Systemic Influence in Systematic Break: Granular Time Series Detection

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Introduction 1.

A new method to detect
individuals of system-wide importance
(granular units),
exploiting a systematic break in panel data.

- Granular units are the main contributors to a systematic break.
- Detection as an initial screening tool.

Modeling Systemic Influence 2.

systematic break a major change in the cross-correlation structure.

- system covariance evolves in discrete steps.
- based on a factor model.
- The change of factor space can capture the dynamics of the correlation structure.

Individuals' systemic influence can be measured by their contribution to the change of the factor space.

The Benchmark Model 3.

$$\mathbf{y}_t^{(N\times 1)} = \boldsymbol{\chi}_{j,t} + \mathbf{u}_t, \quad t \in \mathtt{I}_j,$$

- $j \in \{0,1\}$ for regime 0 (window $\mathtt{I}_0$) / regime 1 (window $\mathtt{I}_1$),
- $\mathbf{u}_t = [\mathbf{g}_t \quad \boldsymbol{\epsilon}_t]'$ for the idiosyncratic components on the granular units($\mathbf{g}_t$) / non-granular units($\boldsymbol{\epsilon}_t$).

Assumption 1. $\Sigma_{y,j} = \Sigma_{\chi,j} + \Sigma_{u,j}$, where $\Sigma_{\chi,j} = P_j \Lambda_{\chi,j} P_j'$, for $\lambda_{\chi,j}^k = O(N), \quad \forall k = 1,\dots,K_j$.
Assumption 2. Independent of N, $\exists q \in [0,1]$, $\exists M > 0$ such that $\max_{i=1,\dots,N} \sum_{i'=1,\dots,N} |\mathrm{cov}(u_{it}, u_{i't})|^q \leq M$.

Assumption 3. $\Sigma_{y,1} = \Sigma_{y,0} + Z$, where $Z = \begin{bmatrix} P_0 & P_0^{\perp} \end{bmatrix} \begin{bmatrix} \mathbf{0}_{K_0} & A \\ A' & \mathbf{0}_{N-K_0} \end{bmatrix} \begin{bmatrix} P_0' \\ P_0^{\perp\prime} \end{bmatrix}$, for some $\|A\|_1 = O(N)$.

- $\mathbf{y}_t$ has factor structure in $\mathtt{I}_1$ as long as it has in $\mathtt{I}_0$ [1].
- The factor space changed $\mathrm{span}(P_0) \to \mathrm{span}(P_1) (= \mathrm{span}(P_0)\cos\Theta + V \sin\Theta)$.

Change of the Factor Space 4.

The projection metric [2] measures the distance between two spaces as

$$d\big(\mathrm{span}(P_0), \mathrm{span}(P_1)\big) \equiv \mathrm{tr}\big[(I_N - P_0P_0')P_1P_1'(I_N - P_0P_0')\big] \big(= \mathrm{tr}\big(P_{0,\perp}{}'P_1P_1'P_{0,\perp}\big)\big).$$

Proposition Under the benchmark model with Assumption1,2 and 3, for large N,

$$d\big(\mathrm{span}(P_0), \mathrm{span}(P_1)\big) = \mathrm{tr}\Big(\Sigma_{u,0}\Sigma_{y,0}^{-1}P_1P_1{}'\Sigma_{y,0}^{-1}\Sigma_{u,0}\Big) + o(1).$$

Measure of Systemic Influence 5.

An individual(i)'s contribution to systematic break can be measured by the first-order effect on the size of the change of the factor space,

$$\mathcal{I}_i \equiv \frac{1}{2}\Big\|\partial_{\boldsymbol{\sigma}_u{}^i} d\big(\mathrm{span}(P), \mathrm{span}(P_1)\big)\Big\| = \Big\|\Sigma_y^{-1}P_1P_1{}'\Sigma_y^{-1}\boldsymbol{\sigma}_u^i\Big\|,$$

where $\boldsymbol{\sigma}_u{}^i = [\sigma_{i1},\dots,\sigma_{iN}]'$, a source of the covariance dynamics.(subscripts '0' omitted.)

