Supplemental Appendix
The Changing Identities of American Wives and Mothers

Jeanne Lafortune* Laura Salisbury† Aloysius Siow‡

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*Instituto de Economia, Pontificia Universidad Catolica de Chile
†York University
‡University of Toronto

ONLINE APPENDIX

A Theoretical Appendix

A.1 A transferable utility marital output function

Chiappori and Gugl (2020) provides a summary of the properties of transferable utility models used in this essay. We consider one case here. Marital output combines private spousal consumption, $c(i,j,r)$, public household consumption, $h(i,j,r)$, such as a house, and a public good, $K(i,j,r)$, which is often thought of as children in this literature. Consider two types of spouses, i and j for the husband and wife respectively, who form a marriage. Let marital output of the couple, given resource allocation r, be:

$$\pi(i,j,r) = \alpha[c(i,j,r) + G(h(i,j,r),k(i,j,r))] + (1-\alpha)c(i,j,r)G(h(i,j,r),k(i,j,r))); \; 0 \leq \alpha \leq 1 \tag{1}$$

$c(i,j,r)$: total private spousal consumption

$h(i,j,r)$: public household consumption

$k(i,j,r)$: public good

When $\alpha = 1$, private spousal consumption is a substitute for public household consumption and the public good. If $\alpha = 0$, private spousal consumption is complementary to public household consumption and the public good. Public household consumption and the public good can be complements or substitutes for each other. Private spousal consumption, $c(i,j,r)$, is divided between the husband and wife:

$$c(i,j,r) = c_i(i,j,r) + c_j(i,j,r) \tag{2}$$

The utility of spouse s is:

$$U_s(i,j,r) = \alpha[c_s(i,j,r) + G(h(i,j,r),k(i,j,r))] + (1-\alpha)c_s(i,j,r)G(h(i,j,r),k(i,j,r))) \tag{3}$$

The marginal utility of private consumption for either spouse is $\alpha + (1-\alpha)G(h(i,j,r),k(i,j,r))$, which is the same for both spouses; and independent of $c_s(i,j,r)$. The gain in utility by the receiving spouse from a transfer of private spousal consumption is equal to the loss in utility from the transferring spouse. In this transferable utility environment, it is efficient for the couple to choose resource allocation which maximizes marital output and allocate private spousal consumption between them to satisfy their relative bargaining power in the marriage. As an extreme example, assume the husband has all the bargaining power in the relationship. The household would assign the wife her reservation utility of U_j^* which would lead to her receiving $c_j(i,j,r)$ in private consumption. Given (3) and her reservation utility, $c_j(i,j,r)$ must satisfy:

$$\alpha c_j(i,j,r) + (\alpha + (1-\alpha)c_j(i,j,r))G(h(i,j,r),k(i,j,r))) = U_j^* \tag{4}$$

His private consumption is $c(i,j,r) - c_j(i,j,r)$. Given (3) and (4), his utility is:

$$U_i(i,j,r) = \alpha(c(i,j,r) - c_j(i,j,r)) + (\alpha + (1-\alpha)(c(i,j,r) - c_j(i,j,r))G(h(i,j,r),k(i,j,r))) \tag{5}$$

$$= (c(i,j,r) - c_j(i,j,r))(\alpha + (1-\alpha)G(h(i,j,r),k(i,j,r)))$$

$$= (c(i,j,r) - \frac{U_j^*}{(\alpha + (1-\alpha)G(h(i,j,r),k(i,j,r)))})(\alpha + (1-\alpha)G(h(i,j,r),k(i,j,r)))$$

$$= \pi(i,j,r) - U_j^* \tag{6}$$

(6) says that his utility is equal to marital output less the reservation utility of his wife. His utility is maximized when he chooses the resource allocation which maximizes marital output, $\pi(i,j,r)$. This conclusion would be the same if the household bargaining was such that the wife would be guaranteed any level of utility. It would always be optimal for the couple to first maximize marital output and then divide it between partners. To further study resource allocation in response to changes in the environment, if an individual works, the individual will incur a fixed commuting time cost which is subtracted from the unit of time. For each spouse, they have to choose time use in the labor market, home production and public good. The time budget constraint for spouse s, $s = i, j$, is:

$$1 = \Phi_s(t_s^{\xi} + t_s^l) + t_s^h + t_s^k;\ s = i,j$$

Φ_s : indicator function takes a value of 1 if $t_s^l > 0$ and 0 otherwise.

t_s^{ξ} : fixed commuting time

t_s^l : time in labor market

t_s^h : time in home production

t_s^k : time in public good provision

They have to allocate labor market earnings to private consumption, home produced goods and public good production. The money budget constraint is at the family level. It satisfies:

$$
\begin{aligned}
N(i,j,r) + w_i t_i^l + w_j t_j^l &= g^c + g^h + g^k \\
N(i,j,r) &: \text{ non-labor income} \\
w_s &: \text{ wage of spouse } s,\ s = i,j \\
g^c &\text{:private consumption goods} \\
g^c &\text{:market goods for producing home production} \\
g^c &\text{:market goods for producing the public good}
\end{aligned}
$$

They also have to choose how many children to have, n, if any. Individual who remain unmatched also have the same choices. Let $k = 1$ if $n = 0$. The production functions for producing marital output is as follows:

$$
\begin{aligned}
c &= g^c \\
h &= H(g^h, t_i^h, t_j^h) \\
k &= K(g^k - g_0^k n, p_i^k(t_i^k - t_0^k n), p_j^k(t_j^k - t_0^k n), n)
\end{aligned}
$$

