe the equilibrium and welfare outcomes of the green-stakeholder equilibrium and compare them to outcomes with the first-best carbon tax.

**Assignment.** In Figure 3a, we plot the assignment function that maps worker skills $s$ to the ranking of firm productivity $k$. The first-best assignment function is denoted by the solid line. In green-stakeholder equilibrium, the green stakeholders are assigned to less productive firms (indicated by the dotted line), while the brown stakeholders are assigned to more productive firms (indicated by the dashed line). The dashed line is only slightly below the solid line since only 33% of the employees are green to start with. For the 33% of the green employees, we can see that the dotted line is significantly above the solid line. The dotted line is roughly 21% ($c\sigma$) higher than the solid line, reflecting the discount on green-stakeholder productivity given any $s$.

**Wages per employee.** In Figure 3b, we plot the wages per employee paid by firms of a given productivity $k$. The solid line represents the wage until the first-best optimal tax. The dashed line represents the wages of brown stakeholders and the dotted line represents the wages of green stakeholders. The wages of the green stakeholders lie below the first-best wages, while the wages of the brown stakeholders lie above the first best wages. There is a significant greenium that reflects both abatement costs and sorting using the discounted productivity index. The ultimate greenium outcomes are highly nonlinear across $k$ depending on the distributions of underlying firm and employee productivity. We see a larger greenium for higher $k$ firms. In other words, more talented brown stakeholders who work for more productive firms do better in the green-stakeholders equilibrium than under first-best carbon taxation.

**Profits per employee.** In Figure 3c, we plot firm profits per employee against firm productivity $k$. The solid line represents firm profits under the first-best carbon tax, while the dashed line represents firm profits in the green-stakeholder equilibrium. The solid line lies below the dashed line and the gap between these two lines increases with $k$. That is, higher $k$ firms do better in the green-stakeholder equilibrium do to their ability to out compete for brown stakeholders and to avoid abatement costs. The economic magnitudes are economically significant for large $k$ firms as we discuss below.

**Emissions per employee.** In Figure 3d, we report the emissions per employee against firm productivity $k$. The dashed line is the emissions of the brown firms. The solid line is the emissions of the green firms. Emissions of brown firms are rising with $k$. The emissions of green firms are constant and equal to the mandate $\zeta$ equal to 15 tons per employee. The emissions of firms under the optimal tax is 13.66 tons per employee.

**Output distortions.** Table 4 below summarizes a number of welfare outcomes of interest. First, we report the ratio of aggregate output under the calibrated mandate $F(\zeta)$ to the aggregate output under the first-best carbon tax $F^{FB}$. This ratio is 99.5%. That is, we find that the distortion of to output from green-stakeholders is relatively small, as $(F^{FB} - F(\zeta))/F^{FB}$ is less than 0.5%.

Why is the output distortion so small? After all, abatement costs represent a non-trivial fraction of firm output. The skills of talented green stakeholders are discounted and their assignment or sorting outcomes are economically significant. Should not this result in large declines in overall output? The answer it turns out is no because firms in the middle to the upper parts of the productivity distribution can always find a comparably productive brown stakeholder. This does not apply to low productivity firms but their output is a small fraction of aggregate output.

Moreover, given that profits are positive for all firms under the calibrated mandate, there is no firm shutdown. Hence, this potential distortion is also absent in our calibrated green-stakeholders equilibrium.

**Distributional distortions.** While the output distortions are small, the distortion to the distribution of firm profits is large. This distortion is simply the difference in the profits of firms across different productivity rankings under the calibrated mandate and under optimal taxation, i.e. the area between the curve for the calibrated mandate

| | Calibrated Mandate |
|---|---|
| $F(\zeta)/F^{FB}$ (%) | 99.5% |
| $M(\zeta)/M^{FB}$ (%) | 28% |
| $cM(\zeta)$/total wage gaps (%) | 89% |

**Table 4:** Aggregate Output Distortions and Welfare of Calibrated Mandate Compared to First-Best Carbon Tax. $F(\zeta)$ denotes aggregate output under calibrated mandate. $F^{FB}$ is aggregate output under first-best carbon tax. $M(\zeta)$ is the abatement under calibrated mandate. $M^{FB}$ is the abatement under first best. Total wage gaps denotes the aggregate difference in wages between green and brown stakeholders under the calibrated mandate.

and the first-best tax from Figure 3c. Recall that firms have to be indifferent between being green or brown and hence the profits are identical given a productivity rank across these these two types of firms. We can see that this difference is rising with rank of firm productivity. For the most productive firms, they have around a $100,000 per employee extra profit.

