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Appendix For "The Consumption Origins of Business Cycles: Lessons from Sectoral Dynamics" by Christian Matthes & Felipe Schwartzman

A Proof of Propositions

A.1 Proof of Proposition 1

Here we give a proof of Proposition 1. For notational convenience, we cast this proof in terms of the time series model developed in Section 3 for our application, but translating it into the notation used in section 2 is straightforward. In particular, we let $u_t = \mathbf{x}_t - E_{t-1}\mathbf{x}_t$, and let ε_t be a vector collecting all structural shocks, and w_t is the idiosyncratic noise. D collects the effects of all structural shocks. In particular, its first column corresponds to $\frac{\partial \mathbf{x}}{\partial c}$. Consider a version of our model without dynamics (to focus our attention on the identification of shocks)[1]:

$$u_t = D\varepsilon_t + w_t \tag{A-1}$$

$$\varepsilon_t = \varepsilon_t \tag{A-2}$$

u_t are the stacked forecast errors at the aggregate level and sectoral level stacked into one vector. The second equation/identity is added to turn our model into a state space model. Because all shocks are Gaussian, we can apply the Kalman filter to calculate filtered estimates of our structural shocks ε_t. Note that because our state ε_t does not feature any dynamics, the application of the Kalman filter does not require specifying initial condition. Likewise, filtered estimates will generally equal smoothed estimates, so there is no need to have a separate treatment for smoothed estimates below.[2] We assume the equations above are the

[1] All VAR-type parameters are identified in our setting, so this is without loss of generality.

[2] Because our state is *iid*, the initial distribution of the state does also not matter for smoothed/filtered estimates of the state. To see this, consider a generic linear Gaussian state space system with observables $\overline{y}_t$ and state $\overline{x}_t$:

$$\overline{y}_t = \overline{A}\overline{x}_t + \overline{u}_t \tag{A-3}$$

$$\overline{x}_t = \overline{C}\overline{x}_{t-1} + \overline{w}_t \tag{A-4}$$

where $\overline{u}_t \sim_{i.i.d.} N(0, \overline{B})$ and $\overline{w}_t \sim_{iid} N(0, \overline{D})$. The one-step ahead conditional expectation and conditional variance of the state are then given by

$$E_{t-1}\overline{x}_t = \overline{C}E_{t-1}\overline{x}_{t-1} \tag{A-5}$$

$$Var_{t-1}\overline{x}_t = \overline{C}Var_{t-1}\overline{x}_{t-1}\overline{C}' + \overline{D} \tag{A-6}$$

true data-generating process. Without loss of generality, we assume that the shock whose responses are not misspecified is the first element of ε_t. The Kalman filter returns a least squares estimate of $E_t\varepsilon_t = \varepsilon_{t|t}$:

$$\varepsilon_{t|t} = \beta u_t$$

The matrix of coefficients β is given by the standard formula linking the covariance matrix of the right hand side variable u_t with the covariance of the right-hand-side variable with the left-hand side variable, the vector of structural shocks ε_t:

$$\beta = E(\varepsilon_t u_t')[E(u_t u_t')]^{-1}$$

The second term on the right hand side, $E(u_t u_t')$, can be identified from the data as the second moment matrix of the observables. As such, it does not depend on whether or not D is correctly specified as long as our choice of D is consistent with the overall variability of the data. Where identification matters is in the first term on the right-hand side:

$$E(\varepsilon_t u_t') = D'$$

Let's now assume that we have a misspecified version of the model where, instead of using the true impact matrix D, we use a matrix $\tilde{D}$ such that the first column of D and $\tilde{D}$ coincide. Therefore, the response to the first element of ε is correctly identified, whereas the others are not. This means that the first row of D' and $\tilde{D}'$ coincide. This in turn, means that the first row of $D'[E(u_t u_t')]^{-1}$ equals the first row of $\tilde{D}'[E(u_t u_t')]^{-1}$ and thus that the first element of the estimated shock series is independent of whether D or $\tilde{D}$ is used to form the estimate.

In terms of the notation in the statement of the proposition, it follows that information on the covariance matrix of $\mathbf{x}_t - E_{t-1}\mathbf{x}_t$ (equal to $[E(u_t u_t')]$) and the vector of effects $\frac{\partial \mathbf{x}}{\partial c}$ (equal to the first column of $\tilde{D}$ and of D) are sufficient for the identification of ε_t^C (the first element of ε_t)

A.2 Proof of Proposition 2

As in the proof of Proposition 1, we use the notation in Section 3. In particular, Let ε_t be the vector of all macroeconomic shocks $\varepsilon_{s,t}$. Let D be a matrix where each row corresponds to an element of $\mathbf{x}_t$ and each column to one of the shocks $\varepsilon_{s,t}$ so that each element has the effect of $\varepsilon_{s,t}$ on $\mathbf{x}_t$. Without loss of generality, we assume that $\varepsilon_{1,t} = \varepsilon_t^C$, in which case the

In our application, $\overline{C} = \mathbf{0}$ (the state is *iid*), and hence the one step ahead expectation and variance do not feature any temporal dependence. This then also means that $E_t\overline{x}_t$ and $Var_t\overline{x}_t$ do not depend on $E_{t-1}\overline{x}_{t-1}$ and $Var_{t-1}\overline{x}_{t-1}$.

first column of D is equal to $\partial \mathbf{x}_t / \partial \varepsilon_t^C$. Also, let $u_t \equiv \mathbf{x}_t - E_{t-1}\mathbf{x}_t$. Finally, let N denote the dimensionality of $\mathbf{x}_t$ or, equivalently, u_t. To prove the proposition, it is sufficient to construct an estimator for ε_t^C and show that it converges asymptotically to its true value as $N \to \infty$.

Step 1 - Obtain estimates of the space spanned by the macroeconomic shocks ε_t: Result A.2(a) in Bai and Ng (2008) states that, given the assumptions in Section 4, as $N \to \infty$, one can estimate a $\widehat{\varepsilon}_t$ such that $\sqrt{N}(\widehat{\varepsilon}_t - H\varepsilon_t) \to N(0, \Xi_t)$ where Ξ_t is a matrix defined in their paper and H is a rotation matrix. The estimation error therefore concentrates around zero as $N \to \infty$. In other words, using factor-analytic methods one can consistently estimate the space spanned by ε_t.

Step 2 - Obtain $\widehat{D} \equiv DH'$

Recall that $u_t = D\varepsilon_t + w_t = DH'\widehat{\varepsilon}_t + w_t$, where we use the fact that, for rotation matrices, $H' = H^{-1}$. As $N \to \infty$, $\widehat{\varepsilon}_t$ is measured without error. Since w_t is orthogonal to $\widehat{\varepsilon}_t$, we can recover $\widehat{D} \equiv DH'$ by regressing u_t on $\widehat{\varepsilon}_t$

Step 3 - Estimate ε_t^C: Given that D has a column matching $\partial \mathbf{x}_t / \partial \varepsilon_t^C$, one can find a rotation matrix $\tilde{H}$ such that (i) $\tilde{D} = \widehat{D}\tilde{H}'$ and (ii) $\tilde{D}$ has its first column equal to $\partial \mathbf{x}_t / \partial \varepsilon_t^C$. Such a matrix exists, since $\tilde{H} = H'$ would satisfy the condition. In general, however, there may be multiple such matrices. We take $\tilde{H}$ to be any matrix of that set.

Let $\bar{u}_t = D\varepsilon_t = \widehat{D}\widehat{\varepsilon}_t = \tilde{D}\tilde{H}\widehat{\varepsilon}_t$ denote the part of u_t explained by ε_t. Note that with $N \to \infty$, one can construct $\bar{u}_t$ given Steps 1 and 2 above. Consider now a projection of $\bar{u}_t$ on $\tilde{D}$. The projection coefficients satisfy

$$\tilde{\varepsilon}_t = (\tilde{D}'\tilde{D})^{-1}\tilde{D}'\bar{u}_t$$

Note that $\tilde{D}'\tilde{D} = \tilde{D}'\tilde{H}'\tilde{H}\tilde{D} = \widehat{D}'\widehat{D} = \widehat{D}'HH'\widehat{D}' = D'D$, so that $(\tilde{D}'\tilde{D})^{-1} = (D'D)^{-1}$ irrespective of H or $\tilde{H}$. Moreover, given that we chose $\tilde{H}$ to ensure that the first column of $\tilde{D}$ is equal to $\partial \mathbf{x}_t / \partial \varepsilon_t^C$, the first row of $\tilde{D}'\bar{u}_t$ will also be the same for all H and for all $\tilde{H}$ satisfying that restriction. In particular, that will be true for $\tilde{H} = H'$, so that $\tilde{D} = D$. It follows that $\tilde{\varepsilon}_{1,t} = \varepsilon_{1,t} = \varepsilon_t^C$.

B Data

B.1 Aggregate Data

See figure 3 for a depiction of the aggregate time-series. The sources and definitions are given below. Growth refers to year over year changes of quarterly data.

- Real GDP growth: Real Gross Domestic Product, Billions of Chained 2012 Dollars Series (FRED Series GDPC1) Quarterly, Seasonally Adjusted Annual Rate. U.S. Bureau of Economic Analysis (f)

- CPI inflation: FRED Series CPIAUCSL, Consumer Price Index for All Urban Consumers: All Items. Quarterly, seasonally adjusted. U.S. Bureau of Labor Statistics (U.S. Bureau of Labor Statistics)

- The effective Federal Funds rate: FRED Series FEDFUNDS, Quarterly, not seasonally adjusted, Percent. Board of Governors of the Federal Reserve System (US) (a)

- Growth rate in real government spending: FRED Series GCEC1, Quarterly, seasonally adjusted, Billions of chained 2009 Dollars. U.S. Bureau of Economic Analysis (e)

- Real PCE consumption growth:FRED Series PCECC96, Quarterly, sea- sonally adjusted, Billions of chained 2009 Dollars. U.S. Bureau of Economic Analysis (g)

- Moody's Seasoned BAA Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity: FRED Series BAA10YM, Quarterly, not seasonally adjusted. Federal Reserve Bank of St. Louis (Federal Reserve Bank of St. Louis)

- Fernald's utility adjusted TFP (Fernald (2014)): Percent Change (natural log difference). John Fernald (2014)

- Inflation based on the relevant producer price index: Producer Prices Index: Economic Activities: Total Energy for the United States, FRED Series PIEAEN01USQ661N. Organization for Economic Co-operation and Development (Organization for Economic Co-operation and Development)

