Online Appendix

MONETARY POLICY IN SUDDEN STOP-PRONE ECONOMIES

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A Proofs

A.1 Proof of Proposition 1

In the absence of credit frictions, the government's optimisation problem reduces to

$$\mathcal{V}(b,s) = \max_{c^T,c^N,\ell,b',p^N,\pi^N} u\left[c\left(c^T,c^N\right),\ell\right] + \beta\mathbb{E}_{s'|s}\mathcal{V}(b',s')$$

subject to

$$c^N = \alpha(p^N)c^T \tag{A.1}$$

$$c^N = \left[1-\frac{\varphi}{2}(\pi^N)^2\right]A\ell \tag{A.2}$$

$$c^T = y^T + b - \frac{b'}{R} \tag{A.3}$$

$$0 = u_T(c^T,c^N) - \beta R\mathbb{E}_{s'|s}u_T\left(\mathcal{C}^T(b',s'),\mathcal{C}^N(b',s')\right) \tag{A.4}$$

$$0 = \varphi\pi^N(1+\pi^N) + (\varepsilon-1)\left[1-z^{-1}(1-\omega)\right] - \varphi\ell^{-1}\mathbb{E}_{s'|s}\Lambda\left[\mathcal{L}(b',s')\mathcal{M}(b',s')\right] \tag{A.5}$$

Let $\iota^* \geq 0$ denote the non-negative Lagrange multiplier on the resource constraint for non-tradable goods (A.2), $\lambda^* \geq 0$ the multiplier on the resource constraint for tradable goods (A.3). δ^*, υ^*, ξ^* are respectively the Lagrange multipliers on the static implementability constraint (A.1) and the dynamic implementability constraints (A.4) and (A.5). We define:

$$\psi \equiv (\varepsilon-1)\left[1-z^{-1}(1-\omega)\right] - \varphi\ell^{-1}\mathbb{E}_{s'|s}\Lambda\left[\mathcal{L}(b',s')\mathcal{M}(b',s')\right]. \tag{A.6}$$

The proof proceeds by analyzing a relaxed problem where the government is not subject to (A.4). Then, we show that a price stability policy solves the relaxed problem of the government, and the omitted implementability condition (A.4) is always satisfied under this policy. Abstracting from (A.4), the first order conditions of the relaxed problem are:

$$p^N :: \quad \delta^* = 0$$

$$c^T :: \quad \lambda^* = u_T(c^T,c^N) + \xi^*\psi_T \tag{A.7}$$

$$b' :: \quad \lambda^* = \beta R\mathbb{E}_{s'|s}\lambda^{*\prime} + \xi^*\psi_{b'} \tag{A.8}$$

$$\ell :: \quad u_\ell(\ell) + \iota^*\left[1-\frac{\varphi}{2}(\pi^N)^2\right]A + \xi^*\psi_\ell = 0 \tag{A.9}$$

$$c^N :: \quad \iota^* = u_N(c^T,c^N) + \xi^*\psi_N \tag{A.10}$$

$$\pi^N :: \quad \xi^* = \frac{\varphi\iota^* y^N}{1+2\pi^N}\pi^N \tag{A.11}$$

where ψ_T, ψ_N and ψ_ℓ represent the derivative of ψ with respect to c^T, c^N and ℓ, respectively. We combine (A.10) and (A.9) to obtain

$$\left[\omega - \frac{\varphi}{2}(\pi^N)^2\right] y^N u_N(c^T, c^N) = -\left(c^N\psi_N + \ell\psi_\ell\right)\xi^* \tag{A.12}$$

We now show that a price stability policy is the solution to the relaxed problem of the government. Consider a price stability policy, that is $\pi^N = \pi^{N\prime} = 0$. The optimality condition (A.11) implies that $\xi^* = 0$. Substituting into $\xi^* = 0$ and $\pi^N = 0$ into (A.12) yields $\omega = 0$. Finally, plugging $\xi^* = 0$ into (A.7) and (A.8), and combining both optimality conditions lead to

$$u_T(c^T, c^N) = \beta R \mathbb{E}_{s'|s} u_T\left(c^{T\prime}, c^{N\prime}\right). \tag{A.13}$$

Thus, $\pi^N = \pi^{N\prime} = 0$ is a solution to the relaxed problem of the government, and the omitted constraint (A.4) is always satisfied. It follows that a price stability policy is the optimal monetary policy in the absence of credit constraints. In addition, this policy perfectly stabilizes the economy ($\omega = 0$).

A.2 Proof of Proposition 2

To characterize the optimal time-consistent monetary policy, we solve for the government's optimization problem (A.14) taking as given policies $\{\mathcal{C}^T(b,s), \mathcal{C}^N(b,s), \mathcal{L}(b,s), \mathcal{V}(b,s)\}$ and $\mathcal{M}(b,s)$. The problem of the government is given by

$$\mathcal{V}(b,s) = \max_{c^T, c^N, \ell, b', p^N, \pi^N, \mu} u\left[c\left(c^T, c^N\right), \ell\right] + \beta \mathbb{E}_{s'|s}\mathcal{V}(b', s') \tag{A.14}$$

subject to

$$c^N = \alpha(p^N)c^T \tag{A.15}$$

$$c^N = \left[1 - \frac{\varphi}{2}(\pi^N)^2\right] A\ell \tag{A.16}$$

$$c^T = y^T + b - \frac{b'}{R} \tag{A.17}$$

$$\frac{b'}{R} \geq -\kappa\left(y^T + p^N c^N\right) \tag{A.18}$$

$$\mu = u_T(c^T, c^N) - \beta R \mathbb{E}_{s'|s} u_T\left(\mathcal{C}^T(b', s'), \mathcal{C}^N(b', s')\right) \tag{A.19}$$

$$\mu \times \left[\frac{b'}{R} + \kappa\left(y^T + p^N c^N\right)\right] = 0 \tag{A.20}$$

$$0 = \varphi\pi^N(1 + \pi^N) + (\varepsilon - 1)\left[1 - z^{-1}(1 - \omega)\right] - \varphi\ell^{-1}\mathbb{E}_{s'|s}\Lambda\left[\mathcal{L}(b', s')\mathcal{M}(b', s')\right] \tag{A.21}$$

Let $\iota^* \geq 0$ denotes the non-negative Lagrange multiplier on the resource constraint for non-tradable goods (A.16), $\lambda^* \geq 0$ the non-negative multiplier on the resource constraint for tradable goods (A.17), and $\mu^* \geq 0$ is the non-negative multiplier on the credit constraint (A.18). $\delta^*, v^*, \nu^*, \xi^*$ are respectively the multiplier on the static implementability constraint (A.15) and the dynamic implementability constraints (A.19), (A.20) and (A.21). ψ is defined as in (A.6).

We denote by $\tilde{\mu}^* \equiv \mu^* + \mu\nu^*$ the government's effective shadow value on the credit constraint. The optimality conditions of the government problem, after eliminating the multiplier δ^*, are:

$$c^T :: \quad \lambda^* = u_T(c^T, c^N) + \tilde{\mu}^* \gamma^{-1} \kappa \frac{p^N c^N}{c^T} - v^* u_{TT}(c^T, c^N) + \xi^* \psi_T \tag{A.22}$$

$$b' :: \quad \lambda^* = \beta R \mathbb{E}_{s'|s} \lambda^{*\prime} + \tilde{\mu}^* + v^* \beta R \mathbb{E}_{s'|s} \frac{\partial u_T \left(\mathcal{C}^T(b', s'), \mathcal{C}^N(b', s')\right)}{\partial b'} + \xi^* \psi_{b'} \tag{A.23}$$

$$\mu :: \quad v^* + \nu^* \left[\frac{b'}{R} + \kappa \left(y^T + p^N c^N\right)\right] + \xi^* \psi_\mu = 0 \tag{A.24}$$

$$\ell :: \quad u_\ell(\ell) + \iota^* \left[1 - \frac{\varphi}{2}(\pi^N)^2\right] A + \xi^* \psi_\ell = 0 \tag{A.25}$$

$$c^N :: \quad \iota^* = u_N(c^T, c^N) + \tilde{\mu}^*(1 - \gamma^{-1}) \kappa p^N - v^* u_{TN}(c^T, c^N) + \xi^* \psi_N \tag{A.26}$$

$$\pi^N :: \quad \xi^* = \frac{\varphi \iota^* y^N}{1 + 2\pi^N} \pi^N \tag{A.27}$$