Observe that the level of the first-best emission will not affect firms' profits. This is because we chose the constant term $\tau(E_t)$ so that a firm's profit is invariant of the level of emission for simplicity. More generally, as discussed earlier, different implementations of the tax revenue redistribution could affect the level of firms' profits by the same constant. Nevertheless, the relative profits across firms with different rankings will always be invariant to the level of the first-best emission, measured by $J_t^M(k) - J_t^M(k_L)$ relative to $J_t^{Tax}(k) - J_t^{Tax}(k_L)$. In other words, our results on the distributional effect remain robust.

**Greenium and productive brown workers.** Another way to see the importance of distributional distortions is through the greenium for an employee of skill $s$, defined as

$$Greenium(s) \equiv \frac{EP(s)}{w_B(s)},$$

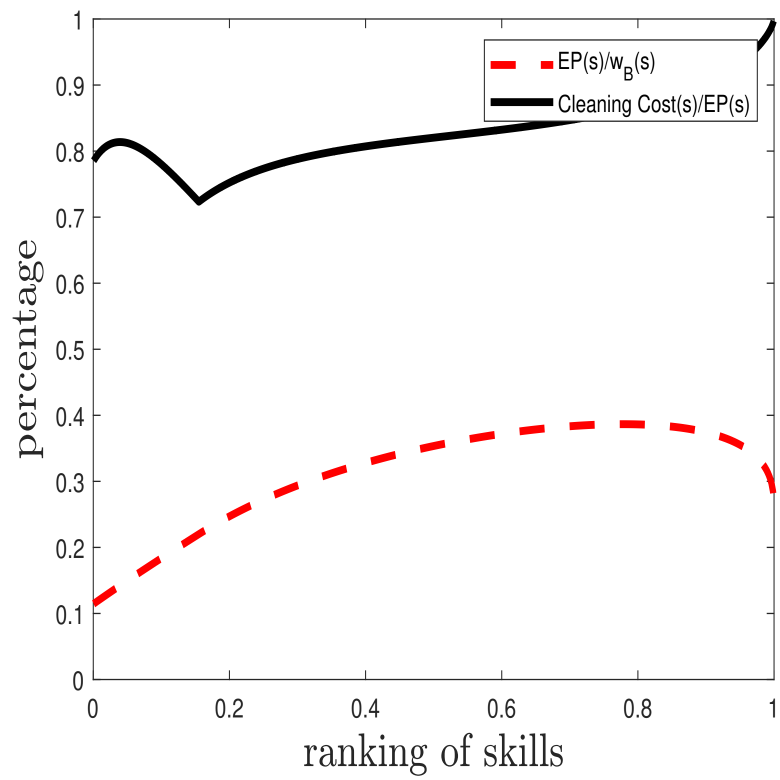



**Figure 4:** Greenium under calibrated mandate. Dashed line denotes the greenium as a function of worker skills. Solid line denotes the fraction of abatement or cleaning costs to the earnings premium $EP(s)$

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where the earnings premium $EP(s)$ is defined in Equation 15 and $w_B(s)$ denotes the wage of the brown stakeholders. We targeted a 28% greenium based on survey data. Notice from Figure 4 that the fraction of the wage gap ($EP$) that is due to cleaning cost is around 80%. The remaining 20 percent is due to the competition for brown stakeholders on the part of productive firms to avoid abatement, i.e. the sorting effect. Moreover, the greenium tends to be higher for more productive workers, though the effect is non-monotonic. The flip side of productive firms doing better in the calibrated mandate equilibrium than the first-best is that productive brown workers also do better.

**Aggregate abatement.** Returning to Table 4, green stakeholders are able to generate abatement $M^*$ that is 28% of the level of the first-best allocation $M^{FB}$. However, the expenditures on abatement is only 89% of the wage gap $EP$ due to sorting effect.

## 5.5 Comparative Statics

We next turn to comparative statics to show that our conclusions regarding output and distributional distortions are robust to the greenium (i.e. the stringency of abatement) that we target and the estimate of the fraction of workers that are green.