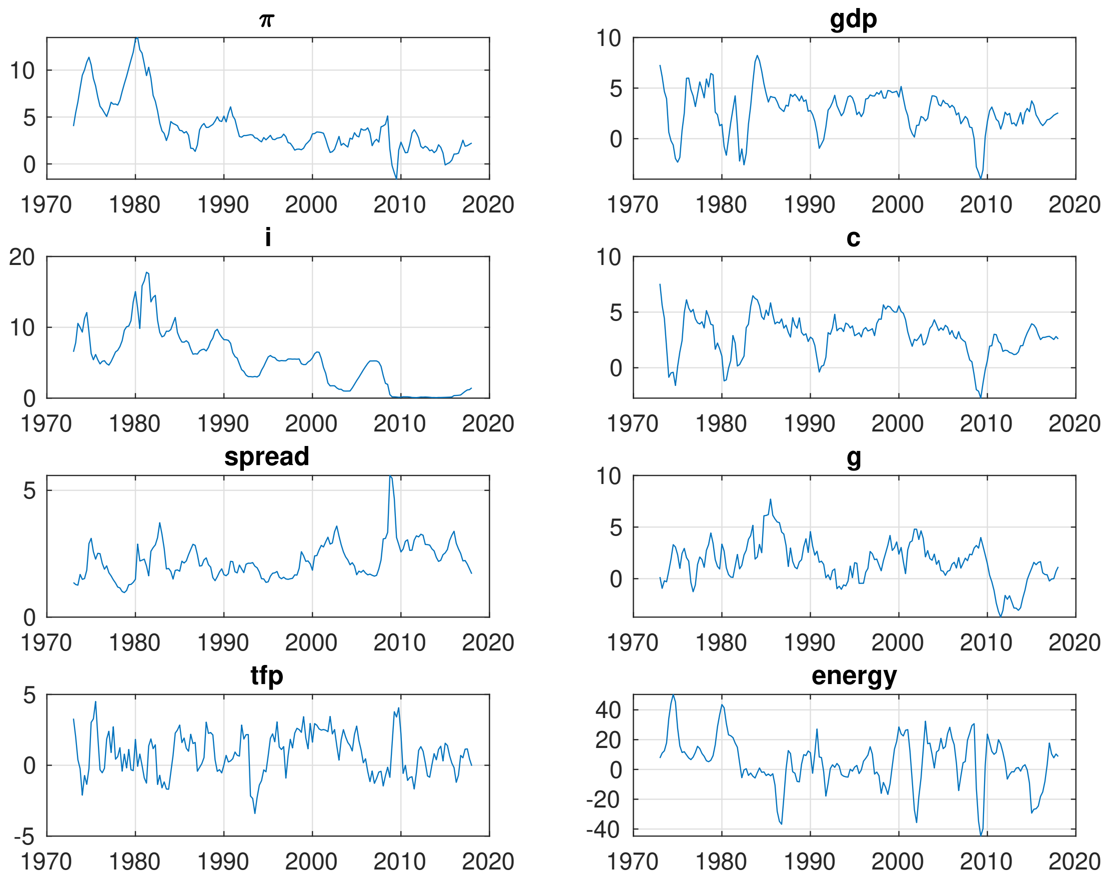

Figure A-1: Aggregate Data

B.2 Sectoral Level Data

We use PCE sectors throughout. For Industrial Production, the data originally was classified by 4-digit 2007 NAICS and was converted to PCE using the 2007 PCE Bridge Table published by the BEA. U.S. Bureau of Economic Analysis (a)

- PCE Price Index (PCEPI): BEA Table 2.4.4U. Price Indexes for Personal Consumption Expenditures by Type of Product. See figure A-2 upper panel for a depiction of the data series. U.S. Bureau of Economic Analysis (d)
- PCE Quantity Index (PCEQI): BEA Table 2.4.3U. Real Personal Consumption Expenditures by Type of Product, Quantity Indexes. See figure A-2 middle panel for a depiction of the data series. U.S. Bureau of Economic Analysis (c)
- Industrial production index: This is the Fed Board of Governor's IP data. One can access the IP data release here: https://www.federalreserve.gov/releases/G17/. See

figure A-2 lower panel for a depiction of the data series. Board of Governors of the Federal Reserve System (US) (b)

- Technology Exposure: Using the BEA Use Table, we take the ratio of intermediate inputs from high technology sectors to total intermediate inputs. High technology sectors are those defined by Heckler (2005) as such. U.S. Bureau of Economic Analysis (b)

- Financial exposure: We take the ratio of intermediate inputs from finance and insurance sectors to total intermediate inputs using the BEA Use Table. Finance sectors are those with 2-digit NAICS code 52.

- Household Consumption Share: We calculate the Household share as the proportion of output that goes to Personal Consumption Expenditures from the BEA IO Use Table.

- Government Consumption Share: We calculate the government share as the total output sold to all federal, state, and local government categories listed in the Use Table, divided by total industry output.

- Energy exposure: We take the ratio of intermediate inputs from energy sectors to total intermediate inputs using the BEA Use Table. Energy sectors are defined as electrical power generation, oil and gas extraction, natural gas distribution, and petroleum and coal manufacturing.

- Price stickiness: The median price adjustment duration from Nakumura Steinsson (2008) across PCE categories. To capture the frequency of price changes within industry, we take the price adjustment durations estimated by Nakamura and Steinsson (2008). Estimates are provided at the Entry Line Item (ELI) level. Using the ELI/PCE crosswalk provided by the BLS, we can transfer these ELI level duration values to the PCE classification. For each PCE category, we assign the average of the duration values for the set of ELIs with which the PCE category is matched. U.S. Bureau of Labor Statistics (2011); Nakamura and Steinsson (2008b).

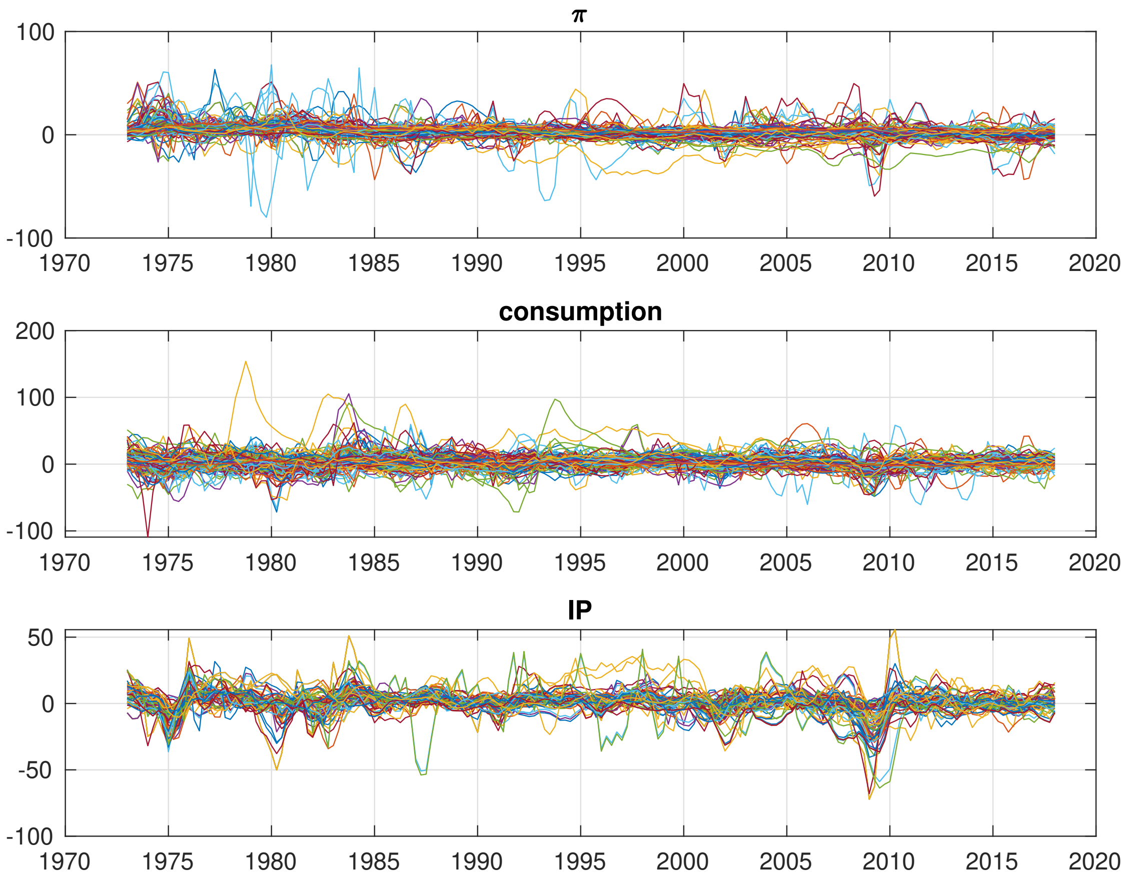

Figure A-2: Sectoral Data

C A Tractable Multi-Sector Model with Nominal Rigidites

We now lay out a tractable, multi-sector model with nominal rigidities to motivate the shock identification scheme. Nominal rigidities allow for a non-trivial "aggregate demand" channel. Since our main focus is in the cross-sectional differences between industries, rather than their individual dynamics, we lay out a static multi-sector economy. This is appropriate for our empirical analysis since we use identifying restrictions (via our priors) on the impact of shocks rather than on the dynamic responses to those. The model shares many elements with the framework developed in Pasten et al. (2020), while also allowing for nominal wage stickiness and for several aggregate shocks.

C.1 Households

There are J sectors, indexed $i \in \{1, ..., J\}$. There is a representative household with Cobb-Douglas preferences over the various goods, with share-parameter α_j for a good of industry i.

$$U = \prod_j C_j^{\alpha_j},$$

where $\sum_j \alpha_j = 1$. The household chooses its the amount it consumes of good i, C_j, to maximize its utility subject to the budget constraint

$$\sum_j P_j C_j + T = WL + \Pi + \sum_j r_j \bar{K}_j,$$

where T is a lump-sum tax levied by the government to finance its consumption, W is the wage rate, Π are profits rebated from firms, $\bar{K}_j$ is the stock of capital specific to sector i owned by the household, with r_j the corresponding rental rate, and $L < 1$ is employment to be determined in equilibrium.

Finally, households supply one unit of labor inelastically, but nominal wages are rigid so that labor is rationed.

Given those constraints, optimal household consumption choice satisfies:

$$P_j C_j = \alpha_j^C PC$$

for $P^C \equiv \prod_j \left(\frac{P_j}{\alpha_j}\right)^{\alpha_j}$ and $C \equiv \prod_j (C_j)^{\alpha_j}$.