Private consumption is produced with market goods alone. Home production is produced with market goods and spousal times where is assumed have equal productivity. The public good depends on the number of children n, market goods g^k, parental time, t_s^k. There is a fixed good cost, g_0^k, and parental time cost per child, t_0^k. For example, diapers cannot be shared between children and each child's diapers have to be changed. Time productivity in producing the public good may differ across parents, p_s^k. The couple will choose resource allocation which maximizes marital output. Consider an (i,j) couple where both spouses work. Their marital output is:

$$
\begin{aligned}
&\pi(i,j,r) = \alpha c(i,j,r) + (\alpha + (1-\alpha) c(i,j,r)) G(h(i,j,r), k(i,j,r))) \\
&= \alpha(N(i,j,r) + \sum_{s\in\{i,j\}} w_s(1 - t_s^\xi - t_s^{hr} - t_s^{kr}) - g^{hr} - g^{kr}) \\
&+ (\alpha + (1-\alpha)(N(i,j,r) + \sum_{s\in\{i,j\}} w_s(1 - t_s^\xi - t_s^{hr} - t_s^{kr}) - g^{hr} - g^{kr}) \\
&\times G(H(g^{hr}, t_i^{hr}, t_j^{hr}), K(g^{kr} - g_0^k n^r, p_i^k(t_i^{kr} - t_0^k n^r), p_j^k(t_j^{kr} - t_0^k n^r), n^r))
\end{aligned}
$$

Consider an (i,j) couple where only the husband works. Let $z_s^{r'}$ be the optimal choice of z by spouse s. Their marital output is:

$$\pi(i,j,r') = \alpha(N(i,j,r') + w_i(1 - t_i^{\xi} - t_i^{hr'} - t_i^{kr'}) - g^{hr'} - g^{kr'})$$
$$+ (\alpha + (1-\alpha)(N(i,j,r') + w_i(1 - t_i^{\xi} - t_i^{hr'} - t_i^{kr'}) - g^{hr\prime} - g^{kr\prime})$$
$$\times G(H(g^{hr'}, t_i^{hr'}, t_j^{hr'}), K(g^{kr'} - g_0^k n^{r'}, p_i^k(t_i^{kr'} - t_0^k n^{r'}), p_j^k(1 - t_j^{hr'} - t_0^k n^{r'}), n^{r'}))$$

We use the above framework to show how a rise in the wife's wage will induce her to enter the marriage market. By the envelope theorem,

$$\frac{\partial \pi(i,j,r)}{\partial w_j} = \alpha(1 - t^{\xi} - t_j^{hr} - t_j^{kr}) + (1-\alpha)(1 - t^{\xi} - t_j^{hr} - t_j^{kr})G(i,j,r) > 0$$
$$\frac{\partial \pi(i,j,r')}{\partial w_j} = 0$$

Since marital output increases when w_j increases if she works and not otherwise, this model predicts that she will be more likely to enter the labor force when w_j increases. Similarly, when her fixed commuting cost falls, she will also be more likely to enter the labor force. Other comparative statics may require further restrictions on the marital output production function as in Greenwood, Guner, and Vandenbroucke (2017). It is hard to check on the validity of many of these restrictions. The empirical framework in this essay estimates $\pi(i,j,r) - \pi(i,0) - \pi(0,j)$ and other aspects of marital output without imposing more restrictions on the marital output production function. As shown in the text, the estimates are informative about the structure of the marital output function. The above model is deterministic which means that if resource allocation r generates a higher utility for the (i,j) household than resource allocation r', we should not observe any (i,j) couple choosing r'. This stark prediction is usually not observed. What we are likely to observe empirically is that more (i,j) couples choose r rather than r'. For this reason, starting with McFadden (1973), empirical economists have turned to using discrete choice models as is done here.

A.2 The CS Framework: Mathematical derivations

The CS model is a frictionless transferable utility model of marriage. Let $g = \{m, f\}$ denote the gender of the individual, m being male and f being female. Let t_g be the type of individual of gender g which depends on the education choice of that individual. t_g^s is the type of the spouse of the individual of type t_g. We will also denote a type of man, t_m, as i and a type of woman, t_f, as j. Let $\mu_g(t_g)$ be the mass of type

t_g marriage market participants. $\mu_g(t_g, 0)$ is the mass of type t_g individuals who enter the marriage market and decide to remain unmatched. $\mu_g(t_g, 1)$ is the mass of married individuals of type t_g. A marriage match or type is denoted by (i, j, r). $\mu(i, j, r)$ denotes the mass of marriages with type i husbands, type j wives in relationship r, where r denotes household decisions such as whether the wife works and/or the number of children. There are R different types of marriages. Consider a particular type t_g person z. The utility that z gets from a particular (i, j, r) match, $U_{t_g z}$, is:

$$
\begin{aligned}
U_{gz}(i,j,r) &= \iota_g((1-\beta(i,j,r))\pi(i,j,r)) + (1-\iota_g)\beta(i,j,r)\pi(i,j,r) + \varepsilon_{gz}(i,j,r) \\
\iota_g &= 1 \text{ if } g \text{ is male and } 0 \text{ otherwise}
\end{aligned}
$$

$\pi(i, j, r)$, marital output, is common to all (i, j, r) matches. This output is divided between the two type of spouses through a share $\beta(i, j, r)$ If i or j remains unmarried, their marital outputs are $\pi(i, 0)$ and $\pi(0, j)$ respectively. $\varepsilon_{gz}(i, j, r)$, is idiosyncratic and particular to that person and the type of match (i, j, r). $\varepsilon_{gz}(t_g, 0)$ denotes $z's$ idiosyncratic component if z chooses to remain unmatched. Every idiosyncratic payoff is assumed to be independently drawn from a Type 1 extreme value distribution. After observing the common and idiosyncratic payoffs from all the potential matches, each individual chooses the match which maximizes their utility. The property of the type-1 extreme value distribution allows us to know that the probability that a relationship (i, j, r) is elected by type i with probability

$$
\frac{\exp((1-\beta(i,j,r))\pi(i,j,r))}{\sum_{j,r}\exp((1-\beta(i,j,r))\pi(i,j,r)) + \exp(\pi(i,0))}
$$

