

**Online Appendix for “A Simple Model of Corporate Tax Incidence”
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A. Analytical results

In the Cobb-Douglas case, we have that $\phi_k = \phi v a k_D^{-1}$ and $\phi_l = \phi v (1-a) l^{-1}$. Combining both first-order conditions yields $al/(1-a)k_D = r^*/(1-t)w$. Then, some simple algebra allows us to compute closed-form solutions for factor demands:

$$\begin{aligned} k_D(w, t) &= (v\psi)^{\frac{1}{1-v}} \left[\frac{a(1-t)}{r^*} \right]^{\frac{1-(1-a)v}{1-v}} \left[\frac{1-a}{w} \right]^{\frac{(1-a)v}{1-v}}, \\ l(w, t) &= (v\psi)^{\frac{1}{1-v}} \left[\frac{a(1-t)}{r^*} \right]^{\frac{av}{1-v}} \left[\frac{1-a}{w} \right]^{\frac{1-av}{1-v}}. \end{aligned}$$

Taking logs and differentiating yields:

$$\begin{aligned} d \log k_D(w, t) &= \frac{1-(1-a)v}{1-v} d \log(1-t) - \frac{(1-a)v}{1-v} d \log w, \\ d \log l(w, t) &= \frac{av}{1-v} d \log(1-t) - \frac{1-av}{1-v} d \log w, \end{aligned}$$

where we assumed $d \log v = d \log \psi = d \log a = d \log r^* = 0$. Let $\epsilon^S = f(w)w/L$ denote the labor supply elasticity. Then, differentiating the labor market equilibrium yields:

$$f(w)dw = N dl(w, t) \quad \Leftrightarrow \quad \epsilon^S d \log w = d \log l(w, t).$$

Replacing in the input demands we get:

$$\begin{aligned} d \log k_D(w, t) &= \frac{1-(1-a)v}{1-v} d \log(1-t) - \frac{(1-a)v}{\epsilon^S(1-v)} d \log l(w, t), \\ d \log l(w, t) &= \frac{av}{1-v} d \log(1-t) - \frac{1-av}{\epsilon^S(1-v)} d \log l(w, t). \end{aligned}$$

Starting from the labor demand equation, we have that:

$$\epsilon_l = \frac{d \log l(w, t)}{d \log(1-t)} = \left(1 + \frac{1-av}{\epsilon^S(1-v)} \right)^{-1} \frac{av}{1-v} = \frac{\epsilon^S av}{\epsilon^S(1-v) + 1 - av},$$

and $\epsilon_w = (\epsilon^S)^{-1} \epsilon_l$. Note that $d \log L = d \log(Nl(w, t)) = d \log N + d \log l(w, t)$, so $\epsilon_l = \epsilon_L$ when N is fixed. Assuming that ϵ^S is locally constant, it follows that:

$$\frac{\partial \epsilon_l}{\partial a} = \frac{\epsilon^S v (\epsilon^S(1-v) + 1)}{(\epsilon^S(1-v) + 1 - av)^2} = \frac{\epsilon_l (\epsilon^S(1-v) + 1)}{a(\epsilon^S(1-v) + 1 - av)} > 0.$$

Regarding capital, using the expressions above, it follows that:

$$\begin{aligned} \epsilon_k &= \frac{d \log k_D(w, t)}{d \log(1-t)} = \frac{1-(1-a)v}{1-v} - \frac{(1-a)v}{\epsilon^S(1-v)} \frac{d \log l(w, t)}{d \log(1-t)}, \\ &= \frac{1}{1-v} \left(1 - (1-a)v - \frac{(1-a)av^2}{\epsilon^S(1-v) + 1 - av} \right), \\ &= \frac{1}{1-v} \left(1 - \frac{(\epsilon^S(1-v) + 1)(1-a)v}{\epsilon^S(1-v) + 1 - av} \right). \end{aligned}$$

Note that $\varepsilon_k > 0$ since $(\varepsilon^S(1-v) + 1)(1-a)v < \varepsilon^S(1-v) + 1 - av$ if and only if $v < 1$. Then:

$$\begin{aligned} \frac{\partial \varepsilon_k}{\partial a} &= \frac{-1}{1-v} \left(\frac{-(\varepsilon^S(1-v) + 1)v(\varepsilon^S(1-v) + 1 - av) + (\varepsilon^S(1-v) + 1)(1-a)v^2}{(\varepsilon^S(1-v) + 1 - av)^2} \right), \\ &= \frac{-(\varepsilon^S(1-v) + 1)v}{1-v} \left(\frac{-(\varepsilon^S(1-v) + 1 - av) + (1-a)v}{(\varepsilon^S(1-v) + 1 - av)^2} \right), \\ &= \frac{-(\varepsilon^S(1-v) + 1)v}{1-v} \left(\frac{-(\varepsilon^S + 1)(1-v)}{(\varepsilon^S(1-v) + 1 - av)^2} \right) > 0. \end{aligned}$$

By comparing the expressions, we can also note that $\varepsilon_k > \varepsilon_l$ if and only if $\varepsilon^S(1-v) + 1 > 0$, a condition that always holds in this model.