C.2 Fiscal Authority

The fiscal authority minimizes the cost of consuming an exogenously given aggregate government consumption G,

$$\min \sum_j P_j G_j$$
$$s.t. : \prod_j (G_j)^{\alpha_j^G} = G,$$

where G is exogenously determined and α_j^G are expenditure shares. The optimality condition for the government is:

$$G_j = \alpha_j^G \frac{P_G}{P_j} G$$

where

$$P_G = \prod_j \left(\frac{G_j}{\alpha_j^G} \right)^{\alpha_j^G}.$$

C.3 Firms

Within each sector there is a continuum of varieties of intermediate products indexed $v \in [0, 1]$. Those varieties are purchased by final goods producers that bundle them into the I goods according to a CES aggregator:

$$Y_j = \left[\int_0^1 Y_j(v)^{\frac{\theta-1}{\theta}} dv \right]^{\frac{\theta}{\theta-1}}$$

The demand for final good producer in sector i for intermediate input of variety v is

$$Y_j(v) = \left(\frac{P_j(v)}{P_j} \right)^{-\theta} Y_j$$

where

$$P_j = \left[\int P_j(v)^{1-\theta} dv \right]^{\frac{1}{1-\theta}}$$

For each variety, production takes place with a Cobb-Douglas production function:

$$Y_j(v) = e^{\epsilon_j} \prod_j \left(X_{j'j}(v)\right)^{\gamma_{j'j}} \times \left(L_j(v)\right)^{\lambda_j} \left(K_j\left(v\right)\right)^{\chi},$$

where $X_{j'j}(v)$ is the quantity of final goods materials produced in sector j used as materials in sector i for variety v, $L_j(v)$ is labor, $K_j(v)$ is sector-specific capital, and ϵ_j is a sector-specific exogenous productivity shock. The share parameter for good j used in sector i is $\gamma_{j'j}$. We assume that $\sum_j \gamma_{j'j} + \lambda_j + \chi = 1$, so that firms in the industry face constant returns to scale.

Producers of varieties are monopolists. Firms differ on the information set available to them regarding prices and the demand for their intermediate input. Letting $\mathbf{s}$ denote the state of the economy, they take the wage rate, final goods prices, and household demand as given and choose their inputs to maximize expected profits.

$$\max_{M_{j'j}} E\left[P_j(v)Y_j(v,\mathbf{s}) - \sum_j P_j(\mathbf{s})X_{j'j}(v,\mathbf{s}) - w(\mathbf{s})L_j(v,\mathbf{s}) - r_j(\mathbf{s})K_j(v,\mathbf{s})|\mathcal{I}_j(v)\right]$$

$$s.t. : Y_j(v,\mathbf{s}) = \left(\frac{P_j(v)}{P_j(\mathbf{s})}\right)^{-\theta} Y_j(\mathbf{s})$$

$$Y_j(v,\mathbf{s}) = e^{\epsilon_j} \prod_j \left(X_{j'j}(v,\mathbf{s})\right)^{\gamma_{j'j}} \left(L_j(v,\mathbf{s})\right)^{\lambda_j} \left(K_j(v,\mathbf{s})\right)^{\chi}$$

where $\mathcal{I}_j(v)$ is the information set for variety v in sector i. For a fraction ϕ_j of variety producers in sector i ($v \in [0, \phi_j]$) the information set does not includes the realized vector of shocks $\mathbf{s}$. For the remainder, the information set does includes it. Yet, firms commit to producing as much as necessary to satisfy demand at the prices that they choose.

Given cost-minimization, marginal cost is

$$\text{mc}_j(\mathbf{s}) = e^{-\epsilon_j} \prod_j \left(\frac{P_j(\mathbf{s})}{\gamma_{j'j}}\right)^{\gamma_{j'j}} \left(\frac{w(\mathbf{s})}{\lambda_j}\right)^{\lambda_j} \left(\frac{\mathbf{r}(\mathbf{s})}{\chi}\right)^{\chi}$$

Firms with full information set prices to

$$P_j(v,\mathbf{s}) = \frac{\theta}{\theta - 1}\text{mc}_j(s)$$

Firms without full information set prices to

$$P_j(v) = \frac{\theta}{\theta - 1} E\left[\frac{P_j(\mathbf{s})^{\theta} Y_j(\mathbf{s})}{E\left[P_j(\mathbf{s})^{\theta} Y_j(\mathbf{s})\right]} mc_j(\mathbf{s})\right]$$

We thus have that the price index for sector i is

$$P_j(\mathbf{s}) = \left[\phi_j \left(\frac{\theta}{\theta-1} E\left[\frac{P_j(\mathbf{s})^\theta Y_j(\mathbf{s})}{E\left[P_j(\mathbf{s})^\theta Y_j(\mathbf{s})\right]} mc_j(\mathbf{s})\right]\right)^{1-\theta} + (1-\phi_j)\left(\frac{\theta}{\theta-1}\mathrm{mc}_j(\mathbf{s})\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

Given that all firms in a sector have the same marginal cost, we can write the average markup as

$$\mu_j = \frac{P_j(\mathbf{s})}{mc_j(\mathbf{s})} = \left[\phi_j \frac{\theta}{\theta-1} E\left[\frac{P_j(\mathbf{s})^\theta Y_j(\mathbf{s})}{E\left[P_j(\mathbf{s})^\theta Y_j(\mathbf{s})\right]} mc_j(\mathbf{s})\right]^{1-\theta} \left(\frac{1}{mc_j(\mathbf{s})}\right)^{1-\theta} + (1-\phi_j)\left(\frac{\theta}{\theta-1}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

C.4 Market Clearing

Market clearing for each sector i, requires that all output is used either as materials, for household consumption or for government consumption:

$$Y_j = \sum_j X_{jj'} + C_j + G_j$$

In addition, there is a fixed stock of capital $\bar{K}_j$ for each sector. Market clearing in capital markets thus requires that the demand for capital in sector i equals supply:

$$K_j = \bar{K}_j$$

The resource constraint in the labor market is

$$\sum_j L_j \leq 1$$

With sticky wages, the inequality need not hold. We assume that wages are stuck at a level high enough that it does not bind. Labor rationing thus implies that

$$L = \sum_j L_j$$

C.5 Shocks

As in Woodford (2003), we assume exogenous processes for nominal aggregates. In particular, we assume that nominal private consumption and nominal government consumption are set exogenously. Specifically, we assume that

$$P^C C = M^C M^Y$$
$$P^G G = M^G M^Y$$

so that nominal private and government consumptions can be affected either by an exogenous component which is specific to each type of final expenditure M^C or M^G, or by a common component M^Y.

Finally, we also allow for industry-level productivity shocks ϵ_j. We assume that $\epsilon_j = \sum_{r=1}^{R} \lambda_{ir}\epsilon_r + \hat{\epsilon}_j$, where ϵ_r are aggregate shocks, F_j captures the sensitivity of various sectors to that shock, and $\hat{\epsilon}_j$ is a sector-specific shock. In our application, we will allow ϵ_r to incorporate shocks to technology and financial shocks.

C.6 Log-linearized system

Up to a first-order approximation the economy is described by the following system of equations (small letters indicate log deviations from steady-state):

$$p^C + c = m^C + m^Y$$
$$p^G + g = m^G + m^Y \tag{A-7}$$
$$w = 0 \tag{A-8}$$
$$g_j - g = p^G - p_j \ \forall i \tag{A-9}$$
$$c_j - c = p^C - p_j \ \forall i \tag{A-10}$$
$$y_j = \epsilon_j + \sum_j \gamma_{j'j} x_{j'j} + \lambda_j l_j + \chi k_j \ \forall i \tag{A-11}$$
$$w + l_j = p_j + y_j - \mu_j \ \forall i \tag{A-12}$$
$$p_j + x_{j'j} = p_j + y_j - \mu_j \ \forall i, j \tag{A-13}$$
$$r_j + k_j = p_j + y_j - \mu_j \ \forall i \tag{A-14}$$
$$k_j = \bar{k}_j \tag{A-15}$$
$$\mu_j = -\phi_j \left(\sum_j \gamma_{j'j} p_j + \lambda_j w + \chi r_j - \epsilon_j \right) \tag{A-16}$$
$$y_j = \sum_j \frac{X_{jj'}}{Y_j} x_{jj'} + \frac{C_j}{Y_j} c_j + \frac{G_j}{Y_j} g_j \tag{A-17}$$

The system can be reduced to:

$$
\begin{aligned}
p_j - (1-\chi)\mu_j &= -\epsilon_j + \sum_j \gamma_{j'j} p_j + \chi \left(p_j + y_j - \bar{k}_j\right) \\
p_j + y_j &= \sum_j \gamma_{jj'} \frac{Y_j}{Y_j}(y_j + p_j - \mu_j) + \frac{C_j}{Y_j}(m^C + m^Y) + \frac{G_j}{Y_j}(m^G + m^Y) \\
\mu_j &= -\frac{\phi_j}{1-\phi_j\chi}\left(-\epsilon_j + \sum_j \gamma_{j'j} p_j + \chi \left(p_j + y_j - \bar{k}_j\right)\right)
\end{aligned}
$$

Or, eliminating μ_j,

$$
\begin{aligned}
p_j &= \frac{1-\phi_j}{1-\chi}\left(-\epsilon_j + \sum_j \gamma_{j'j} p_j + \chi \left(y_j - \bar{k}_j\right)\right) \\
p_j + y_j &= \sum_j \gamma_{jj'} \frac{Y_j}{Y_j}(y_j + \frac{1}{1-\phi_j} p_j) + \frac{C_j}{Y_j}(m^C + m^Y) + \frac{G_j}{Y_j}(m^G + m^Y)
\end{aligned}
$$

The system can be rewritten as

$$
\begin{aligned}
p_j = &\frac{1-\phi_j}{1-\chi}\chi\left[(1-\chi\Phi_j)\left[\sum_j f_{jj'}(y_j + \frac{1}{1-\phi_j} p_j) + \frac{C_j}{Y_j}(m^C + m^Y) + \frac{G_j}{Y_j}(m^G + m^Y)\right] + \Phi_j\left(\epsilon_j + \chi\bar{k}_j\right)\right] \\
&- \Phi_j\left(\epsilon_j + \chi\bar{k}_j\right) + \Phi_j \sum_j b_{j'j} p_j
\end{aligned}
$$

$$
y_j = (1-\chi\Phi_j)\left[\sum_j f_{jj'}(y_j + \frac{1}{1-\phi_j} p_j) + \frac{C_j}{Y_j}(m^C + m^Y) + \frac{G_j}{Y_j}(m^G + m^Y)\right] + \Phi_j\left(\epsilon_j + \chi\bar{k}_j\right) - \Phi_j \sum_j b_{j'j} p_j
$$

with $f_{jj'} = \gamma_{jj'} \frac{Y_j}{Y_j}$ capturing forward links and $b_{j'j} = \gamma_{j'j}$ capturing backward links

After log-linearizing and rearranging, the model can be reduced to:

$$p_j = \frac{1-\phi_j}{1-\chi}\left(-\epsilon_j + \sum_j \gamma_{j'j}p_j + \chi\left(y_j - \bar{k}_j\right)\right)$$

$$p_j + y_j = \sum_j \gamma_{jj'}\frac{Y_j}{Y_j}\left(y_j + \frac{1}{1-\phi_j}p_j\right) + \frac{C_j}{Y_j}(m^C + m^Y) + \frac{G_j}{Y_j}(m^G + m^Y)$$

where small caps letters denote log deviations from a reference level. The first set of equations are "sectoral supply" equations, relating marginal production cost to prices. The second set of equations are "sectoral demand" equations, relating nominal expenditures to sectoral prices. The last set of equations links nominal consumption expenditures and exogenous demand shocks.