In large populations, the fraction of the population of type i that will select relationship j, r will thus be given by $\mu_m(i, j, r)/\mu_m(i)$ and will be equal to the above equation. Similarly, the fraction that remains single will be equal to

$$
\frac{\mu_m(i,0)}{\mu_m(i)} = \frac{\exp(\pi(i,0)}{\sum_{j,r}\exp((1-\beta(i,j,r))\pi(i,j,r)) + \exp(\pi(i,0))}
$$

Dividing one by the other, we obtain the result that

$$
\frac{\mu_m(i,j,r)}{\mu_m(i,0)} = \frac{\exp((1-\beta(i,j,r))\pi(i,j,r))}{\exp(\pi(i,0)}
$$

Taking logs, we obtain that

$$\ln \frac{\mu_m(i,j,r)}{\mu_m(i,0)} = (1-\beta(i,j,r))\pi(i,j,r) - \pi(i,0)$$

which gives us the individual log odds ratio, $l_m^I(i,j,r)$ presented in the text. A similar equation can be obtained for women of type j:

$$\ln \frac{\mu_f(i,j,r)}{\mu_f(j,0)} = \beta(i,j,r)\pi(i,j,r) - \pi(0,j)$$

If we sum the two individual log odds ratio for a man of type i and a woman of type j, we obtain

$$\ln \frac{\mu_m(i,j,r)}{\mu_m(i,0)} + \ln \frac{\mu_f(i,j,r)}{\mu_f(j,0)} = \pi(i,j,r) - \pi(i,0) - \pi(0,j)$$

By the properties of the logs, we obtain

$$\ln \frac{\mu_m(i,j,r)\mu_f(i,j,r)}{\mu_m(i,0)\mu_f(j,0)} = \pi(i,j,r) - \pi(i,0) - \pi(0,j)$$

Given that the number of men of type i married to type j in relationship type r must be equal to the number of women of type j married to type i in relationship type r, this implies that $\mu_m(i,j,r) = \mu_f(i,j,r) = \mu(i,j,r)$, which allows us to simplify the expression to

$$\ln \frac{\mu(i,j,r)^2}{\mu_m(i,0)\mu_f(j,0)} = \pi(i,j,r) - \pi(i,0) - \pi(0,j)$$

Finally, taking the square root inside the logarithm gives us the marriage log odds:

$$l^M(i,j,r)\sqrt{\mu_m(i,0)\mu_f(j,0)} = \frac{\pi(i,j,r) - \pi(i,0) - \pi(0,j)}{2} \tag{7}$$

We can also compute the within-household binary log odds using the same computation but this time looking at a type of relationship $r = 1$ versus $r = 0$ for a given couple i, j.

A.2.1 Who matches with whom and in what type of relationship

We can then compute the marriage log odds for four types of relationships or individuals.Let i or j equal 1 if the individual is a college graduate, and equal zero otherwise, we will obtain the following four marriage log odds:

$$\ln \frac{\mu(0,0,r)}{\sqrt{\mu_m(0,0)\mu_f(0,0)}} = \frac{\pi(0,0,r) - \pi(0,0) - \pi(0,0)}{2}$$

$$\ln \frac{\mu(1,0,r)}{\sqrt{\mu_m(1,0)\mu_f(0,0)}} = \frac{\pi(1,0,r) - \pi(1,0) - \pi(0,0)}{2}$$

$$\ln \frac{\mu(0,1,r)}{\sqrt{\mu_m(0,0)\mu_f(0,1)}} = \frac{\pi(0,1,r) - \pi(0,0) - \pi(0,1)}{2}$$

$$\ln \frac{\mu(1,1,r)}{\sqrt{\mu_m(1,0)\mu_f(1,0)}} = \frac{\pi(1,1,r) - \pi(1,0) - \pi(0,1)}{2}$$

Taking the difference between the first two and the last two and then subtracting the second result from the first one, we obtain the matching log odds:

$$\ln \frac{\mu(0,0,r)\mu(1,1,r)}{\mu(1,0,r)\mu(0,1,r)} = \frac{\pi(0,0,r) + \pi(1,1,r) - \pi(0,1,r) - \pi(1,0,r)}{2} \tag{8}$$

Similarly, couples may choose relationships based on more than one characteristic. Let $r_1 = 1$ if the couple chooses decision 1 for the first characteristic of the relationship and $r_2 = 1$ if the couple chooses decision 1 for the second characteristic. In this case, each couple choose two activities in their relationship, (r_1, r_2). We can define again four marriage logs based on those two relationship characteristics

$$\ln \frac{\mu(i,j,0,0)}{\sqrt{\mu_m(i,0)\mu_f(0,j)}} = \frac{\pi(0,0,0,0) - \pi(i,0) - \pi(0,j)}{2}$$

$$\ln \frac{\mu(i,j,0,1)}{\sqrt{\mu_m(i,0)\mu_f(0,j)}} = \frac{\pi(0,0,1,0) - \pi(i,0) - \pi(0,j)}{2}$$

$$\ln \frac{\mu(i,j,1,0)}{\sqrt{\mu_m(i,0)\mu_f(0,j)}} = \frac{\pi(0,0,1,0) - \pi(i,0) - \pi(0,j)}{2}$$

$$\ln \frac{\mu(i,j,1,1)}{\sqrt{\mu_m(i,0)\mu_f(0,j)}} = \frac{\pi(0,0,0,0) - \pi(i,0) - \pi(0,j)}{2}$$

If we sum the first and the last one and subtract the middle two, we obtain the within multinomial local log odds:

$$\ln \frac{\mu(i,j,1,1)\mu(i,j,0,0)}{\mu(i,j,1,0)\mu(i,j,0,1)} = \frac{\pi(i,j,1,1) + \pi(i,j,0,0) - \pi(i,j,1,0) - \pi(i,j,0,1)}{2} \qquad (9)$$