Combining (A.26) and (A.25), we obtain

$$\left[\omega_t - \frac{\varphi}{2}(\pi^N)^2\right] u_N(c^T, c^N) y^N - u_{TN}(c^T, c^N) c^N v^* + \left(1 - \gamma^{-1}\right) \kappa p^N c^N \tilde{\mu}^* = -\xi^* \left(c^N \psi_N + \ell \psi_\ell\right) \tag{A.28}$$

Substituting (A.27) into (A.28) and letting $\tilde{\iota}^* \equiv \frac{\iota^*}{(1+2\pi^N) u_N}$, we get

$$\varphi \left[-(c^N \psi_N + \ell \psi_\ell) \tilde{\iota}^* + \frac{\pi^N}{2}\right] u_N(c^T, c^N) y^N \pi^N = u_N(c^T, c^N) y^N \omega + \left(\sigma - \gamma^{-1}\right) \frac{c_T(c^T, c^N)}{c} u_N(c^T, c^N) c^N v^* + \left(1 - \gamma^{-1}\right) \kappa p^N c^N \tilde{\mu}^* \tag{A.29}$$

where $c_T(c^T, c^N)$ is the derivative of the consumption function $c(c^T, c^N)$ with respect to c^T.

To calculate $(c^N\psi_N + \ell\psi_\ell)$ in (A.29), note that

$$1 - z^{-1}(1-\omega) = 1 + \left[u_N(c^T, c^N) + \kappa\mu\left(\frac{1-a}{a}\right)\left(\frac{c^T}{c^N}\right)^{1/\gamma}\right]^{-1}\frac{u_\ell(\ell)}{A} \tag{A.30}$$

Plugging (A.30) into the expression of the auxiliary variable ψ_t and taking the derivative of ψ_t with respect to c_t^N and ℓ_t, we get

$$c_t^N\psi_{N,t} + \ell_t\psi_{\ell,t} = -(\varepsilon-1)z_t^{-1}\left[\frac{-c_t^N u_{NN}(t) + \gamma^{-1}\kappa\mu_t p_t^N}{z_t u_N(t)} + \frac{\ell_t u_{\ell\ell}(t)}{u_\ell(t)}\right]\frac{-u_\ell(t)}{Au_N(t)} + \left[\frac{c_t^N u_{NN}(t)}{u_N(t)} + 1\right]\frac{\varphi}{\ell_t}\mathbb{E}_t\left[\Lambda_{t,t+1}\mathcal{L}(t+1)\mathcal{M}(t+1)\right] \tag{A.31}$$

Finally, substituting (A.31) into (A.29) yields (in sequential form)

$$\varphi\left[\Delta_0 + \mathbb{E}_t(\Delta_1\pi_{t+1}^N) + \frac{\pi_t^N}{2}\right]y_t^N\pi_t^N = y_t^N\omega_t + (\sigma-\gamma^{-1})\frac{c_T(t)}{c_t}c_t^N v_t^* + (1-\gamma^{-1})\frac{\kappa p_t^N c_t^N}{u_N(t)}\tilde{\mu}_t^* \tag{A.32}$$

where Δ_0 and Δ_1 are given by

$$\Delta_0 \equiv (\varepsilon-1)z_t^{-1}\left[\frac{-c_t^N u_{NN}(t) + \gamma^{-1}\kappa p_t^N\mu_t}{z_t u_N(t)} + \ell_t\frac{u_{\ell\ell}(t)}{u_\ell(t)}\right]\tilde{\imath}_t^*\frac{-u_\ell(t)}{Au_N(t)} > 0, \tag{A.33}$$

$$\Delta_1 \equiv \left[\frac{-c_t^N u_{NN}(t)}{u_N(t)} - 1\right]\frac{\tilde{\imath}_t^*}{\ell_t}\Lambda_{t,t+1}\ell_{t+1}(1+\pi_{t+1}^N). \tag{A.34}$$

We now describe the optimal policy in deviation from the efficient allocation described in proposition 1 using the first order Taylor series expansions. Recall that for some arbitrary function $f(x)$, where $x \equiv \{x_i\}_i$, the first order Taylor series expansions evaluated at the point $\bar{x} = \{\bar{x}_i\}_i$ is given by

$$f(x) \approx f(\bar{x}) + \sum_i f'_{x_i}(\bar{x})(x_i - \bar{x}_i)$$

f'_{x_i} denotes the partial derivative of the function with respect to x_i. We apply the Taylor expansion to (A.32) evaluated at the efficient allocation (described in proposition **??**). Letting $\hat{x} \equiv x - \bar{x}$ indicate the deviation of a variable x from its value in the efficient allocation (described in proposition 1), it is straightforward to see that $\hat{\pi}^N = \pi^N$, $\hat{\omega} = \omega$, $\hat{\mu}^* = \mu^*$, $\hat{\mu} = \mu$ and $\hat{v}^* = v^*$. Recall that $\tilde{\mu}^* \equiv \mu^* + \mu v^*$. Thus, to a first order, $\hat{\tilde{\mu}} = \hat{\mu}^* = \mu$.

Letting $\phi \equiv \frac{\ell_t u_{\ell\ell}(t)}{u_\ell(t)}$ denotes the inverse of the Frisch elasticity of labor supply, the first order approximation of (A.32) yields

$$\varphi(\varepsilon-1)\left[\frac{-\bar{c}_t^N \bar{u}_{NN}(t)}{\bar{u}_N(t)}+\phi\right]\pi_t^N=\omega_t+(\sigma-\gamma^{-1})\frac{\bar{c}_T(t)}{\bar{c}_t}v_t^*+(1-\gamma^{-1})\frac{\kappa}{\bar{u}_T(t)}\mu_t^* \quad \text{(A.35)}$$

where (A.26) is used to obtain $\bar{\iota}^*=\bar{u}_N$ which implies that $\bar{\tilde{\iota}}^*=1$ (at the efficient allocation). Finally, denoting by $a_t \equiv \frac{c^T}{c^T+p^N c^N}$ the share of expenditures in tradables, we have

$$\frac{-c_t^N u_{NN}(t)}{u_N(t)}=(1-a_t)\sigma+a_t\gamma^{-1} \quad \text{and} \quad \frac{1}{c_t}c_T(t)=\frac{1}{c_t^T}a_t$$

This completes the proof of proposition 2.

To understand why $v_t^*>0$ implies a non-binding credit constraint under a price stability policy, we start by first showing that $v_t^*>0$. To see this, combine (A.22) and (A.23) to obtain

$$\left[-u_{TT}(t)-\beta R_t\mathbb{E}_t\frac{\partial u_T(t+1)}{\partial b_{t+1}}\right]v_t^*$$
$$=\beta R_t\mathbb{E}_t\left[\gamma^{-1}\kappa\frac{p_{t+1}^N c_{t+1}^N}{c_{t+1}^T}\mu_{t+1}^*\right]+\beta R_t\mathbb{E}_t\left[-v_{t+1}^* u_{TT}(t+1)\right] \quad \text{(A.36)}$$

Then, we iterate (A.36) forward and use the transversality condition to get

$$v_t^*=\sum_{j=1}^{\infty}\beta^j\mathbb{E}_t\left[\left(\prod_{k=0}^{j-1}\frac{R_{t+k}}{Q_{t+k}}\right)\Theta_{t+j}\mu_{t+j}^*\right]>0$$

where $\Theta_t \equiv \gamma^{-1}\kappa p_t^N c_t^N/c_t^T>0$ and $Q_t=1+\frac{\beta R_t\partial u_T(t+1)/\partial b_{t+1}}{u_{TT}(t)}>1$. The optimality condition with respect to μ_t (A.24) can be rewritten as

$$v_t^*=-v_t^*\left[\frac{b_{t+1}}{R_t}+\kappa\left(y_t^T+p_t^N c_t^N\right)\right] \quad \text{(A.37)}$$

Therefore, $v_t^*>0$ implies that $\frac{b_{t+1}}{R_t}>-\kappa\left[y_t^T+p_t^N c_t^N\right]$, that is the credit constraint does not bind or equivalently $\mu_t^*=0$. Conversely, a binding credit constraint $\frac{b_{t+1}}{R_t}=-\kappa\left[y_t^T+p_t^N c_t^N\right]$ (or equivalently $\mu_t^*>0$) implies that $v_t^*=0$.

