Scale-Dependent Returns and the Interest Rate

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Abstract

I revisit the relationship between household wealth and returns on wealth in the United States over the past 70 years. While recent studies find that wealthier households earn higher returns, I show that this pattern is specific to the post-1980 period. Before 1980, the relationship was reversed: returns declined at the top of the wealth distribution, and the bottom 90 percent earned higher returns than the top 10 percent. I attribute this reversal to differences in exposure to interest rate risk. Wealthier households hold longer-duration assets, such as stocks and private businesses, whose valuations (and hence returns) are more sensitive to changes in real interest rates. Rising real rates before 1980 depressed their returns, whereas the post-1980 decline in real rates boosted them. To explain why richer households hold longer-duration portfolios, I develop a model in which households choose asset duration to hedge income risk. Because their income is more correlated with short-term interest rates, wealthier households optimally select longer-duration (countercyclical) assets to offset this exposure.

JEL Classification: G51; D31; G11.

Keywords: scale-dependent returns, dynamics of inequality, returns to wealth, heterogeneous returns, duration of assets, portfolio choice.

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1 Introduction

There is growing empirical evidence from different countries that households' returns on wealth increase with wealth; that is, if you have more wealth, you have higher returns on it (e.g., see Fagereng, Guiso, Malacrino, and Pistaferri (2020); Bach, Calvet, and Sodini (2020)). This phenomenon is referred to as the scale-dependence property of returns on wealth.

As Gabaix, Lasry, Lions, and Moll (2016) argue, scale-dependent returns is of high importance for explaining the rapid rise in wealth inequality that we have observed in the last 30 years (after 1980), which is otherwise hard to explain by just relying on random returns.[1]

In this paper, I address the question of whether scale-dependent returns have always existed or whether they are specific to the recent period of rapid rise in inequality. I study how scale-dependent returns change over time and what the main driver of these changes is. I measure the differences in households' returns on wealth using household micro data over a longer period of time, starting from 1949 to 2022, covering both the pre-1980 period of declining wealth inequality and the post-1980 period of increasing wealth inequality (see Figure 1). I show that, unlike the post-1980 period, in the pre-1980 period, the top 10 percent of the wealth distribution had lower returns than the bottom 90 percent. I then argue how changes in the real interest rate can explain this reversal in scale-dependent returns. Finally, I explore the causes that lead to the different household portfolio choices and assess their welfare implications.

The paper proceeds in four steps. First, I provide evidence on the distribution of returns for the period before 1980, using the extended version of the survey of consumer finances (SCF+, 1949-2022) for U.S. households (Kuhn, Schularick, and Steins (2020)). I find that during that 1949-1980 period, returns were not increasing with scale. Specifically, the top ten percent of the wealth distribution had, on average, lower returns than the bottom 90 percent (1.4 percentage points less), while for the post-1980 period, the top ten percent of the wealth distribution had, on average, higher returns than the bottom 90 percent (1.7 percentage points more).

Second, I connect this observed change in the scale-dependency of returns to the changes in the real risk-free interest rate (which are taken exogenously in this paper). Real risk-free interest rates increased in the postwar period, from 1949 until 1980, after which they started to decline (see Figure 11). As richer households hold more of assets that are of a higher duration, like stocks and private businesses, their returns

[1]The paper by Gabaix et al. (2016), mainly focuses on income inequality. However, in Appendix E, they discuss a similar concept for the dynamics of wealth.

are more exposed to changes in the interest rate (Greenwald, Leombroni, Lustig, and Van Nieuwerburgh (2023), Catherine, Miller, Paron, and Sarin (2023)).

Third, I look at the human capital asset of households. I proxy return to human capital with the growth rate of labor income and show that, in both sub-periods of pre-1980 and post-1980, the growth rate of labor income is, on average, increasing in wealth. That is, the labor income of the rich always grows faster than that of the non-rich. Furthermore, building on Guvenen, Schulhofer-Wohl, Song, and Yogo (2017), I measure the growth beta of labor income across wealth percentiles and show that the top 10 percent households are much more exposed to (GDP) growth shocks.

Finally, I build a model to determine why wealthy households choose assets with a longer duration. I develop a partial equilibrium portfolio choice model, based on the framework introduced by Campbell and Viceira (2002). In this portfolio choice model, interest rate follows an exogenously determined process, and households' choice of their duration depends on the cyclicality of their labor income growth (returns on human capital). As the wealthiest households have a highly pro-cyclical labor income, they choose a high duration to hedge their human capital risk. The rationale behind that is that long-duration assets are counter-cyclical, and it is a hedging strategy for them to do so. With this optimal choice, when the interest rate has an unexpected positive shock, the return on long-duration assets decreases while the return on short-duration assets increases. This explains the observed difference in returns among the rich and non-rich. The opposite happens when a negative shock occurs. When taking the model to the data, the model explains most variations in the duration choice of households with positive wealth (far from the borrowing constraint).

Related literature This paper contributes to the rapidly growing literature on heterogeneous household returns on wealth and its implications for the dynamics of wealth inequality (Benhabib, Bisin, and Luo (2019)). It explores an important aspect of it, that is, scale-dependent returns, and the mechanisms underlying it. In recent years, there has been much evidence from different countries on the scale-dependency of returns: Fagereng et al. (2020) show that returns on wealth are increasing in wealth using administrative data in Norway. Similarly, Bach et al. (2020) use Sweden's administrative data and find similar results. As Norway and Sweden have wealth tax data, measuring returns on wealth is easier and more precise using their data, but their data are available only for recent years. Elsewhere, some papers have used U.S. survey data to show this scale-dependency property of households' return on wealth (Cao and Luo (2017), Gaillard and Wangner (2021), Snudden (2019), Snudden (2023), and Xavier (2021)). A further example is Brunner, Meier, and Naef (2020), which uses Swiss administrative data to achieve this task.

Like all these papers, I measure returns on wealth of households. However, in contrast to these papers, I will use a significantly longer horizon, going back to 1949 (until the SCF+ data allows) until 2022, to cover periods that were much different than the recent period in terms of economic dynamics. Furthermore, I also study the return on human capital, which is an important asset for all households and is not much studied in this scale-dependent returns literature, although it is an important asset for all households.

Regarding the relevance of scale dependency of returns, the most significant implication is in explaining the rapid dynamics of inequality. Along with the seminal paper of (Gabaix et al. (2016)), who show that scale-dependence of returns will help to explain the sharp observed increase in inequality, Hubmer, Krusell, and Smith (2021) develop a quantitative model with different features, including scale-dependent returns[2], to see how it can help, along with other factors, to explain the observed changes in inequality. This paper offers new insights, suggesting that, for the pre-1980 period, the same logic, albeit in an opposite direction (negative scale-dependence), can help explain the decline in inequality.

Rather than the dynamics of inequality, there are other implications for the scale-dependency of returns. In the realm of optimal taxation, Gaillard and Wangner (2021) show that the macroeconomic implications of wealth taxation depend on the degree of scale-dependency of returns, along with some other factors. Also, Zanoutene (2023) studies a similar question. With my new evidence, we should rethink the optimal taxation system, as returns at the top of the wealth distribution are closer to neutral-to-scale in the long run. Especially when considering taxing capital gains, this paper sheds light on their long-term behavior, extending our perspective from the recent period of significant revaluation benefiting wealthy households to a time of substantial losses for them.

In another application of scale-dependency of returns, related to the Parato-tail order puzzle. Gaillard, Hellwig, Wangner, and Werquin (2021) argue that scale-dependent returns, along with other features, is needed to match the empirically observed order of tail indices for consumption, income, and wealth in recent years. My paper challenges the relevance of this idea to solve this puzzle for the pre-1980 period (if indeed it holds in longer horizons).

Regarding the interest rate mechanism, which I argue can explain a large part of the changes in scale-dependent returns, this paper also brings new insights to the literature. The idea of a connection between interest rate dynamics and revaluation of households' wealth is not new. It was first pointed out by Greenwald et al. (2023) that the dynamics of the interest rate predict the dynamics of wealth inequality. They build a model that captures the effect of changes in interest rates on wealth

[2]see Fig. 6 in their paper.

revaluations and connect this to the dynamics of wealth inequality. However, I directly measure households' returns using household microdata. Aligned with their paper, I build on the interest rate mechanism and the fact that richer households hold assets with longer duration. However, unlike it, I directly measure returns on wealth using households' balance sheets and use the same mechanism to explain the changes in the scale-dependency of returns. In other words, I show that this mechanism is indeed appearing in household returns on wealth: when interest rate changes, it affects household returns on wealth and, through that, it affects wealth inequality.

On the portfolio choice side, this paper contributes to the literature on why richer households choose assets with longer durations. Catherine et al. (2023) build a portfolio choice model in which households choose their interest rate exposure based on their age (through human capital) and social security share in wealth. They also have bequest motives in their model. This paper brings a new insight to this literature by looking to this problem from the angle of labor income risk and portfolio choice.

In the literature on how different labor income risk explains households' different portfolio choices, most of the papers focus on how background labor income risk affects stock market participation and stock holdings of households. Viceira (2001), Campbell and Viceira (2002), and Benzoni, Collin-Dufresne, and Goldstein (2007) study in a tractable way how the correlation of labor income growth and the return of the stocks affects stock holdings. Cocco, Gomes, and Maenhout (2005) studies how background risk in a life-cycle setting effect households portfolio choice. Catherine, Sodini, and Zhang (2024) and Catherine (2021) study the effects of higher moments of labor income risk, like variance and skewness. Unlike most of these papers, I focus on the aggregate component of labor income fluctuations (the first moment of it) and argue how holding long-duration assets can hedge this risk.

On the welfare consequences of changes in asset prices, Fagereng et al. (2025) and Greenwald et al. (2023) study the asset price redistribution across the life cycle. However, this paper studies its connection with changes in labor income.

Outline In section 2, I will describe my approach for measuring the returns and the data I use. Then, I compare empirical evidence on the scale-dependency of returns before and after 1980 and establish the empirical findings on changes in scale-dependent returns. In section 3, I will discuss the interest rate channel and how it can explain the empirical findings. Then, in section 4, I discuss returns on human capital and how they change across the wealth percentiles. In section 5, I build a parsimonious model, which is a theory of duration choice, and take the model to the data. Finally, I conclude in section 6. The paper also includes an Appendix with further empirical details and results, as well as proofs and solutions for the model.

2 Households' returns on wealth

This section documents the changes in the cross-section of households' returns on wealth over time. I start by discussing why it is important to look at other periods as well, rather than only the recent period, to study the relative difference in household returns on wealth. After that, first, I define returns on wealth and its components. Next, I will explain the available data. Then I will establish the results on the changes in scale-dependency of returns over time.

2.1 Why a longer horizon is key?

The existing evidence for scale-dependent returns from different countries is limited to post-1980, and mostly very recent years. As shown in Figure 1, inequality decreased prior to 1980 and increased post-1980.[3] This aligns closely with arguments concerning the relation between scale-dependent returns and the dynamics of inequality (Gabaix et al. (2016)). However, there is a lack of evidence for scale-dependent returns for older years, such as those before 1980, which this paper now sheds light on.

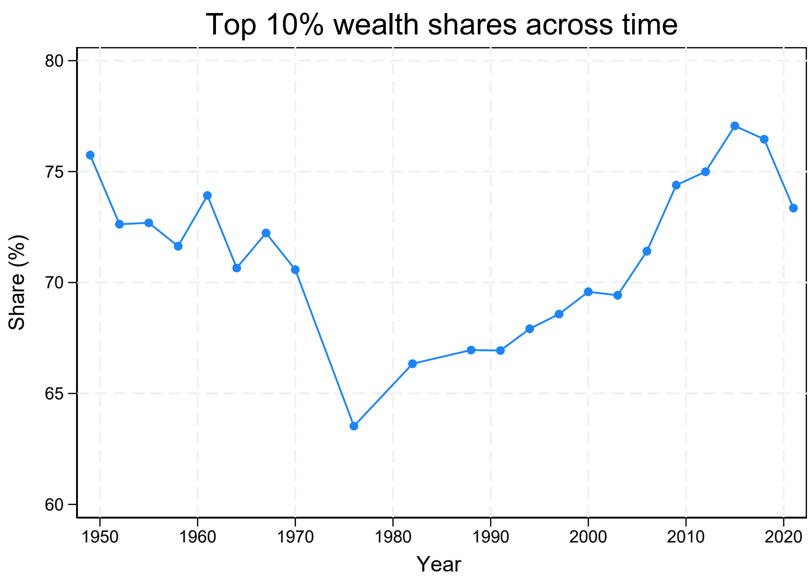

Figure 1: Wealth inequality in terms of top 10 percent wealth share across time using SCF+ data (1949-2022).[4]The graphs shows that over the pre-1980 period, wealth inequality on average was decreasing while for the post-1980 period, it was increasing.

Furthermore, post-1980 is a special period accompanied by low economic growth and a decline in interest rates, while it has not always been like that. Table 1 sum-

[3]Other measures of wealth inequality, like the Gini coefficient (for wealth), follow the same pattern. (See Greenwald et al. (2023), Figure 1c in Appendix B)

[4]Other data sets and methods find quite similar results. (see Saez and Zucman (2016), Smith, Zidar, and Zwick (2022))

marizes the growth rate in key economic variables for the two sub-periods of pre-1980 and post-1980, and in the long-term (1949-2022).

Table 1: Growth rate of key variables

	GDP	r^{short}	r^{long}	Labor income
Pre-1980	3.5	+0.08	+0.15	5.4
Long-term	3.1	-0.02	-0.02	3.3
Post-1980	2.6	-0.08	-0.13	1.9

Notes: The table shows the growth rate of key economic variables. The pre-1980 period was a period of high economic growth, while the post-1980 period was a period of lower growth.

2.2 Measurement of returns

I follow a definition of returns based on return on assets (ROA) in accounting.[5] Most other papers studying household returns on wealth use the same definition as well.[6] The net of debt repayment return to wealth, or simply net return of each household i at year t is defined as:

$$r_{it} = \frac{y_{it}^{div} + y_{it}^{kg} - y_{it}^{b}}{w_{it} + \dfrac{f_{it}}{2}} \tag{1}$$

where, y_{it}^{div} is the dividend income from wealth, which includes all received interests, rents (inclusive of imputed rents of households who live in their own house), and stock dividends or income from private business; y_{it}^{kg} is the capital gains (regardless of it being realized or not) related to the period t; y_{it}^{b} is cost of debt (interest payment on loans) during period t; w_{it} is the wealth at the beginning of the period t; and finally, f_{it} is the net flow of (active) investment into gross wealth in period t.[7]

Note the presence of net flow or active savings f_{it} in this formula. As Fagereng et al. (2020) shows, it is because the income from the asset in the numerator is not only from the wealth at the beginning of the period w_{it}, but also from the new flow of assets during that period f_{it}, and on average, half of them as they occur in different

[5]This paper and most of the other papers mentioned in the literature are about *actual* returns. However, there is evidence that scale-dependency of returns also extends to *expected* returns (Bach et al. (2020)).

[6]Some papers use other definitions (like Xavier (2021), which subtracts debt in the denominator. However, the ROA is more natural and intuitive to be returns on wealth.

[7]Note the difference between active saving and gross saving. The former does not include savings through capital gains on asset holdings: $w_{i,t+1} = w_{it} + y_{it}^{kg} + f_{it}$. That is, your wealth in the next period is equal to your wealth in this period plus the amount of capital gains, plus the net flows to your wealth, which is the net money that you actively invest (or divest). The sum of capital gains and flows is called gross savings, in contrast to net savings, which only includes the flows.

times of that period.[8]

In this paper, I measure pre-tax real returns. Hence, all variables are in real terms and measured before (individual) tax. Depending on the data structure, one can use other equivalent formulas (see Appendix A.1). Specifically, if the data has a panel dimension, then one can use this equivalent formula:

$$r_{it} = \frac{y_{it}^{div} + y_{it}^{kg} - y_{it}^{b}}{\frac{w_{it} + w_{it+1}}{2} - \frac{y_{it}^{kg}}{2}} \tag{3}$$

which does not explicitly require knowledge of flows.

