Online Appendix to "Relative Wealth Concerns, Executive Compensation, and Managerial Risk-Taking" by Qi Liu and Bo Sun

Online appendix A: Performance-based outside option

Since firm performance is realized at the end of a period and affects the manager's outside options in the next period, it is necessary to investigate the impact of performance-dependent outside options on risk-taking in a two-period setup. Suppose that there are two periods: $\tau = 1, 2$. At the beginning of each period τ, manager i in firm i is offered a contract that matches his outside option in period τ. We assume that the manager i's (certainty-equivalent) outside option in period 1 is $\bar{u}_{i1}$, and his outside option in period 2 depends on the firm performance at the end of period 1, which we will discuss below in details. The firm performance in each period is given by

$$V_{i\tau} = a_{i\tau}(\pi + \tilde{m}_\tau) + \eta_{i\tau}.$$

This is the same specification as in the baseline model, and $V_{i\tau}$ will be realized at the end of both periods $\tau = 1$ and $\tau = 2$. Similarly, at the beginning of each period, the manager is offered a contract in the following form: $w_{i\tau} = \alpha_{i\tau} + \beta_{i\tau}[V_{i\tau} - \bar{a}_{i\tau}\tilde{m}_\tau] + \gamma_{i\tau}\tilde{m}_\tau$. Similar to Oyer (2004), we interpret $V_{i\tau}$ as the profit earned in period τ, and at the end of the period, $V_{i\tau} - w_{i\tau}$ and $w_{i\tau}$ will be paid out to shareholders and managers, respectively. That is, for tractability, the optimization problems in the two periods are independent of each other, except that the manager's outside option in period 2 depends on firm performance in period 1.

At the beginning of the first period, given the compensation contract, the manager i's objective is to maximize

$$E\left[-exp\left[-\lambda\left(w_{i1} - \tilde{w}_{i1} - \frac{1}{2}a_{i1}^2 + w_{i2} - \tilde{w}_{i2} - \frac{1}{2}a_{i2}^2\right)\right]\right],$$

where $w_{i\tau}, \tilde{w}_{i\tau}, a_{i\tau}$ refer to the manager i's compensation, relative wealth concerns, and effort in period τ for $\tau = 1, 2$.

Internalizing that period-2 managerial pay is positively related to period-1 firm performance, a manager may have incentives to exert higher efforts to increase the firm performance in period 1. As firm performance can be measured by two observable signals, and we are mainly interested in how the effort-related outside options change a manager's incentives to exert effort, we assume that period-2 outside option is given by $\mu(V_{i1} - \bar{a}_{i1}\tilde{m}_1) + \bar{u}_{i2}$.

We can then calculate managerial utility in period 1 as

$$
\begin{aligned}
& E\left[-exp\left[-\lambda\left(w_{i1} - \tilde{w}_{i1} - \frac{1}{2}a_{i1}^2 + w_{i2} - \tilde{w}_{i2} - \frac{1}{2}a_{i2}^2\right)\right]\right] \\
= \; & E\left[E\left[-exp\left[-\lambda\left(w_{i1} - \tilde{w}_{i1} - \frac{1}{2}a_{i1}^2 + w_{i2} - \tilde{w}_{i2} - \frac{1}{2}a_{i2}^2\right)\right]|I_1\right]\right] \\
= \; & E\left[-exp\left[-\lambda\left(w_{i1} - \tilde{w}_{i1} - \frac{1}{2}a_{i1}^2 + \mu(V_{i1} - \bar{a}_{i1}\tilde{m}_1) + \bar{u}_{i2}\right)\right]\right]
\end{aligned}
$$

It immediately follows that manager i chooses effort a_{i1} to maximize his certainty-equivalent expected utility $CE_{i1} = \alpha_{i1} + (\beta_{i1} + \mu)\pi a_{i1} - \frac{1}{2}\lambda[(\beta_{i1} + \mu)(a_{i1} - \bar{a}_{i1}) + \gamma_{i1} - h_i M_1]^2\sigma_m^2 - \frac{1}{2}\lambda(\beta_{i1} + \mu)^2\sigma_\eta^2 - \frac{1}{2}a_{i1}^2$, where $M_1 = \int_0^1 \gamma_{k1} dk$ refers to the average exposure to period-1 luck shock $\tilde{m}_1$ in other executives' compensation. The first-order condition implies that optimal a_{i1} satisfies

$$\frac{\partial CE_{i1}}{\partial a_{i1}} = (\beta_{i1} + \mu)\pi - \lambda(\beta_{i1} + \mu)[(\beta_{i1} + \mu)(a_{i1} - \bar{a}_{i1}) + \gamma_{i1} - h_i M_1]\sigma_m^2 - a_{i1} = 0.$$