The system has the form

$$Z = AZ + b = A^N Z + \sum_{n=0}^{N-1} A^n b$$

with Z including prices and quantities in all sectors, b including the direct impact of all exogenous shocks, and A including the indirect impact of shocks through linkages.

Lemma 1 characterizes the direct and indirect impacts of the shocks on prices, output and consumption:

Lemma 1 *The direct impact of shocks is given by* $b = \left[\mathbf{p}^{Direct}, \mathbf{y}^{Direct}, \mathbf{c}^{Direct}\right]^T$*, where*

$$p_j^{Direct} = \Phi_j\chi\left[\frac{C_j}{Y_j}m^C + \frac{G_j}{Y_j}m^G + m^Y\right] - \Phi_j\left(\epsilon_j + \chi\bar{k}_j\right) \quad \text{(A-18)}$$

$$y_j^{Direct} = (1-\Phi_j\chi)\left[\frac{C_j}{Y_j}m^C + \frac{G_j}{Y_j}m^G + m^Y\right] + \Phi_j\left(\epsilon_j + \chi\bar{k}_j\right) \quad \text{(A-19)}$$

$$c_j^{Direct} = \left(1 - \Phi_j\chi\frac{C_j}{Y_j}\right)m^C + (1-\Phi_j\chi)m^Y - \Phi_j\chi\frac{G_j}{Y_j}m^G + \Phi_j\left(\epsilon_j + \chi\bar{k}_j\right) \quad \text{(A-20)}$$

and

$$\Phi_j \equiv \frac{1-\phi_j}{\chi(1-\phi_j) + 1 - \chi}$$

is inversely related to ϕ_j*. Indirect effects are* $AZ = \left[\mathbf{p}^{Indirect}, \mathbf{y}^{Indirect}, \mathbf{c}^{Indirect}\right]^T$*, where*

$$p_j^{Indirect} = \Phi_j \sum_j \left(\chi \frac{f_{jj'}}{1-\phi_j} + b_{j'j} \right) p_j + \chi \Phi_j \sum_j f_{jj'} y_j \tag{A-21}$$

$$y_j^{Indirect} = (1 - \chi\Phi_j) \sum_j f_{jj'} y_j + \sum_j \left[\frac{1-\chi\Phi_j}{1-\phi_j} f_{jj'} - \Phi_j b_{j'j} \right] p_j \tag{A-22}$$

$$c_j^{Indirect} = -p_j^{Indirect} \tag{A-23}$$

where $f_{jj'} = \gamma_{jj'} \frac{Y_j}{Y_j}$ *capture forward linkages and* $b_{j'j} = \gamma_{j'j}$ *captures backward linkages.*

Lemma 1 implies that the direct impact of a consumption shock m^C on prices increases in $\Phi_j \chi \frac{C_j}{Y_j}$

D Dynamic Model

In what follows, we present a dynamic model with multiple sectors, sticky nominal prices and sticky nominal wages. The exposition largely follows Justiniano et al. (2010), with some simplifications (we omit markup shocks) and extensions where needed.

D.1 Final good producers

There are J sectors (indexed $j \in [1, ..., J]$). In each of these sectors there are perfectly competitive firms producing final goods Y_t^j combining a continuum of intermediate goods $\{Y_t(i)\}_r$, $i \in [0, 1]$, according to the technology

$$Y_t^j = \left[\int_0^1 Y_t^j(i)^{\frac{\epsilon^p - 1}{\epsilon^p}} di \right]^{\frac{\epsilon^p}{\epsilon^p - 1}}$$

From profit maximization and zero profit conditions we have that

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon^p} Y_t^j$$

where P_t is the price of final good j and satisfies

$$P_t = \left[\int_0^1 P_t(i)^{\frac{1}{1-\epsilon^p}} di \right]^{1-\epsilon^p}$$

D.2 Intermediate good producers

A monopolist produces the intermediate good i in sector j according to the production function

$$Y_t^j(i) = \max\left\{\left(\frac{K_t^j(i)}{(1-\gamma^j)\omega^j}\right)^{(1-\gamma^j)\omega^j}\left(\frac{A_t^j L_t^j(i)}{(1-\gamma^j)\left(1-\omega^j\right)}\right)^{(1-\gamma^j)\left(1-\omega^j\right)}\prod_{j'}\left(\frac{M_t^{j'j}(i)}{\gamma^{j'j}}\right)^{\gamma^{j'j}} - F^j, 0\right\}$$

where $K_t^j(i)$, $L_t^j(i)$ denote the amounts of capital and labor emploiyed by firm i in sector j, $M_t^{j'j}(i)$ is the amount of materials produced in sector j' used by firm i in sector j and F^j is a fixed cost of production, chosen so that profits are zero in stead state. A_t^j respresents exogenous technological progress in sector j. We assume that it consists of a combination of aggregate and sector specific components:

$$A_t^j = A_t \widehat{A}_t^j$$

where

$$\ln A_t = \rho^A \ln A_{t-1} + \varepsilon_t^A$$

where ϵ_t^A is *iid* with standard deviation σ^A

Furthermore,

$$\ln \widehat{A}_t^j = (1-\rho^{A^j}) \ln \widehat{A}^j + \rho^{A^j} \ln \widehat{A}_{t-1}^j + \varepsilon_t^{A,j}$$

where $\varepsilon_t^{A,j}$ has, likewise, standard deviation σ^{A^j} Every period in each sector j, a fraction ξ^{pj} of intermediate firms cannot choose its price optimally, and as in Smets and Wouters (2003), they reset it according to the indexation rule

$$P_t(i) = P_{t-1}(i)\left(\Pi_{t-1}^j\right)^{\iota^p} \Pi^{1-\iota^p},$$

where $\pi_t^j = \frac{P_t^j}{P_{t-1}^j}$ is gross sector j inflation and π is its steady state. The remaining fraction of firms chooses its price $P_t(i)$ optimally, by maximizing the present discounted value of future profits

$$E_t\left\{\sum_{s=0}^{\infty}\left(\xi^{pj}\right)^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t}\left[P_t(i)\left(\Pi_{t,t+s}^j\right) Y_{t+s}(i) - W_{t+s}^j L_{t+s}(i) - R_{t+s}^{k,j} K_{t+s}(i) - \sum_{j'} P_{t+s}^{j'}(i) M_{t+s}^{j'}(i)\right]\right\}$$

where

$$\Pi^j_{t,s} \equiv \prod_{k=1}^{s} \left(\Pi^j_{t+k-1}\right)^{\iota^p} \Pi^{(1-\iota^p)k} \text{ for } s \geq 1$$
$$\Pi^j_{t,t} = 1$$

and

$$Y_{t+s}(i) = \left(\frac{P_{t+s}(i)}{P_{t+s}}\right)^{-\epsilon^p} Y^j_{t+s}$$

subject to the demand function and to cost minimization. In this objective, Λ_t is the marginal utility of nominal income for the representative household that owns the firm, while W_t and $r^{k,j}_t$ are the nominal wage and the rental rate of capital specific to sector j.

Cost minimization by firms implies that

$$\frac{K^j_t(i)}{L^j_t(i)} = \frac{W^j_t}{R^{k,j}_t}\frac{\omega^j}{1-\omega^j}$$

and

$$\frac{M^{j'j}_t(i)}{L^j_t(i)} = \frac{W^j_t}{P^{j'}_t}\frac{\gamma^{j'j}}{(1-\gamma^j)(1-\omega^j)},$$

so that nominal marginal cost in sector j is common to all firms and given by

$$MC^j_t = \left(R^{k,j}_t\right)^{(1-\gamma^j)\omega^j} \left(\frac{W^j_t}{A^j_t}\right)^{(1-\gamma^j)\left(1-\omega^j\right)} \prod_{j'} \left(P^{j'}_t\right)^{\gamma^{j'j}}.$$

Substituting back input choices, and ignoring the fixed costs, yields employment in each variety as a function of sectoral output and the price of the variety,

$$L^j_t(i) = (1-\gamma^j)(1-\omega^j)\frac{MC^j_t}{W^j_t}\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon^p} Y^j_t.$$

Integrating both sides yields sectoral employment:

$$L^j_t = (1-\gamma^j)(1-\omega^j)\frac{MC^j_t}{W^j_t}P^{\epsilon^p}_t Y^j_t \int P_t(i)^{-\epsilon^p}di.$$

From the intermediate input demand function,

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon^p} Y_t^j.$$

Given that, with our production function, average variable costs and marginal costs coincide, the objective function for firms setting prices optimally can be rewritten as

$$\max_{P_t(i)} E_t \left[\sum_{s=0}^{\infty} \left(\xi^{pj}\right)^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left[\left(P_t^j(i)\Pi_{t,t+s}^j - MC_t\right) Y_{t+s}(i)\right]\right]$$
$$s.t. : Y_{t+s}^j(i) = \left(\frac{P_t(i)\Pi_{t,t+s}^j}{P_{t+s}}\right)^{-\epsilon^p} Y_{t+s}^j$$

The first-order condition can then be written as

$$\tilde{P}_t^j = \frac{\epsilon^p}{\epsilon^p - 1} \sum_{s=0}^{\infty} \frac{E_t \left\{\left(\beta \xi^{pj}\right)^s \Lambda_{t+s} \tilde{Y}_{t+s}^j MC_{t+s}^j\right\}}{\sum_{s=0}^{\infty} E_t \left\{\left(\beta \xi^{pj}\right)^s \Lambda_{t+s} \tilde{Y}_{t+s}^j \Pi_{t,t+s}^j\right\}}$$

where $\tilde{P}_t^j$ is the optimally chosen price for all firms i choosing their prices in period t (so that $P_t^j(i) = \tilde{P}_t^j$), and $\tilde{Y}_{t+s}$ is the demand they face in $t+s$.

Alternatively,

$$\frac{\tilde{P}_t^j}{P_t} = \frac{\epsilon^p}{\epsilon^p - 1} \sum_{s=0}^{\infty} \frac{E_t \left\{\left(\beta \xi^{pj}\right)^s \Lambda_{t+s} P_{t+s} \left(\tilde{Y}_{t+s}^j\right) \frac{MC_{t+s}^j}{P_t^j}\right\}}{\sum_{s=0}^{\infty} E_t \left\{\left(\beta \xi^{pj}\right)^s \Lambda_{t+s} P_{t+s} \left(\tilde{Y}_{t+s}^j\right) \left(\Pi_{t,t+s}^j / \Pi_{t,t+s}\right)\right\}}$$

where

$$\Pi_{t,s} \equiv \prod_{k=1}^{s} \Pi_{t+k} \text{ for } s \geq 1$$
$$\Pi_{t,t} = 1$$

D.3 Employment Agencies

Workers have monopoly power over their labor supply. There is a competitive employment agency which combines specialized household labor into a homogeneous labor input sold to firms in sector j according to

$$L_t^j = \left[\int L_t^j(h)^{\frac{\epsilon^w - 1}{\epsilon^w}} dh \right]^{\frac{\epsilon^w}{\epsilon^w - 1}}.$$

Profit maximization implies that

$$L_t^j(h) = \left(\frac{W_t^j(h)}{W_t^j} \right)^{-\epsilon^w} L_t^j,$$

and the wage paid by firms for homogeneous labor input is

$$W_t^j = \left[\int_0^1 W_t^j(h)^{1-\epsilon^w} dh \right]^{\frac{1}{1-\epsilon^w}}$$

D.4 Households

Each household (h) has labor which is specific to some sector j and utility function given by

$$U_t = \sum_s E_t \beta^s b_{t+s} \left[\ln \left[X_{t+s}(h) \right] - \sum_j \frac{\varphi^j}{1+\nu} L_t^j(h)^{1+\nu} \right],$$

where

$$X_{t+s}(h) = \prod_j \left(C_{t+s}^j(h) - \eta C_{t+s-1}^j \right)^{\alpha_t^j},$$

and where $C_{t+s}^j(i)$, $L_t(i)$ and $X_{t+s}(i)$ are household choices and X_{t+s} and C_{t+s}^j are equilibrium objects that the household takes as given. The formulation corresponds to allowing for habits to consumption of particular goods.