A.2.2 Log odds of marriage

By the properties of the type-1 extreme value distribution, the expected utility experienced by an individual in the marriage market is the log of the sum of the exponential of the systematic utilities she receives in each type of match and when she remains unmatched plus Euler's constant, here referred to as k, namely:

$$EU_f(j) = k + \ln[\exp(\pi(0,j)) + \sum_{i,r} \exp(\beta(i,j,r)\pi(i,j,r))]$$

For men, the equivalent expression is

$$EU_m(i) = k + \ln[\exp(\pi(i,0)) + \sum_{j,r} \exp((1-\beta(i,j,r))\pi(i,j,r))]$$

We can then replace in the above expression what $\beta(i,j,r)\pi(i,j,r)$ represents in $l_f^I(i,j)$ to obtain

$$EU_f(j) = k + \ln[\exp(\pi(0,j)) + \sum_{i,r} \exp(\ln \frac{\mu_f(i,j,r)}{\mu_f(j,0)} + \pi(0,j))]$$

which, by the rules of the exponents then simplifies to

$$EU_f(j) = k + \ln[\exp(\pi(0,j)) + \sum_{i,r} \exp(\pi(0,j)) \frac{\mu_f(i,j,r)}{\mu_f(j,0)}]$$

and

$$EU_f(j) = k + \ln[\exp(\pi(0,j)) + \exp(\pi(0,j)) \frac{\sum_{i,r} \mu_f(i,j,r)}{\mu_f(j,0)}]$$

$$EU_f(j) = k + \ln[\exp(\pi(0,j)) \frac{\mu_f(j)}{\mu_f(j,0)}]$$

Finally, we obtain

$$EU_f(j) = k + \pi(0,j) + \ln \frac{\mu_f(j)}{\mu_f(j,0)}$$

Rearranging terms, we obtain

$$l_f^I(j,\mu) \equiv \ln\left[\frac{\mu_f(j)}{\mu_f(j,0)}\right] = \ln\left[1+\frac{\mu_f(j,1)}{\mu_f(j,0)}\right] = EU_f(j) - \pi(0,j) - k \tag{10}$$

A similar derivation can be made to obtain the log odds of marriage of men of type i.

A.2.3 Premarital investments

Finally, to derive the equation for the log odds of college attainment, we must think that an individual z can choose to go to college and obtain a utility of

$$EU_g(1) - C_g + \varepsilon_{z1}$$

or not go and obtain a utility of

$$EU_g(0) + \varepsilon_{z0}$$

where the utility shocks are once again assumed to be from an extreme value type 1 distribution. By the same argument as before, the fraction of individuals who will go to college will ge given by

$$\frac{\mu_g(1)}{\mu_g(1)+\mu_g(0)} = \frac{\exp(EU_g(1)-C_g)}{\exp(EU_g(0))+\exp(EU_g(1)-C_g)}$$

Taking the log of that fraction and dividing it by the log of the fraction of those who do not attend college, we obtain

$$\ln\frac{\mu_g(1)}{\mu_g(0)} = -C_g + EU_g(1) - EU_g(0)$$

which is the log odds of attending college. Now, we can compute the difference in the log odds of marriage for those who go to college and those who do not and this will give us, based on the derivations of the previous subsection,

$$l_g^I(1,\mu) - l_g^I(0,\mu) = EU_g(1) - EU_g(0) - (\pi_g(1,0) - \pi_g(0,0)) \tag{11}$$

Finally, we can obtain the sequential log odds by combining the elements above.

A.3 Behavioral interpretations of log odds: comparision between Choo and Siow (2006; CS) and Chiappori, Salanie and Weiss (2017; CSW)

The behavioral interpretations are qualitatively similar in both models for most of the log odds considered here. To make the comparison easier, we follow the notation in Mourifié and Siow (2021) which encompasses both models rather than what we used in the text. This forces us to have transfers between spouses instead of a sharing rule. Once can think of the amount $\beta(i,j)\pi(i,j)$ that the woman receives in the model in the main text as being equal to τ_{ij} which corresponds to her share of the resources in this version of the model. The utility of a match (i,j) may differ depending on whether they choose a relationship of type r and r'. Let the utility of male g of type i who matches a female of type j in a relationship r be:

$$U^r_{gj} = \tilde{u}^r_{ij} + \phi^r_i \ln \mu^r_{ij} - \tau^r_{ij} + \varsigma^r_{gj}, \quad r \in \{\mathcal{M}, \mathcal{C}\} \tag{12}$$

where $\tilde{u}^r_{ij} + \phi^r_i \ln \mu^r_{ij}$ denote the systematic gross return to a man of type i matching to a woman of type j in relationship r. $\phi^r_i \in [0,1]$ are the peer effect parameters for relationship r, μ^r_{ij} denote the equilibrium number of (r,i,j) relationships, τ^r_{ij} is the share of the household surplus that the woman j receives which is thus deducted from what the man of type i obtains in relationship r, and finally ς^r_{gj} denote the errors terms (idiosyncratic payoffs) which are assumed to be i.i.d. random variables distributed according to the extreme value Type-I distribution. Due to the peer effects, the net systematic return is increased when more type i men are in the same relationships. It is smaller when women receive a higher part of the household surplus τ^r_{ij}. The utility of being unmatched for a male g of type i is given by

$$U_{g0} = \tilde{u}_{i0} + \phi^0_i \ln \mu_{i0} + \varsigma_{g0},$$

where $\widetilde{u}_{i0} + \phi^0_i \ln \mu_{i0}$ is the systematic payoff that type i men get from remaining unmatched and where μ_{i0} denotes the number of men of type i who are unmatched.