Regarding effects on pre-tax profits, introducing the optimal factor demands in the pre-tax profits function yields, after some algebra:

$$\pi_D(w, t) = \left(\frac{a(1-t)}{r^*} \right)^{\frac{av}{1-v}} \left(\frac{1}{w} \right)^{\frac{v(1-a)}{1-v}} \Omega,$$

where $\Omega = \psi(v\psi)^{\frac{v}{1-v}}(1-a)^{\frac{v(1-a)}{1-v}} - (v\psi)^{\frac{1}{1-v}}(1-a)^{\frac{1-av}{1-v}}$ is a constant. Then:

$$d \log \pi_D(w, t) = \frac{av}{1-v} d \log(1-t) - \frac{v(1-a)}{1-v} d \log w,$$

so

$$\varepsilon_\pi = \frac{d \log \pi_D(w, t)}{d \log(1-t)} = \frac{av}{1-v} - \frac{v(1-a)}{\varepsilon^S(1-v)} \varepsilon_l.$$

Then:

$$\begin{aligned} \frac{\partial \varepsilon_\pi}{\partial a} &= \frac{v}{1-v} + \frac{v\varepsilon_l}{\varepsilon^S(1-v)} - \frac{v(1-a)}{\varepsilon^S(1-v)} \frac{\partial \varepsilon_l}{\partial a}, \\ &= \frac{v}{1-v} \left(1 + \frac{av}{\varepsilon^S(1-v) + 1 - av} - \frac{(1-a)v(\varepsilon^S(1-v) + 1)}{(\varepsilon^S(1-v) + 1 - av)^2} \right). \end{aligned}$$

Then, $\partial \varepsilon_\pi / \partial a > 0$ if:

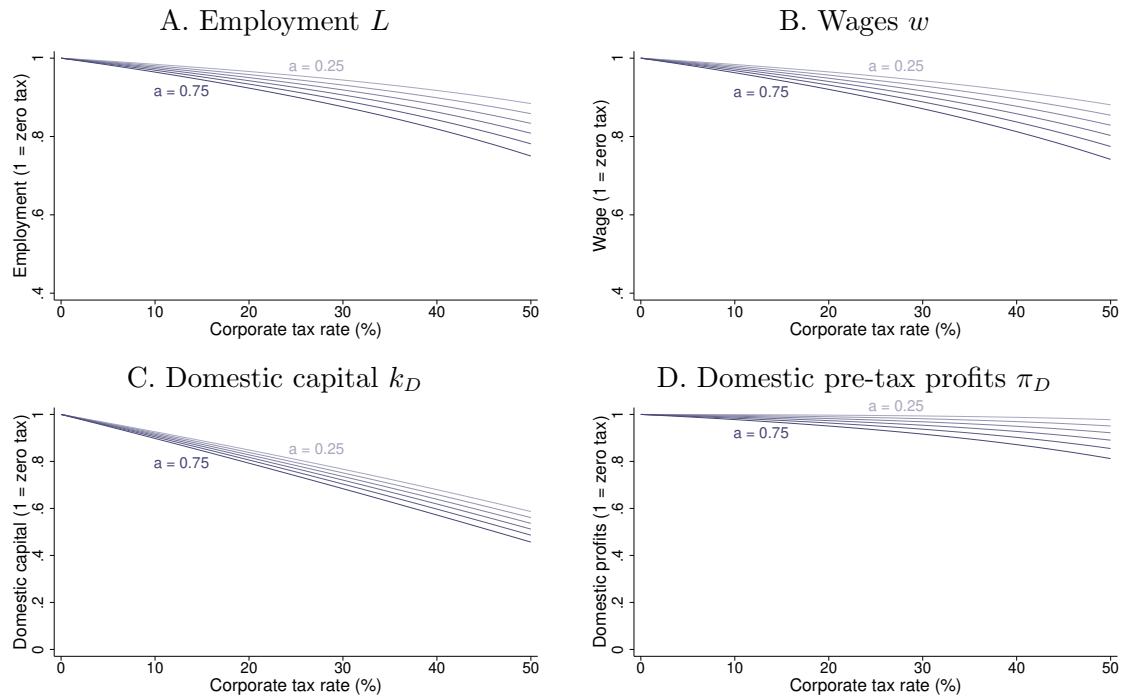
$$1 + \frac{av}{\varepsilon^S(1-v) + 1 - av} - \frac{(1-a)v(\varepsilon^S(1-v) + 1)}{(\varepsilon^S(1-v) + 1 - av)^2} > 0,$$

which holds if $\varepsilon^S(1-v) + 1 > 0$, a condition that is always true. Then, $\partial \varepsilon_\pi / \partial a > 0$.