**The stringency of abatement mandate $\zeta$.** In our calibrated benchmark, $\zeta = 15$, which implies that the average earning premium is 31%. Table 5 below summarizes the effect when varying the level of the mandate $\zeta$. Intuitively, a tighter the mandate (i.e., a lower $\zeta$) implies higher clean up; hence, it becomes relatively costly to match with green stakeholders. As a result, the wage for non-green stake-holder increases, as firms are willing to pay more for them.

Moreover, recall that the level of $\zeta$ will not affect the sorting, hence the output distortion $(F(\zeta)/F^{FB})$ remains the same as we vary $\zeta$. Thus, a tighter mandate (a lower value of $\zeta$) will increase the *level* of the wage for all non green stake-holders by having a higher initial condition $W_t(z_L) = w_0 - c\zeta$. The wage for green workers, on the other hand, remain the same.

As a result, firms now pay higher cleaning costs when hiring green workers or higher wages for brown workers. Thus, the value of $c\zeta$ determines the level of the profits for all firms. A tighter mandate effectively implies moving surplus from firms to brown workers. This can be seen in Figure 5 by comparing the profit curves as we vary $\zeta$ relative to the baseline $\zeta = 15$. A higher $\zeta = 30$, which is associated with a less tight mandate, results in higher profit difference for firms between the calibrated mandate

| | $\zeta = 0$ | $\zeta = 15$ | $\zeta = 30$ |
|---|---|---|---|
| average $EP(\%)$ | 38% | 31% | 19% |
| average brown wage | $1.60 \cdot 10^5$ | $1.55 \cdot 10^5$ | $1.49 \cdot 10^5$ |
| average green wage | $1 \cdot 10^5$ | $1 \cdot 10^5$ | $1 \cdot 10^5$ |
| $F(\zeta)/F^{FB}$ (%) | 99.5% | 99.5% | 99.5% |
| $M(\zeta)/M^{FB}$ (%) | 31.8% | 28% | 25% |
| $cM(\zeta)$/total wage gaps (%) | 90% | 89% | 88% |

**Table 5:** Welfare outcomes comparing calibrated mandate to first-best carbon tax. Comparative statics around $\zeta$. $\zeta = 15$ is the calibrated mandate.

| | $\lambda = 10\%$ | $\lambda = 33\%$ | $\lambda = 50\%$ |
|---|---|---|---|
| average $EP(\%)$ | 22% | 31% | 32% |
| average brown wage | $1.47 \cdot 10^5$ | $1.55 \cdot 10^5$ | $1.6 \cdot 10^5$ |
| average green wage | $9.53 \cdot 10^4$ | $1 \cdot 10^5$ | $1.04 \cdot 10^5$ |
| $F(\zeta)/F^{FB}$ (%) | 99.8% | 99.5% | 99.4% |
| $M(\zeta)/M^{FB}$ (%) | 8.2% | 28% | 44.7% |
| $cM(\zeta)$/total wage gaps (%) | 88% | 89% | 90% |

**Table 6:** Welfare outcomes comparing calibrated mandate to first-best carbon tax. Comparative statics around $\lambda$. $\lambda = 33\%$ is the calibrated mandate.

and the first-best carbon tax. The converse is true for a more stringent mandate $\zeta = 0$.

**The measure of green stakeholders** The effects of the $\lambda$ comparative statics can be understood in Lemma 2. A higher measure of green stakeholders implies that non-green stakeholders become relatively scarce and thus higher wages for non-green stakeholders. On the other hand, it also implies more abatement and higher output distortions. Nonetheless, the quantitative effects are small. Output distortions increase slightly as we increase $\lambda$. Moreover, as a higher $\lambda$ increases the wages for non-green stakeholders, it makes it more expensive for larger firms to avoid the clean-up, which decreases the distortion of firms' profits relative to the first-best, as illustrated in Figure 6. Nonetheless, the size of the profit distortions are sizeable regardless of the $\lambda$.