A.3 Proof of Proposition 3

Taking as given policies $\{\mathcal{C}^T(b,s), \mathcal{C}^N(b,s), \mathcal{L}(b,s), \mathcal{V}(b,s)\}$ and $\mathcal{M}(b,s)$, the optimization problem of the government when capital controls are available is given by

$$\mathcal{V}(b,s) = \max_{\tau, c^T, c^N, \ell, b', p^N, \pi^N, \mu} u\left[c\left(c^T, c^N\right), \ell\right] + \beta \mathbb{E}_{s'|s} \mathcal{V}(b', s') \tag{A.38}$$

subject to

$$c^N = \alpha(p^N) c^T \tag{A.39}$$

$$c^N = \left[1 - \frac{\varphi}{2}(\pi^N)^2\right] A\ell \tag{A.40}$$

$$c^T = y^T + b - \frac{b'}{R} \tag{A.41}$$

$$\frac{b'}{R} \geq -\kappa\left(y^T + p^N c^N\right) \tag{A.42}$$

$$\mu = u_T(c^T, c^N) - \beta R(1+\tau) \mathbb{E}_{s'|s} u_T\left(\mathcal{C}^T(b', s'), \mathcal{C}^N(b', s')\right) \tag{A.43}$$

$$\mu \times \left[\frac{b'}{R} + \kappa\left(y^T + p^N c^N\right)\right] = 0 \tag{A.44}$$

$$0 = \varphi \pi^N (1 + \pi^N) + (\varepsilon - 1)\left[1 - z^{-1}(1-\omega)\right] - \varphi \ell^{-1} \mathbb{E}_{s'|s} \Lambda\left[\mathcal{L}(b', s') \mathcal{M}(b', s')\right] \tag{A.45}$$

Once again, let δ_t^*, ι_t^*, λ_t^*, μ_t^*, v_t^*, ν_t^* and ξ_t^* be the Lagrange multipliers on constraints (A.39)-(A.45). Notice how τ only appears in (A.43). The first-order condition with respect to τ yields, $v^* = 0$. In other words, τ can be dropped from the government's problem along with the implementability constraint (A.43). Defining ψ as in (A.6), the optimality conditions of the problem of the government are given by:

$$c^T :: \quad \lambda^* = u_T(c^T, c^N) + \mu^* \gamma^{-1} \kappa \frac{p^N c^N}{c^T} + \xi_t^* \psi_T \tag{A.46}$$

$$b' :: \quad \lambda^* = \beta R \mathbb{E}_{s'|s} \lambda^{*\prime} + \mu^* + \xi^* \psi_{b'} \tag{A.47}$$

$$\ell :: \quad u_\ell(\ell) + \iota^* \left[1 - \frac{\varphi}{2}(\pi^N)^2\right] A + \xi^* \psi_\ell = 0 \tag{A.48}$$

$$c^N :: \quad \iota^* = u_N(c^T, c^N) + \tilde{\mu}^*(1 - \gamma^{-1}) \kappa p^N + \xi^* \psi_N \tag{A.49}$$

$$\pi^N :: \quad \xi^* = \frac{\varphi \iota^* y^N}{1 + 2\pi^N} \pi^N \tag{A.50}$$

We combine (A.49) and (A.48) to obtain

$$\left[\omega_t - \frac{\varphi}{2}(\pi^N)^2\right] u_N(c^T, c^N) y^N + \left(1 - \gamma^{-1}\right) \kappa p^N c^N \mu^* = -\xi^* \left(c^N \psi_N + \ell \psi_\ell\right) \tag{A.51}$$

Then, substituting (A.50) into (A.51), we get

$$\varphi\left[-(c^N\psi_N+\ell\psi_\ell)\tilde{\iota}^*+\frac{\pi^N}{2}\right]u_N(c^T,c^N)y^N\pi^N=u_N(c^T,c^N)y^N\omega+\left(1-\gamma^{-1}\right)\kappa p^N c^N\mu^* \tag{A.52}$$

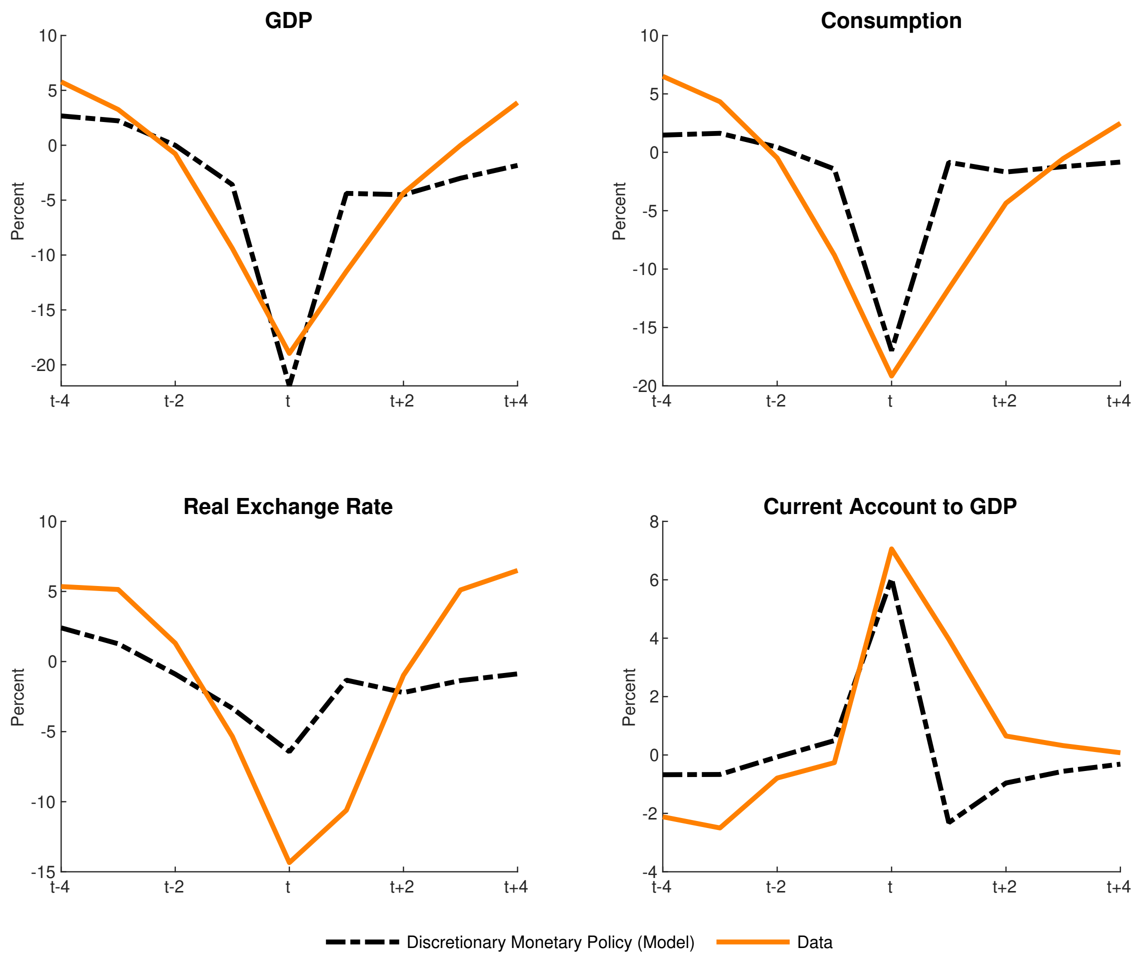
where the expression for $(c_t^N\psi_{N,t}+\ell_t\psi_{\ell,t})$ is given by (A.31), which evaluated at the efficient allocation is given by $\varphi\bar{\ell}_t\bar{u}_N(t)(\varepsilon-1)\left[(1-\bar{a}_t)\sigma+\bar{a}_t\gamma^{-1}+\phi\right]$. Thus, taking the first order approximation of (A.52) around the efficient allocation, we arrive to