2.3 Data

I use the extended version of the Survey of Consumer Finances (SCF+) from 1949 to 2022. which will be a span of 70 years from the post-war period until now. SCF+ connects modern SCF with old SCF (Kuhn et al. (2020)) in a consistent and comparable way.[9] The harmonized version is trieanual for most of the years in the sample.

SCF+ includes asset holdings for each household for almost all of the major asset classes. It also includes dividend income generated by their assets. Interest payment on debt is also included, but mostly for modern SCF. Here, I briefly review the available variables in SCF+, needed for measuring returns.

Asset holdings Liquid assets (checking, savings, call/money market accounts, and certificates of deposits.), housing and other real estate, bonds, stocks, and business equity. Mutual funds and (defined contribution) pensions are also included, mostly for modern SCF.

Divined income from assets Rental income (the imputed rental income of homeowners is separately added), interest and dividends income, as well as business and farm income.

Cost of debt For most of the years in the modern SCF, the data on the interest rate of mortgages and some other debt classes exist in the survey. Otherwise, the

[8] Fagereng et al. (2020) show (in their appendix) that if the multiplier of flows in the denominator is not $\frac{1}{2}$, the measurement of returns will be biased. As in the equation below, if we denote returns on wealth as a generic function of wealth $R(w_{it})$, and λ as a coefficient on flows in the denominator,

$$r_{it} = \frac{R(w_{it}) * (w_{it} + \frac{1}{2}f_{it})}{w_{it} + \lambda f_{it}} = R(w_{it}) + \frac{(\frac{1}{2} - \lambda) * f_{it}R(w_{it})}{w_{it} + \lambda f_{it}} = R(w_{it}) + (\frac{1}{2} - \lambda)\frac{R(w_{it})}{\frac{w_{it}}{f_{it}} + \lambda}, \tag{2}$$

one can see that if $\lambda \neq \frac{1}{2}$, the formula is potentially biased.

[9] The modern SCF is from 1983 to 2022, and its historical SCF is from 1949 to 1977. For more details on how it is constructed, see Appendix A.3.

average interest rate for mortgage or non-mortgage debt for each year is imputed using other data sets (see Appendix A.4 for the details).

Rather than these variables, one also needs to know the capital gains and flows, which do not exist in SCF+.[10] Here, I explain how I tackle this challenge.

Capital gains For approximating the capital gains, I use average asset class capital gains for different asset classes (see Appendix A.4 for more details). This approach captures across asset class differences in capital gains and misses the within asset class differences. As a robustness check, I use approximated flows and a pseudo-panel assumption to recover a series of capital gains that includes both across and within asset class differences in returns. This robustness makes the results stronger (see Appendix A.5).

Flows I assume for a pseudo-panel, i.e., treat each percentile of net wealth as an individual that does not change over time. This way, leveraging the panel dimension, one can measure the returns without explicitly knowing the flows.[11] As a robustness check for this approach, I approximate the individual flow using different sets of assumptions, and the results remain similar (see Appendix A.5).

2.4 Returns across wealth over time

Figure 2 plots the average net (of debt repayment) return on wealth for U.S. households, for the two periods 1949 to 1980 and 1980 to 2022. This new evidence shows that returns have not always been scale-dependent or increasing in wealth in the way they have been in recent years, as documented by other papers. Specifically, if we look at the long-term average, before 1980, they are not increasing with scale, especially at the top, where returns are decreasing with scale.

The surprising fact for the pre-1980 period is that during that period, the richest people had a return which is similar to the lower middle class. Although, this finding seems a bit counter-intuitive in the beginning, but is in a way consistent with the observed changes in inequality measures in these two periods: before 1980, top10-% wealth share decreased and after 1980 it increased (Saez and Zucman (2016); Kuhn et al. (2020)), as it is plotted in Figure 1.

[10] For flows, a few years (1949, 1950, 1951) of the historical waves included questions about active savings, but for other years, it is not asked for. For capital gains, modern SCF includes realized capital gains, but it is not enough as all the capital gains (regardless of whether it being realized or unrealized should be included in the measurement of returns.

[11] This is a good assumption in this paper as the percentiles are not defined so small, as otherwise, the effect of households moving between them may become larger. For example, Gomez (2023) shows that the role of individuals moving between very small percentiles is high. That paper uses Forbes 400 list data, which is about a small fraction (top 3 percent of households) in the top 0.01 percent of the US wealth distribution. In my paper, the smallest growth is the top 1 percent, which is much larger.

(a) pre-1980 (b) post-1980

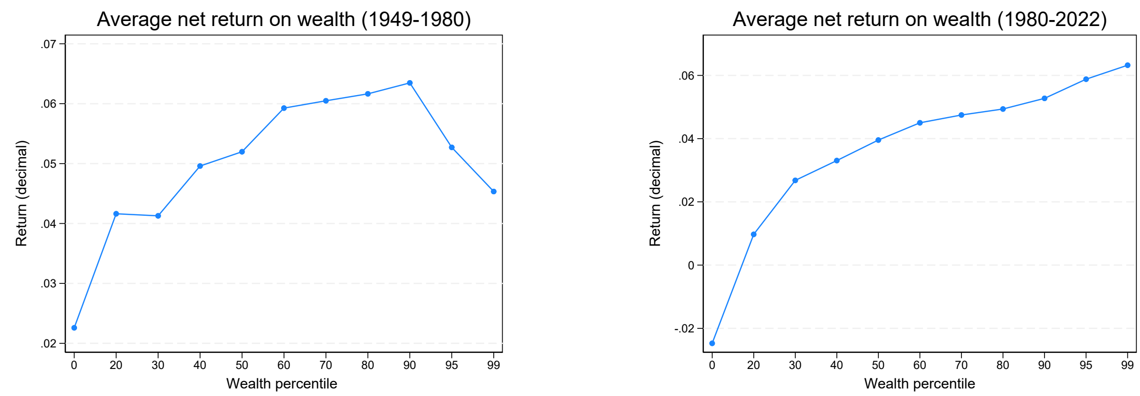

Figure 2: Average realized net (of debt repayment) return on wealth for the years before and after 1980 using SCF+ data set. This graph shows that, the returns on wealth of households was not always increasing in wealth. Especially, the richest people had returns similar to those of l to households in the lower middle-class.

Figure 3a plots the cross-section of returns for the two periods in one graph. As it is clear from this graph, the bottom-90 had higher return in the pre-1980 period compared to bottom-90 in the post-1980 period. This is while for the top-10 percent, it is the opposite. Figure 3b plots the long-term average over the whole sample, which shows returns on wealth are increasing with wealth, but for the top-10 percent are almost similar to a flat curve.

To understand these graphs better, it is noteworthy to know that the average annual growth rate of the economy (real GDP) in the pre-1980 period was 3.74 percent, while for the post-1980 period it was 2.64 percent, which is a difference if 1.1 percent, close to the average gap between the two lines in Figure 3a.

2.5 Top 10 to bottom 90 return ratio

To see the huge possible implications of this difference in returns on wealth, it is better to measure the wealth-weighted returns of the top-10 and the bottom-90 percent of the wealth distribution. This is because the wealth distribution in the U.S. is highly skewed, as it is plotted in Figure 4, and the top 10 percentiles own around 70 percent of total household wealth.

Table 2 reports the wealth-weighted average of net returns on wealth for top-10 percent of the wealth distribution over the bottom-90 percent for the two periods 1949-1980 and 1980-2022 using SCF+ data. During the pre-1980 period, the top 10 had on average **1.4% less** compared to the bottom 90, while during the post-1980 period, they had on average **1.7% more** in returns.

(a) pre-1980 and post-1980 in one graph (b) long-term average

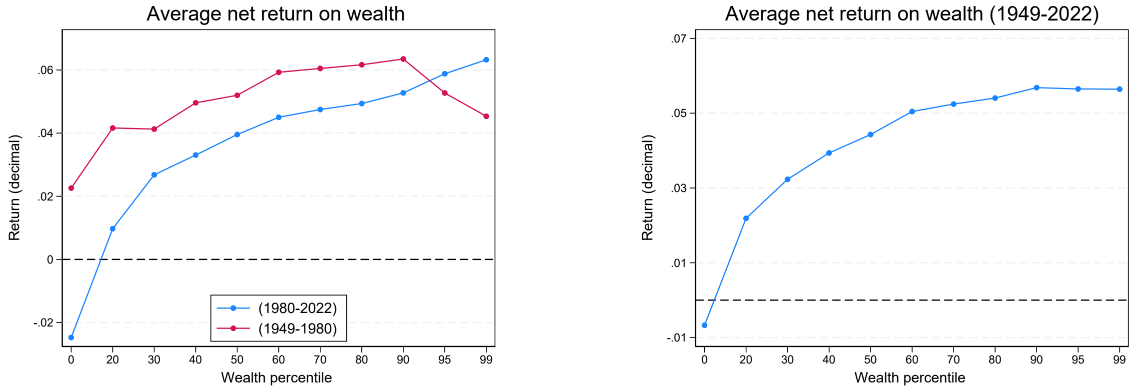

Figure 3: Average realized net (of debt repayment) return on wealth for the years before and after 1980 using SCF+ data set. The left graph shows that returns on wealth of households were higher in the pre-1980 sample, compared to the post-1980, except for the very rich households (mostly in top percent of the wealth distribution). The right graph shows the long-term average of returns on wealth over the whole sample 1949-2020, which shows returns are not increasing in the very top percentiles of the wealth distribution in the long-term, though they are increasing for the bottom 90 percent of households.

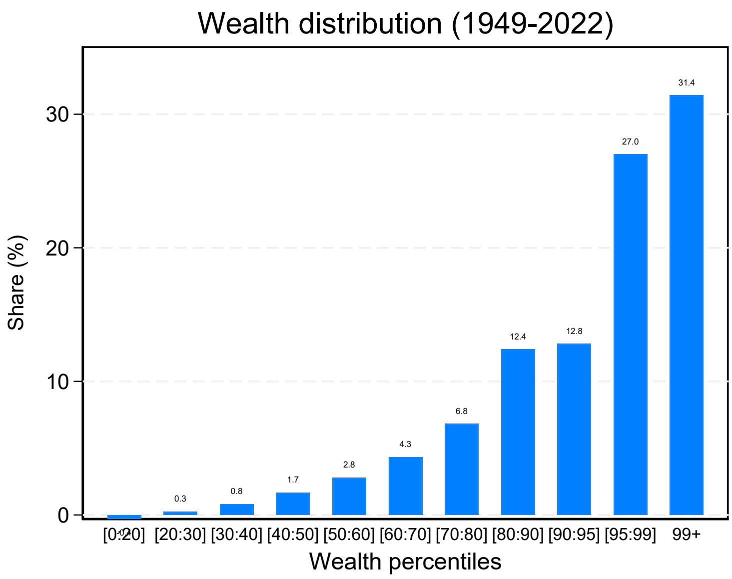

Figure 4: Long-term wealth distribution of U.S. households (1949-2022) using SCF+ data set. This grpah shows that wealth is highly concentrated at the top. Over that last 70 years in the U.S., the top 1 percentile of the wealth distribution alone, on average hold around 30 percent of the total households' wealth, and the top 10 percentiles together owned around 71 percent of the total households' wealth.

	pre-1980	post-1980
$\frac{R_{\text{top }10}}{R_{\text{bottom }90}}$	0.9859	1.017

Table 2: Wealth-weighted average of net returns on wealth for top-10 percent of the wealth distribution over the bottom-90 percent for the two periods 1949-1980 and 1980-2022 using SCF+ data. Note that $R_{\text{x}} = 1 + r_{\text{x}}$. This table shows that the top-10 to bottom-90 return ratio was below one for the pre-1980 period and above one for the post-1980 period.

Furthermore, the result of $(\frac{R_{top10}}{R_{bottom90}})_{pre-1980} < (\frac{R_{top10}}{R_{bottom90}})_{post-1980}$ is significant at 95% confidence interval. On the economic significance of these numbers, note that this is an average over 30 years, and it can compound to huge differences in the dynamics of inequality. Although the connection between returns and inequality is not the main focus of this paper, it is worth briefly mentioning. Figure 1 plots the dynamics of wealth inequality in the U.S. across time, measured as the wealth share of the top 10 percent households in the wealth distribution.

2.6 Components of return on wealth

To better understand the difference in returns among households, it is quite helpful to look at the components of returns. In this section, I discuss two natural decompositions of returns.

2.6.1 Gross return and cost of debt

An insightful approach to better understand the dynamics of net returns on wealth is to decompose it into gross return and cost of debt:

$$r_{it} = r_{it}^{gross} - r_{it}^{b} \tag{4}$$

where $r_{it}^{gross} = r_{it}^{div} + r_{it}^{kg}$. Figure 5 plots the gross return and the cost of them across wealth for the pre-1980 and post-1980 periods in one graph. As is seen in this graph, in the pre-1980 period, the top 10 percent of the wealth distribution had lower gross returns than the bottom 90 percent, while the opposite happened in the post-1980 period. In terms of the cost of debt (leverage times interest payment of debt), the pre-1980 period was better for almost everyone, as they had a lower cost of debt, except for the bottom 20 percent.

In fact, Figure 5 reveals an important fact: the gross return is almost the same for households in the 30th percentile to 90th percentile, which can be thought of as broadly defined middle class. In other words, the reason returns are increasing for

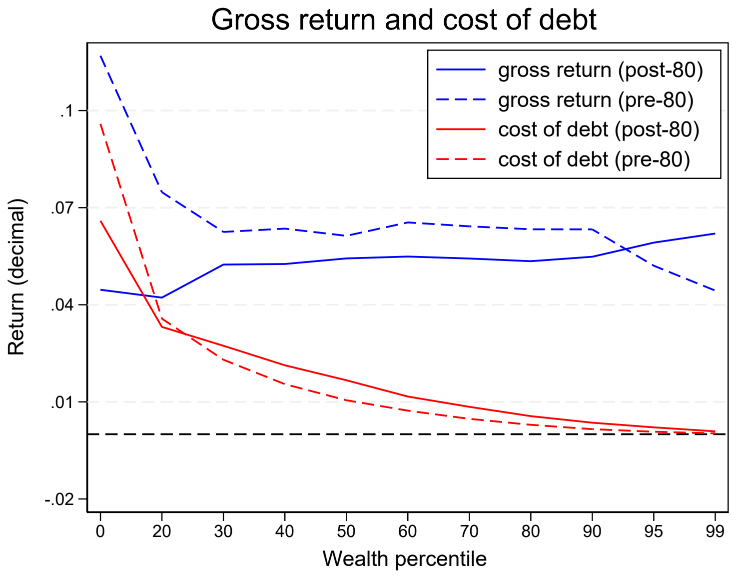

Figure 5: Gross return and cost of debt for two periods of 1949-1980 and 1980-2022 using SCF+ data set. The graph compares the gross return and the cost of debt across wealth for the two periods.

middle-class households in Figure 2 is the decline in the cost of debt.

The cost of debt (which is the multiplication of leverage and interest rate on debt) is decreasing in wealth because of two reasons: leverage is on average decreasing in wealth (see Figure 9), and the interest rate on debt is also on average decreasing in wealth (see Figure A.3)

2.6.2 Dividend and capital gain returns

One can decompose net return on wealth to dividend return, capital gain return, and cost of debt:

$$r_{it} = r_{it}^{div} + r_{it}^{kg} - r_{it}^{b} \tag{5}$$

Figure 6 plots this decomposition for the pre- and post-1980 samples.

As we can see, dividend returns are always on average decreasing in wealth, which comes from the fact that richer people have more of assets that not only pay through dividend cash flows, but also through capital gains, like stocks and businesses. For example, stocks and businesses usually have a payout ratio well below 1 and retain their profits within the firm as retained earnings, which show up as capital gains. Table 3 shows the average payout ratio for public stocks and private businesses.