To allow for sector-specific demand shocks, we allow consumption shares, α_t^j to be time-varying. Specifically[3]

$$\ln \alpha_t^j = (1 - \rho^\alpha)\alpha^j + \rho^\alpha \ln \alpha_{t-1}^j + \varepsilon_t^{\alpha,j}$$

where ε_t^α is a random normal variable with standard deviation σ^{α^j}. The time-varying parameter b_t is a shock to the discount factor, affecting both the marginal utility of consumption and the marginal disutility of labor. This intertemporal preference shock follows the stochastic process

$$\Delta \log b_t = \rho^b \Delta \log b_{t-1} + \varepsilon_{b,t}$$

[3]While this formulation constrains share parameters to be positive, it does not constrain them to add up to 1. Allowing for this degree of freedom is necessary to give the ability to match the full set of sector-specific variables.

where Δ is the time-difference operator and $\varepsilon_{b,t}$ is an *iid* random normal variable with mean zero and standard deviation σ^b. There are state contingent securities ensuring that in equilibrium consumption and asset holdings are the same for all households. As a result, the household's flow budget constraint is

$$\sum_j P_t^j C_t^j + \sum_{j,j'} P_t^{j'} I_t^{j'j} + T_t + B_t \leq R_{t-1}B_{t-1} + Q_t(j) + \Pi_t + W_t^j(j)L_t(j) + \sum_j R_t^{k,j} K_{t-1}^j,$$

where $I_t^{j'j}$ is investment in good j' to form capital in sector j, T_t is lump-sum taxes, B_t is holdings of government bonds, R_t is the gross nominal interest rate, $Q_t(j)$ is the net cas flow from household's j portfolio of state contingent securities, and Π_t is the per-capital profit accruing to households from ownership of the firms.

Consumption Given interest rates on riskless debt R_t, the problem induces the Euler equation:

$$\Lambda_t = \beta R_t E_t \Lambda_{t+1},$$

where $P_t = \prod_j \left(\frac{P_t^j}{\alpha_t^j}\right)^{\alpha_t^j}$ is the consumption price index and $\Lambda_t \equiv \frac{b_t}{P_t X_t}$ is the "nominal" marginal utility of consumption. Given that we get the intra-temporal allocation across industries:

$$C_t^j(h) = \alpha_t^j \frac{P_t}{P_t^j} X_t(h) + \eta C_{t-1}^j.$$

The model features a representative household, so that in equilibrium, $C_t^j = C_t(h)$.

Capital accumulation Households own capital specific to each sector j and rent them to firms at the rate $R_t^{k,j}$. The physical capital accumulation equation is

$$K_t^j = (1-\delta)K_{t-1}^j + \left(1 - S\left(\frac{I_t^j}{I_{t-1}^j}\right)\right) I_t^j,$$

where δ is the depreciation rate and is the investment in sector j. The function S captures the presence of adjustment costs in investment, as in Christiano, Eichenbaum, and Evans (2005). In steady state, $S = S' = 0$ and $S'' > 0$.

Production of investment goods in sector j require using goods produced by other sectors according to the production function

$$I_t^j = B_t^j \prod_{j'} \left(\frac{I_t^{j'j}}{\gamma_I^{j'j}} \right)^{\gamma_I^{j'j}}$$

where $I_t^{j'j}$ is the quantity of goods produced in sector j' used for investment in sector j. The production function for investment in each sector is scaled by an investment-specific productivity shock B_t^j. Like the labor-augmenting productivity shock A_t^j, B_t^j has both aggregate and an idiosyncratic components:

$$B_t^j = B_t \widehat{B}_t^j$$

where

$$\ln B_t = \rho^B \ln B_{t-1} + \varepsilon_t^B$$

and

$$\ln \widehat{B}_t^j = \rho^B \ln \widehat{B}_{t-1}^j + \varepsilon_t^{Bj}$$

where ε_t^B and ε_t^{Bj} are *iid* normal variables with zero mean and variance σ^B and σ^{Bj}, respectively. We assume that they have a common persistence parameter ρ^B.

The optimal choice of physical capital stock for sector j satisfies the optimality conditions:

$$\chi_t^j = \beta E_t \left[R_{t+1}^{k,j} \Lambda_{t+1} + (1-\delta) \chi_{t+1}^j \right],$$

$$P_t^{j'} \Lambda_t = \gamma_I^{j'j} \frac{I_t^j}{I_t^{j'j}} \left[\chi_t^j \left[1 - S\left(\frac{I_t^j}{I_{t-1}^j} \right) - S'\left(\frac{I_t^j}{I_{t-1}^j} \right) \frac{I_t^j}{I_{t-1}^j} \right] + \beta S'\left(\frac{I_{t+1}^j}{I_t^j} \right) \left(\frac{I_{t+1}^j}{I_t^j} \right)^2 \chi_{t+1} \right],$$

where χ_t is the multiplier on the capital accumulation equation. Defining Tobin's q for sector j as $Q_t^j = \frac{\chi_t^j}{P_t^{I,j} \Lambda_t} = \frac{P_t \chi_t^j}{P_t^{I,j} b_t [X_t(h)]^{-\sigma}}$, where $P_t^{I,j} = \prod \left(P_t^{j'} \right)^{\gamma^{j'j}}$, the relative marginal value of installed capital with respect to consumption, we can also write

$$Q_t^j = \beta E_t \left[\frac{R_{t+1}^{k,j} \Lambda_{t+1}}{P_t^{I,j} \Lambda_t} + \frac{P_{t+1}^{I,j} \Lambda_{t+1}}{P_t^{I,j} \Lambda_t} (1-\delta) Q_{t+1}^j \right],$$

$$1 = \left[Q_t^j \left[1 - S\left(\frac{I_t^j}{I_{t-1}^j} \right) - S'\left(\frac{I_t^j}{I_{t-1}^j} \right) \frac{I_t^j}{I_{t-1}^j} \right] + \beta \frac{\Lambda_{t+1} P_{t+1}^{I,j}}{\Lambda_t P_t^{I,j}} S'\left(\frac{I_{t+1}^j}{I_t^j} \right) \left(\frac{I_{t+1}^j}{I_t^j} \right)^2 Q_{t+1}^j \right].$$

Wage setting Every period a fraction ξ^w of households cannot freely set its wage, but follows the indexation rule

$$W_t^j(j) = W_{t-1}^j(j) \left(\pi_{t-1} e^{z_{t-1}}\right)^{\iota^w} (\pi)^{1-\iota^w}.$$

The remaining fraction of households chooses instead an optimal wage $W_t(j)$ by maximizing

$$E_t \left\{ \sum_{s=0}^{\infty} \xi^{ws} \beta^s \left[-b_{t+s} \varphi^j \frac{L_{t+s}^j(h)^{1+\nu}}{1+\nu} + \Lambda_{t+s} \Pi_{t,t+s}^w W_t^j(h) L_{t+s}^j(h) \right] \right\},$$

where

$$\Pi_{t,t+s}^w = \prod_{v=1}^{s} \left(\Pi_{t+v-1} e^{z_{t+v-1}}\right)^{\iota^w} (\Pi)^{v(1-\iota^w)} \text{ if } s \geq 1$$
$$\Pi_{t,t}^w = 1$$

subject to the labor demand function of the employment agencies.

The F.O.C. for a wage chosen by household h to work in industry j is to maximize

$$E_t \left\{ \sum_{s=0}^{\infty} \xi^{ws} \beta^s \left[-b_{t+s} \varphi \frac{L_{t+s}^j(h)^{\nu}}{1+\nu} + \Lambda_{t+s} \Pi_{t,t+s}^w W_t^j(h) L_{t+s}^j(h) \right] \right\},$$

subject to the demand of the employment agency,

$$L_t^j(h) = \left(\frac{W_t^j(h)}{W_t^j} \right)^{-\epsilon^w} L_t^j,$$

The F.O.C. is

$$E_t \left\{ \sum_{s=0}^{\infty} \xi^{ws} \beta^s \left[b_{t+s} \varphi \left[\left(\frac{\Pi_{t,t+s}^w W_t^j(h)}{W_{t+s}^j} \right)^{-\epsilon^w} L_{t+s}^j \right]^{1+\nu} \frac{1}{W_t^j(h)} \right] \right\}$$
$$= E_t \left\{ \sum_{s=0}^{\infty} \xi^{ws} \beta^s \left[\Lambda_{t+s} \Pi_{t,t+s}^w \left[\left(\frac{\Pi_{t,t+s}^w W_t^j(h)}{W_{t+s}^j} \right)^{-\epsilon^w} L_{t+s}^j \right] \right] \right\},$$

which can be rewritten as

$$\left(\tilde{W}_t^j\right)^{1+\nu\epsilon^w} = \frac{\epsilon^w}{\epsilon^w - 1} \frac{E_t\left\{\sum_{s=0}^{\infty} \xi^{ws}\beta^s \left[b_{t+s}\varphi^j \left[\left(\frac{\Pi^w_{t,t+s}}{W^j_{t+s}}\right)^{-\epsilon^w} L^j_{t+s}\right]^{1+\nu}\right]\right\}}{E_t\left\{\sum_{s=0}^{\infty} \xi^{ws}\beta^s \Lambda_{t+s}\Pi^w_{t,t+s}\left(\frac{\Pi^w_{t,t+s}}{W^j_{t+s}}\right)^{-\epsilon^w} L^j_{t+s}\right\}}$$

D.5 The government

A monetary policy authority sets the nominal interest rate following a feedback rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho^R} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_X}\right]^{1-\rho^R} \eta_{mp,t},$$

where R is the steady-state of the gross nominal interest rate. As in Smets and Wouters (2003), interest rates responds to deviations of inflation from its steady state, as well as to the level and growth rate of the GDP ($Y_t = \sum \gamma^j \frac{P_t^j}{P_t} Y_t^j$). The monetary policy rule is also perturbed by a monetary policy shock $\eta_{mp,t}$, is *iid* $N(0, \sigma^2_{mp})$.