$$\varphi(\varepsilon-1)\left[(1-\bar{a}_t)\sigma+\bar{a}_t\gamma^{-1}+\phi\right]\pi_t^N=\omega_t+(1-\gamma^{-1})\frac{\kappa}{\bar{u}_T(t)}\mu_t^* \tag{A.53}$$

This completes the proof of Proposition 3.

B Crises Dynamics: Model and Data

Figure 1 shows that the model predictions for the aggregate dynamics of key macroeconomics variables (GDP, consumption, real exchange rate, and current-account to GDP ratio) are aligned with the dynamics of these variables during sudden-stop episodes observed in Argentina data. In the model, GDP, consumption, and real exchange rate are expressed in percentage deviations from averages in the ergodic distribution. In the data, GDP, consumption, and the real exchange rate are expressed in log deviations from a log-linear trend.



Note: Data corresponds to the average of two sudden stop episodes, in which $t = 0$ is set to 1990 and 2002.

Figure 1: Comparison of crises dynamics.

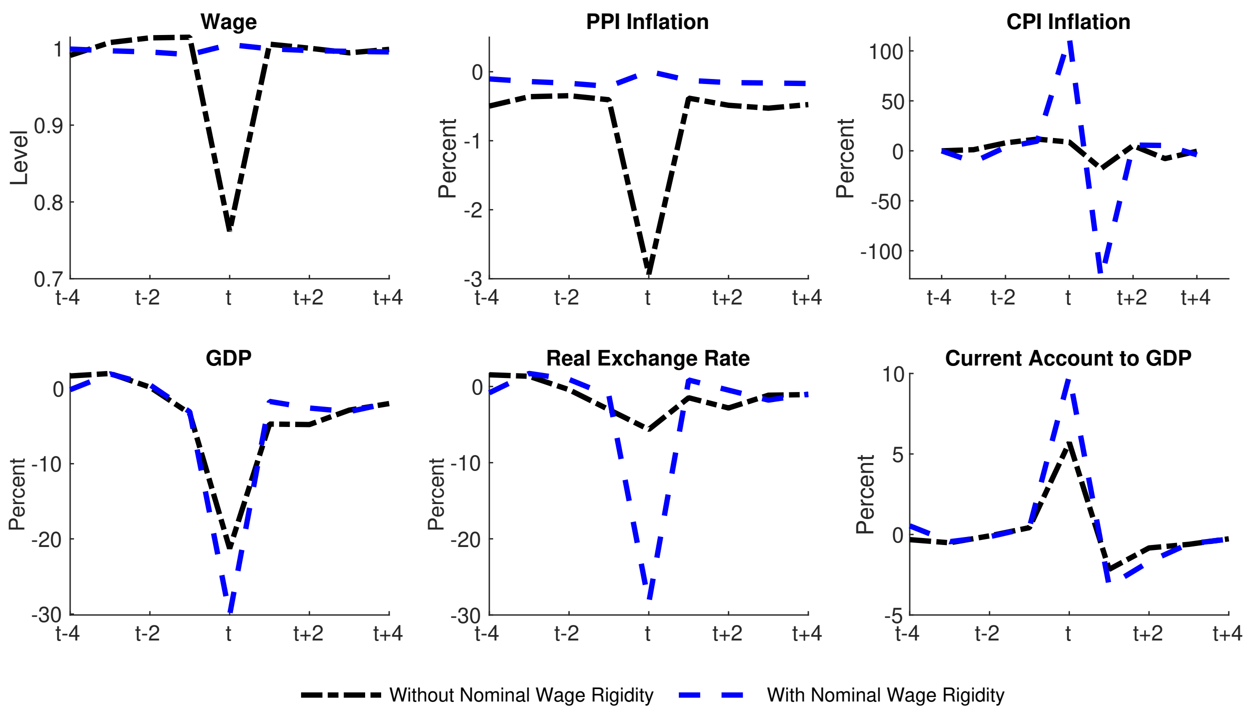
C Robustness and Extensions

C.1 Downward Wage Rigidity

This section discusses the qualitative insights of the model in the presence of nominal wage rigidities. Specifically, we assume there exists a minimum wage in nominal terms, $\bar{W}$, such that

$$W_t \geq \bar{W}$$

In Schmitt-Grohé and Uribe (2016), this minimum wage corresponds a fraction γ of the previous period wage, γW_{t-1}. The estimated value of the parameter γ, governing the degree of downward nominal wage rigidity, is found to be close to unity in emerging market economies ($\gamma = 0.99$). The next figure presents the quantitative implications of the model for $\bar{W} = \tilde{W}_{t-1}$ where $\tilde{W}_{t-1}$ corresponds to the previous period's full-employment nominal wage, that is the nominal wage that would prevail under flexible price allocation at period $t-1$.[1]



Note: Wages are nominal wages expressed in units of nontradables. GDP and real exchange rate are expressed in percentage deviations from averages in the ergodic distribution.

Figure 2: Comparison of crisis dynamics.

[1] The use of the previous period full-employment nominal wage $\tilde{W}_{t-1}$ has the advantage of not introducing an additional endogenous state variable in the model and makes the quantitative results easy to compare with the quantitative findings in the absence of downward nominal wage rigidity.

Figure 2 shows that financial crises are more severe in an economy in which nominal wages are downwardly rigidity relative to an economy with flexible wages considered. The blue dashed line represents the crisis dynamics in the former economy, while the black solid line represents the crisis dynamics in the absence of wage rigidity. When wages are downwardly rigid, firms face a higher marginal cost in midst of a financial crisis (top-left panel) leading to a larger decline in production (total output drops by 30% vs 21.9% when wages are flexible). The sharp decline in total output reduces significantly the value of the collateral which in turn induces more capital outflows (bottom-right panel) and larger real exchange depreciation (-25% vs -5% when wages are flexible). The CPI inflation turns out to be more important in this economy during sudden stops as the stickiness of nominal wages (downward rigidity) prevents firms from reducing the prices of non-tradable goods.

C.2 Discretionary Monetary Policy with GHH Preferences

This section derives the discretionary monetary policy in an environment in which households' preferences are specified following Greenwood et al. (1988), where utility is defined in terms of the excess of consumption over the disutility of labor $u(c, \ell) = u(c - g(\ell))$, with $g' > 0$ and $g'' > 0$. With these preferences, the marginal utility of tradables depends also on leisure and we have for the Euler equation

$$u_T(c^T, c^N, \ell) - \beta R \mathbb{E}_{s'|s} u_T\left(\mathcal{C}^T(b', s'), \mathcal{C}^N(b', s'), \mathcal{L}(b', s')\right) = \mu_t \tag{A.54}$$

The proof follows the same steps as in Appendix A.2. We focus here only on the key equations. The government's problem is similar to the government's problem in appendix A.2 with the The discretionary monetary policy solves (A.14) subject to (A.15), (A.16), (A.17), (A.18), (A.54), (A.20) and (A.21).