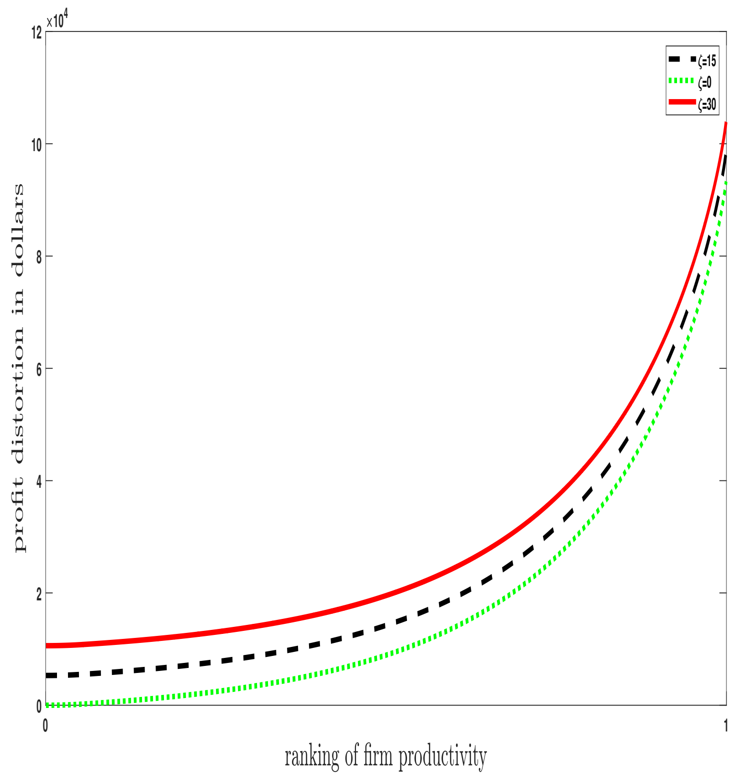


**Figure 5:** Profit distribution distortions (calibrated mandate to first-best carbon tax). Plot of the difference in firm profits under the calibrated mandate and the first-best carbon tax again ranking of firm productivity. Comparative statics around $\zeta$. $\zeta = 15$ tons per employee is the calibrated mandate.

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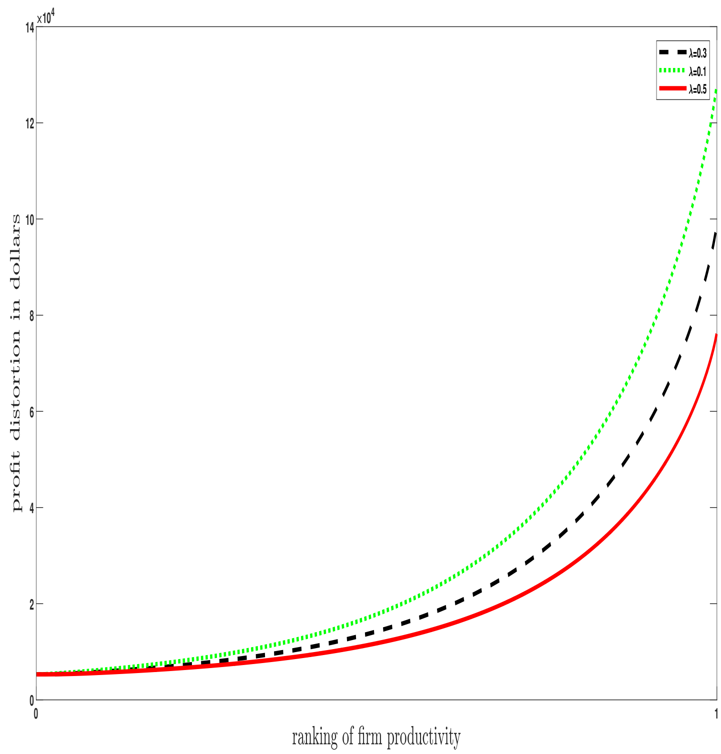


**Figure 6:** Profit distribution distortions (calibrated mandate versus first-best carbon tax). Plot of the difference in firm profits under the calibrated mandate and the first-best carbon tax again ranking of firm productivity. Comparative statics around $\lambda$. $\lambda = 33\%$ is the calibrated mandate.

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## 5.6 Additional Discussions

Finally, we address two additional issues regarding our analysis. The first is that our conclusions are likely extrapolate to other settings such as bank lending. For instance, Chang, Gomez, and Hong 2023 and Schwert 2018 estimate matching models for bank lending with heterogeneous firm risk and heterogeneity in bank size or ability to absorb risk. Given that data from Kacperczyk and Peydró 2022 banks globally follow their green mandates, one can conduct a similar quantitative exercise as the one we have done for the US labor market.