Fiscal policy is fully Ricardian. The government finances its budget deficit by issuing short term bonds. Public spending is determined exogenously as a time varying fraction of output:

$$G_t = \left(1 - \frac{1}{\zeta_t}\right) Y_t$$

where the government spending shock ζ_t follows the stochastic process

$$\log \zeta_t = (1 - \rho^G)\zeta + \rho^G \log \zeta_{t-1} + \varepsilon^G_t.$$

where ε^G_t is *iid* normal random variable with standard deviation σ^G.

Public spending is a Cobb-Douglas aggregate of spending in different sectors. The government chooses sector-specific spending to minimize the cost of G_t:

$$\left\{G_t^j\right\}_j = \arg\min \sum_j P_t^j G_t^j$$
$$s.t. : \prod \left(G_t^j\right)^{\alpha_G^j} = G_t$$

so that

$$G_t^j = \alpha_G^j \frac{P_t^G}{P_t^j} G_t$$

where $P_t^G = \prod \left(\frac{P_t^j}{\alpha_j^G} \right)^{\alpha_j^G}$

D.6 Market clearing

The aggregate resource constraint for each sector j is

$$C_t^j + \sum_{j'} I_t^{jj'} + \sum_{j'} M_t^{jj'} + G_t^j = Y_t^j$$

D.7 Model Solution and Calibration

To solve the model, we first write it in terms of stationary variables (detrended the permanent part of TFP for real output variables and by the price level for nominal variables), log-linearize it and find the rational expectations equilibrium using Dynare.

The calibration is based on Justiniano et al. (2010) and Carvalho et al. (2021). Furthermore, we use information from sectoral linkages and consumer shares obtained from the input-output tables made available by the BEA and on sector-specific price stickiness from Nakamura and Steinsson (2008a). Tables A-1 and A-2 list the calibrated parameters together with their sources.

Parameter	Description	Value	Source
N	Number of Sectors	52	
ζ	1/steady-state government share of output	2.70	G/Y = 37%
δ	Capital depreciation	0.05	Justiniano et al. (2010)
β	Discount Factor	1.00	Justiniano et al. (2010)
ν	Inverse Frisch elasticity of labor supply	3.79	Justiniano et al. (2010)
η	Consumption habit parameter	0.78	Justiniano et al. (2010)
ϵ^w	Elasticity of substition for employment	1.87	Justiniano et al. (2010)
ϵ^p	Elasticity of substition for goods	1.81	Justiniano et al. (2010)
ξ^w	Calvo parameter (wages)	0.70	Justiniano et al. (2010)
ι^p	Indexation coefficient for prices	0.24	Justiniano et al. (2010)
ι^w	Indexation coefficient for wages	0.11	Justiniano et al. (2010)
I''	Investment adjustment cost parameter	2.85	Justiniano et al. (2010)
ϕ_x	Taylor rule, coefficient on output	0.24	Justiniano et al. (2010)
ϕ_π	Taylor rule, coefficient on inflation	2.09	Justiniano et al. (2010)
Π	steady-state inflation rate	0.03	Justiniano et al. (2010)
ρ^R	Taylor rule, smoothing parameter	0.82	Justiniano et al. (2010)
ρ^A	Persistence aggregate TFP	0.99	Carvalho et al. (2019)
ρ^{A^j}	Persistence sectoral TFP shock	0.93	Carvalho et al. (2019), average persistence for sectoral demand shock
ρ^G	Persistence government spending shock	0.99	Justiniano et al. (2010)
ρ^b	Persistence intertemporal preference shock	0.94	Carvalho et al. (2019), average persistence for sectoral demand shock
ρ^B	Persistence investment-specific TFP	0.72	Justiniano et al. (2010)
ρ^α	Persistence sectoral Demand shock	0.94	Carvalho et al. (2019), average persistence for sectoral demand shock
σ^η	Volatility to monetary shock	0.001	Carvalho et al. (2019), adjusted for iid monetary shocks
σ^A	Volatility, aggregate TFP	0.003	Carvalho et al. (2019)
σ^{A^j}	Volatility, sectoral TFP	0.003	Carvalho et al. (2003), average across sectors
σ^G	Volatility, government shock	0.00	Justiniano et al. (2010)
σ^B	Volatility, investment-specific TFP	0.06	Justiniano et al. (2010)
σ^{B^j}	Volatility, sectoral investment productivity	0.06	Same proportion to aggregate as A shock
σ^b	Volatility, preference shock	0.15	see text
σ^{α^j}	Volatility, consumption share	0.019	Carvalho et al. (2003), average across sectors

Table A-1: Calibration of Aggregate Parameters

Parameter	Description	Source
α^j	steady-state consumption share	BEA use tables
α_G^j	government consumption share	BEA use tables
φ_j	Disutility of labor of type j parameter	calibrated so steady-state wage is the same for all sectors
ω_j	Capital Share (by sector)	BEA use tables
γ_j	Materials Share (by sector)	BEA use tables
$\gamma_I^{j'j}$	Share of sector j' in sector j investment	Capital flow table
ξ_j^p	Calvo parameter (prices)	Nakamura and Steinsson (2008)

Table A-2: Sectoral Parameters

E Selected Impulse Responses for the Simulation-based experiment

Our model has variables for 182 sectors as well as 8 aggregate variables. This leads us to focus on the estimated shock series as a low dimensional check in the main text. Nonetheless, we want to give readers a sense of the estimated impulse responses. Below we plot the responses of GDP and consumption. The true impulse responses of those variables in the DSGE model are very similar. As expected, the estimated impulse responses in our model are then very similar across these two aggregate variables. As Figure A-3 shows, we are able to replicate the patterns of the true impulse responses.

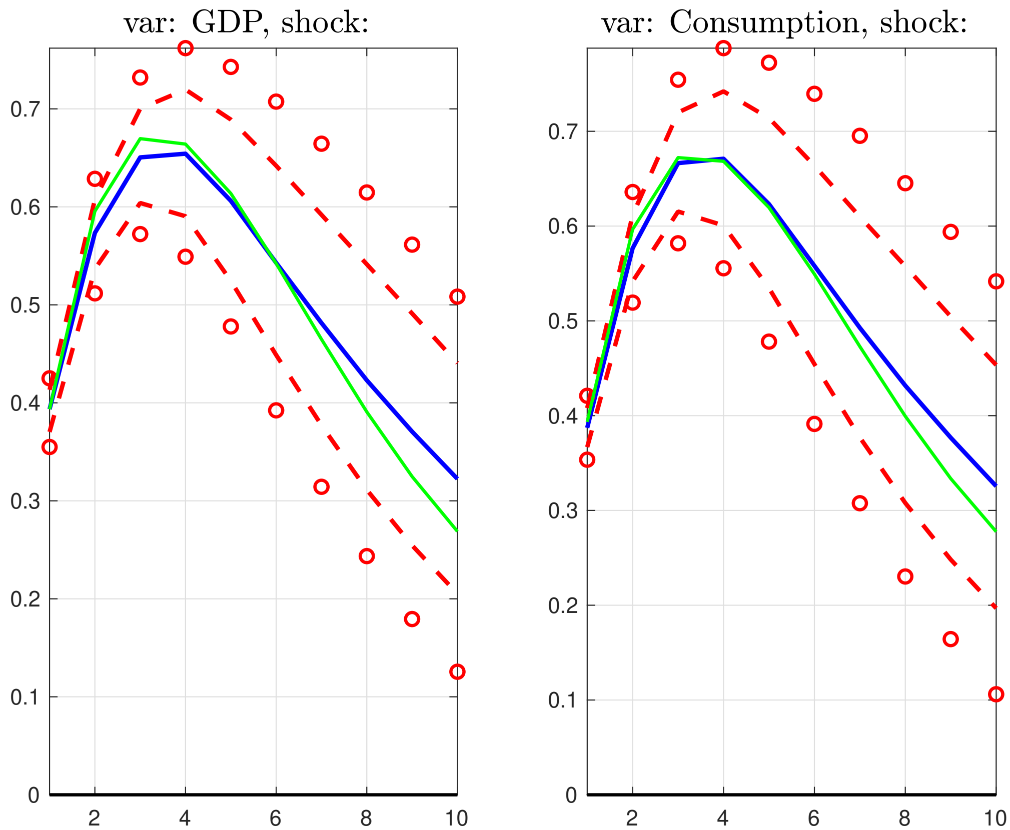

Figure A-3: Responses to Household Demand Shock for consumption and GDP in Monte Carlo exercise. Dashed lines are 16th and 84th Posterior Percentile Bands, Dots are 5th and 95th Posterior Percentiles. The x-axis shows time in quarters. DSGE-model based IRF in green (normalized to coincide with the median estimated IRF on impact).

F Results with $T = 1,000$

We simulate 1,000 observations from our benchmark DSGE model. As can be seen from Figure A-4, the results are similar to the results in the main text. This confirms that with a macro standard sample size we already achieve what is possible with our specific identification assumptions (as we discuss in the main text, if a researcher had more detailed information on the sectoral responses, that researcher could improve on our benchmark approach using sectoral differences in C/Y ratios, but that is practically infeasible).

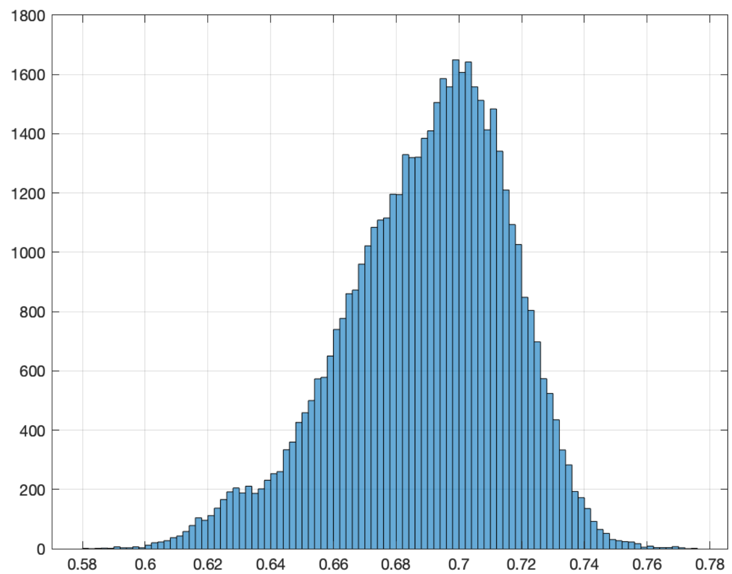

Figure A-4: Posterior of β^i, DGP with 1,000 observations.

G The Prior for the Household Shock

Table A-3 shows the percentiles (across sectors) of the prior mean of the relevant entries of D^i for the household shock. We focus on sectoral inflation and consumption since those variables are available for all sectors. The prior means completely characterize the Gaussian priors since we set the prior standard deviation equal to a fixed fraction of the absolute value of the prior mean.