A.3.1 Individual log odds

Using the same results derived above, we know that the log of the ratio of men of type i married to type j women in relationship of type r to the number of unmarried men of type i is equal to the difference in the average utility of both outcomes:

$$ln\frac{(\mu_{ij}^{r})^{d}}{(\mu_{i0})^{d}} = \tilde{u}_{ij}^{r} - \tilde{u}_{i0} + \phi_{i}^{r}\ln\mu_{ij}^{r} - \phi_{i}^{0}\ln\mu_{i0} - \tau_{ij}^{r}. \quad (13)$$

Under CS, $\phi_{i}^{0} = \phi_{i}^{r} = 0$, and the individual log odds become

$$ln\frac{(\mu_{ij}^{r})^{d}}{(\mu_{i0})^{d}} = \tilde{u}_{ij}^{r} - \tilde{u}_{i0} - \tau_{ij}^{r} \quad (14)$$

Which is equivalent to the expression we have in the main text where $\tilde{u}_{ij}^{r} - \tau_{ij}^{r} = (1 - \beta(i,j,r))\pi(i,j,r)$ and where $\tilde{u}_{i0} = \pi(i,0)$ Under CSW, $\phi_{i}^{0} = \phi_{i}^{r} = \phi_{i}$ and

$$\ln\frac{(\mu_{ij}^{r})}{(\mu_{i0})} = \frac{\tilde{u}_{ij}^{r} - \tilde{u}_{i0} - \tau_{ij}^{r}}{1-\phi_{i}}. \quad (15)$$

If ϕ_i is fixed over time, then changes over time in the individual log odds will have to come from the same sources of variations in both models. The quasi-supply equation of type j women for (r,i,j) relationships is given by:

$$\ln\frac{(\mu_{ij}^{r})^{s}}{(\mu_{0j})^{s}} = \tilde{v}_{ij}^{r} - \tilde{v}_{0j} + \Phi_{j}^{r}\ln\mu_{ij}^{r} - \Phi_{j}^{0}\ln\mu_{0j} + \tau_{ij}^{r}. \quad (16)$$

Under CS, $\Phi_{j}^{0} = \Phi_{j}^{r} = 0$ and equilibrium, the individual log odds become

$$\ln\frac{(\mu_{ij}^{r})}{(\mu_{0j})} = \tilde{v}_{ij}^{r} - \tilde{v}_{0j} + \tau_{ij}^{r}$$

which is the same as in the main paper where $\tilde{v}_{ij}^{r} + \tau_{ij}^{r} = \beta(i,j,r)\pi(i,j,r)$ and $\tilde{v}_{0j} = \pi(0,j)$ Under CSW, $\Phi_{j}^{0} = \Phi_{j}^{r} = \Phi_{j}$ and equilibrium, the individual log odds become

$$\ln\frac{(\mu_{ij}^{r})^{s}}{(\mu_{0j})^{s}} = \frac{\tilde{v}_{ij}^{r} - \tilde{v}_{0j} + \tau_{ij}^{r}}{1-\Phi_{j}}$$

A.3.2 Matching log odds

As before, the matching log odds can be obtained by adding both individual log odds:

$$\ln \mu_{ij}^r = \frac{1-\phi_i^0}{2-\phi_i^r-\Phi_j^r}\ln \mu_{i0} + \frac{1-\Phi_j^0}{2-\phi_i^r-\Phi_j^r}\ln \mu_{0j} + \pi_{ij}^r \tag{17}$$

where $\pi_{ij}^r = \frac{\tilde{u}_{ij}^r - \tilde{u}_{i0} + \tilde{v}_{ij}^r - \tilde{v}_{0j}}{2-\phi_i^r-\Phi_j^r}$. The matching function is:

$$\ln \mu_{ij}^r = \alpha_{ij}^r \ln \mu_{i0} + \beta_{ij}^r \ln \mu_{0j} + \pi_{ij}^r \tag{18}$$

Models and restrictions on α^r and β^r of CD marriage matching function				
Model	α_{ij}^r	β_{ij}^r	π_{ij}^r	Restrictions
CS	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{\tilde{u}_{ij}^r-\tilde{u}_{i0}+\tilde{v}_{ij}^r-\tilde{v}_{0j}}{2}$	$\alpha^r = \beta^r = \frac{1}{2}$
CSW	$\frac{\sigma_i}{\sigma_i+\Sigma_j}$	$\frac{\Sigma_j}{\sigma_i+\Sigma_j}$	$\frac{\tilde{u}_{ij}^r-\tilde{u}_{i0}+\tilde{v}_{ij}^r-\tilde{v}_{0j}}{\sigma_i+\Sigma_j}$	$\alpha_{ij}, \beta_{ij} > 0; \alpha_{ij} + \beta_{ij} = 1$
CS with peer effects	$\frac{1-\phi_i^0}{2-\phi_i^r-\Phi_j^r}$	$\frac{1-\Phi_j^0}{2-\phi_i^r-\Phi_j^r}$	$\frac{\tilde{u}_{ij}^r-\tilde{u}_{i0}+\tilde{v}_{ij}^r-\tilde{v}_{0j}}{2-\phi_i^r-\Phi_j^r}$.	$\alpha_{ij}^r, \beta_{ij}^r \geq 0$, $\frac{\alpha_{ij}^{\mathcal{M}}}{\alpha_{ij}^{\mathcal{C}}} = \frac{\beta_{ij}^{\mathcal{M}}}{\beta_{ij}^{\mathcal{C}}}$

The CSW marriage matching function is observationally equivalent to peer effects coefficients that are relationship independent. i.e., $\phi_i^0 = \phi_i^r$, $\Phi_j^0 = \Phi_j^r$. Indeed, if you consider that peer effects are gender-specific, we can write $\sigma_i \equiv 1-\phi_i^0 = 1-\phi_i^r$, and $\Sigma_j \equiv 1-\Phi_j^0 = 1-\Phi_i^r$ where σ_i, Σ_j can be interpreted as the standard deviations of idiosyncratic payoffs of type i men and type j women, respectively. In this case, we recover the CSW marriage matching function.