Similarly, supply-chain relationships for firms can also be modeled using matching models where co-investments are important for relationship outcomes (see Cen and Dasgupta 2021 for survey). Given that large consumer-facing companies such as Apple are pledging to work with green suppliers, our conclusions ought to port over to this setting as well.

The second is that we have assumed that the emissions transformation $\sigma$ is the same for all firms by targeting the average $\sigma$ from the population of firms using data from Table 1. But clearly, energy firms have much a larger $\sigma$. We can introduce this additional heterogeneity by allowing for two groups of firms – a low versus a high $\sigma$ group. This would accentuate the sorting effect and greenium that is at the center of our analysis since green stakeholder are more likely to sort to the low $\sigma$ group. However, this is unlikely to affect our quantitative conclusions regarding output and distributional distortions as long as productive firms with more emissions intensity are still be able to find productive brown workers.

# 6 Conclusion

We model the welfare effects of green stakeholders in competitive two-sided markets. Examples we have in mind include markets for bank lending, workers and suppliers. We solve for the green-stakeholders equilibrium and characterize the assignment, stakeholder earnings and firm profit outcomes. Compared to the first-best carbon-emissions tax, we identify three distortions that arise in the green-stakeholders equilibrium. The first distortion is that despite complementarities, productive firms need not hire productive stakeholders due to abatement costs. The second distortion is that the green-stakeholders equilibrium requires firms abate for others, which might be infeasible. The third distortion is distributional in nature — brown stakeholders earn more and productive firms do better than under an emissions tax. We calibrate our model to the employee setting using US data. We find that the first two distortions pertaining to output and potential firm shutdown are small, while the third distortion pertaining to distribution of firm profits is sizeable.

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# A Appendix

## A.1 Omitted Proofs

### A.1.1 Proof for Lemma 2

*Proof.* For any $s' > s$,we have

$$EP(s',t) - EP(s,t) = \int_{(1-c\sigma)s'}^{s'} a(\kappa^*(z))dz - \int_{(1-c\sigma)s}^{s} a(\kappa^*(z))dz > 0,$$

as $Q(s) \equiv \int_{(1-c\sigma)s}^{s} a(\kappa^*(z))dz$ and $Q'(s) \equiv a(\kappa^*(s)) - a(\kappa^*((1-c\sigma)s))(1-c\sigma) > 0$ since $\kappa^*(z)$ increases in $z$. □

### A.1.2 Derivation for the un-constrained value function

Define $\hat{V}(A_t, E_{t-}|\tilde{F})$ the value function given any sorting that gives the production $\tilde{F}$ when the no-shut down constraint is not binding.

$$\begin{aligned} r\hat{V}(A_t, E_{t-}|\tilde{F}) = \max_{M_t} A_t F - \frac{d}{1+\chi}\left(E_{t-} + \left(\sigma A_t \tilde{F} - M_t\right)\right)^{\chi+1} - cM_t \\ + \mu\left\{\int V(A', E_{t-}|\tilde{F})dG(A') - V(A_t, E_{t-}|\tilde{F})\right\} + \frac{\partial \hat{V}(A_t, E_{t-}|\tilde{F})}{\partial E_t}dE_t, \end{aligned} \tag{A1}$$

which can be understood as choosing the optimal clean-up given any $F$. The lemma below first provides the analytical solution for $\hat{V}(A_t, E_{t-}|\tilde{F})$.

**Lemma A1.** *Given any $\tilde{F}$,*

$$\hat{V}(A_t, E_t|\tilde{F}) = \left(\frac{(1-c\sigma)\tilde{F}}{r}\right)\left(\frac{rA_t + \mu\bar{A}}{r+\mu}\right) + \gamma_E E_t + v_0. \tag{A2}$$

*where $\gamma_E \equiv -\left(\frac{c}{1+r+\delta}\right)$ and $v_0 = \frac{d}{r}\left(\frac{\chi}{1+\chi}\right)\left(\frac{c}{d}\left(\frac{r+\delta}{1+r+\delta}\right)\right)^{\frac{\chi+1}{\chi}}$. The socially optimal cleanup* *is given by*

$$M^*(A_t, E_t|\tilde{F}) = \left(\sigma A_t\tilde{F} + E_{t-}\right) - \left(\frac{c+\gamma_E}{d}\right)^{\frac{1}{\chi}}. \tag{A3}$$