	5th Percentile	Median	95th Percentile
Inflation	0.1	0.7	1.7
Consumption	0.1	1.2	2.4

Table A-3: Prior on the Impact of the Household Shock.

H Asymptotic Posterior Distribution of D^Z

We can make some progress toward characterizing the asymptotic behavior of the marginal posterior of D. Our prior $p(D^Z, \theta)$ is absolutely continuous with respect to the likelihood function $\mathcal{L}(D^Z, \theta|Z)$ where Z is the array of all observations on Z_t and θ is the vector of all parameters except D^Z.[4] VAR and factor model identification arguments imply that under standard regularity conditions (including linearity and Gaussian innovations) all parameters except D^Z are identified - even with infinite data we can only identify $D^Z D^{Z\prime}$. All other parameters converge to a unique limiting value θ^* such that the asymptotic posterior $p^*(D^Z, \theta|Z)$ (with conditional distribution $p^*(D^Z|Z, \theta)$ and marginal distribution $p^*(D^Z|Z)$) is given by

$$p^*(D^Z, \theta^*|Z) = p^*(D^Z|Z, \theta = \theta^*) = p^*(D^Z|Z)$$

This equivalence between joint, conditional, and marginal asymptotic posterior is due to the fact that asymptotically the marginal posterior for θ will be degenerate and only have mass at θ^*.

Let's define the limit of $D^Z D^{Z\prime}$ as the sample size T grows large:

$$\lim_{T \to \infty} D^Z D^{Z\prime} = \phi$$

where this limit should be understood to mean that asymptotically the joint posterior $p(D^Z, \theta|Z)$ will be equal to 0 except when $\theta = \theta^*$ and $D^Z D^{Z\prime} = \phi$. Then the asymptotic marginal posterior of D^Z (denoted by $p^*(D^Z|Z)$) is the prior restricted to those values of D^Z consistent with ϕ:

$$p^*(D^Z|Z) = p(D^Z|D^Z D^{Z\prime} = \phi)$$

Applying Bayes' rule to the conditional prior yields:

$$p(D^Z|D^Z D^{Z\prime} = \phi) = \frac{p(D^Z D^{Z\prime} = \phi|D^Z)p(D^Z)}{p(D^Z D^{Z\prime} = \phi)}$$

The first term in the numerator $p(D^Z D^{Z\prime} = \phi|D^Z)$ can be interpreted as an indicator function because it will only be non-zero when a value for D^Z is consistent with $D^Z D^{Z\prime} = \phi$. The second term in the numerator is just the prior $p(D^Z)$. The term in the denominator is a normalizing constant that will be independent of D^Z for all values of D^Z such that $D^Z D^{Z\prime} = \phi$.

[4] Since our priors on blocks of parameters are either Gaussian or inverse Wishart this assumption is satisfied in our model.

I Validating our approach: A Monte Carlo experiment with a Hi-VAR DGP

This section describes the results of an experiment that is meant to highlight the amount of additional information that sectoral information brings to bear on identifying structural shocks of interest. We simulate one dataset[5] of 170 observations (roughly the size of our actual sample) and discuss results for two sets of priors. We assume there are 4 aggregate variables, 180 sectors (in line with the number of sectors in our actual sample), and 2 observables per sector. All lag lengths (in both the data-generating process and the estimated model) are set to 1 for simplicity. The aggregate VAR coefficients in the data-generating process are set so that all variables are stationary, but persistent. The VAR coefficient matrices for each sector are drawn at random subject to the constraint that dynamics are stationary. We set the values of Ω, Ω^i, and the loadings on the two structural shocks for all variables in such a way that the structural shocks explain a small fraction of the variance at the sectoral level, as depicted in Figure A-5. These fractions are substantially smaller than what we find with our posterior estimates, both at the aggregate and sectoral level, so we are tying our hands with this conservative choice - we are consciously making this exercise hard for our approach. Furthermore, to mimic our empirical setting, we allow the loading on the structural shocks to be correlated within sectors across variables and across sectors.[6] The priors for the shock loadings are centered at the true value. The variance is set in the same fashion as in the empirical analysis of the main text.

We now ask two related questions: (i) How well does the posterior median of the structural shock series line up with the true value? and (ii) Is the estimation uncertainty small enough to draw meaningful conclusions from such an estimation?

We first set the prior means of the effects equal to their true value, and their standard deviations as in the empirical analysis, to be half the absolute values of the prior means.

Figure A-6 plots the true shock series, the posterior medians as well as 98 percent posterior bands centered at the median. We see that the posterior median capture the true evolution of the shock very well (the correlations are 0.93 for both shocks) and the posterior uncertainty surrounding the estimates are small. Why is the posterior uncertainty small? While each piece of identification information we use is not very informative, with a large number of sectors, the set of identification restrictions implicit in our priors is actually informative. This is reminiscent of results in standard dynamic factor models, where the model can

[5] We show that even with one datatset the evidence in favor of using sectoral information is so strong that we don't need to simulate a larger number of samples.

[6] We draw all these sectoral coefficients jointly from a multivariate Gaussian distribution with correlation coefficient 0.5.

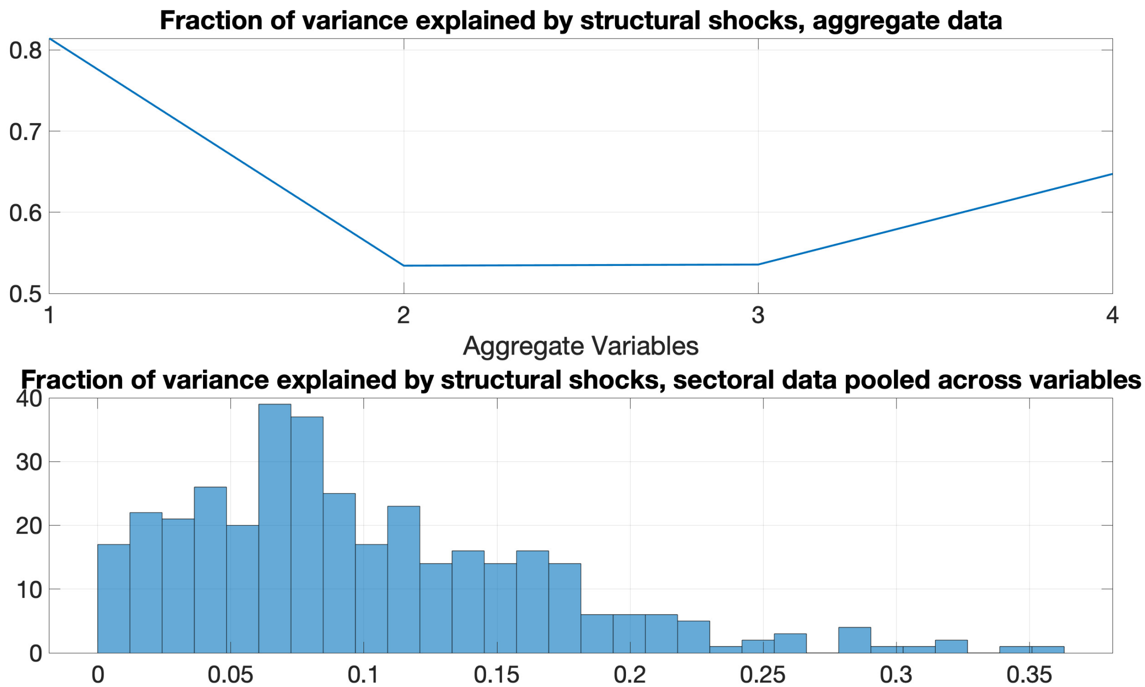

Figure A-5: Fraction of variance explained by structural shocks in our simulation exercise.

become exactly identified even when using standard sign restrictions when the number of sign restrictions grows to infinity (Amir-Ahmadi and Uhlig (2015)). On top of that we get additional identification strength from using information on magnitudes, as highlighted by Amir-Ahmadi and Drautzburg (2021).

As depicted in Figure A-6, we can identify the structural shocks with great accuracy. In the main text we discuss that knowledge of loadings of *other shocks* is not necessary to identify the loadings of one specific shock. To highlight this feature, we now re-estimate our model with the same simulated data, but setting the prior on all shock loadings of the second shock to a Gaussian distribution with mean 0 and standard deviation 0.25. Figure A-7 shows the results. Two results stand out: first, the first shock is still estimated precisely (the correlation of the posterior median with the true shock series is now 0.78), whereas the estimated second shock series does not match the truth at first sight. However, a further look reveals that the correlation between the posterior median and the true series is actually high in absolute value (-0.89). What happens? Our model correctly estimates the space spanned by the two shocks (i.e. the overall effect of the two shocks). But without any identification information on the second shock (in particular on the sign of the effects of this shock), the algorithm cannot pin down the shock exactly, but only the space spanned by this second shock. In this run of the posterior sampler, it concentrated on the part of the posterior distribution where