Shareholders choose period-1 compensation contract to maximize $E[V_{i1}] - E[w_{i1}] + E[V_{i2}] - E[w_{i2}]$. In our two-period model setup, $E[V_{i2}] = \pi a_2$ is not affected by a_{i1}, but the level of w_{i2} will be increased by $\mu(V_{i1} - \bar{a}_{i1}\tilde{m}_1)$, so the shareholders will maximize $\pi a_{i1} - E[w_{i1}] - E[\mu(V_{i1} - \bar{a}_{i1}\tilde{m}_1)]$, which is equivalent to (note that $a_{i1} = \bar{a}_{i1}$ in equilibrium)

$$\max_{\beta_{i1}, \gamma_{i1}, a_{i1}} \pi a_{i1} - \frac{1}{2}\lambda[\gamma_{i1} - h_i M_1]^2\sigma_m^2 - \frac{1}{2}\lambda(\beta_{i1} + \mu)^2\sigma_\eta^2 - \frac{1}{2}a_{i1}^2,$$

subject to $(\beta_{i1} + \mu)\pi - \lambda(\beta_{i1} + \mu)(\gamma_{i1} - h_i M_1)\sigma_m^2 - a_{i1} = 0$. If we define $\hat{\beta}_{i1} = \beta_{i1} + \mu$, then the principal-agent model is simplified to

$$\max_{\hat{\beta}_{i1}, \gamma_{i1}, a_{i1}} \pi a_{i1} - \frac{1}{2}\lambda[\gamma_{i1} - h_i M_1]^2\sigma_m^2 - \frac{1}{2}\lambda\hat{\beta}_{i1}^2\sigma_\eta^2 - \frac{1}{2}a_{i1}^2,$$

subject to $\hat{\beta}_{i1}\pi - \lambda\hat{\beta}_{i1}(\gamma_{i1} - h_i M_1)\sigma_m^2 - a_{i1} = 0$.

It is straightforward to see that the shareholders' optimization problem reduces to the same principal-agent model in Proposition 2. It implies that incorporating performance-dependent outside option does not necessarily affect managerial risk-taking. The intuition is that as both the shareholders and the manager expect that additional μ shares of period-1 profit will be paid out to the manager due to performance-dependent outside option in period 2, shareholders optimally adjust the shares paid to the manager at the beginning of period 1, and the manager will behave the same as in the case without performance-dependent outside option.

Corollary 1. *Suppose that period-2 outside option for manager i is given by $\mu(V_{i1} - \bar{a}_{i1}\tilde{m}_1) + \bar{u}_{i2}$. Then period-1 optimal effort is not affected by μ for any h_i.*

Since the signal $\tilde{m}_1$ is beyond managerial control, and the signal $V_{i1} - \bar{a}_{i1}\tilde{m}_1$ can reflect the managers' effort and productivity, we have assumed that the performance-dependent outside option depends on $V_{i1} - \bar{a}_{i1}\tilde{m}_1$ rather than $\tilde{m}_1$ in Corollary 1. In Oyer (2004), the dependence of managers' outside option on period-1 luck shock $\tilde{m}_1$, combining with costly compensation adjustment, implies that tying managerial pay to $\tilde{m}_1$ (i.e., $\gamma_{i1} > 0$) can be optimal. In our model with relative wealth concerns, if period-2 outside option depends on $\tilde{m}_1$, shareholders will optimally adjust γ_{i1} to absorb the effect. Therefore, even if managers' outside option depends on $\tilde{m}_1$, incorporating the contingent reservation utility may not affect the optimal effort and risk-taking in the pay-for-luck equilibrium in our setup. To that end, relative wealth concerns on the part of managers remain crucial in producing excessive risk-taking that is associated with pay-for-luck.

Online appendix B: Shareholders with relative wealth concerns

In this appendix, we study how a manager's pay-for-luck will be affected if shareholders have relative wealth concerns. In the baseline model, the shareholders in firm i are risk-neutral, and have an objective to maximize their expected payoff $E[P_i]$, where $P_i = V_i - w_i$. In this appendix, we assume that shareholders are risk averse with an exponential utility, and they are also concerned about their peers' payoff. Specifically, the shareholders' objective is now given by

$$\max_{w_i} E[-exp(-\lambda_P(P_i - \tilde{P}_i))]. \tag{1}$$

$\tilde{P}_i$ reflects the shareholders' relative wealth concerns. It can be related to the average of other shareholders' payoff (i.e., $h_{Pi}\int_0^1 P_i di$). Similar to the baseline model, we are always able to rewrite $\tilde{P}_i$ as $\tilde{P}_i = h_{Pi}(W_P + M_P\tilde{m})$ by the law of large numbers. All the other specifications are the same as in the baseline model except that the managers have no relative wealth concerns now, because we focus on studying how shareholders' relative wealth concerns affect managers' pay-for-luck. The manager i's utility is now given by $u(w_i, a_i) = -exp\left[-\lambda\left(w_i - \frac{1}{2}a_i^2\right)\right]$.