- For a given structure of partial correlation (Σ_y^{-1}), the factor space changes in the directions(V) such that the partial effects of granular units are the largest.

Detection Criteria of Granular Units 6.

$\mathcal{G}$: a set of granular units.
Assumption The factor space changes in $|\mathcal{G}|$ independent directions.

Number of granular units $|\mathcal{G}| = \mathrm{rank}\big(P_{0,\perp}{}'P_1P_1{}'P_{0,\perp}\big)$.
Membership The $|\mathcal{G}|$ individuals of the highest column norms $\mathcal{I}_i = \big\|\Sigma_y^{-1}P_1P_1{}'\Sigma_y^{-1}\boldsymbol{\sigma}_u^i\big\|$ are the granular units.

Estimation 7.

For a known breakpoint, the ingredients of $\hat{\mathcal{I}}_i = \big\|\hat{\Sigma}_y^{-1}\hat{P}_1\hat{P}_1{}'\hat{\Sigma}_y^{-1}\hat{\boldsymbol{\sigma}}_u^i\big\|$ are consistently estimated under standard regularity conditions ([3]).

An unknown breakpoint can be detected via $d\big(\mathrm{span}(P), \mathrm{span}(P_1)\big)$, e.g.,

$$S \equiv \ln \mathrm{tr}\big[\hat{P}_\perp{}'\hat{P}_1\hat{P}_1{}'\hat{P}_\perp\big] - 2\ln G - \ln E,$$

where G and E penalize estimation errors of the factor spaces.

Simulation: A breakpoint detection 8.

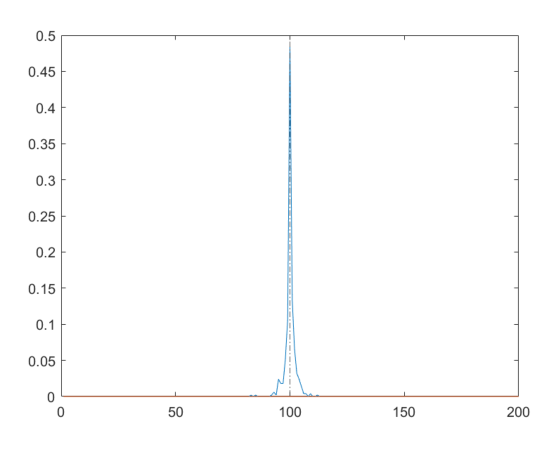

Fig. 1: A single breakpoint at $t^* = 100$

$j \in \{0,1\}$. $\sigma_\chi = 5$ and $\sigma_u = \sqrt{2}$.

- $N = 100$, $T_j = 100$, $t^* = 100$.
- $P_0^{N\times K_0}$ column orthonormal, $K_j = 3$.
- $\mathbf{f}_t \sim \mathcal{N}(\mathbf{0}_{3\times 1}, \sigma_\chi^2 I_3)$, $\mathbf{u}_t \sim \mathcal{N}(\mathbf{0}_{N\times 1}, \sigma_u^2 I_N)$.
- S_b from $\{Y_{1:t_b}, Y_{t_b+1:T_0+T_1}\}$, where
  $G = g_0 g_1$, for $g_j \equiv \frac{1}{N_j}\Big(\frac{1}{\sqrt{N_j}} + \sqrt{\frac{\ln N_j}{T_j}}\Big)$ and
  $E = e_0 e_1$, for $e_j \equiv \frac{1}{N_jT_j}\|U_jU_j{}'\|$.
  $\hat{t}^* = \mathrm{argmax}_{t_b}\{S_b\}$.

Simulation: Granular units 9.