Let δ_t^*, ι_t^*, λ_t^*, μ_t^*, v_t^*, ν_t^* and ξ_t^* be the Lagrange multipliers on implementability constraints (A.15)-(A.18), (A.54), (A.20) and (A.21) respectively. We follow the same steps as in appendix A.2 and focus here on the key equations. Combining the first order conditions with respect to labor and non-tradable consumption, we obtain

$$\left[\omega_t - \frac{\varphi}{2}(\pi^N)^2\right] u_N(c^T, c^N) y^N + \left[\left(\sigma q - \gamma^{-1}\right) + \sigma q \frac{\ell g'(\ell)}{c^N c_N(\cdot)}\right] \frac{c_T(c^T, c^N)}{c} u_N(c^T, c^N) c^N v^* \\ + \left(1 - \gamma^{-1}\right) \kappa p^N c^N \tilde{\mu}^* = -\xi^* \left(c^N \psi_N + \ell \psi_\ell\right) \tag{A.55}$$

with $q_t \equiv \frac{c_t}{c_t - g(\ell_t)} > 1$ and σ is the relative rsik-aversion coefficient $\sigma \equiv \frac{-cu''(c-g(\ell))}{u'(c-g(\ell))}$. The expression for $(c_t^N \psi_{N,t} + \ell_t \psi_{\ell,t})$ is given by (A.31). We then use the first order condition with respect to π_t^N

$$\xi_t^* = \frac{\varphi \iota_t^* y_t^N}{1 + 2\pi_t^N} \pi_t^N$$

to substitute for ξ_t^* in (A.55) and after some algebraic manipulation – see appendix A.2 for more details – we arrive to

$$\varphi \left[\Delta_0 + \mathbb{E}_t \left(\Delta_1 \pi_{t+1}^N \right) + \frac{\pi_t^N}{2} \right] y_t^N \pi_t^N = y_t^N \omega_t$$
$$+ \left[\left(\sigma q_t - \gamma^{-1} \right) + \sigma q_t \frac{\ell_t g'(\ell_t)}{c_t^N c_N(t)} \right] \frac{c_T(t)}{c_t} v_t^* + (1 - \gamma^{-1}) \frac{p_t^N c_t^N}{u_N(t)} \kappa \tilde{\mu}_t^* \quad \text{(A.56)}$$

where Δ_0 and Δ_1 are defined in (A.33) and (A.34). Using a first order Taylor series expansions, the optimal discretionary monetary policy (A.56) becomes

$$\omega_t = \varphi(\varepsilon - 1) \left[(1 - \bar{a}_t)\sigma + \bar{a}_t \gamma^{-1} + \phi \right] \pi_t^N$$
$$- \left[(\sigma \bar{q}_t - \gamma^{-1}) \bar{a}_t + \sigma \phi \bar{q}_t \frac{g''(\bar{\ell}_t)}{\bar{c}_t} \right] \frac{1}{\bar{c}_t^T} v_t^* + (\gamma^{-1} - 1) \frac{\kappa}{\bar{u}_T(t)} \mu_t^*$$

where ϕ is the inverse of the Frisch elasticity of labor supply. Because for $1/\sigma < \gamma < 1$, we have $\left[(\sigma \bar{q}_t - \gamma^{-1}) \bar{a}_t + \sigma \phi \bar{q}_t \frac{g''(\bar{\ell}_t)}{\bar{c}_t} \right] > 0$, we have, as discussed in Proposition 2, that when $1/\sigma < \gamma < 1$ the discretionary monetary policy is procyclical.

C.3 Commitment

The focus throughout has been on the government's optimal time-consistent plans, and we thus studied the discretionary monetary policy (when the government lacks commitment). We show here how the ability to commit to future policies affects qualitatively the design of the optimal monetary policy. Under commitment, policy rules are chosen by the government at date 0 in a once-and-for-all fashion. The next proposition characterizes the optimal monetary policy when the government can commit to future policies (see proof below).

Proposition 1 (Optimal Monetary Policy without Capital Flow Taxes)**.** *Under commitment,* *to a first-order the optimal monetary policy targets*

$$\omega_t = \varphi(\varepsilon - 1)\left[(1-\bar{a}_t)\sigma + \bar{a}_t\gamma^{-1} + \phi\right]\hat{p}_t^N - (\sigma - \gamma^{-1})\frac{\bar{a}_t}{\bar{c}_t^T}(v_t^* - R_{t-1}v_{t-1}^*) + (\gamma^{-1} - 1)\frac{\kappa}{\bar{u}_T(t)}\mu_t^* \tag{A.57}$$

where $\hat{p}_t^N = \log(P_t^N / P_{-1}^N)$ *is the log deviation between the price level and an "implicit target" given by the price level prevailing one period before the government chooses its optimal plan.*

Equation (A.57) can be viewed as the targeting rule that the government must follow period-by-period to implement the optimal policy under commitment. There are two key differences between (A.57) and the corresponding targeting rule for the discretionary monetary policy (28) that are worth pointing out. First, given the financial conditions, the discretionary monetary policy (28) requires that the government keep the labor wedge positive (negative), or equivalently that it implement a contractionary (expansionary) policy as long as inflation is positive (negative). By way of contrast, under the optimal policy with commitment (A.57), the government sets the sign and size of the labor wedge in proportion to the deviations of the price level from its implicit target. Second, the presence of the lagged multipliers in the third term, v_{t-1}^*, results from the time-inconsistency of the government's problem under commitment. According to (A.57), the government internalizes how a more contractionary monetary policy when the credit constraint binds at time t helps relax the borrowing constraint at time t and also encourages borrowing at time $t-1$.

Note that with capital controls, from the first order condition with respect to τ_t the Lagrange multiplier on households' Euler equation for bonds is zero at all times t. The proposition below describes the optimal monetary policy with capital controls.

Proposition 2 (Optimal Monetary Policy with Capital Flow Taxes)**.** *Under commitment, to a first-order, the optimal monetary policy targets*

$$\omega_t = \varphi(\varepsilon - 1)\left[(1-\bar{a}_t)\sigma + \bar{a}_t\gamma^{-1} + \phi\right]\hat{p}_t^N + (\gamma^{-1} - 1)\frac{\kappa}{\bar{u}_T(t)}\mu_t^*$$

where $\hat{p}_t^N = \log(P_t^N / P_{-1}^N)$ *is the log deviation between the price level and an "implicit target" given by the price level prevailing one period before the government chooses its optimal plan.*

Proof of Proposition 1 and 2 The problem of the government under commitment consists in choosing $\{c_t^T, c_t^N, \ell_t, b_{t+1}, p_t^N, \pi_t^N, \mu_t\}_{t\geq 0}$ at date 0 to solve

$$\max \ \mathbb{E}_t \sum_{t=0} \beta^t u \left[c\left(c_t^T, c_t^N\right), \ell_t \right]$$

subject to

$$c_t^N = \alpha(p_t^N) c_t^T \tag{A.58}$$

$$c_t^N = \left[1 - \frac{\varphi}{2}(\pi_t^N)^2\right] A\ell_t \tag{A.59}$$

$$c_t^T = y_t^T + b_t - \frac{b_{t+1}}{R_t} \tag{A.60}$$

$$\frac{b_{t+1}}{R_t} \geq -\kappa\left(y_t^T + p_t^N c_t^N\right) \tag{A.61}$$

$$\mu_t = u_T(c_t^T, c_t^N) - \beta R_t (1+\tau_t) \mathbb{E}_t u_T\left(c_{t+1}^T, c_{t+1}^N\right) \tag{A.62}$$

$$\mu_t \times \left[\frac{b_{t+1}}{R_t} + \kappa\left(y_t^T + p_t^N c_t^N\right)\right] = 0 \tag{A.63}$$

$$0 = \varphi(1+\pi_t^N)\pi_t^N \ell_t u_N(t) + \psi^o - \varphi\beta\mathbb{E}_t\left[u_N(t+1)\ell_{t+1}(1+\pi_{t+1}^N)\pi_{t+1}^N\right] \tag{A.64}$$