Finally, to see the role of wage adjustments in mediating factor demands, we have that:

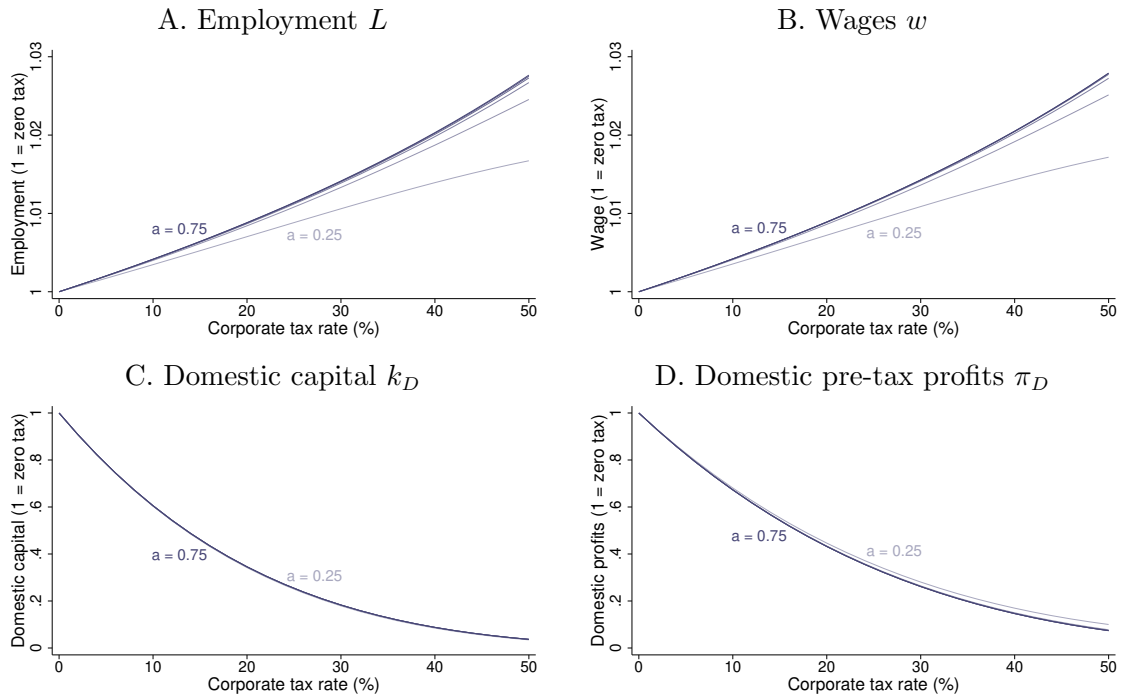
$$\begin{aligned} \frac{\partial \varepsilon_l}{\partial \varepsilon^S} &= \frac{1 - av}{(\varepsilon^S(1-v) + 1 - av)^2} > 0, \\ \frac{\partial \varepsilon_k}{\partial \varepsilon^S} &= \frac{(1-a)av^2}{(\varepsilon^S(1-v) + 1 - av)^2} > 0. \end{aligned}$$

B. Additional Figures

FIGURE B.1. COMPARATIVE STATICS WITH RESPECT TO THE CORPORATE TAX RATE, t , LOW CAPITAL-LABOR SUBSTITUTION ($\rho = -1$)

Note: This figure shows how the employment, wage, domestic capital, and domestic pre-tax profit responses to the corporate tax depend on the capital intensity, a , of the firm. In each figure, the outcome is normalized to be equal to 1 under $t = 0$, and the different lines represent different values of a , from $a = 0.25$ (lighter) to $a = 0.75$ (darker). These figures use $\rho = -1$, $v = 0.79$, $r^* = 0.042$, $N = 10$, $\psi = 0.15$, and $c \sim \exp(0.2)$.

FIGURE B.2. COMPARATIVE STATICS WITH RESPECT TO THE CORPORATE TAX RATE, t , HIGH CAPITAL-LABOR SUBSTITUTION ($\rho = 0.8$)



Note: This figure shows how the employment, wage, domestic capital, and domestic pre-tax profit responses to the corporate tax depend on the capital intensity, a , of the firm. In each figure, the outcome is normalized to be equal to 1 under $t = 0$, and the different lines represent different values of a , from $a = 0.25$ (lighter) to $a = 0.75$ (darker). These figures use $\rho = 0.8$, $v = 0.79$, $r^* = 0.042$, $N = 10$, $\psi = 0.15$, and $c \sim \exp(0.2)$.