A.3.3 Positive assortative mating (PAM)

Let the heterogeneity across males (females) be one dimensional and ordered. Without loss of generality, let male (female) ability be increasing in i (j). We consider type-independent peer effects:

$$\phi_i^0 = \phi^0; \Phi_j^0 = \Phi^0; \phi_i^r = \phi^r; \Phi_j^r = \Phi^r. \tag{19}$$

Then using (17), the local log odds for (r, i, j) is:

$$\begin{aligned} l(r,i,j) = \ln \frac{\mu_{ij}^r \mu_{i+1,j+1}^r}{\mu_{i+1,j}^r \mu_{i,j+1}^r} &= \pi_{ij}^r + \pi_{i+1,j+1}^r - \pi_{i+1,j}^r - \pi_{i,j+1}^r \\ &= \frac{\tilde{u}_{ij}^r + \tilde{v}_{ij}^r + \tilde{u}_{i+1,j+1}^r + \tilde{v}_{i+1,j+1}^r - (\tilde{u}_{i+1,j}^r + \tilde{v}_{i+1,j}^r) - (\tilde{u}_{i,j+1}^r + \tilde{v}_{i,j+1}^r)}{2-\phi-\Phi} \end{aligned} \tag{20}$$

As a measure of PAM, the local log odds is the same under CS or CSW.

A.3.4 Bargaining log odds

Assuming relationship independent peer effects. Subtract the wife's individual log odds of marriage from the husband's to get:

$$\ln \frac{(\mu_{0j})}{(\mu_{i0})} = \frac{\tilde{u}^r_{ij} - \tilde{u}_{i0}}{1 - \phi_i} - \frac{\tilde{v}^r_{ij} - \tilde{v}_{0j}}{1 - \Phi_j} - \frac{2 - \phi_i - \Phi_j}{(1 - \phi_i)(1 - \Phi_j)} \tau^r_{ij}. \tag{21}$$

In CS, $\phi_i = \Phi_j = 0$, which implies that the log is given by $\tilde{u}^r_{ij} - \tilde{u}_{i0} - \tilde{v}^r_{ij} - \tilde{v}_{0j} - 2\tau^r_{ij}$ which is equivalent to what we presented in the main text. If ϕ_i and Φ_j do not change over time, the evolution over time in bargaining log odds will depend in the same way in how much the marital surplus of men and women has evolved over time.

A.3.5 Log odds of choice of within household choice of activity

Consider two within household activity, r versus r'. Under CS, and CSW, the binary within household log odds is:

$$\ln \frac{\mu^r_{ij}}{\mu^{r'}_{ij}} = \pi^r_{ij} - \pi^{r'}_{ij}$$

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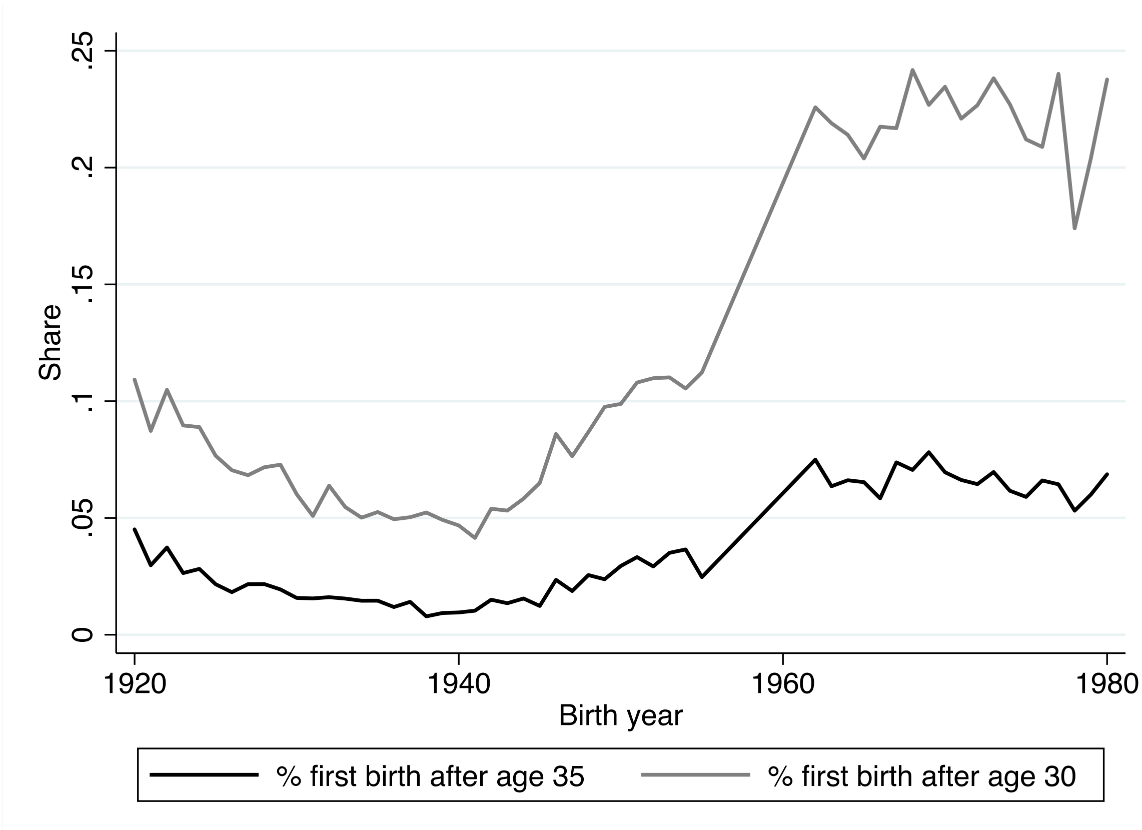
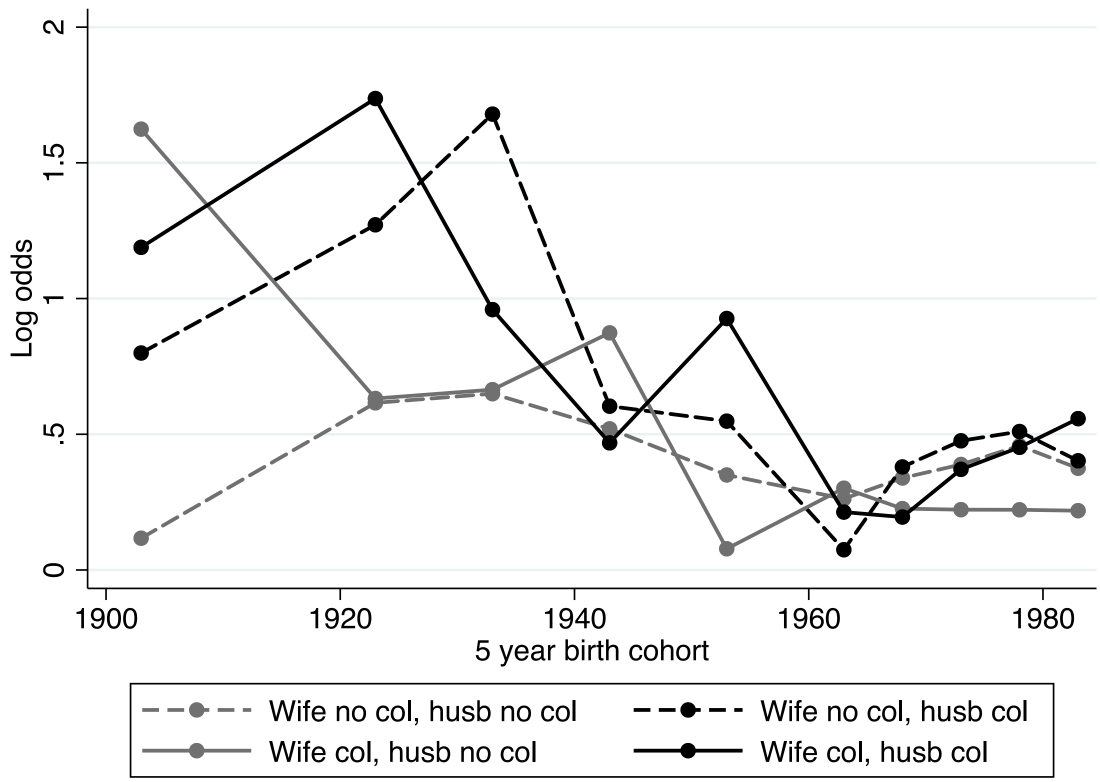
B Additional Figures