where ψ^o is defined as $\psi^o \equiv (\varepsilon - 1)\left[1 - z_t^{-1}(1-\omega_t)\right]\ell_t u_N(t)$. Let $\iota_t^* \geq 0$, $\lambda_t^* \geq 0$, $\mu_t^* \geq 0$, δ_t^*, v_t^*, ν_t^* and ξ_t^* denote the non-negative Lagrange multiplier on (A.59), (A.60), (A.61), (A.58), (A.62), (A.63) and (A.64). The proof proceeds by deriving the optimal monetary policy under commitment when capital controls are not used and then deriving the optimal monetary policy with capital controls under commitment. The optimality conditions, after eliminating the multiplier δ_t^*, are:

$$\pi_t^N :: \quad -\iota_t^* y_t^N \varphi \pi_t^N + \varphi(\xi_t^* - \xi_{t-1}^*)(1+2\pi_t^N) u_N(t)\ell_t = 0 \tag{A.65}$$

$$\mu :: \quad v_t^* + \nu_t^* \times \left[b_{t+1} + \kappa(p_t^N c_t^N + y_t^T)\right] + \xi_t^* \psi_\mu^o(t) = 0 \tag{A.66}$$

$$\ell_t :: \quad u_\ell(t) + \iota_t^* A\left[1 - \frac{\varphi}{2}(\pi_t^N)^2\right] + \varphi(\xi_t^* - \xi_{t-1}^*) u_N(t)(1+\pi_t^N)\pi_t^N + \xi_t^* \psi_\ell^o(t) = 0 \tag{A.67}$$

$$\begin{aligned} c_t^N :: \quad \iota_t^* = {} & u_N(t) + \tilde{\mu}_t^*(1-\gamma^{-1})\kappa p_t^N - \left[v_t^* - v_{t-1}^* R_{t-1}\right] u_{TN}(t) \\ & + \varphi(\xi_t^* - \xi_{t-1}^*) u_{NN}(t)\ell_t(1+\pi_t^N)\pi_t^N + \xi_t^* \psi_N^o(t) \end{aligned} \tag{A.68}$$

$$\begin{aligned} c_t^T :: \quad \lambda_t^* = {} & u_T(t) + \tilde{\mu}_t^* \gamma^{-1}\kappa \frac{p_t^N c_t^N}{c_t^T} - [v_t^* - v_{t-1}^* R_{t-1}] u_{TT}(t) \\ & + \varphi(\xi_t^* - \xi_{t-1}^*)\ell_t(1+\pi_t^N)\pi_t^N u_{NT}(t) + \xi_t^* \psi_T^o(t) \end{aligned} \tag{A.69}$$

$$b' :: \quad \lambda_t^* = \beta R_t \mathbb{E}_t \lambda_{t+1}^* + \tilde{\mu}_t^* \tag{A.70}$$

where $\tilde{\mu}_t^* \equiv \mu_t^* + \mu_t v_t^*$. At the optimum $\tilde{\mu}_t^* = \mu_t^*$ (the proof here follows the one in section A.2). Combining (A.68) and (A.67), We obtain

$$\Big[\omega_t - \frac{\varphi}{2}(\pi_t^N)^2\Big] y^N u_N(t) + \mu_t^*(1-\gamma^{-1})\kappa p_t^N c_t^N - \left[v_t^* - v_{t-1}^* R_{t-1}\right] c_t^N u_{TN}(t) \\ = -\varphi \ell_t [u_N(t) + c^N u_{NN}(t)](1+\pi_t^N)(\xi_t^* - \xi_{t-1}^*)\pi_t^N - (c_t^N \psi_{N,t}^o + \ell_t \psi_{\ell,t}^o)\xi_t^* \quad \text{(A.71)}$$

From equation (A.65), we have

$$\xi_t^* - \xi_{t-1}^* = A\tilde{\iota}_t^* \pi_t^N \quad \text{which implies that} \quad \xi_t^* = A\sum_{s=0}^{t} \tilde{\iota}_s^* \pi_s^N \qquad \text{(A.72)}$$

Moreover, differentiating ψ_t^o with respect to c_t^T and c_t^N leads to

$$c_t^N \psi_{N,t}^o + \ell_t \psi_{\ell,t}^o = -\varphi(\varepsilon-1)\ell_t u_N(t) \left\{ \left[1 - z_t^{-1}(1-\omega_t)\right] \left[\frac{-c_t^N u_{NN}(t)}{u_N(t)} - 1\right] \right. \\ \left. + \frac{-u_\ell(t)}{A u_N(t)} \left[\frac{-c_t^N u_{NN}(t) + \kappa \mu_t \gamma^{-1} p_t^N}{z_t u_N(t)} + \frac{\ell_t u_{\ell\ell}(t)}{u_\ell(t)}\right] \right\} \quad \text{(A.73)}$$

which evaluated at the efficient allocation reduces to $\varphi \bar{\ell}_t \bar{u}_N(t)(\varepsilon-1)\left[(1-\bar{a}_t)\sigma + \bar{a}_t\gamma^{-1} + \phi\right]$. The first order Taylor series expansions of (A.71) (after substituting (A.72) and (A.73)) evaluated at the efficient allocation, yields

$$\varphi\chi\pi_t^N = -\varphi\chi\sum_{s=1}^{t-1}\pi_s^N + \omega_t + (\sigma - \gamma^{-1})\frac{\bar{c}_T(t)}{\bar{c}_t}(v_t^* - R_{t-1}v_{t-1}^*) + (1-\gamma^{-1})\frac{\kappa}{\bar{u}_T(t)}\mu_t^* \quad \text{(A.74)}$$

where $\chi = (\varepsilon-1)\left[(1-\bar{a}_t)\sigma + \bar{a}_t\gamma^{-1} + \phi\right]$. Noting that $\sum_{s=1}^t \pi_s^N = \log P_t^N - \log P_{-1}^N = \hat{p}_t^N$ and rearranging (A.74) we obtain the first order approximation of the optimal monetary policy under commitment (in the absence of capital flow taxes)

$$\omega_t = \varphi(\varepsilon-1)\left[(1-\bar{a}_t)\sigma + \bar{a}_t\gamma^{-1} + \phi\right]\hat{p}_t^N - (\sigma-\gamma^{-1})\frac{\bar{a}_t}{\bar{c}_t^T}(v_t^* - R_{t-1}v_{t-1}^*) + (\gamma^{-1}-1)\frac{\kappa}{\bar{u}_T(t)}\mu_t^*.$$

The proof of proposition 2 is then straightforward. It follows from the fact that with capital flow taxes, the first-order condition with respect to τ_t is given by

$$v_t = 0 \quad \text{for all } t.$$

Thus, when capital controls are used optimally the optimal monetary policy under commitment targets

$$\omega_t = \varphi(\varepsilon - 1)\left[(1-\bar{a}_t)\sigma + \bar{a}_t\gamma^{-1} + \phi\right]\hat{p}_t^N + (\gamma^{-1}-1)\frac{\kappa}{\bar{u}_T(t)}\mu_t^*.$$

C.4 Future-Income Constraint

This section shows that a model in which future income is used as collateral cannot rationalize the observed procyclicality of monetary policy in emerging market economies.

Consider, in the spirit of Ottonello, Perez and Varraso (2021), an alternative formulation of the credit constraint in which future income, not current income, is used as collateral:

$$\frac{b_{t+1}^*}{R_t} \geq -\kappa \mathbb{E}_t\left[y_{t+1}^T + w_{t+1}\ell_{t+1} + \phi_{t+1}^N\right] \tag{A.75}$$

This constraint limits total debt denominated in units of tradables to a fraction of the minimum value of the household's future income, which is referred to as the future-income constraint. The microfoundations of this collateral constraint can be found in Ottonello, Perez and Varraso (2021). It is derived from an environment in which borrowers (households) lack commitment and can default in the repayment period, and in which foreign lenders can seize a fraction κ of the household's income if households default.