*Proof.* We guess and verify that $\hat{V}(A_t, E_t|\tilde{F}) = \gamma_E E_t + \gamma_A A + \tilde{v}_0$, and thus

$$M^*(A_t, E_t|\tilde{F}) = \left(\sigma A_t\tilde{F} + E_t\right) - \left(\frac{c+\gamma_E}{d}\right)^{\frac{1}{\chi}}.$$

Plugging into Equation A1, we thus have

$$r\left(\gamma_E E_t + \gamma_A A_t + \tilde{v}_0\right) = A_t\tilde{F} - \frac{d}{1+\chi}\left(\frac{\gamma_E+c}{d}\right)^{\frac{\chi+1}{\chi}} - c\left\{\left(\sigma A_t\tilde{F} + E_t\right) - \left(\frac{\gamma_E+c}{d}\right)^{\frac{1}{\chi}}\right\} + \gamma_E\left\{\left(\frac{\gamma_E+c}{d}\right)^{\frac{1}{\chi}}\right.$$

and thus

$$r\gamma_E = -\left\{c + \gamma_E(1+\delta)\right\} \Rightarrow \gamma_E = \frac{-c}{1+r+\delta}$$

and

$$r\gamma_A = (1-c\sigma)\tilde{F} - \mu\gamma_A \Rightarrow \gamma_A = \frac{(1-c\sigma)\tilde{F}}{r+\mu}$$

and hence

$$\begin{aligned}
r\tilde{v}_0 &= -\frac{d}{1+\chi}\left(\frac{\gamma_E+c}{d}\right)^{\frac{\chi+1}{\chi}} + c\left(\frac{\gamma_E+c}{d}\right)^{\frac{1}{\chi}} + \gamma_E\left(\frac{\gamma_E+c}{d}\right)^{\frac{1}{\chi}} + \mu\gamma_A\bar{A} \\
&= \left\{-\frac{d}{1+\chi}\left(\frac{\gamma_E+c}{d}\right) + c + \gamma_E\right\}\left(\frac{\gamma_E+c}{d}\right)^{\frac{1}{\chi}} + \mu\gamma_A\bar{A} \\
&= (c+\gamma_E)\left(\frac{\chi}{1+\chi}\right)\left(\frac{\gamma_E+c}{d}\right)^{\frac{1}{\chi}} + \mu\gamma_A\bar{A} \\
&= d\left(\frac{\chi}{1+\chi}\right)\left(\frac{\gamma_E+c}{d}\right)^{\frac{1+\chi}{\chi}} + \mu\gamma_A\bar{A}
\end{aligned}$$

We thus have

$$\hat{V}(A_t, E_{t-}|\tilde{F}) = \left(\frac{(1-c\sigma)\tilde{F}}{r}\right)\left(\frac{rA_t+\mu\bar{A}}{r+\mu}\right) + \gamma_E E_t + \frac{d}{r}\left(\frac{\chi}{1+\chi}\right)\left(\frac{c}{d}\left(\frac{r+\delta}{1+r+\delta}\right)\right)^{\frac{\chi+1}{\chi}}$$

Since $\zeta_t$ only affects the total cleanup, the optimal mandate when the no-shut down constraint is not binding thus solves

$$M(\zeta_t) = \int_{s_L}^{s_H} \{A_t f(\kappa^*((1-c\sigma)s), s) - \zeta_t\} g_w(s,1)ds = F^g - \lambda\zeta_t,$$
$$= M^*(A_t, E_{t-}|F^*) = (\sigma A_t F^* + E_{t-}) - \left(\frac{c+\gamma_E}{d}\right)^{\frac{1}{\chi}}.$$

□

### A.1.3 Proof for Lemma 3

*Proof.* We first show that, $F = F^{FB}$, under the first-best allocation. Suppose $(m, F)$ is such that $F < F^{FB}$, by increasing the production to $F^{FB}$ while increasing the removal by $\sigma A\left(F^{FB} - F\right)$ so that $\sigma AF - m = \sigma AF^{FB} - \left(m + \sigma A\left(F^{FB} - F\right)\right)$. Hence, the gain by doing so is positive when $1 - c\sigma > 0$, as

$$\left\{A\left(F^{FB} - F\right) - c\sigma\left(A\left(F^{FB} - F\right)\right)\right\} = (1-c\sigma)A\left(F^{FB} - F\right) \geq 0.$$

Hence, we have $V^{FB}(A, E) = \hat{V}(A_t, E_t|F^{FB})$.