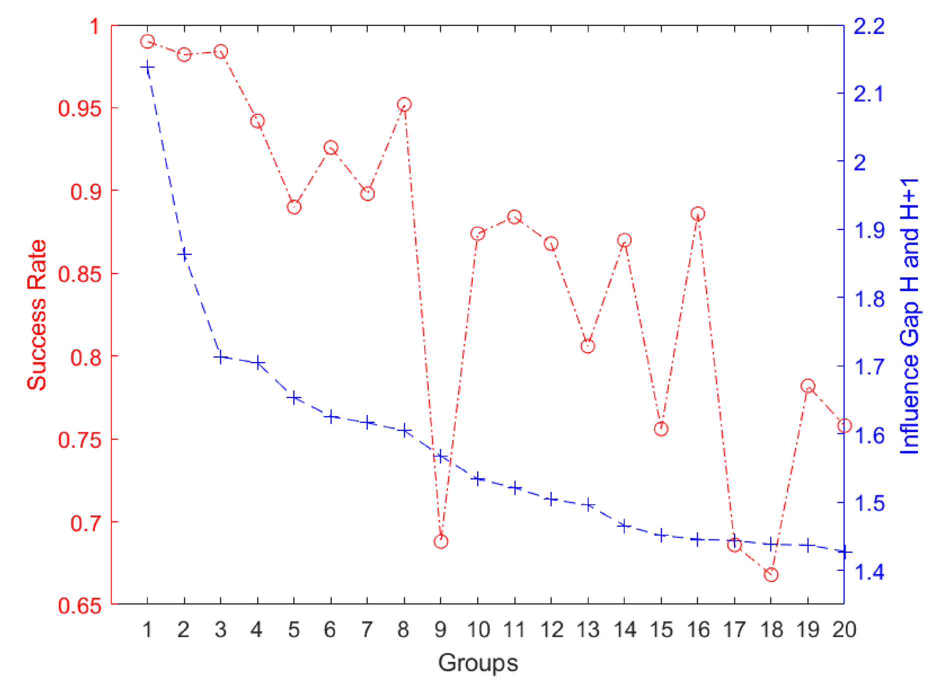

Fig. 2: Success Rate (known BP)

$|\mathcal{G}| \equiv H$.

- $N_j, T_j, K_j, \mathbf{f}_t, \mathbf{u}_t, P_0$: identical as the BP simulation.
- P_1 (or P_1P_1') $\longleftrightarrow \mathcal{G}$. 20 different dynamics / groups ($H = 3$).
  - identical $d\big(\mathrm{span}(P_0), \mathrm{span}(P_1)\big)$.
  - the influence gap $\mathcal{I}_H/\mathcal{I}_{H+1}$ varies (descending order).
- Repeat $M = 500$ for each group and count the number of successful detection of the true membership.
- As long as the least essential granular unit is 40% more influential than the non-granular units, the success rate is higher than 66.8%.

Application 10.

Daily S&P 100 stock return (log price differences) data and low dimensional component breakpoints from [4].

- The proposed method($\mathcal{I}$) is able to detect reasonable early sources of the known crisis.

Dot-Com Bubble $\mathtt{I}_0$ Oct26,00 - Apr25,01 / $\mathtt{I}_1$ Apr26,01 - Jun14,02
$\mathcal{I}$ **Three tech companies (DELL, EMC, TXN) and other two (AMZ, HD).**
Σ^{-1} Two energy(CVX, XOM), two finance(C, SPG), one Utility(SO).($\mathtt{I}_0$ energy, $\mathtt{I}_1$ finance.) [5]

Financial Crisis $\mathtt{I}_0$ Jul20,07 - Sep10,08 / $\mathtt{I}_1$ Sep10,08 - Dec11,08
$\mathcal{I}$ **Financial sector (MS, AIG, C, GS, SPG).**
Σ^{-1} Consumer discretionary(PG), Energy(CVX), Health care(JNJ), Industrials(UPS), Utility(SO).

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