The cost of debt, as already discussed, is decreasing in wealth because leverage is on average decreasing in wealth (see Figure 9), and the interest rate on debt is also on average decreasing in wealth (see Figure A.3). Finally, capital gain returns are always

(a) pre-1980 (b) post-1980

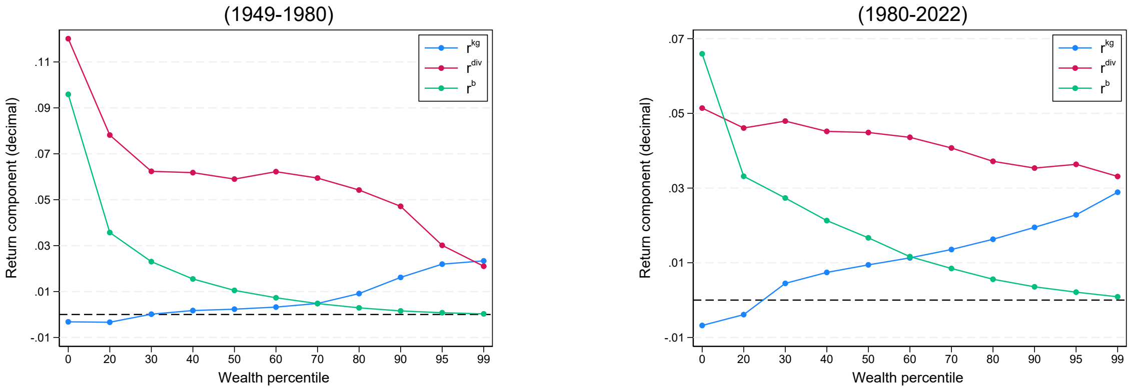

Figure 6: Components of return on wealth (dividend return (r^{div}), capital gain return (r^{kg}), and cost of debt (r^{b})) for two periods of 1949-1980 and 1980-2022 using the SCF+ data set. As is shown, the dividend return and the cost of debt are on average decreasing in wealth, while the capital gain return is increasing in wealth for both sub-periods.

	pre-1980	post-1980
Stocks (only dividends)	0.51	0.49
Stocks (with share repurchases)	0.44	0.77
Businesses	0.41	0.58

Table 3: This table reports the payout ratio for publicly traded stocks and for businesses in general. Because publicly traded stocks, especially after 1980, have paid dividends indirectly through share repurchases and buybacks, they are reported with two payout ratios. For details on the measurement of payout ratio, see Appendix A.7.

increasing wealth, because of the same reason that dividend returns are decreasing in wealth: the wealthy have assets that pay lower dividends and pay capital gains to the owner. (See Appendix A.2 for more graphs on components of returns.)

2.7 Households' portfolios

For explaining the observed changes in the relative return of the top 10 percent relative to the bottom 90 percent, the first thing to check is to see if portfolio shares have changed before and after 1980. For this aim, I plot the portfolio share graphs for the asset and liability sides of households' balance sheets before and after 1980.

On the assets side, Figures 7 and 8 plot the portfolio shares of households for before and after 1980 across wealth percentiles. As one can see in these figures, the general pattern is that the very rich households tend to have more public stocks and private businesses in their portfolio, as for the middle class, it is mostly housing which attracts them, and for the poor, mostly their vehicle and liquid assets make up the majority of their asset portfolio. Note that there are many subtle differences; however, this pattern is seen in both post-1980 and pre-1980.

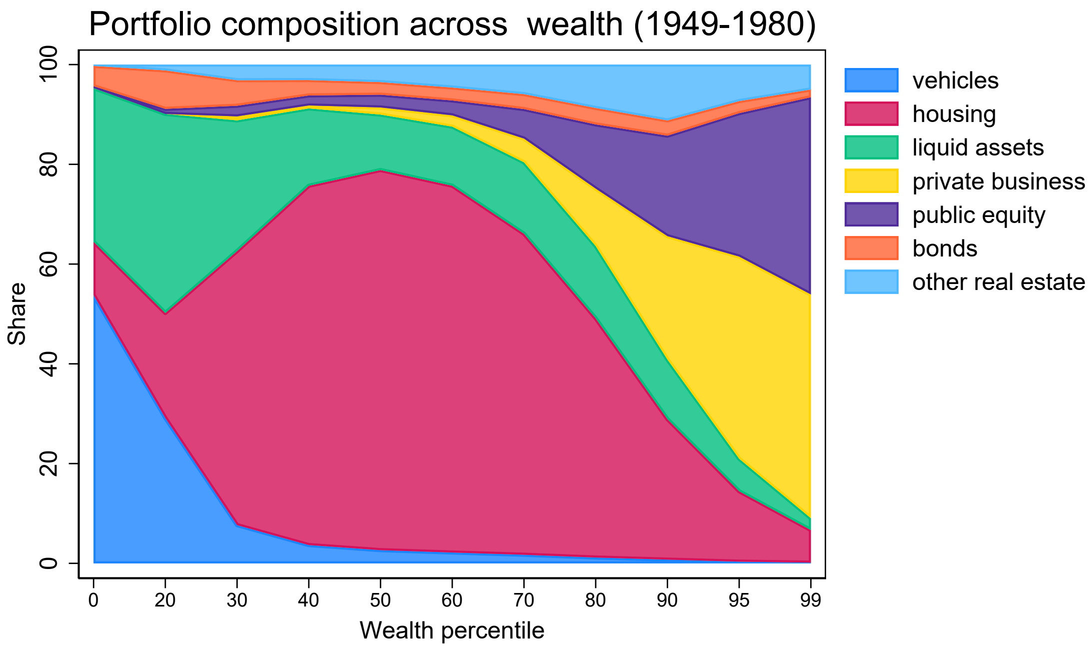

Figure 7: Average portfolio share of different asset classes for each wealth percentile for the years 1949-1980 using SCF+. Households in lower percentiles mostly have liquid assets and vehicles, while the middle class have housing as their main asset, and the rich hold more of equity: both public equity (stocks) and private businesses.

On the liabilities side, Figures 9 and 10 plot the leverage shares of households and also the decomposition of their debt for before and after 1980 across wealth percentiles. As we can see in these figures, again, the general pattern of the liabilities side across wealth is quite similar before and after 1980.

In this section, I have documented the changes in the scale-dependence of returns. I also show that, despite the small differences, the general pattern in the portfolio share across the wealth has not changed. But, how can we explain the changes in scale-dependent returns? Next section will deal with this question.

3 Interest rate and returns

In this section, after refreshing the fact about trends in the interest rate, I will review the concept of the duration of an asset and how it relates to interest rate exposure and changes in returns. Then, I will explain the empirical evidence on the heterogeneity of the duration of asset holdings of households across the wealth distribution. And finally, I will argue how changes in the interest rate can explain the changes in scale-dependent returns.

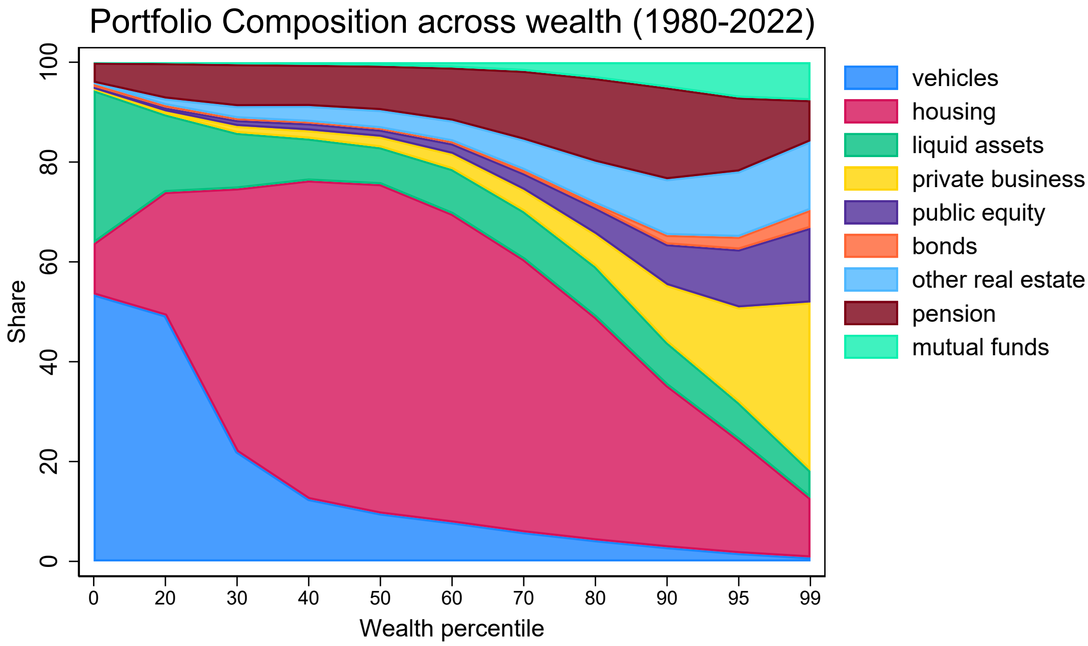

Figure 8: Average portfolio share of different asset classes for each wealth percentile for the years 1980-2022 using SCF+. Households in lower percentiles mostly have liquid assets and vehicles, while the middle class have housing as their main asset, and the rich hold more of equity: stocks (directly in public equity and indirectly through mutual funds) and private businesses.

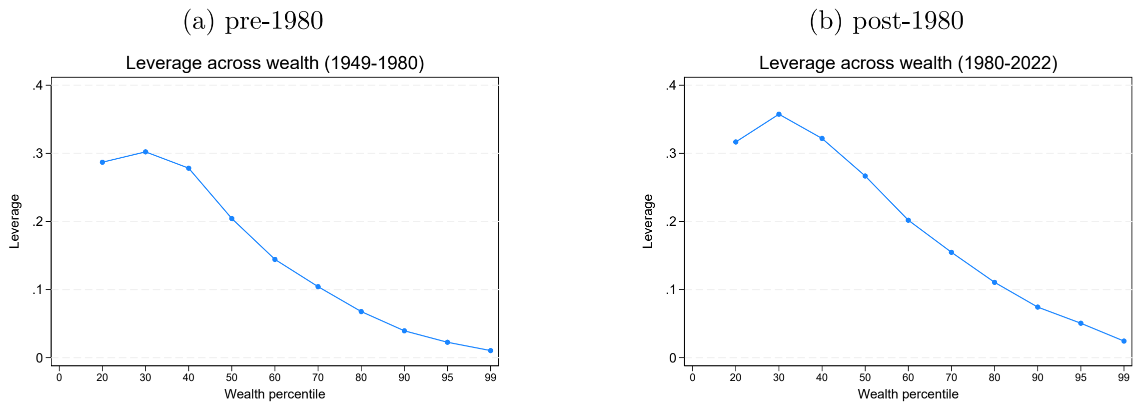

Figure 9: Average leverage of different wealth percentiles for the periods 1949-1980 and 1980-2022 using SCF+. Leverage is defined as total debt divided by gross wealth. Note that the leverage of the lowest group (households between 0 to 20th percentile) is not plotted for a better exposition of the rest of the graph. The average leverage of the lowest group for the pre-1980 period is 7.1 and for the post-1980 period is 5.6.

(a) pre-1980 (b) post-1980

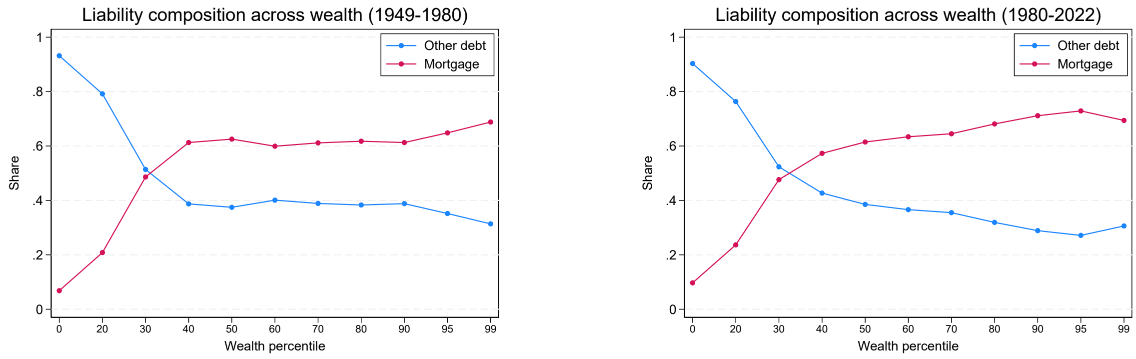

Figure 10: Average decomposition of households' total debt to mortgage and other debt of different wealth percentiles for the periods 1949-1980 and 1980-2022 using SCF+. As is shown, in both sub-periods, the richer households tend to have more mortgage debt and less from other debt (including vehicle debt, credit card debt, student loans, etc.).

3.1 Interest rate trends

An important economic variable that relates to changes in the scale-dependency of returns is the long-term interest rate. Changes in the long-term interest rate effects the valuation of assets, especially those assets that most of their present value comes from distant future cash flows, like stocks and private businesses. Changes in the long-term interest rate will significantly change the valuation of these assets, and hence will affect their capital gains.

Figure 11 plots the dynamics of the interest rate. As shown in this figure, the real interest rate was on an increasing trend from 1950 until 1980, when it started to show a decrease until 2020.

In the rest of this section, I will explain how these changes in the interest rate affect asset returns through revaluations.

3.2 Duration and interest rate exposure

The easiest setting to study how changes in the interest rate affect asset prices and hence returns, is the deterministic case where the interest rate is deterministic and is the same at all maturities (flat term-structure), and assets have deterministic cash-flows. Note that this is a partial equilibrium setting, and demand and supply do not play any role in prices (returns) as they are given.

[12] As long-term real interest rate is a forward looking variable, one needs to know the expected inflation to be able to calculate real rate from nominal rate and data on expected inflation do not exist at such a long horizon. Greenwald et al. (2023) build an empirical asset pricing model to back out the real interest rate based on that. Hall, Payne, Sargent, and Szőke (2019) use another model and find similar results.

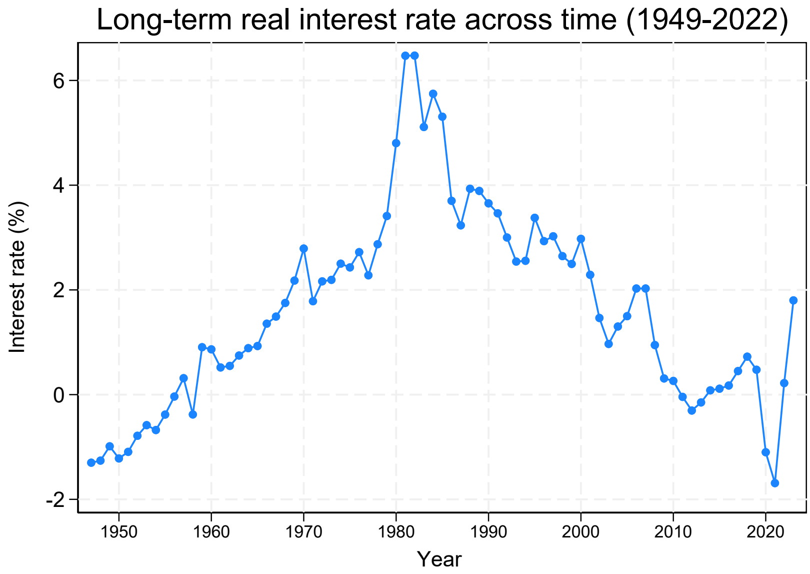

Figure 11: 5-year real interest rate based on the methodology in Greenwald et al. (2023).[12] As is shown in this figure, the real interest rate was on average increasing from 1950 until 1980, and decreasing from 1980 until 2022.