Figure B1. Share of mothers having a first birth after age 30 or 35



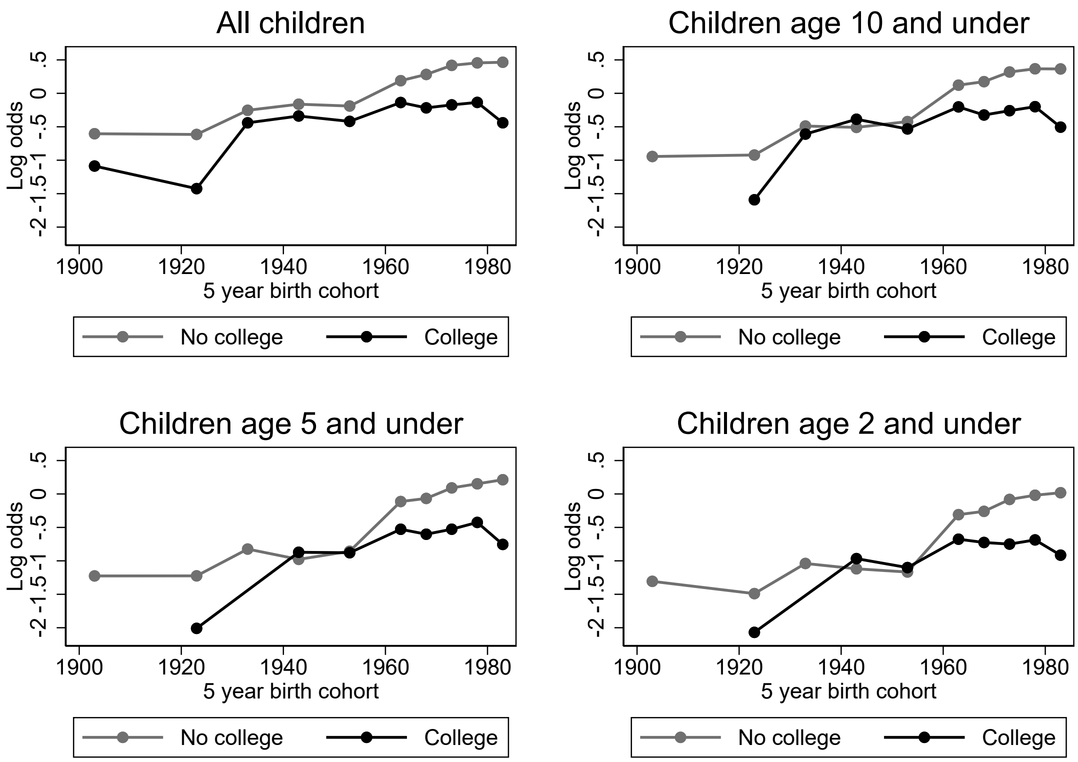
Source: Current Population Survey, June Fertility Supplement, 1977-2020 (Flood et al., 2022). Figure plots the number of women having a first birth after age 30 or 35, as a share of all mothers, observed at age 40 or later. Age at first birth is inferred from the FRBIRTHY1 variable available from IPUMS CPS.