Now, we prove that $T(e)$ can implement the first-best. Firm's optimization under $T(e)$ yields,

$$J_t(k) = \max_{s,m} A_t f(k,s) - T_t\left(\sigma A_t f(k,s) - m\right) - w_t(s) - cm.$$

Note that, given $(s, \theta)$, the FOC of clean-up for firm $k$ yields

$$T'(\sigma A_t f(k,s) - m) = d\left((\sigma A_t f(k,s) - m) + (E_{t-} + \hat{e}_t)\right)^{\chi} + \left(\frac{c}{1+r+\delta}\right) = c,$$

and thus

$$m(k,s) = \sigma A_t f(k,s) + (E_{t-} + \hat{e}_t) - \left(\frac{c}{d}\left(\frac{r+\delta}{1+r+\delta}\right)\right)^{\frac{1}{\chi}}. \tag{A4}$$

The we have

$$J_t(k) = \max_s A_t f(k,s) - C_t(f(k,s)) - w_t(s),$$

where $E_t = E_{t-} + \hat{e}_t$ represents the stock at period $t$ given the new emission $\hat{e}_t$,

$$T_t(e) = \left\{\frac{d}{1+\chi}(e+E_t)^{\chi+1} - \gamma_E e\right\} + \tau(E_t),$$

$$\begin{aligned}
C_t(f) &\equiv \min{}_m T_t(f-m) + cm \\
&= \frac{d}{1+\chi}\left(\frac{\gamma_E + c}{d}\right)^{\frac{\chi+1}{\chi}} - \gamma_E\left\{\left(\frac{c+\gamma_E}{d}\right)^{\frac{1}{\chi}} - E_t\right\} + \tau(E_t) - c\left\{\sigma A_t f(k,s) + E_t - \left(\frac{c+\gamma_E}{d}\right)^{\frac{1}{\chi}}\right\} \\
&= c\sigma A f(k,s(k)) + (c+\gamma_E)(E_t) - d\left(\frac{\chi}{1+\chi}\right)\left(\frac{\gamma_E + c}{d}\right)^{\frac{\chi+1}{\chi}} + \tau_t(E_t) \\
&= c\sigma A f(k,s(k)),
\end{aligned}$$

using the fact that $\tau_t(E_t) = d\left(\frac{\chi}{1+\chi}\right)(\gamma_E + c)^{\frac{\chi+1}{\chi}} - (c+\gamma_E)E_t$. Hence, given any $s$, the problem can be rewritten as

$$J(k) = \max_s (1-c\sigma)A_t f(k,s) - w(s),$$

hence emission tax lower the marginal value of production but no distortion on $F$. All workers and firms' profit then decreases by the factor $(1-c\sigma)$ equally and wages solve

$$\{1-c\sigma\} A f_s(k,s) - w'(s) = 0.$$

□

### A.1.4 Proof for Proposition 5

*Proof.* Given that $\frac{dJ^M{}_t(k)}{dk} = A_t z^*(k)$ and $\frac{dJ_t^{Tas}(k)}{dk} = A_t(1 - c\sigma)s^*(k)$,we thus have

$$D(k) \equiv J_t^M(k) - J_t^{Tax}(k) = A_t \int_{k_L}^{k} \left( z^*(\tilde{k}) - (1 - c\sigma)s^*(\tilde{k}) \right) d\tilde{k} \geq 0,$$

where inequality uses the fact that $z^*(\tilde{k}) - s^*(\tilde{k}) \geq 0$. This is because that $(1-c\sigma)s^*(k)$ is equivalent to the case with $\lambda = 1$. Hence for any $\lambda < 1$, all firms must now hire agents with higher index $z$. Moreover, for the same reason, $D'(k) = z^*(k) - (1 - c\sigma)s^*(k) \geq 0$. □