To see how changes in the interest rate effects asset prices, it is useful to think of the concept of duration. The Macually duration of an asset that generates cash flows in the future is generally defined as the weighted average of the times until those cash flows are received. More formally, Macaulay duration (Macaulay (1938)) in the deterministic setting described above for an asset with future cash flows $\{x_t\}$ is defined as:

$$D := \frac{\sum_{t=0}^{\infty} t \times R^{-t} x_t}{P_0}. \tag{6}$$

Where $R = 1 + r$ and r is the annualized discount rate and is constant at all maturities. Note that

$$P_0 = \sum_{t=0}^{\infty} R^{-t} x_t. \tag{7}$$

Duration is closely related to how sensitive an asset's price is to changes in interest rates. In different settings, various measures of price sensitivity to interest rate changes can be defined, although they are quite similar. For the simple deterministic environment, one can show that:

$$\frac{\partial \log P_0}{\partial \log R} = -D \tag{8}$$

Proof.

$$\frac{\partial P_0}{\partial R} = \sum_{t=0}^{\infty} -t \times R^{-t-1} x_t = \frac{-1}{R} \sum_{t=0}^{\infty} t \times R^{-t} x_t = \frac{P_0}{R} \times -D \tag{9}$$

which gives us:

$$\frac{\partial P_0}{\partial R} \times \frac{R}{P_0} = \frac{\partial \log P_0}{\partial \log R} = -D \tag{10}$$

□

The notion of duration and its relation to interest rate exposure can be extended to more complicated settings. I follow Catherine et al. (2023), in which the interest rate exposure of bonds is extended to the case where the interest rate follows an AR(1) process. Denote the short-term real interest rate with r_{ft}, then:

$$r_{f,t+1} = (1-\varphi)\bar{r}_f + \varphi r_{ft} + \sigma_r \epsilon_{r,t+1} \tag{11}$$

is the evolution of short-term real interest rate across time with a mean reversion coefficient of φ and a standard deviation of σ_r, where $\epsilon_{r,t+1} \sim N(0,1)$.

If we assume the expectation hypothesis holds, then the return on a zero-coupon bond with maturity n, will be:

$$r_{n,t+1} = r_{ft} + \mu_n - \sigma_n \epsilon_{r,t+1}, \tag{12}$$

where $\sigma_n = \frac{1-\varphi^{n-1}}{1-\varphi}\sigma_r$ and μ_n is the term premium of the n-period bond. I define interest rate exposure (IRE) as the percentage change in the price of an asset caused by an unexpected change in the interest rate:

$$IRE\left(P_t\right) := -\frac{\log P_{t+1} - \mathbb{E}_t \log P_{t+1}}{r_{f,t+1} - \mathbb{E}_t r_{f,t+1}} \tag{13}$$

It follows that the interest rate exposure of a long-term bond with maturity n is

$$IRE\left(P_{nt}\right) = \frac{1-\varphi^{n-1}}{1-\varphi} \tag{14}$$

Note that this is a correction of the duration for the expected changes in interest rate. In fact, if $\phi \to 1$ then $IRE\left(P_{nt}\right) \to n-1$, which is similar to the deterministic case of the Macually duration.

3.3 Interest rate exposure of assets

There are different ways to define and measure the duration of an asset. Greenwald et al. (2023) provides measures of duration for different asset classes using different methods. In this paper, I mostly follow their approach. Table 4 reports interest rate exposure of different asset classes. Appendix A.7 discusses other methods as robustness checks. As seen in this table, assets that are of interest to wealthy households have a high interest rate exposure.

Asset/Liability class	Interest rate exposure
Real Estate	9.8
Public Equity	20.4
Private Business (average)	16.3
Private Business (Corporations)	24.3
Private Business (Non-Corporations)	16.3
Vehicle	3.4
Bonds	5.0
Liquid Assets	0.3
Mortgage Debt	8.2
Other Debt	2.6

Table 4: Asset/liability class interest rate exposure following Catherine et al. (2023). Asset classes that their cash flows realize on average further in the future from now have a longer duration and hence a higher interest rate exposure.

3.4 Households' interest rate exposure

I follow an approach similar to those and extend their results to the pre-1980 period. There are some differences in the results, which could be because of less detailed data availability (especially for calculating the duration of liabilities). In doing so, I first measure interest rate exposure for each asset class similar to Catherine et al. (2023), and then for each household with a portfolio of different asset classes indexed by j, I calculate the interest rate exposure as:

$$IRE_t := \sum_j \omega_t(j) IRE_t(j)$$

Where $\omega_t(j)$ is the portfolio weight of each asset (liability) class j.

Figure 12 plots the interest rate exposure of households in wealth percentiles for two periods before 1980 and after 1980. As is depicted in this figure, as richer households hold assets with longer durations, they are highly exposed to interest rate changes.

3.5 Interest rate exposure and returns

If HHs have different durations, changes in interest rates affect them differently. Table 5 uses the formula $\Delta R' = -\Delta IRE \times \Delta r_t^{long}$ to see how much difference in returns this different exposure can generate. As shown in this table, during the pre-1980 period, the difference in interest rate exposure explains almost all of the difference between

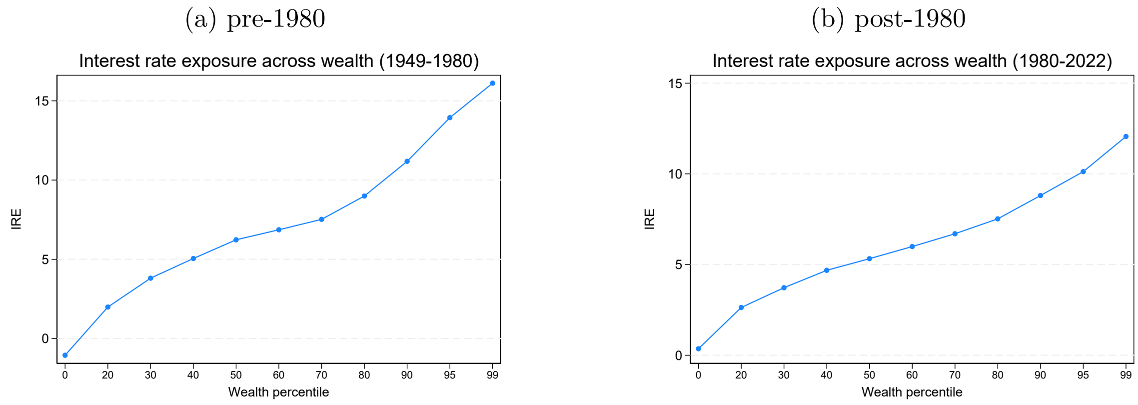

Figure 12: Interest rate exposure of households across the wealth distribution for two periods of 1949-1980 and 1980-2022 using the SCF+ data set. As is shown in these graphs, interest rate exposure is increasing in wealth and the richest households always hold more interest rate exposure.

the top 10 percent and the bottom 90 percent. Note that there are other reasons like different risk premiums that the top 10% earn due to the other risks that they take (like the cash flow risk, liquidity risk, etc.)

	pre-1980	post-1980
$R_{\text{top 10}} - R_{\text{bottom 90}}$	-1.4%	+1.7%
$IRE_{\text{top 10}} - IRE_{\text{bottom 90}}$	6.62	4.25
$R'_{\text{top 10}} - R'_{\text{bottom 90}}$	-1.46%	+0.47%

Table 5: Difference in interest rate exposure and actual returns vs. the implied difference in returns due to changes in the interest rate using the formula $\Delta R'_i = -\Delta IRE_i \times \Delta r_t^{long}$.

3.6 Recent increase in interest rates

As it is seen in Figure 11, interest rates had a sharpe increase after 2020 (Post-COVID). Although, the sample is small (basically one year until now is available from the SCF), but one can already see the prediction of this paper's argument in the distribution of returns in the post-COVID sample. One can also see in Figure 1 that inequality has started to decline.

Figure 13 plots the returns in wealth of households across wealth.

4 Return on human capital

The other asset that is of high importance for households is their human capital, which I did not discuss so far. Human capital is a special asset as it is non-tradable. This

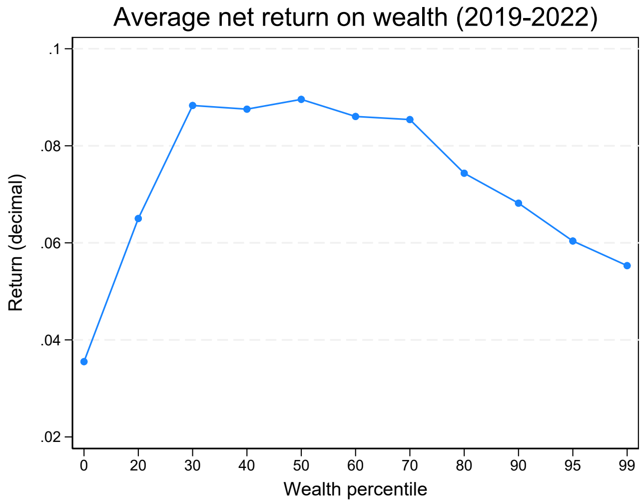

Figure 13: Post-COVID distribution of returns using SCF. As this paper predicts, after the sharp increase in interest rates after COVID, the cross-section of returns of households looks similar to the pre-1980 period, and the top-10 percent of the wealth distribution have less returns.

non-tradibility makes it difficult to measure its price and true return. However, labor income growth can be used as a proxy for measuring the return to human capital (see, for example, Campbell (1996) and Lustig and Van Nieuwerburgh (2006)) . Figure 14 plots labor income growth for the pre-1980, post-1980, and the long-term sample.

Figure 14 shows that, the growth rate of labor income as a proxy for human capital return is always increasing in wealth. It also shows that, in the pre-1980 period, the growth rate of labor income was higher than its long-term average and in the post-1980 period, lower than its long-term average.

4.1 Importance of labor income

Figure 15 depicts the labor income share in total (cash flow) income. As shown, labor income decreases in wealth and is the main source of income for households in the lower percentiles. However, it is still a large portion of total (cash flow) income for the very rich (almost 40 percent).

4.2 Growth exposure of labor income

In the previous section, the notion of interest rate exposure for returns on wealth was introduced, and we saw how changes in interest rates affect different households differently. We saw that the rich households have a highly negative interest rate exposure in their returns on wealth. In this section, the exposure of labor income to GDP growth shocks for households in different wealth percentiles is measured.

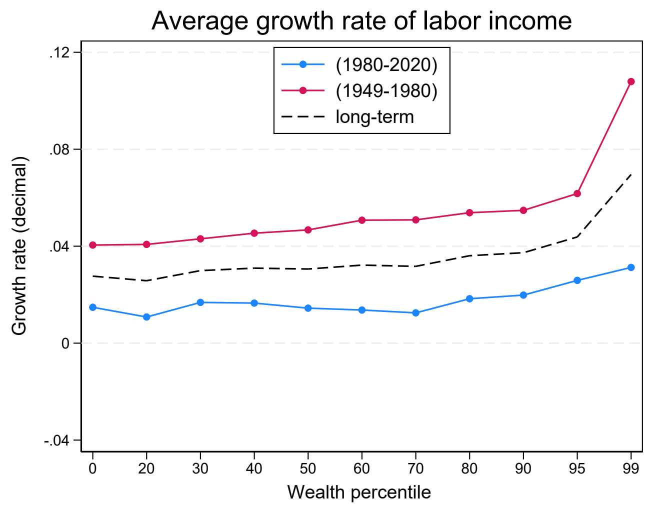

Figure 14: Growth rate of labor income in terms of changes in the logarithm of labor income for pre-1980, post-1980, and long-term samples using the SCF+ data set. Growth rate of labor income in both periods has been increasing in wealth.

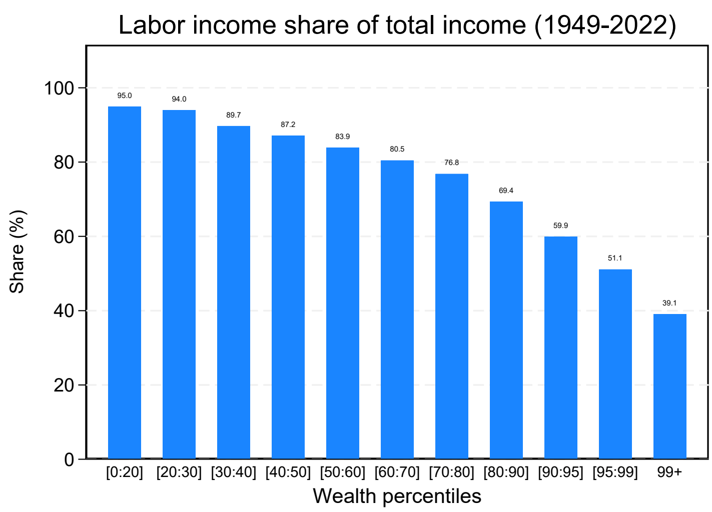

Figure 15: Share of labor income in total cash flow income using the SCF+ data set. Labor income plays a bigger role for households in lower deciles and a smaller role for the higher deciles as a source of cash flow income.

Denote the logarithm of labor income for each percentile of households at period t with l_{it}. As in Guvenen et al. (2017), the GDP beta of labor income is defined as the sensitivity of growth rate of labor income to growth rate of GDP. Simply defined in a regression:

$$\Delta l_{it} = \alpha_i + \beta_{GDP,i} \Delta \log(GDP_t) + \zeta_{i,t} \tag{15}$$

where $\beta_{GDP,i}$ is the called the GDP beta of labor income for households in percentile i.

Figure 16 plots the GDP beta of households' labor income across the wealth percentiles.[13] As is shown in this figure, the rich households are more exposed to GDP growth shocks.

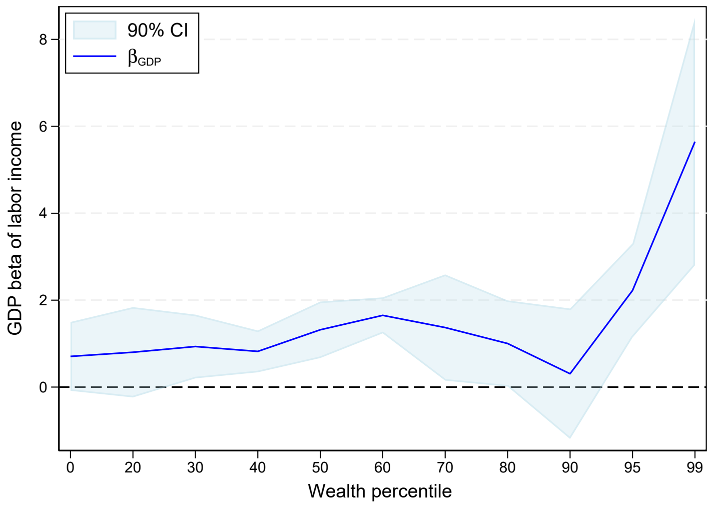

Figure 16: GDP beta of labor income across wealth percentiles. The growth shows that labor income of the rich households (top 5 percent) is more sensitive to growth shocks in GDP, than that of the non-rich.

So far, we saw the difference between returns on wealth and the human capital of households in different percentiles of wealth and we also saw how their are exposed to growth shocks. The interest rate exposure for wealth and GDP exposure of human capital. We also saw how it had changed across time, especially over the high-growth period of pre-1980 and the low-growth period of post-1980. Now, it is time to study

[13] Appendix A.8 plots the GDP beta of households' earnings growth across the earning percentiles as it is done originally in Guvenen et al. (2017). The reason why it looks different when it is plotted across wealth than when plotted across earnings is that some of the very rich households with very high *betas* and high *growth* rates of labor income have low *levels* of labor income, as they get compensated through stocks and options that if they do not cash it during the year that they receive it, will not be in the SCF+ data. So, they will be detected as in lower percentiles if the graph is plotted across earnings percentiles (labor income). However, if we plot it across wealth percentiles, they are in the right place.

the welfare and hedging implications that we can learn from this data.

5 A theory of hedging growth

In this section, I will address the potential welfare consequences of the observed difference in returns. I present a model to rationalize the findings that wealthy individuals exhibit a desire for high exposure to interest rate risk, and how this fact, combined with the realization of interest rate risk, has led to the observed changes in scale-dependence of returns.

I first provide empirical evidence on the cyclicality of labor income risk (returns on human capital) and how this risk is more cyclical for the super-rich. Then, I argue that return on long-duration assets is countercyclical.

Then I will put this evidence into a portfolio choice model with risky human capital and establish the result that richer people choose a higher duration because of their higher cyclicality of labor income.

5.1 All returns in one graph

Figure 17 plots the growth rate of labor income and the return on wealth next to each other. As shown in these figures, rich households are quite hedged to the growth shocks: while their labor income earnings is having a high realization (pre-1980) their returns on wealth are low and while their labor income earnings is having having a low realization (post-1980) their returns on wealth is high. This hedging helps them smooth their consumption possibilities.