The household's problem in this environment can be formulated in recursive form as follows

$$\mathcal{V}(b,B,s) = \max_{c^T,c^N,\ell,b'} u\big(c(c^T,c^N),\ell\big) + \beta\mathbb{E}_{s'|s}V(b',B',s')$$

subject to

$$\begin{aligned} c^T + p^N(B,s)c^N + \frac{1}{1+\tau}\frac{b'}{R} &= y^T + w(B,s)\ell + \phi(B,s) + b + T \\ \frac{b'}{R} &\geq -\kappa\ \mathbb{E}_{s'|s}\Big[y^{T'} + w(B',s')\ \ell(b',B',s') + \phi(B',s')\Big] \\ B' &= \Gamma(B,s) \end{aligned}$$

The optimality conditions of the household's problem are

$$\frac{-u_\ell(t)}{u_N(t)} = \frac{w_t}{p_t^N} \tag{A.76}$$

$$u_T(t) = \beta R_t(1+\tau_t)\mathbb{E}_t u_T(t+1) + \mu_t\left[1 + \kappa\mathbb{E}_{t+1}w_{t+1}\frac{\partial \ell_{t+1}}{\partial b_{t+1}}\right] \tag{A.77}$$

The remaining optimality conditions of the household's problem, (6), (7), (8) and the problem of the firms remain unchanged, while the optimality condition is given by (12). The next proposition describes the optimal monetary and capital flow management policies in this environment.

Proposition 3 (Under Discretion)**.** *With future-income constraint, a price stability policy, $\pi_t^N = 0$ for all t, is the discretionary monetary policy. Furthermore, the optimal policy under discretion doesn't involve capital flow taxes $\tau_t = 0$ for all t.*

As is well understood in the literature, there is no pecuniary externality and the value of the collateral is not affected by current choices in an environment in which future income is used as collateral. Thus, under discretion, the current government focuses on stabilizing prices, $\pi_t^N = 0$ for all t, and replicating the flexible price allocation. In addition, capital flow taxes are not used, $\tau_t = 0$ for all t, since private agents' borrowing decisions coincide with the socially desirable level of debt from the viewpoint of the current government.[2]

Proof. Under discretion, the government takes as given future policies and solves

$$\mathcal{V}(b,s) = \max_{\pi^N,\tau,c^T,c^N,\ell,b',p^N,\mu} u\left[c\left(c^T,c^N\right),\ell\right] + \beta\mathbb{E}_{s'|s}\mathcal{V}(b',s')$$

subject to

$$c^N = \alpha(p^N)c^T \tag{A.78}$$

$$c^N = \left[1 - \frac{\varphi}{2}(\pi^N)^2\right] A\ell \tag{A.79}$$

$$c^T = y^T + b - \frac{b'}{R} \tag{A.80}$$

$$\frac{b'}{R} \geq -\kappa\, \mathbb{E}_{s'|s}\left[y^{T'} + \mathcal{P}^N(b',s')\mathcal{C}^N(b',s')\right] \tag{A.81}$$

$$\mu\Delta = u_T(c^T,c^N) - \beta R(1+\tau)\mathbb{E}_{s'|s}u_T\left(\mathcal{C}^T(b',s'),\mathcal{C}^N(b',s')\right) \tag{A.82}$$

$$\mu\Delta \times \left[\frac{b'}{R} + \kappa\, \mathbb{E}_{s'|s}\left(y^{T'} + \mathcal{P}^N(b',s')\mathcal{C}^N(b',s')\right)\right] = 0 \tag{A.83}$$

$$0 = \varphi\pi^N(1+\pi^N) + (\varepsilon-1)\omega - \varphi\ell^{-1}\mathbb{E}_{s'|s}\Lambda\left[\mathcal{L}(b',s')\mathcal{M}(b',s')\right] \tag{A.84}$$

where $\Delta \equiv 1 + \partial\ell(b',s')/\partial b'$. Let δ^*, ι^*, λ^*, μ^*, v^*, ν^* and ξ^* be the Lagrange multipliers on constraints (A.78)-(A.84). From the first-order condition with respect to τ, we have $v^* = 0$ and the optimality condition for μ then yields $\nu^* = 0$. The remaining optimality

[2]Devereux et al. (2019) shows that when there is a working motive for borrowing and the future price of capital enters the collateral, a policymaker under discretion will also tax capital inflows in a crisis. But this is not optimal from an ex-ante social welfare perspective.

conditions of the problem of the government are then:

$$c^T :: \quad \lambda^* = u_T(c^T, c^N) + \xi_t^* \psi_T \tag{A.85}$$

$$b' :: \quad \lambda^* = \beta R \mathbb{E}_{s'|s} \lambda^{*\prime} + \mu^* \left[1 + \kappa \mathbb{E}_{s'|s} \frac{\partial Y(b', s')}{\partial b'}\right] + \xi^* \psi_{b'} \tag{A.86}$$

$$\ell :: \quad u_\ell(\ell) + \iota^* \left[1 - \frac{\varphi}{2}(\pi^N)^2\right] A + \xi^* \psi_\ell = 0 \tag{A.87}$$

$$c^N :: \quad \iota^* = u_N(c^T, c^N) + \xi^* \psi_N \tag{A.88}$$

$$\pi^N :: \quad \xi^* = \frac{\varphi \iota^* y^N}{1 + 2\pi^N} \pi^N \tag{A.89}$$

where the function $Y(b', s') \equiv y^{T'} + \mathcal{P}^N(b', s')\mathcal{C}^N(b', s')$ determines the income in the next period as a function of the stock of net foreign assets in the next period b' and the future exogenous state variable s'.

We now show that the optimal solution features $\pi^N = 0$ and $\tau = 0$. First, note that $\pi^N = 0$ implies $\xi^* = 0$ which in turn implies that $\iota^* = u_N(c^T, c^N)$ by equation (A.88). Substituting both $\xi^* = 0$ and $\iota^* = u_N(c^T, c^N)$ into (A.87) yields $\omega = 0$. From $\pi^N = 0$ and $\omega = 0$ it follows that the implementability constraint (A.84) is satisfied. Now pick $\mu^* = \mu \Delta \left[1 + \kappa \mathbb{E}_{s'|s} \frac{\partial Y(b', s')}{\partial b'}\right]^{-1}$. It is straightforward to see that $\mu^* \geq 0$ and that $\mu^* = 0$ when the credit constraint does not bind. Then, combining (A.85) and (A.86) and using $\xi^* = 0$ we arrive to

$$u_T(c^T, c^N) = \beta R \mathbb{E}_{s'|s} u_T\left(\mathcal{C}^T(b', s'), \mathcal{C}^N(b', s')\right) + \mu$$

and the implementability constraint (A.82) is also satisfied (with $\tau = 0$). Therefore, $\pi^N = 0$ and $\tau = 0$ is. the solution of the government's problem. Hence, under discretion, the optimal monetary policy in a model in which future income is used as collateral is a price stability policy $\pi^N = 0$ and the government does not use capital controls $\tau = 0$. □

D Numerical Solution Method (Algorithm)