Since w_i is linear in $\tilde{m}$ and η_i, $P_i = V_i - w_i$ is also in $\tilde{m}$ and η_i. Thus, the shareholders' objective is to maximize $E[P_i - \tilde{P}_i] - \frac{1}{2}\lambda_P Var[P_i - \tilde{P}_i]$. For manager i, we still have the binding participation constraint and incentive compatibility constraint, which imply that in equilibrium (note that $\bar{a}_i = a_i$), we must have $\alpha_i + \beta_i a_i\pi - \frac{1}{2}\lambda\gamma_i^2\sigma_m^2 - \frac{1}{2}\lambda\beta_i^2\sigma_\eta^2 - \frac{1}{2}a_i^2 = \bar{u}_i$, and $\beta_i\pi - \lambda\beta_i\gamma_i\sigma_m^2 - a_i = 0$. So it is straight-forward to derive that the shareholders' optimization problem is

$$\max_{\beta_i,\gamma_i,a_i} a_i\pi - \frac{1}{2}\lambda_P[(a_i - \gamma_i - h_{Pi}M_P)^2\sigma_m^2 + (1-\beta_i)^2\sigma_\eta^2] - \frac{1}{2}\lambda(\gamma_i^2\sigma_m^2 + \beta_i^2\sigma_\eta^2) - \frac{1}{2}a_i^2,$$

subject to $\beta_i = \frac{a_i}{\pi - \lambda\gamma_i\sigma_m^2}$. Substituting $\beta_i = \frac{a_i}{\pi - \lambda\gamma_i\sigma_m^2}$ into the objective function and taking first-order condition with respect to a_i and γ_i yield that

$$\pi - \lambda_P\left[(a_i - \gamma_i - h_{Pi}M_P)\sigma_m^2 - \frac{1}{\pi - \lambda\gamma_i\sigma_m^2}\left(1 - \frac{a_i}{\pi - \lambda\gamma_i\sigma_m^2}\right)\sigma_\eta^2\right] - \lambda\frac{a_i}{(\pi - \lambda\gamma_i\sigma_m^2)^2}\sigma_\eta^2 - a_i = 0,$$
$$\lambda_P\left[(a_i - \gamma_i - h_{Pi}M_P)\sigma_m^2 + \frac{\lambda\sigma_m^2 a_i}{(\pi - \lambda\gamma_i\sigma_m^2)^2}\left(1 - \frac{a_i}{\pi - \lambda\gamma_i\sigma_m^2}\right)\sigma_\eta^2\right] - \lambda\left[\gamma_i\sigma_m^2 + \frac{\lambda\sigma_m^2 a_i^2}{(\pi - \lambda\gamma_i\sigma_m^2)^3}\sigma_\eta^2\right] = 0.$$

To obtain a closed-form solution, we study a special case where σ_η^2 approaches to zero. In this case, we will have $\pi - \lambda_P(a_i - \gamma_i - h_{Pi}M_P)\sigma_m^2 - a_i = 0$ and $\lambda_P(a_i - \gamma_i - h_{Pi}M_P) - \lambda\gamma_i = 0$. Then we obtain that in equilibrium, we have $a_i^* = \dfrac{(\lambda_P + \lambda)\pi + \lambda_P\lambda\sigma_m^2 h_{Pi}M_P}{(\lambda_P + \lambda) + \lambda_P\lambda\sigma_m^2}$ and $\gamma_i^* = \dfrac{\lambda_P(\pi - h_{Pi}M_P)}{(\lambda_P + \lambda) + \lambda_P\lambda\sigma_m^2}$.

Proposition 1. *Suppose that shareholders have relative wealth concerns as represented by (1). If* σ_η^2 *approaches to zero, then in equilibrium, we have* $a_i^* = \dfrac{(\lambda_P + \lambda)\pi + \lambda_P\lambda\sigma_m^2 h_{Pi}M_P}{(\lambda_P + \lambda) + \lambda_P\lambda\sigma_m^2}$ *and* $\gamma_i^* = \dfrac{\lambda_P(\pi - h_{Pi}M_P)}{(\lambda_P + \lambda) + \lambda_P\lambda\sigma_m^2}$. *This implies that (1) if* $\lambda_P = 0$*, then* $\gamma_i^* = 0$*; (2)* $\lambda_P > 0$*, then* γ_i^* *is decreasing in* h_{Pi}.

If the shareholders are risk neutral, it is costless for them to bear the risk, and they are willing to be exposed to the luck shock as much as possible. So they will always make managers have zero exposure to the luck shock regardless of their relative wealth concerns.

If the shareholders are risk averse, then γ_i is positive. This is due to the risk-sharing effect as shown in Ozdenoren and Yuan (2016). When the shareholders have relative wealth concerns, then similarly this implies that shareholders are now more willing to be exposed to the luck shock to keep up with their peers. In other words, it is now less costly for shareholders to bear the risk associated with the luck shock. As a result, they will increase their exposure to the luck shock, and reduce the managers' exposure to the luck shock.