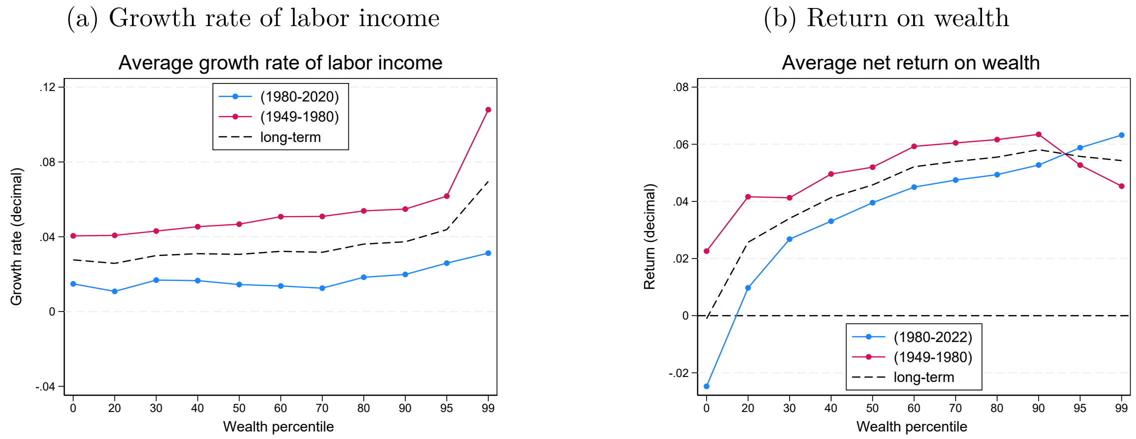

Figure 17: The figures plot the growth rate of labor income (left) and net return on wealth (right) using the SCF+ data for pre-1980 period (in red), post-1980 (in blue), and long-term (dashed).

5.2 Model

In this section, I connect the choice of interest rate exposure to labor income risk. To get the idea, based on the Campbell and Viceira (2002) framework, I provide a simple model in which the households should choose their optimal interest rate exposure.

5.3 Environment

There are two types of households: rich and non-rich. Each household has a time-separable CRRA utility. During their life, they can have two states: employment or retirement.[14] During employment, they receive some labor income, and should decide on their consumption and portfolio choice for their savings. During retirement, they have zero labor income, but still save and consume. Furthermore, they face a probability of death [15].

Each household during its life has access to two financial assets:

- Short-term bond (risk-free, 1-period duration)
- Long-term bond (n-period duration)

The assets are designed in a way so the household can choose any duration through its portfolio choice between 1 and n. The Risk-free rate, which will be the return on the short-term bond, follows an AR(1) process:

$$r_{f,t+1} = (1 - \varphi)\bar{r}_f + \varphi r_{ft} + \sigma_r \epsilon_{r,t+1} \tag{16}$$

where φ is the mean reversion coefficient, σ_r is the standard deviation of interest rate, and $\epsilon_{r,t+1} \sim N(0, 1)$ is the unexpected shock.

For the pricing of the long-term bond, I assume the expectation hypothesis holds, which pins down the term structure and the return of the long-term bond[16]:

$$r_{n,t+1} = r_{ft} + \mu_n - \sigma_n \epsilon_{r,t+1}, \tag{17}$$

where, $\sigma_n = \frac{1-\varphi^{n-1}}{1-\varphi}\sigma_r$ and μ_n is the term premium of the n-period bond.

The other feature of the model is that household's labor income is risky. Following

[14]Having the retirement state helps with the tractability of the solution as it induces a nonzero probability of zero labor income, which forces households to avoid zero wealth.

[15]This assumption helps with having the model stationary.

[16]See Catherine et al. (2023) Appendix B.2 for the proof. Also, note that is shouldn't be mistaken with long-term ex ante interest rate. This is just a one-period expected return on an n-period bond and is quite different from the n-period ex ante yield or the expected long-term interest rate.

Viceira (2001), I assume the income process:

$$L_{t+1} = L_t \exp(g_i + \beta_i \epsilon_{l,t+1}) \quad ; \quad i \in \{\text{rich, nonrich}\} \tag{18}$$

Or in the log format:

$$l_{t+1} = l_t + g_i + \epsilon_{l,t+1} \quad ; \quad \epsilon_{l,t+1} \sim NIID\left(0, \sigma_l^2\right) \tag{19}$$

where g_i is the growth rate of labor income and $\epsilon_{l,t+1}$ is the unexpected shocks to the logarithm of labor income and β_i is their exposure to the aggregate shock (similar to the GDP beta of labor income). Furthermore, I assume that the shock to the labor income is correlated with the shock to the interest rate:

$$\text{Cov}_t\left(\epsilon_{l,t+1}, \epsilon_{r,t+1}\right) = \sigma_{rl} > 0 \tag{20}$$

The intuition for this assumption is through the cyclicality of labor income and the cyclicality of the interest rate, which makes them correlated (both with GDP). Table 6 summarizes the correlation of the growth rate of key economic variables.

Table 6: Correlation of key growth variables

	GDP	r^{short}	r^{long}	Labor income
GDP	1.00			
r^{short}	0.54***	1.00		
r^{long}	0.34***	0.72***	1.00	
Labor income	0.35***	0.12*	0.02*	1.00

Notes: The table shows pairwise correlations. *, **, *** denote significance at the 10%, 5%, and 1% levels, respectively. As the table shows, growth rates of the key variables of the economy are positively correlated.

5.4 Duration choice

Households optimization problem during employment is:

$$\max_{\{C_t, \alpha_t\}_{t=0}^{\infty}} \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \frac{C_t^{1-\gamma}}{1-\gamma} \tag{21}$$

$$s.t. \quad W_{t+1} = \left(W_t + L_t - C_t\right) R_{w,t+1} \tag{22}$$

where, $R_{w,t+1} = \alpha_{it}\left(R_{n,t+1} - R_f\right) + R_f$.

Proposition 1 The approximate portfolio choice of the employed household in this model will be

$$\alpha_i^e = \underbrace{\frac{1}{\gamma b_1}\frac{\mu_n + \frac{1}{2}\sigma_n^2}{\sigma_n^2}}_{\text{myopic demand}} + \underbrace{(1-\frac{1}{\gamma})\frac{\bar{b}_2}{\gamma b_1}\frac{\sigma_r}{\sigma_n}}_{\text{hedging demand}} + \underbrace{\frac{(1-b_1)}{\gamma b_1}\frac{\sigma_{rl}\beta_i}{\sigma_n^2}}_{\substack{\text{human capital} \\ \text{substitution}}} \tag{23}$$

where $0 < b_1 < 1$ and $\bar{b}_2$ are constants defined in the appendix.

Proof. See appendix B.

As we can see from this equation, choice of duration has three components. The first one is myopic demand, which comes from the fact that the long-term asset has an expected return premium and the risk-averse household will demand this asset. The second term, the hedging demand, is coming from the fact that the return on long-term bonds is time-variyng and so is the investment opportunity set. Any househould (with $\gamma \neq 1$) will try to take advantage of this change through his or her hedging demand. The last component, which is of key importance in this paper, is the human capital substitution demand.

Human capital substitution term in the duration choice is telling us that the household will take into account his or her human capital asset (which is a non-tradable asset) when choosing for duration. Especially, if his labor income is in a way that is very much correlated whith the return on the short therm asset, he will choose a higher duration to hedge that risk.This term

$$\frac{(1-b_1)}{\gamma b_1}\frac{\sigma_{rl}\beta_i}{\sigma_n^2} \tag{24}$$

is proportional to the regression hedge ratio of labor income ($\frac{\sigma_{rl}\beta_i}{\sigma_r^2}$), which is the slope in the regression of labor income shocks onto unexpected interest rate shocks.[17]

5.5 Results

The approximate analytical solution provides a great tool to understand the implications of differences in exposure of labor income to growth shocks on duration choice which has not been studied so far.

Result I: $\beta_{rich} > \beta_{nonrich} \Rightarrow \alpha^e_{rich} > \alpha^e_{nonrich}$

The first result states that if two agents are only different in their exposure of labor income to aggregate shocks, then the one with a higher exposure will choose a higher duration.

[17]Note that since in this model, log labor income is an AR(1) process with fully persistent shock (random walk), for the empirical measurement of σ_{rl} we simply have $Cov(u_t, \epsilon_t) = Cov(\Delta l_t, \Delta r_t) = \sigma_{rl}$.

Result II: $\beta_{rich} > \beta_{nonrich} \quad (\mu_n = 0)$*:*

- $\epsilon_{r,t+1} > 0 \Rightarrow Return_{rich} < Return_{nonrich}$
- $\epsilon_{r,t+1} < 0 \Rightarrow Return_{rich} > Return_{nonrich}$

The second result states that if there is no term premium ($\mu_n = 0$), then an unexpected positive shock to interest rate makes the rich have less returns than the non-rich.

Result III: $\beta_{rich} > \beta_{nonrich} \quad (\mu_n > 0)$*:*

- $\epsilon_{r,t+1} > \delta \Rightarrow Return_{rich} < Return_{nonrich}$
- $\epsilon_{r,t+1} < \delta \Rightarrow Return_{rich} > Return_{nonrich}$

The third result states that if there is a positive term premium ($\mu_n > 0$), then an unexpected positive shock to interest rate that is bigger than a threshold, makes the rich have lower returns than the non-rich.

These results show that, even in a simple frictionless model with just the features of correlated growth shocks and different exposures to labor shocks, we can get the different returns between the rich and the non-rich.

6 Conclusions

In this paper, I have studied how the scale-dependent property of returns on wealth of households has changed over time. I have used a long-horizon micro data set called SCF+, going back until 1949, to measure households' returns on wealth to study how rich and non-rich households' returns differ from each other and how this difference has changed over time. I have shown that before 1980, households in the top 10 percent of the wealth distribution had lower returns on their wealth than the rest of the population. However, this pattern reversed after 1980. I have shown that during the pre-1980 period, the rich had, on average, lower returns than the non-rich.

I have argued that this reversal in the scale dependency of returns is explained by the changes in the long-term real interest rate. As richer households choose assets with longer durations, they have a greater exposure to changes in interest rates. As the interest rate increases, like in the pre-1980 period, their assets are revaluated to a lower value, and this lowers their returns. If the long-term interest rate decreases, as it did in the post-1980 period, it revaluates their assets to a higher value, and the rich will have higher returns.

I have also separately looked at the human capital asset. I have proxied the return to human capital with the growth rate of labor income and shown that in both periods the growth rate of labor income is, on average, increasing in wealth.

Finally, I have built a model to explain the fact that richer people optimally choose a longer duration. I have shown that, as the labor income of the very rich is highly cyclical, and the long-duration assets are counter cyclical, they optimally hedge their human capital by investing in long-duration assets.

These findings provide novel insights into differences in households' portfolio characteristics and returns, and how these differences play a role in the dynamics of inequality and also welfare. In future research, one might explore the quantitative welfare consequences of changes in returns and also their implications for tax system design.

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Appendix

This appendix includes two sections. In Appendix A, I will cover details about the empirical measurements discussed in the paper. In Appendix B, I will prove the solutions for the model.

A Data and measurements

In this section of the appendix, I will first explain the measurements of returns and the the data I use for that. Then, I will explain the measurement of duration.

A.1 Measuring returns: equivalent formulas

The main definition of returns that I use, following Fagereng et al. (2020), is:

$$r_{it} = \frac{y_{it}^{div} + y_{it}^{kg} - y_{it}^{b}}{w_{it} + \dfrac{f_{it}}{2}}. \tag{A.1}$$

This formula is based on the wealth at the beginning of the period t. However, if one knows only the wealth at the end of the period, it is still possible to measure the returns by using the identity $w_{i,t+1} = w_{it} + y_{it}^{kg} + f_{it}$, where $w_{i,t+1}$ is the wealth at the end of period t (or at the beginning of period $t+1$). Applying this identity with the above equation for returns, one can have:

$$r_{it} = \frac{y_{it}^{div} + y_{it}^{kg} - y_{it}^{b}}{w_{i,t+1} - \dfrac{f_{it}}{2} - y_{it}^{kg}}, \tag{A.2}$$

for the case where we observe the end-of-period wealth.

For a panel data set, similarly, by applying the same identity, one can get:

$$r_{it} = \frac{y_{it}^{div} + y_{it}^{kg} - y_{it}^{b}}{\dfrac{w_{it} + w_{it+1}}{2} - \dfrac{y_{it}^{kg}}{2}}. \tag{A.3}$$

The advantage of this equation is that it does not need the explicit knowledge of the flows f_{it}.

All these three equations for measuring returns are equivalent, and one can use any of them depending on the available data.

A.2 Decomposition of returns

To analyze which components are serving the scale-dependency of returns more, it is a good idea to decompose the returns into their components:

a) Dividend return

$$r_{it}^{div} = \frac{y_{it}^{div}}{w_{it} + \frac{f_{it}}{2}} \tag{A.4}$$

b) Capital gain return

$$r_{it}^{kg} = \frac{y_{it}^{kg}}{w_{it} + \frac{f_{it}}{2}} \tag{A.5}$$

c) Cost of debt

$$r_{it}^{b} = \frac{y_{it}^{b}}{w_{it} + \frac{f_{it}}{2}} \tag{A.6}$$

Where one can write the $r_{it} = r_{it}^{div} + r_{it}^{kg} - r_{it}^{b}$. Figure A.1 plots these components for the two sub-periods in one graph.

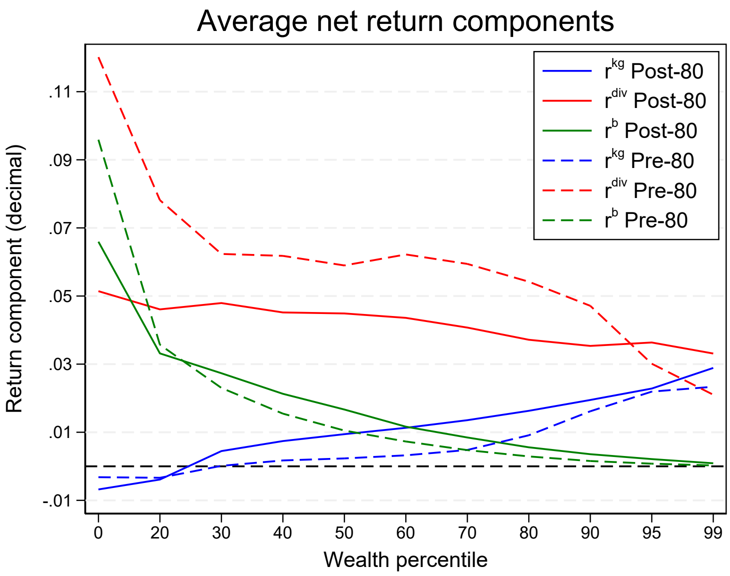

Figure A.1: Components of net return on wealth (dividend return (r^{div}), capital gain return (r^{kg}), and cost of debt (r^{b})) for two periods of 1949-1980 and 1980-2022 using the SCF+ data set.

Another useful decomposition is gross return and cost of debt:

a) Gross return

$$r_{it}^{gross} = \frac{y_{it}^{div} + y_{it}^{kg}}{w_{it} + \dfrac{f_{it}}{2}} \tag{A.7}$$

b) Cost of debt

$$r_{it}^{b} = \frac{y_{it}^{b}}{w_{it} + \dfrac{f_{it}}{2}} \tag{A.8}$$

Where one can write the $r_{it} = r_{it}^{gross} - r_{it}^{b}$. Figure A.2 plots these components for the two sub-periods in one graph.