D.1 For Competitive Equilibrium under a Price Stability Policy

This algorithm is build on Bianchi (2011)'s algorithm that incorporates the occasionally binding endogenous constraint, modified to account for the nominal rigidities. Formally, the computation of the competitive equilibrium operates directly on the first-order conditions and requires solving for functions $\{\mathcal{B}(b,s), \mathcal{L}(b,s), \mathcal{C}^T(b,s), \mathcal{P}^N(b,s), \mu(b,s)\}$ such that:

$$\mathcal{C}^T(b,s) + \frac{\mathcal{B}(b,s)}{R} = y^T + b \tag{D.1}$$

$$\alpha\left(\mathcal{P}^N(b,s)\right)\mathcal{C}^T(b,s) = A\,\mathcal{L}(b,s) \tag{D.2}$$

$$\frac{\mathcal{B}(b,s)}{R} \geq -\kappa\left(A\,\mathcal{P}^N(b,s)\mathcal{L}(b,s) + y^T\right) \tag{D.3}$$

$$\begin{aligned} &u_T\left(c(b,s) - g\left(\mathcal{L}(b,s)\right)\right) \\ &\qquad = \beta R \mathbb{E}_{s'|s}\left\{u_T\left(c(\mathcal{B}(b,s),s') - g\left(\mathcal{L}(\mathcal{B}(b,s),s)\right)\right)\right\} + \mu(b,s) \end{aligned} \tag{D.4}$$

$$u_N\left(c(b,s) - g\left(\mathcal{L}(b,s)\right)\right) + \frac{1}{A}u_\ell\left(c(b,s) - g\left(\mathcal{L}(b,s)\right)\right) = -\kappa\mathcal{P}^N(b,s)\mu(b,s) \tag{D.5}$$

where $c(b,s) \equiv c\left(\mathcal{C}^T(b,s), A\,\mathcal{L}(b,s)\right)$. The steps for the algorithm are the following:

1. Generate discrete grids $G_b = \{b_1, b_2, ..., b_M\}$ for the bond position and $G_s = \{s_1, s_2, ..., s_N\}$ for the shock state space, and choose an interpolation scheme for evaluating the functions outside the grid of bonds. The piecewise linear approximation is used to interpolate the functions and the grid for bonds contains 200 points.

2. Conjecture $\mathcal{B}_h(b,s)$, $\mathcal{L}_h(b,s)$, $\mathcal{C}_h^T(b,s)$, $\mathcal{P}_h^N(b,s)$, $\mu_h(b,s)$ at time H, $\forall b \in G_b$ and $\forall s \in G_s$.

3. Set $i = 1$

4. Solve for the values of $\mathcal{B}_{h-i}(b,s)$, $\mathcal{L}_{h-i}(b,s)$, $\mathcal{C}_{h-i}^T(b,s)$, $\mathcal{P}_{h-i}^N(b,s)$, $\mu_{h-i}(b,s)$ at time $h-i$ using (D.1)-(D.5) and $\mathcal{B}_{h-i+1}(b,s)$, $\mathcal{L}_{h-i+1}(b,s)$, $\mathcal{C}_{h-i+1}^T(b,s)$, $\forall b \in G_b$ and $\forall s \in G_s$:

    (a) First, assume that the credit constraint (D.3) is not binding. Set $\mu_{h-i}(b,s) = 0$ and using (D.4), (D.5) and a root finding algorithm solve for $\mathcal{C}_{h-i}^T(b,s)$ and $\mathcal{L}_{h-i}(b,s)$. Solve for $\mathcal{B}_{h-i}(b,s)$ and $\mathcal{P}_{h-i}^N(b,s)$ using (D.1) and (D.2).

(b) Check whether $\frac{\mathcal{B}_{h-i}(b,s)}{R} \geq -\kappa\left(A\mathcal{P}^N_{h-i}(b,s)\mathcal{L}_{h-i}(b,s) + y^T\right)$ holds. If the credit constraint is satisfied, move to the next grid point.

(c) Otherwise, using (D.1), (D.3), (D.4), (D.5) and a root finding algorithm solve for $\mu_{h-i}(b,s)$, $\mathcal{B}_{h-i}(b,s)$, $\mathcal{C}^T_{h-i}(b,s)$ and $\mathcal{L}_{h-i}(b,s)$ and using (D.2) solve for $\mathcal{P}^N_{h-i}(b,s)$.

5. Convergence. The competitive equilibrium is found if $\left\|\sup_{B,s} x_{h-i}(b,s) - x_{h-i+1}(b,s) < \epsilon\right\|$ for $x \in \{\mathcal{B},,\mathcal{L},\mathcal{C}^T\}$. Otherwise, set $x_{h-i}(b,s) = x_{h-i+1}(b,s)$, $i \approx i+1$ and go to step 4.

D.2 For Optimal Time-Consistent Monetary Policy

The solution method proposed here uses a nested fixed point algorithm to solve for optimal time-consistent monetary policy and is related to the literature using Markov perfect equilibria (e.g. Klein et al. (2008) and Bianchi and Mendoza (2018)). In the inner loop, using the Bellman equation and value function iteration, solve for value function and policy functions taking as given future policies. Formally, given functions $\{\mathcal{C}^T(b,s), \mathcal{P}^N(b,s), \mathcal{B}(b,s), \mathcal{L}(b,s), \mathcal{M}(b,s)\}$, the Bellman equation is given by:

$$\mathcal{V}(b,s) = \max_{c^T,\ell,b',p^N,\pi^N,\mu} u\left[c\left(c^T, \alpha(p^N)c^T\right) - g(\ell)\right] + \beta\mathbb{E}_{s'|s}\mathcal{V}(b',s') \tag{D.6}$$

$$s.t. \quad \alpha(p^N)c^T = \left[1 - \frac{\varphi}{2}(\pi^N)^2\right]A\ell \tag{D.7}$$

$$c^T = y^T + b - \frac{b'}{R} \tag{D.8}$$

$$\frac{b'}{R} \geq -\kappa\left(p^N A\ell + y^T\right) \tag{D.9}$$

$$\mu = u_T(c,\ell) - \beta R\mathbb{E}_{s'|s}u_T\left(c\left(\mathcal{C}^T(b',s'), \mathcal{P}^N(b',s')\right) - g\left(\mathcal{L}(b',s')\right)\right) \tag{D.10}$$

$$\mu \times \left[b' + \kappa\left(p^N A\ell + y^T\right)\right] = 0 \tag{D.11}$$

$$\varphi\pi^N(1+\pi^N) - (\epsilon - 1)\left[z^{-1}(1-\omega) - 1\right] - \varphi\ell^{-1}\mathbb{E}_{s'|s}\Lambda\left[\mathcal{L}(b',s')\mathcal{M}(b',s')\right] = 0 \tag{D.12}$$

Given the solution to the Bellman equation, update future policies as the outer loop. The steps for the algorithm are the following:

1. Generate discrete grids $G_b = \{b_1, b_2, ..., b_M\}$ for the bond position and $G_s = \{s_1, s_2, ..., s_N\}$

for the shock state space, and choose an interpolation scheme for evaluating the functions outside the grid of bonds. The piecewise linear approximation is used to interpolate the functions and the grid for bonds contains 200 points.

2. Guess policy functions $\mathcal{B}$, $\mathcal{C}^T$, $\mathcal{P}^N$, $\mathcal{M}$ at time H, $\forall b \in G_b$ and $\forall s \in G_s$.

3. For given $\mathcal{L}$, $\mathcal{C}^T$, $\mathcal{P}^N$, $\mathcal{M}$ solve the recursive problem using value function iteration to find the value function and policy functions:

   (a) First, assume that the credit constraint (D.9) is not binding. Set $\mu = 0$ – (D.11) is thus satisfied – and solve the optimization problem (D.6) subject to (D.7), (D.8), (D.10), (D.12) using a Newton type algorithm and check whether (D.9) holds.

   (b) Second, assume that the credit constraint (D.9) is binding – (D.11) is thus satisfied. Solve the optimization problem (D.6) subject to (D.7)-(D.10), (D.12) using a Newton type algorithm.

   (c) Compare the solutions in (a) and (b).The optimal choices in each state is the best solution. Denote $\{x^i\}_i$, with $x^i \in \{b', \ell, c^T, p^N, \pi^N\}$, the associated policy functions.

4. Evaluate convergence. Compute the sup distance between $\mathcal{B}$, $\mathcal{C}^T$, $\mathcal{P}^N$, $\mathcal{M}$ and $\{x^i\}$, with $x^i \in \{b', c^T, p^N, \pi^N\}$. If the sup distance is not smaller enough (higher than $\epsilon = 1e-7$), update $\mathcal{B}$, $\mathcal{C}^T$, $\mathcal{P}^N$, $\mathcal{M}$ and solve again the recursive problem.