Figure B2. Surplus for pairing childbearing/non-childbearing with men's labor force participation/non-participation, married men 35-39



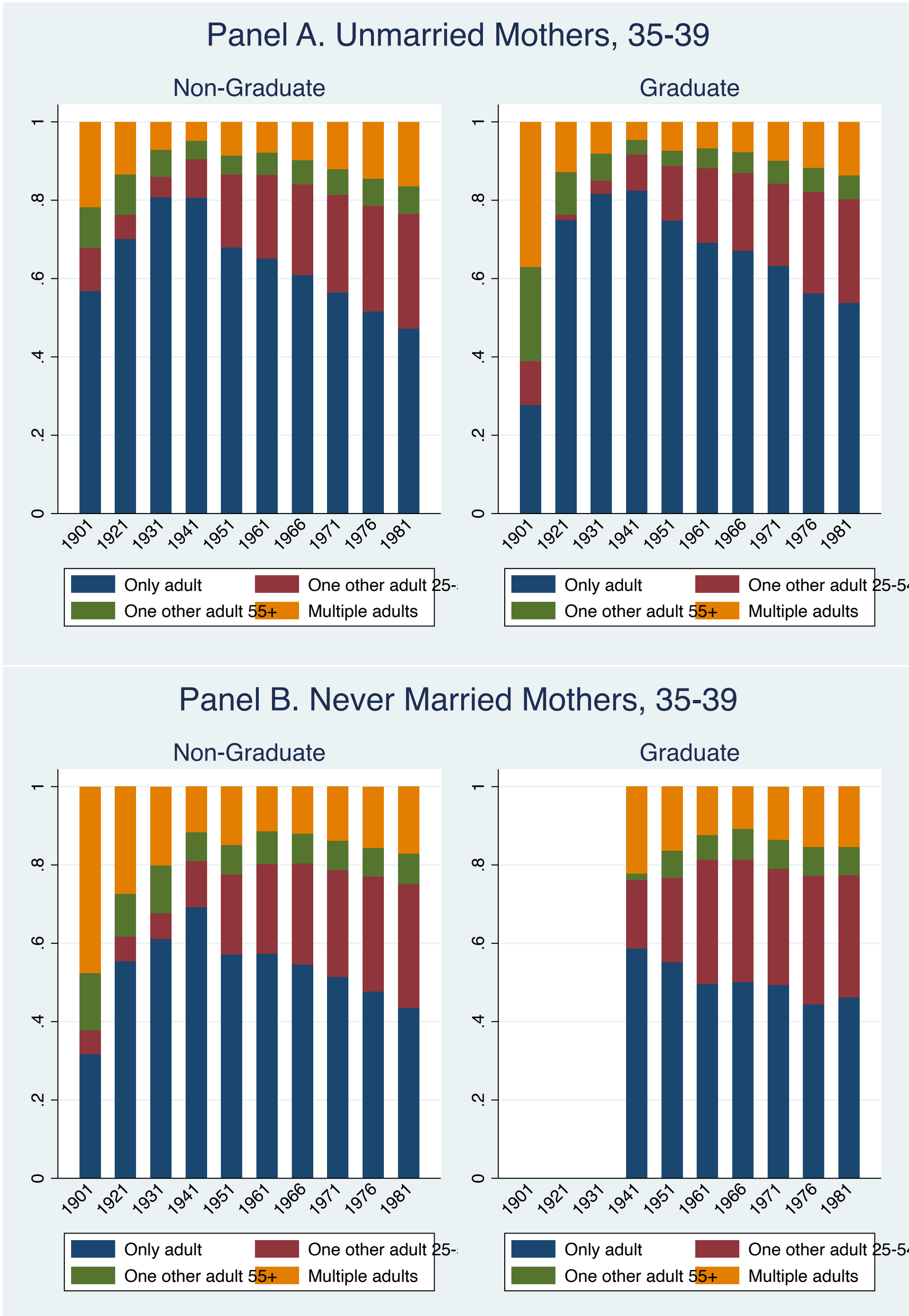
Source: Decennial U.S. Federal Census, 1940-1990; American Community Survey, 2000-2017 (Ruggles et al., 2021). Figure plots the within multinomial log odds (Eq. **??**) of childbearing and labor force participation for married men between the ages of 35 and 39. Log odds are computed for consecutive five-year birth cohorts, starting with 1901-1905. Couples including a man born in year t are observed in census/ACS years that fall between $t + 35$ and $t + 39$, e.g., a couple with the man born in 1901 is part of the 1901-1905 cohort and is observed in 1940. Childbearing is inferred from co-residence with a man's own child under the age of 18. Father-child pairs are observed using the "POPLOC" variable generated by IPUMS. Labor force status is observed using the "EMPSTAT" variable in IPUMS. Log odds are reported separately for all four possible combinations of spousal college graduation status. This figure reproduced Figure **??** for men.

Figure B3. Surplus from pairing childbearing/non-childbearing with living in a multiple adult/single adult household, unmarried women 35-39



Source: Decennial U.S. Federal Census, 1940-1990; American Community Survey, 2000-2017 (Ruggles et al., 2021). Figure plots the within multinomial log odds (Eq. **??**) of childbearing and labor force participation for unmarried college and non-college women between the ages of 35 and 39. Log odds are computed for consecutive five-year birth cohorts, starting with 1901-1905. A woman born in year t is observed in census/ACS years that fall between $t + 35$ and $t + 39$, e.g., a woman born in 1901 is part of the 1901-1905 cohort and is observed in 1940. In the top left panel, childbearing is inferred from co-residence with a woman's own child under the age of 18. In other panels, only children under a given age cut off are included; mothers who only have children above this age cutoff are dropped from the sample. Mother-child pairs are observed using the "MOMLOC" variable generated by IPUMS. Labor force status is observed using the "EMPSTAT" variable in IPUMS.

Figure B4. Living arrangements of unmarried and never-married mothers, ages 35-39



Source: Decennial U.S. Federal Census, 1940-1990; American Community Survey, 2000-2017 (Ruggles et al., 2021). Figure plots the share of currently unmarried and never-married mothers in different living arrangements, by educational attainment. Motherhood is inferred from co-residence with a woman's own child under the age of 18. Mother-child pairs are observed using the "MOMLOC" variable generated by IPUMS.