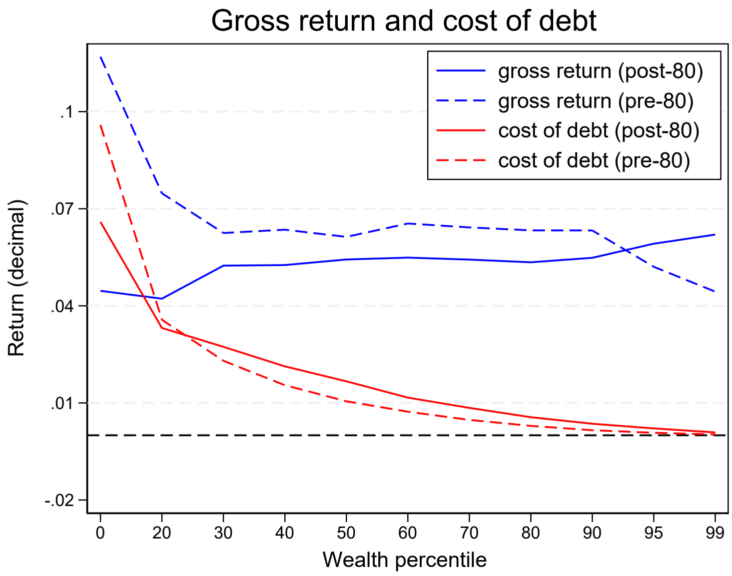

Figure A.2: Gross return and cost of debt for two periods of 1949-1980 and 1980-2022 using SCF+ data set.

A.3 Main data set: SCF+

In this section, I explain my main source of data, the SCF+ data set. SCF+, introduced in Kuhn et al. (2020), consists of two data sets: the modern SCF and the historical waves of SCF. The modern SCF, or Survey of Consumer Finances is a triennial cross-sectional household survey of U.S. households created by the Board of Governors of the Federal Reserve Board from 1983 to 2022. It covers many useful variables like asset holdings from different asset classes and the income generated by those. It covers many variables, such as asset holdings from different asset classes and the income generated by them. The old SCF is similar to the modern SCF, but with some differences like the variables included in the survey, sampling scheme, etc. Furthermore, it is done annually or biannually for most of the period, but it sometimes has larger gaps.

Kuhn et al. (2020) developed SCF+ by reharmonizing the historical waves to have them comparable to modern SCF. For construction of SCF+, old surveys and other databases are used to impute the missing variables, harmonize, and re-weight the historical data to create this extension in a way that represents US households and is comparable with the modern SCF. Kuhn et al. (2020) show that the SCF+, is compatible with the other micro data that we have, like the IRS data in the U.S.

A.4 Other variables

In this section, I will explain all other data sets, approximation and assumptions rather than the SCF+ that I have used in the paper.

Owner-occupied housing rent I use the time series of housing yield from Jordà-Schularick-Taylor Macrohistory Database (Jord'a, Knoll, Kuvshinov, Schularick, and Taylor (2019)). I also extend it for the post-2020 period, using the dynamics of the housing price to rent ratio index.

Vehicles dividend and capital returns: I assume a 0.05 rate of service flow (like dividend return) and -0.05 rate of capital gains (dues to depreciation).

Interest rate on households' debt For early years in the sample where there is no data on individual interest rate on each household's debt, I use the average interest rate for each debt class (mortgage and non-mortgage, and smaller classes if the data on a finer debt class is available for that year). For doing so, I use different data sets from FRED:

- Monetary interest paid: Households: Owner-occupied housing (W498RC1A027NBEA),

- Monetary interest paid: Households (W292RC1A027NBEA),

- Households and Nonprofit Organizations; Total Mortgages; Liability, Level (HNOTMLQ027S),

- Households and Nonprofit Organizations; Total Liabilities, Level (TLBSHNO).

The rates on FRED for different debt classes are for newly issued debts, but my methodology is the average of all the debts, new or old.

Figure A.3 plots the average interest rate on debt for households.

Average asset-class capital gains For stocks (public equity), real estate, businesses (private equity), bonds, and mutual funds, I approximate the average capital gains. For stocks and real estate, I use changes in price indeces, for businesses, I use data from flow of funds to back out capital gains. Specifically, I use the series available on FRED:

- Households and Nonprofit Organizations; Corporate Equities; Asset (Level and Transactions: HNOCEAQ027S and HNOCESQ027S)

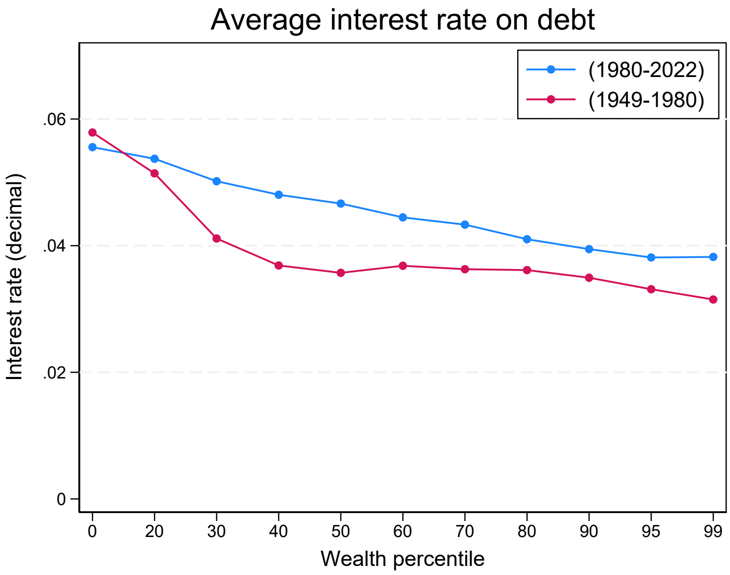

Figure A.3: Debt interest rate

- Households and Nonprofit Organizations; Proprietors' Equity in Noncorporate Business (Level and Transactions: HNOPEBA027N and HNOPEUQ027S). [18] For bonds, I assume and average duration of 5 years and use Duration formula to back out the capital gains, and finally for mutual funds, I use the average of stocks and bonds capital gains with respective weights of 60 and 40 percent. Figure A.4 plots average asset class capital gain returns for major asset classes across time.

Return on defined contribution pension For measuring the return on defined contribution pension, I use the balance sheet data of the private defined contribution pension funds available on FRED to get their average asset class portfolio shares and then use asset class returns to estimate their returns. The used series are:

- Private Defined Contribution Pension Funds; Total Financial Assets, Level (BOGZ1FL574090055A)

- Private Defined Contribution Pension Funds; Corporate Equities Held Directly and Indirectly Through Mutual Funds; Asset, Market Value Levels (BOGZ1LM573064175Q)

- Private Defined Contribution Pension Funds; Debt Securities Held Indirectly Through Mutual Funds; Asset, Level (BOGZ1FL573064223Q)

- Private Defined Benefit Pension Funds; Real Estate, Level (BOGZ1FL575035045A)

For asset class returns, I use Damodaran's data. For other asset classes, I use the baseline risk-free return (3-month T-bill), and for debt securities I use a 10-year

[18] I use the formula $r^{KG} = \dfrac{w_{t+1} - w_t - f_t}{w_t + \dfrac{f_t}{2}} = \dfrac{w_{t+1} - \dfrac{f_t}{2}}{w_t + \dfrac{f_t}{2}} - 1$, where w_t is the level of holdings at time t and f_t is the flow (transactions) at time t

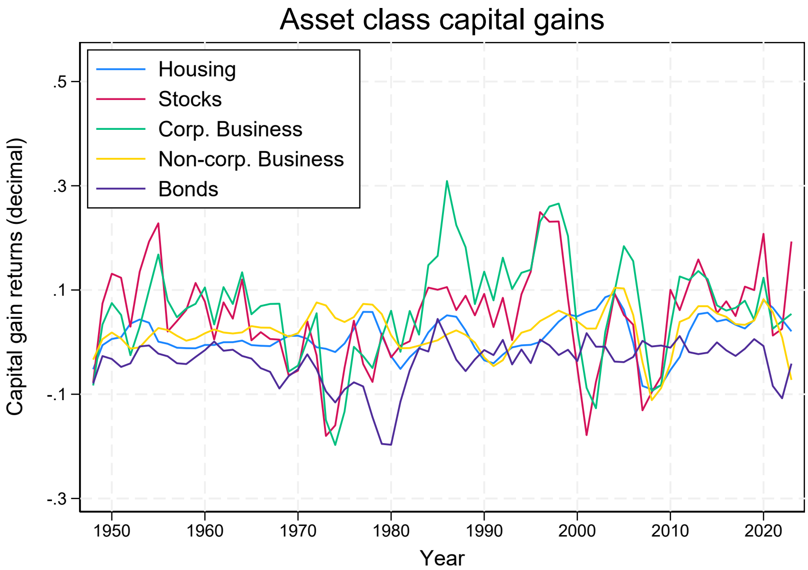

Figure A.4: Asset class capital gains. A three-year moving average filter is used to smooth the extreme fluctuations.

T-bonds.

Return on life insurance assets Similar to pensions, I use the data available on Fred on the balance sheet of life insurance companies to get their average asset class portfolio shares and then use asset class returns to estimate their returns. The used series are:

- Life Insurance Companies; Total Financial Assets, Level (BOGZ1FL544090005Q)

- Life Insurance Companies; Debt Securities and Loans; Asset, Level (LICTCMAHDFS)

- Life Insurance Companies; Corporate Equities Held Directly and Indirectly Through Mutual Funds; Asset, Market Value Levels (BOGZ1LM543064153Q)

For asset class returns, I use Damodaran's data. For other asset classes, I use the baseline risk-free return (3-month T-bill), for debt securities I use 10-year T-bonds.

Interest rates on government bonds For the nominal short-term interest rate, I use the series of 3-Month Treasury Bill Secondary Market Rate, Discount Basis (TB3MS) from Fred. For backing out the real short-term interest rate, I use the 1-year inflation expectation series estimated by Hall et al. (2019) (and the series 1-Year Expected Inflation (EXPINF1YR) from Fred for the few recent years that are not included in the former). For the long-term real interest rate, I use the results of the empirical estimation of 5-year real interest rate of Greenwald et al. (2023) (and for the few recent years that are not included there, I use the series Market Yield on U.S. Treasury Securities at 5-Year Constant Maturity, Quoted on an Investment Basis, Inflation-Indexed (DFII5) from Fred). For the nominal long-term interest rate, I use the yield on 10-year US treasury bond from Damodaran's data set. Figure A.5

plots all these interest rates in one graph.

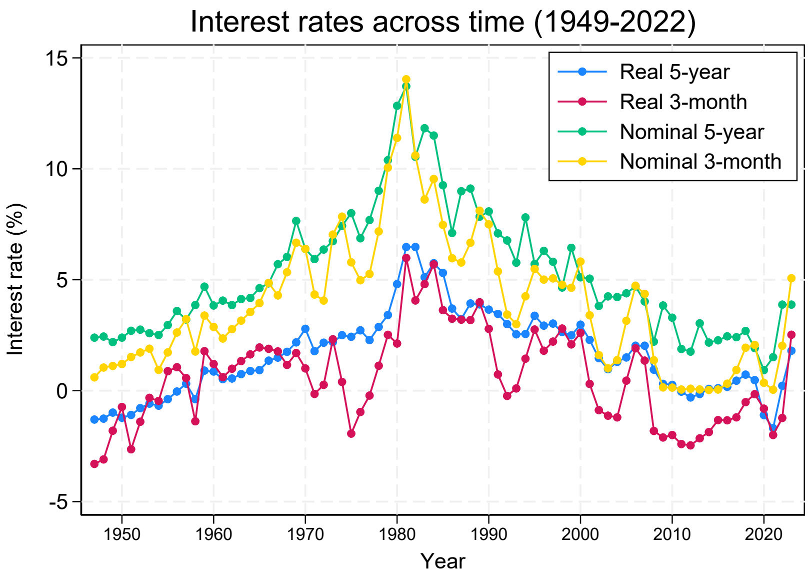

Figure A.5: All discussed interest rates in one graph. In the pre-1980 period, all notions of the interest rate were on average increasing, while for the post-1980 period, they are on average decreasing.

Approximating individual flows I use the approximation of:

$$Individual\ flow_i \approx Saving\ Rate \times Disposable\ Income_i \tag{A.9}$$

which is in sprit of the findings of Fagereng, Blomhoff Holm, Moll, and Natvik (2019), and in lines with the flow data that I extract from historical waves of SCF.

Saving rate out of disposable income For measuring the aggregate active saving rate, I use the ratio of aggregate personal savings to aggregate disposable income. More precisely, I use the series

- Households and Nonprofit Organizations; Personal Saving Excluding Consumer Durables and Federal Government Life Insurance Reserves and Railroad Retirement Board and National Railroad Retirement Investment Trust Pension Fund Reserves (NIPA), Transactions (BOGZ1FA156007015Q)

divided by the series

- Households and Nonprofit Organizations; Disposable Income, Net (IMA), Transactions (BOGZ1FU156012095Q)

as saving rate, which matches quite well the Personal Saving Rate (PSAVERT), but has a longer duration as I need it here.

Tax The SCF+ data is already prior to (individual) tax payments and so is my measurement of returns. For approximating individual flows using my approach, one needs to approximated net disposable income, which is post-tax. SCF+ does not include data on how much tax each household pays. To approximate the tax payment, I use Distributional National Accounts (DINA), which is a synthetic data based on Internal Revenue Service (IRS) data introduced in Piketty, Saez, and Zucman (2017) and develop linear models based on observed income variables in SCF, to predict each households paid tax based on their observed income. I use these models to approximate household tax payments.

(Net) Disposable income Household disposable income, following Fagereng et al. (2019), is defined as the sum of labor income, business income, capital income, transfers, and housing service flows, minus taxes[19]. The income which is left in equity assets (either public or private) should also be considered as Fagereng et al. (2019) argues. So, using the dividend payout ratio, I approximate the total earnings.

Robustness check capital gains For the approximation of the rate of return of capital gains, I assume the Pseudo-panel assumption on wealth percentiles. That is, treating the average observations of each percentile (or decile) as a single observation in a panel setting. Then, it is possible to approximate the rate of return on capital gains for each percentile (or decile) using my approximation of flows.[20]

Outlier removal I trim the distribution of returns in each year and for each wealth decile at the top and the bottom by 3%. This ensures that there are no outliers polluting the estimates of the regression of returns and aims to reduce measurement errors.

Adjusting the weights adjusted weights from SCF+ and divide by the total number of yearly observations

A.5 Robustness checks for measuring returns

I do a few robustness checks in the appendix. The first one is for the approximation of flows which we can see how well it works for the years that we know the flows. The second robustness check is for the way I measure the capital gains. The third one is about including other assets like pension funds, or vehicles. The results are robust to all of them.

Flows To solve the challenge with flows, I go back to historical waves of SCF and look for years that the questionnaire includes questions about active savings in different asset classes. Fortunately, for four years, we have the flow data as well as the other needed variables in pre 1980 data.[21] These data on active savings discover

[19] For some years when DINA is not available, I use data for close years to do the approximation of the linear model.

[20] As the data is not annual for most of the years, I use linear approximation for the flows of the years in between.

[21] There is flow data for a few more years, but unfortunately, we do not have asset class holdings

an important fact: active savings out of personal income (for people with positive wealth) is uncorrelated with their wealth. This finding is in lines with Fagereng et al. (2019) who use administrative data for more recent year to show this fact using Norwegian households. Using these insights from the data, I approximate individual flows as a constant (to be the average saving rate of that year) times individual personal income.

Capital gains Capital gains are always difficult to measure, even using very high-quality administrative data. That is because it contains both realized and unrealized capital gains, and measuring things that are not realized can be tricky. Thanks to the data on flows, I am able to use an approximate measure for capital gain's returns across the wealth distribution using a pseudo-panel technique and use it for calculating gross ROA. [22] There is another way that I use as a robustness check and that is using average asset class capital gains and using the portfolio shares to measure the capital gains.

Social security Catherine, Miller, and Sarin (2025) argue that adding social security to wealth while measuring wealth dynamics might change the results. Fagereng et al. (2020) argues that including pension wealth in measuring returns does not affect return inequality for people above the median wealth. (section 3.3.4 in their paper. They conclude: As expected, the adjustment reduces inequality in returns (and wealth) by increasing the return at the bottom of the distribution (where pension wealth is a quantitatively important wealth component), but it has virtually no effect above median wealth.) SCF+ data set has the pension variable from 1983 onward. Kuhn et al. (2020) argues (in section 2.1, footnote 8) that according to the financial accounts of the United States, this variable makes up a small part of household wealth before the 1980s, so missing information before 1983 is unlikely to change the picture meaningfully.

Try to follow Fagereng et al. (2020) method in the mentioned section and use SCF+ data to implement this robustness check. In case, you can use the methods in Catherine et al. (2025).

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for those years. There is also a short panel survey for the years 1962 and 1963 that I have used.

[22] If we assume that the number of households that move between deciles of wealth in one year is negligible, we can aggregate households at the decile level and then treat the data as panel data and leverage the knowing of flows to measure return on capital gains:

$$r_{it}^{kg} = \frac{w_{i,t+1} - w_{it} - f_{it}}{w_{it} + \frac{f_{it}}{2}} \tag{A.10}$$

A.6 Confidence intervals for returns

Figure A.6 plots the returns graphs that are in the main text, but with the 90 percent confidence intervals. The reason that confidence intervals are a bit wide is that the data is used in a pseudo-panel approach, which decreases the number of observations to one observation per year for each percentile noted in the graph. This small sample issue makes the confidence intervals wide.

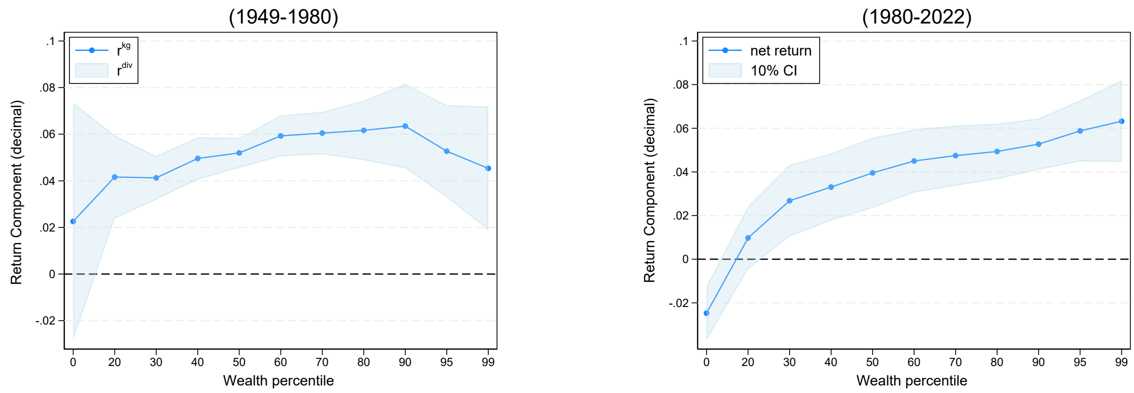

Figure A.6: Average realized net return on wealth for the years before and after 1980 using SCF+ data set. The shaded area is a 90% confidence interval.

A.7 Measuring duration and interest rate exposure

In this section, I will first explain the measurement of duration that I use. I will also comment on if it has changed on average in in two sub-periods before and after 1980. Then, I will explain an alternative approach for measuring interest rate exposure of asset classes without relying on measurements of duration.

A.7.1 Annual series for return ratio

Figure A.7 plots the time series for the top-10 to bottom-90 return ratio.

With the annual series, one can run a regression to see what the main drivers of the changes in return ratio are, which is a measure of the relative return of rich vs non-rich households.

A.7.2 Measuring asset class duration

Real estate

Public equity

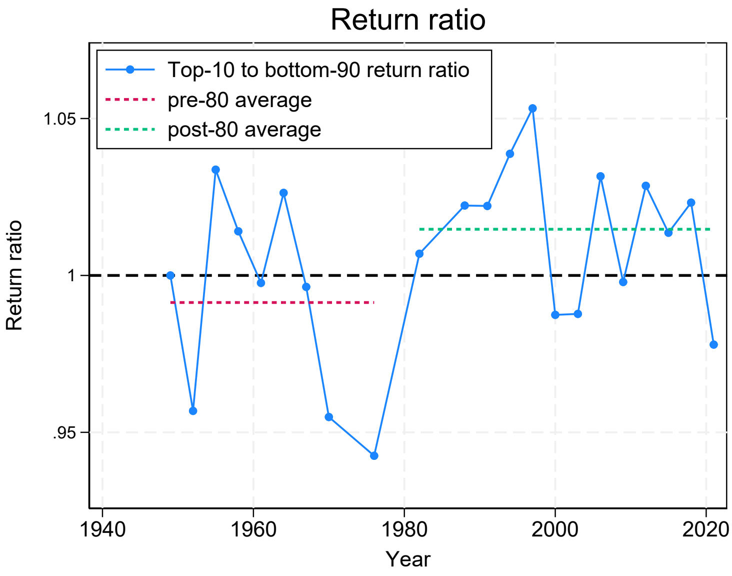

Figure A.7: Annual return ratio

Corporate equity private business I use a Gordon growth model implied approximation[23], which give the duration as price to total earnings ratio (dividends × 1/payout). I use dividends from:

-Households and Nonprofit Organizations; Net Dividends Received, Transactions, Millions of Dollars, Annual, Not Seasonally Adjusted, BOGZ1FU156121175Q

and prices from

- Households and Nonprofit Organizations; Corporate Equities; Asset, Market Value Levels, Millions of Dollars, Annual, Not Seasonally Adjusted BOGZ1LM153064105A

Payout ratio For measuring the corporate equity payout ratio, I use

Net value added of nonfinancial corporate business: Corporate profits with IVA and CCAdj: Profits after tax with IVA and CCAdj: Net dividends (B467RC1Q027SBEA)

Net value added of nonfinancial corporate business: Corporate profits with IVA and CCAdj: Profits after tax with IVA and CCAdj (W328RC1Q027SBEA)

For measuring the payout ratio of stocks, I use Robert Shiller's data for the S&P 500. For the repurchase payout ratio, I do an approximation, using the net issuance and net profit data on FED.

- Nonfinancial Corporate Business; Corporate Equities; Liability, Transactions (NCBCEBQ027S)

[23]Gordon growth model implied duration: If we assume that the cash flows grow at a constant rate g and the interest rate is constant $R = 1 + r$:

$$D_t = \frac{1+r}{r-g} = 1 + \frac{P_t}{Cashflow_t} = 1 + \frac{P_t}{\frac{Div_t}{Payout_t}}$$

Non-corporate equity private business Nonfinancial Noncorporate Business; Proprietors' Equity in Noncorporate Business, Market Value Levels (BOGZ1LM112090205Q)

Nonfinancial Noncorporate Business; Net Income with IVA and CCAdj, Transactions (BOGZ1FA116110005Q).

A.7.3 Duration before vs after 1980

Table A.1 reports the average duration of risky asset classes. The methodology for real estate and public equity is based on an asset pricing model, and for private businesses, it is the Gordon growth implied duration. (See Appendix A.7 for a more detailed discussion and robustness checks.)

	Duration (years)		
Risky asset class	**pre-1980**	**post-1980**	**long-run**
Real Estate	15.3	15.8	15.6
Public Equity	30.3	34.5	32.7
Private Business (average)	23.8	24.9	24.4
Private Business (corp.)	33.6	33.3	33.6
Private Business (non-corp.)	15.0	14.3	14.7

Table A.1: Asset class durations for risky assets for periods pre-1980 (1949-1980), post-1980 (1980-2022), and long-run (1949-2022).

As it is clear from Table A.1, although the measured duration changes with time, the long-term averages are quite stable, and so is the ordering of them. Table **??** reports the average duration for other asset and liability classes. The numbers are from Greenwald et al. (2023), where they are mostly done through simple approximation or assumptions.

A.7.4 Alternative measures for interest rate exposure

To address the concerns raised by the paper Gormsen and Lazarus (2025), regarding the issue that the measured duration, might be different than interest rate exposure. Using the capital gain returns, one can directly measure duration:

$$\frac{\partial \log P_0}{\partial \log R} = -D \rightarrow r^{kg} = -D \times \Delta r \tag{A.11}$$

Estimate:

$$\Delta r_t^{kg} = \alpha_0 + \alpha_1 \Delta r_t + \alpha_2 GDPGrowth_t + \alpha_3 RealizedVolatility_t + \epsilon_t \tag{A.12}$$

Then $D = -\alpha_1$

Asset class	Interest rate exposure Method I	Method II	Method III
Real Estate	15.6		
Public Equity	32.7		
Private Business (average)	24.4		
Private Business (corp.)	33.6		
Private Business (non-corp.)	14.7		

Table A.2: Average interest rate exposure of different risky asset classes based on long-run sample (1949-2022). Method I used simple duration as in Greenwald et al. (2023), method II, uses adjusted and adjustment to duration to take into account expected (unpriced) movements as in Catherine et al. (2023), and method III uses a regression model (see Appendix).

A.8 GDP beta across earnings

Figure A.8 plots the GDP beta of households earnings growth across the earning percentiles. It basically captures how much income change correlates with changes in GDP. If it is higher, it means the cyclicality of the labor income in higher. As we can see, it is a U-shaped curve.

(a) GDP beta of labor income

(b) GDP beta of total income

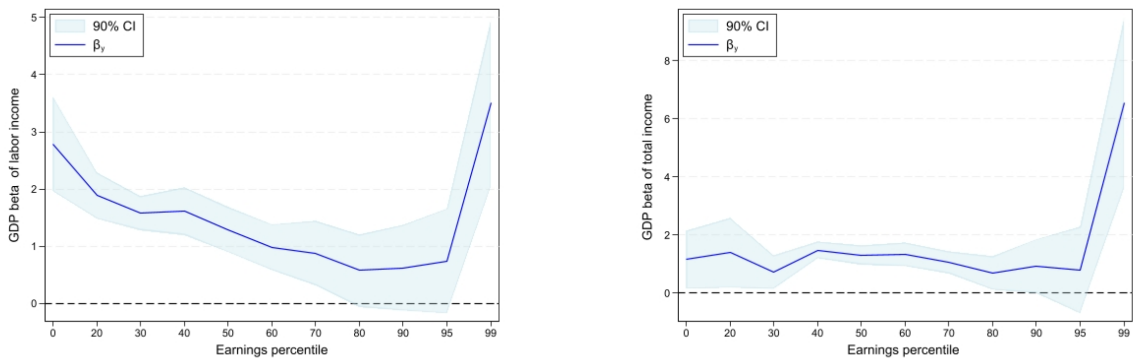

Figure A.8: The figures plot the GDP sensitivity of income across earning percentiles. The shaded area is a 90% confidence interval.

Although my data set for this task is a pseudo panel, the results are quite comparable to Guvenen et al. (2017), who uses admin panel data of social security. And also to Amberg, Jansson, Klein, and Picco (2022) (Appendix C), who uses admin data in Sweden. [24]

[24] Another seemingly similar to the GDP beta results, is Parker and Vissing-Jorgensen (2009), but note they regress income fluctuation on aggregate income fluctuation and not GDP, which is a different thing, and is more related to the question of which groups' fluctuations explain more of the aggregate fluctuations. They also have different results for before and after 1980, but the GDP beta looks almost the same before and after 1980.

(a) Interest rate beta of labor income (b) Interest rate beta of total income

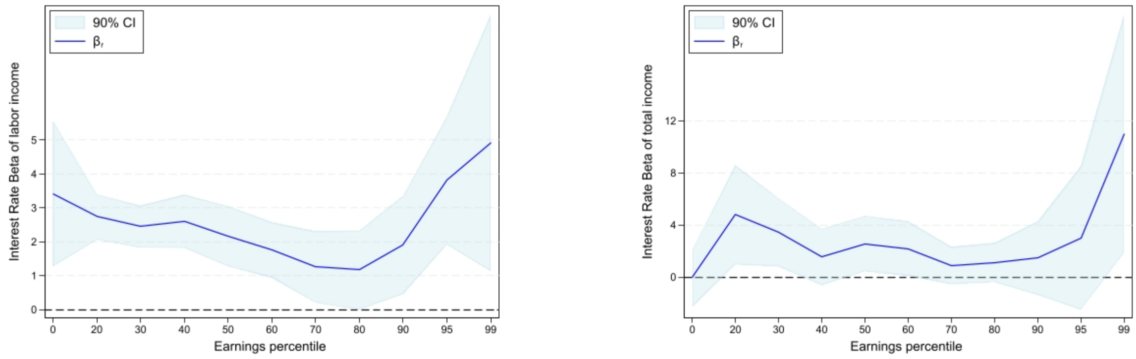

Figure A.9: The figures plot the interest rate sensitivity of income across earning percentiles. The shaded area is a 90% confidence interval.

B Model Solution

This section of the appendix contains the solution to the model.

HHs optimization problem with constant relative risk aversion (CRRA) utility function, γ the coefficient of relative risk aversion:

$$\max_{\{C_t,\pi_t\}_{t=0}^{\infty}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \tag{B.1}$$

$$\begin{aligned} s.t. \quad & W_{t+1} = (W_t + L_t - C_t)\, R_{w,t+1}, \\ & W_{t+1} > 0. \end{aligned} \tag{B.2}$$

where, the gross return on wealth is $R_{w,t+1} = \pi_t\,(R_{n,t+1} - R_f) + R_f$. I will denote the logarithm of gross returns with small letter: $r_{w,t} = \log(R_{w,t})$ So, for the return of the short-term bond, we have:

$$r_{f,t+1} = (1-\varphi)\bar{r}_f + \varphi r_{ft} + \sigma_r \epsilon_{r,t+1} \tag{B.3}$$

The return of the long-term bond :

$$r_{n,t+1} = r_{ft} + \mu_n - \sigma_n \epsilon_{r,t+1}, \tag{B.4}$$

(Where, $\sigma_n = \frac{1-\varphi^{n-1}}{1-\varphi}\sigma_r$). And, for the return of the wealth, we will have:

$$r_{w,t+1} \approx r_{f,t} + \pi_t\,(r_{n,t+1} - r_{f,t}) + \frac{1}{2}\pi_t\,(1-\pi_t)\,\mathrm{Var}_t\,(r_n) \tag{B.5}$$

which will be precise if time is continuous. (See Campbell and Viceira (2002), Appendix, pages 2-5.)

B.1 HHs FOCs

I start by writing the Lagrangian:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} + \lambda_{t+1}[(W_t - C_t)\, R_{w,t+1} - W_{t+1}] \right\} \tag{B.6}$$

First order conditions (FOCs) will be: [25]

$$\begin{aligned} [C_t]: &\qquad \beta^t U'(C_t) - E_t\left[\lambda_{t+1} R_{w,t+1}\right] = 0 \\ [\pi_t]: &\qquad (W_t - C_t)\, E_t\left[\lambda_{t+1}(R_{n,t+1} - R_{f,t})\right] = 0 \\ [W_{t+1}]: &\qquad -\lambda_{t+1} + E_{t+1}\left(\lambda_{t+2} R_{w,t+2}\right) = 0 \end{aligned} \tag{B.7}$$

Simplifying the FOCs will give us three Euler Equations (for consumption and for asset holdings)[26]:

$$1 = \beta E_t \left[\frac{U'(C_{t+1})}{U'(C_t)} R_{j,t+1} \right] \quad ; \quad \textit{for } j \in w, n, f \tag{B.8}$$

B.2 Approximating the FOCs and Budget Constraint

General note on linearizing logarithms of expectation: There are two approaches. Either we rely on the normality of the distribution of random variables and use the fact that for a Normally distributed random variable x, we have $\log E_t\left[e^x\right] = \mu_t + \frac{1}{2}\sigma_t^2$. Or more generally, one can use a second-order Taylor approximation around the mean, if x is close to its mean ($\text{Var}(x)$ is close to zero):

$$\begin{aligned} \log E_t\left(e^x\right) &\approx \log E_t \left(e^{\bar{x}} + e^{\bar{x}}(x - \bar{x}) + \frac{1}{2} e^{\bar{x}} (x - \bar{x})^2 \right) \\ &\approx \log \left(e^{\bar{x}} + \frac{1}{2} e^{\bar{x}}\, \text{Var}_t(x) \right) \\ &\approx \log \left(e^{\bar{x}} \left(1 + \frac{1}{2}\, \text{Var}_t(x) \right) \right) \\ &\approx \bar{x} + \frac{1}{2}\, \text{Var}_t(x) \end{aligned} \tag{B.9}$$

Approximating EEs: For the EEs, we have the approximation (taking the

[25]Note: have we to take derivative with respect to W_{t+1} as well, since it is a function of controls (C_t, π_t) and state variable (W_t).

[26]Note that two of the three equations above will give the third one as a result, and any two of them are enough for finding the solution.

logarithm of both sides and then a second-order Taylor expansion) will give us:

$$0 = \log\beta + E_t\left[-\gamma\Delta c_{t+1} + r_{j,t+1}\right] + \frac{1}{2}\operatorname{Var}_t\left(-\gamma\Delta c_{t+1} + r_{j,t+1}\right); \quad \textit{for} \quad j = w, n, f \tag{B.10}$$

(Note that when j=f, it should be $r_{f,t}$) If we subtract the above equations for n and f (n minus f), we will get:

$$E_t\left[r_{n,t+1} - r_{f,t}\right] + \frac{1}{2}\operatorname{Var}_t\left(r_{n,t+1}\right) = \gamma \operatorname{Cov}\left(r_{n,t+1}, \Delta c_{t+1}\right) \tag{B.11}$$

Approximating the budget constraint: The budget constraint is:

$$W_{t+1} = \left(W_t + L_t - C_t\right) R_{w,t+1} \tag{B.12}$$

divide both sides by L_{t+1}, to get:

$$\frac{W_{t+1}}{L_{t+1}} = \left(\frac{W_t}{L_t} + 1 - \frac{C_t}{L_t}\right)\frac{L_t}{L_{t+1}} R_{w,t+1} \tag{B.13}$$

Then I take log of both sides, denoting log variables with small letters ($w_t = \log(W_t)$):

$$w_{t+1} - l_{t+1} = \log\left(1 + \exp\left(w_t - l_t\right) - \exp\left(c_t - l_t\right)\right) - \Delta l_{t+1} + r_{w,t+1} \tag{B.14}$$

We can linearize the above equation by taking applying first-order Taylor expansion around $\mathrm{E}\left[c_t - l_t\right]$ and $\mathrm{E}\left[w_t - l_t\right]$. This gives:

$$w_{t+1} - l_{t+1} \approx \kappa + \rho_w\left(w_t - l_t\right) - \rho_c\left(c_t - l_t\right) - \Delta l_{t+1} + r_{w,t+1} \tag{B.15}$$

where

$$\begin{aligned} \rho_w &= \frac{\exp\left\{\mathrm{E}\left[w_t - l_t\right]\right\}}{1 + \exp\left\{\mathrm{E}\left[w_t - l_t\right]\right\} - \exp\left\{\mathrm{E}\left[c_t - l_t\right]\right\}} \\ \rho_c &= \frac{\exp\left\{\mathrm{E}\left[c_t - l_t\right]\right\}}{1 + \exp\left\{\mathrm{E}\left[w_t - l_t\right]\right\} - \exp\left\{\mathrm{E}\left[c_t - l_t\right]\right\}} \end{aligned} \tag{B.16}$$

and

$$\kappa = -\left(1 - \rho_w + \rho_c\right)\log\left(1 - \rho_w + \rho_c\right) - \rho_w \log\left(\rho_w\right) + \rho_c \log\left(\rho_c\right). \tag{B.17}$$

Note that $\rho_w, \rho_c > 0$. (*Proof*: Since along the optimal path we need to have $W_{t+1} >$ and so $W_t + L_t - C_t > 0$. This is equivalent to $1 + \frac{W_t}{L_t} - \frac{C_t}{L_t} > 0$ or $1 + \exp(w_t - l_t) - \exp(c_t - l_t) > 0$. And as our variables here are continuous, we will have $1 + \exp(E(w_t - l_t)) - \exp(E(c_t - l_t)) > 0$, and this immediately results in $\rho_w, \rho_c > 0$.)

Also, note that the definition of ρ_w and $\rho_c > 0$ depend on the values of w_t and l_t, which should be added to the final system of equations to be solved simultaneously. (Viceira (2001) section IV.A)

B.3 Solving the approximated system

Proof of proposition 1: The system of equations that we should solve now is: I use the EE for $j = w$ and substitute for π_t.

$$E_t\left[\Delta c_{t+1}\right] = \frac{1}{\gamma}\log\beta + \frac{1}{\gamma}E_t\left[r_{w,t+1}\right] + \frac{\gamma^2}{2\gamma}\operatorname{Var}_t\left[\Delta c_{t+1} - \frac{1}{\gamma}r_{w,t+1}\right] \tag{B.18}$$

and another equation:

If we subtract the above equations for n and f (n minus f), we will get:

$$E_t\left[r_{n,t+1} - r_{f,t}\right] + \frac{1}{2}\operatorname{Var}_t\left(r_{n,t+1}\right) = \gamma\operatorname{Cov}_t\left(r_{n,t+1}, \Delta c_{t+1}\right) \tag{B.19}$$

These are the two equilibrium conditions for finding our two unknowns c_t and π_t.

For using the first equation, we should first find the values of $E_t\left[\Delta c_{t+1}\right]$ and $\operatorname{Var}_t\left[\Delta c_{t+1} - \frac{1}{\gamma}r_{w,t+1}\right]$, and for the second equation, we need $\operatorname{Cov}_t\left(r_{n,t+1}, \Delta c_{t+1}\right)$.

$$\begin{aligned} E_t\left[\Delta c_{t+1}\right] &= E_t\left[c_{t+1} - l_{t+1} - (c_t - l_t) + \Delta l_t\right] \\ &= E_t[c_{t+1} - l_{t+1}] - (c_t - l_t) + E_t\left(\Delta l_t\right) \\ &= b_0 + b_1 E_t[w_{t+1} - l_{t+1}] + b_2 E_t\left(r_{f,t+1}\right) - (c_t - l_t) + E_t\left(\Delta l_t\right) \end{aligned} \tag{B.20}$$

One need to calculate $\operatorname{Cov}_t\left(r_{n,t+1}, \Delta c_{t+1}\right)$ for EEs. I use the identity

$$\Delta c_{t+1} = c_{t+1} - l_{t+1} - (c_t - l_t) + \Delta l_t \tag{B.21}$$

and guess and verify

$$c_{t+1} - l_{t+1} = b_0 + b_1\left(w_{t+1} - l_{t+1}\right) + b_2 r_{f,t+1} \tag{B.22}$$

Replacing these two amount step by step, we will have:

$$\begin{aligned}
\operatorname{Cov}_t\left(r_{n,t+1}, \Delta c_{t_{+1}}\right) &= \operatorname{Cov}_t\left(r_{n,t+1}, c_{t+1} - l_{t+1} - (c_t - l_t) + \Delta l_t\right) \\
&= \operatorname{Cov}_t\left(r_{n,t+1}, c_{t+1} - l_{t+1}\right) + \operatorname{Cov}_t\left(r_{n,t+1}, l_{t+1}\right) \\
&= \operatorname{Cov}_t(r_{n,t+1}, b_0 + b_1\left(w_{t+1} - l_{t+1}\right) + b_2 r_{f,t+1}) - \sigma_n\sigma_{rl} \\
&= \operatorname{Cov}_t(r_{n,t+1}, b_1\left(w_{t+1} - l_{t+1}\right)) + \operatorname{Cov}_t(r_{n,t+1}, +b_2 r_{f,t+1}) - \sigma_n\sigma_{rl} \\
&= -b_1 \operatorname{Cov}_t\left(r_{n,t+1}, \Delta l_t\right) + b_1 \operatorname{Cov}_t\left(r_{n,t+1}, r_{w_1 t+1}\right) - b_2\sigma_n\sigma_r - \sigma_n\sigma_{rl} \\
&= -\left(1 - b_1\right)\sigma_n\sigma_{rl} + b_1\pi_t\sigma_n^2 - b_2\sigma_n\sigma_r
\end{aligned} \tag{B.23}$$

Now, I replace this in the second EE to solve for π_t. We will have:

$$\pi_t = \frac{E_t\left[r_{n,t+1} - r_{f,t}\right] + \frac{1}{2}\operatorname{Var}_t\left(r_{n,t1}\right)}{\gamma b_1\sigma_n^2} + \frac{b_2\sigma_r\sigma_n}{\gamma b_1\sigma_n^2} + \frac{\left(1 - b_1\right)\sigma_{rl}}{\gamma b_1\sigma_n^2} \tag{B.24}$$

Note that π_t is time invariant as expected, since we do not have life-cycle or anything time-varying parameters in this model.

Now, we have found the solution for π_t. Now, we need to pin down the coefficients b_0, b_1, and b_2. For doing so, I use the first EE (for $j = w$) and substitute for π_t.

$$E_t\left[\Delta c_{t+1}\right] = \frac{1}{\gamma}\log\beta + \frac{1}{\gamma}E_t\left[r_{w,t+1}\right] + \frac{\gamma^2}{2\gamma}\operatorname{Var}_t\left[\Delta c_{t+1} - \frac{1}{\gamma}r_{w,t+1}\right] \tag{B.25}$$

For using this equation, we should first find the values of $E_t\left[\Delta c_{t+1}\right]$ and $\operatorname{Var}_t\left[\Delta c_{t+1} - \frac{1}{\gamma}r_{w,t+1}\right]$.

$$\begin{aligned}
E_t\left[\Delta c_{t+1}\right] &= E_t\left[c_{t+1} - l_{t+1} - (c_t - l_t) + \Delta l_t\right] \\
&= E_t[c_{t+1} - l_{t+1}] - (c_t - l_t) + E_t\left(\Delta l_t\right) \\
&= b_0 + b_1 E_t[w_{t+1} - l_{t+1}] + b_2 E_t\left(r_{f,1+t}\right) - (c_t - l_t) + E_t\left(\Delta l_t\right) \\
&= b_0 + b_1\kappa + b_1\rho_w\left(w_t - l_t\right) + \left(-b_1\rho_c - 1\right)\left(c_t - l_t\right) + b_2\left((1-\varphi)\bar{r}_f + \varphi r_{f,t}\right) \\
&+ r_{f,t} + \pi_t\mu_n + \frac{1}{2}\pi_t(1 - \pi_t)\sigma_n^2
\end{aligned} \tag{B.26}$$

and

$$
\begin{aligned}
\mathrm{Var}_t\left[\Delta c_{t+1}-\frac{1}{\gamma}r_{w,t+1}\right] &= \mathrm{Var}_t\left[c_{t+1}-l_{t+1}-(c_t-l_t)+\Delta l_t-\frac{1}{\gamma}r_{w,t+1}\right] \\
&= \mathrm{Var}_t\left[(c_{t+1}-l_{t+1})+l_{t+1}-\frac{1}{\gamma}r_{w,t+1}\right] \\
&= \mathrm{Var}_t\left[b_0+b_1\left(w_{t+1}-l_{t+1}\right)+b_2 r_{f,t+1}+l_{t+1}-\frac{1}{\gamma}r_{w,t+1}\right] \\
&= \mathrm{Var}_t\left[\left(1-b_1\right)l_{t+1}+\left(1-\frac{1}{\gamma}\right)r_{w,t+1}+b_2 r_{f,t+1}\right] \\
&= \mathrm{Var}_t\left[\left(1-b_1\right)l_{t+1}\right]+\mathrm{Var}_t\left[\left(1-\frac{1}{\gamma}\right)r_{w,t+1}\right]+\mathrm{Var}_t\left[b_2 r_{f,t+1}\right] \\
&+2\,\mathrm{Cov}_t\left(\left(1-b_1\right)l_{t+1},\left(1-\frac{1}{\gamma}\right)r_{w_1 t_{+1}}\right)+2\,\mathrm{Cov}_t\left(\left(1-b_1\right)l_{t+1},b_2 r_{f,t+1}\right) \\
&+2\,\mathrm{Cov}_t\left(\left(1-\frac{1}{\gamma}\right)r_{n,t+1},b_2 r_{f,t+1}\right) \\
&= \left(1-b_1\right)^2\sigma_l^2+\left(1-\frac{1}{\gamma}\right)^2\pi^2\sigma_n^2+b_2^2\sigma_r^2 \\
&+2\left(1-b_1\right)\left(1-\frac{1}{\gamma}\right)\pi\left(-\sigma_n\sigma_{r_l}\right) \\
&+2\left(1-b_1\right)b_2\sigma_r\sigma_{rl}+2\left(1-\frac{1}{\gamma}\right)b_2\left(-\sigma_r\sigma_n\right)=V
\end{aligned}
\tag{B.27}
$$

As the above term does not depend on the state variables of the model, I have denoted with V, which is a function of parameters.

Now, back to the first EE, we replace the equivalents:

$$
\begin{gathered}
b_0+b_1\kappa+b_1\rho_w\left(w_t-l_t\right)+\left(-b_1\rho_c-1\right)\left(c_t-l_t\right)+b_2\left((1-\varphi)\bar{r}_f+\varphi r_{f,t}\right) \\
+r_{f,t}+\pi_t\mu_n+\frac{1}{2}\pi_t(1-\pi_t)\sigma_n^2=\frac{1}{\gamma}\log\beta+\frac{1}{\gamma}\left(r_{f_1 t}+\pi\mu_n+\pi(1-\pi)\sigma_n^2\right)+\frac{\gamma}{2}V
\end{gathered}
\tag{B.28}
$$

With a bit of rearranging, we will have:

$$
\begin{aligned}
&b_0+b_1\kappa+b_2(1-\varphi)\bar{r}_f+\pi_t\mu_n+\frac{1}{2}\pi_t(1-\pi_t)\sigma_n^2-\frac{1}{\gamma}\log\beta-\frac{1}{\gamma}\left(\pi\mu_n+\frac{1}{2}\pi\left(1-\pi\right){\sigma_n}^2\right)-\frac{\gamma}{2}V \\
&+b_1\rho_w\left(w_t-l_t\right)+\left(-b_1\rho_c-1\right)\left(c_t-l_t\right)+b_2\left(\varphi r_{f,t}\right)+r_{f,t}-\frac{1}{\gamma}r_{f,t}=0
\end{aligned}
\tag{B.29}
$$

Now, I replace $(c_t - l_t)$ from the guess:

$$\begin{aligned} & b_0 + b_1\kappa + b_2(1-\varphi)\bar{r}_f + \pi_t\mu_n + \frac{1}{2}\pi_t(1-\pi_t)\sigma_n^2 - \frac{1}{\gamma}\log\beta \\ & - \frac{1}{\gamma}\left(\pi\mu_n + \frac{1}{2}\pi\left(1-\pi\right){\sigma_n}^2\right) - \frac{\gamma}{2}V + \left(-b_1\rho_c - 1\right)b_0 \\ & + \left\{b_1\rho_w + \left(-b_1\rho_c - 1\right)b_1\right\}\left(w_t - l_t\right) \\ & + \left\{b_2\left(-b_1\rho_c - 1\right) + b_2\varphi - \frac{1}{\gamma} + 1\right\} r_{f,t} = 0 \end{aligned} \tag{B.30}$$

This equation should hold for all t. Equating the coefficients to zero, we get[27]:

$$b_0 = \frac{1}{b_1\rho_c}\{b_1\kappa + b_2(1-\varphi)\bar{r}_f - \frac{1}{\gamma}\log\beta + (1-\frac{1}{\gamma})\left(\pi\mu_n + \frac{1}{2}\pi\left(1-\pi\right){\sigma_n}^2\right) - \frac{\gamma}{2}V\} \tag{B.31}$$

$$b_1 = \frac{\rho_w - 1}{\rho_c} \tag{B.32}$$

$$b_2 = \frac{1-\frac{1}{\gamma}}{\rho_w - \varphi} = (1-\frac{1}{\gamma})\bar{b}_2 \tag{B.33}$$

Note that the definitions of ρ_w and ρ_c and the fact that $W_t + L_t - c_t > 0$ imply that they are both positive and $0 < 1 - \rho_w + \rho_c$, which implies $\frac{\rho_w - 1}{\rho_c} < 1$. For provint that $b_2 > 0$, note that from the solution to for optimal consumption:

$$c_{t+1} - l_{t+1} = b_0 + b_1\left(w_{t+1} - l_{t+1}\right) + b_2 r_{f,t+1} \tag{B.34}$$

If $b_2 < 0$, it implies that consumption is a decreasing function of wealth for all income levels. That is, the individual is better off with less wealth, which is a contradiction.

This completes the derivation of the approximate solution. □

[27] One can rule out the case where $b_1 = 0$, for which b_0 gets arbitrary, and b_3 will have no solution.