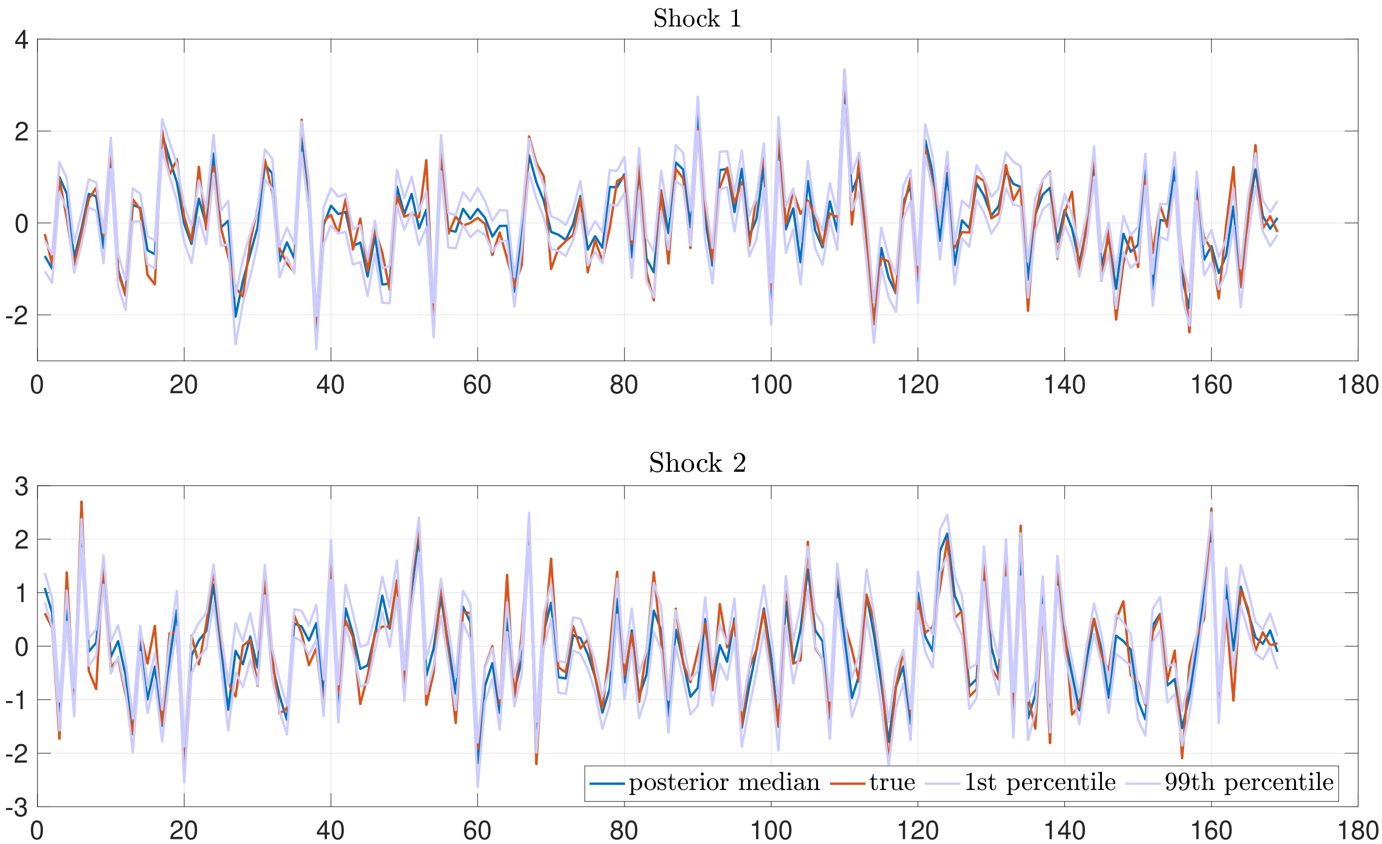

Figure A-6: Estimated and true shocks, Monte Carlo Exercise.Prior centered at true values for both shocks.

the sign of the effects and the actual shocks is flipped relative to the true values. [7]

[7]We run the posterior sampler for only 20,000 draws, half of which are discarded, in this simulation exercise. Even with this small amount of draws we can already see that our algorithm performs well. Such a small number of draws is generally not enough to fully capture severe multi-modality of the posterior distribution. In our empirical analyses we use 150,000 draws.

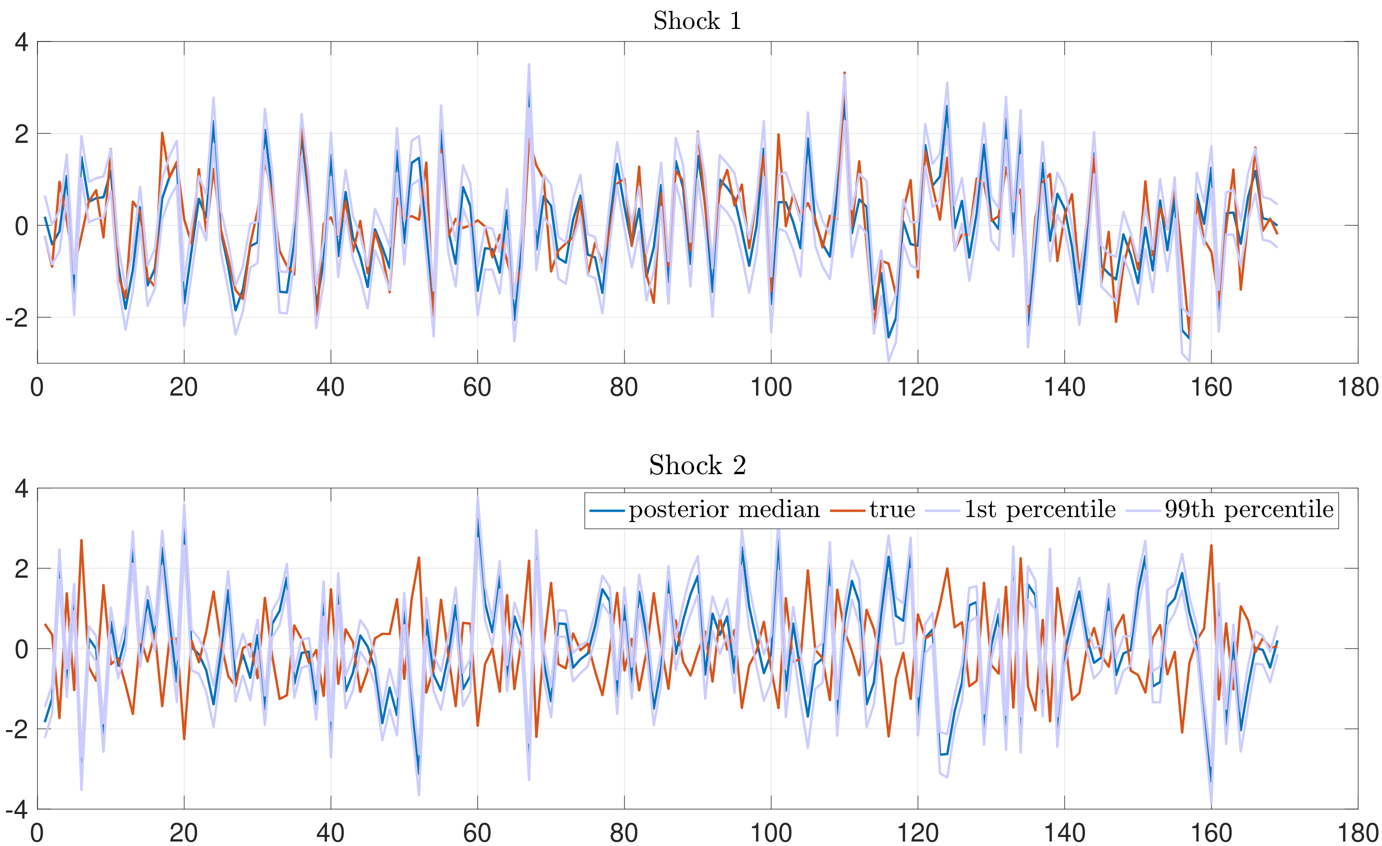

Figure A-7: Estimated and true shocks. Uninformative prior on effects of second shock.

J Why don't we use more aggregated sectoral data?

Sectoral data are available at various levels of aggregation. We choose to use data that is as disaggregated as possible. To justify this choice, we will study a very simple example. Consider an economy consisting of two equally sized sectors (we could easily generalize this argument to more sectors, but this extension would not add anything to our argument). We disregard aggregate variables here because they are not important for the argument. We also consider one observable per sector. So the state space system we study is

$$u_t^1 = \varepsilon_t + w_t^1 \tag{A-24}$$

$$u_t^2 = \varepsilon_t + w_t^2 \tag{A-25}$$

$$\varepsilon_t = \varepsilon_t \tag{A-26}$$

where $w_t^1 \sim (N(0, \Sigma^1)$ and $w_t^2 \sim (N(0, \Sigma^2)$ are two independent Gaussian processes, and, as before, $\varepsilon_t \sim (N(0, 1)$. For simplicity, we have normalized D to 1 in this example in both sectors. Alternatively, we could study a system where we aggregate the two sectors (we use equal weights here because we have assumed for simplicity that the sectors have equal size):

$$\overline{u}_t = \varepsilon_t + \overline{w}_t \tag{A-27}$$

$$\varepsilon_t = \varepsilon_t \tag{A-28}$$

Here we have $\overline{w}_t = \frac{1}{2}(w_t^1 + w_t^2)$ and thus $\overline{w}_t \sim N(0, \frac{1}{4}(\Sigma^1 + \Sigma^2))$. First note that we abstract in this example from two aspects that would make a researcher want to use more disaggregated data:

1. We don't model any dynamics in the sector. It is well known in the time series literature that aggregating VAR processes generally leads to VARMA processes for the aggregated variables. To at the very least be able to approximate these VARMA dynamics in our framework we would need to incorporate more lags of observables into the sectoral equations when using more aggregated data.

2. Here, we focus on the case of one aggregate shock. If there is more than one shock and different sectors have heterogeneous exposures to the different shocks, then averaging over this heterogeneous exposure can lead to a substantial loss of information.

Returning to our example, we can ask which of the two systems leads to a more precise estimate of the shock ε_t. We focus here on the variance of the estimation error for ε_t [8]. While

[8] To be precise, we study $var(\varepsilon_t|I_t)$ where I_t is the information set including time t observations

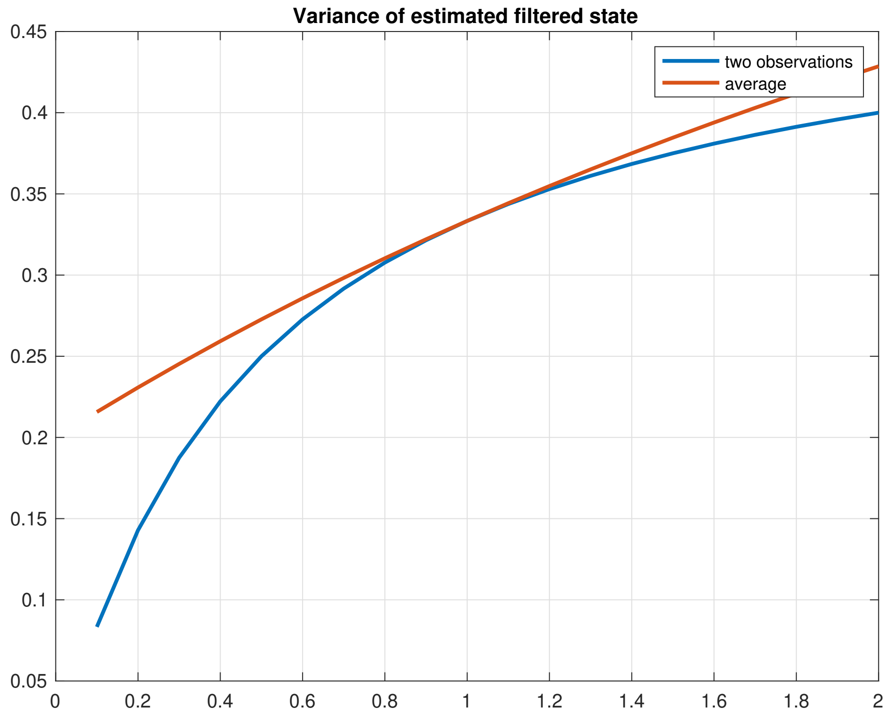

Figure A-8: Variance of estimation error.

it is easy to derive the formulas for the variance in closed form in our simple examples, we can already illustrate the main point with a numerical example. We fix the variance of w_t^1 at 1 and vary the variance of w_t^2 from 0.1 to 2. We then compute the estimated variance for both environments (one with two observables, one with the average observable). Figure A-8 shows our main result: it is always preferable to use more disaggregated data. The only point of indifference occurs when the variances of the w shocks are exactly equal. Turning to the analytical solutions, $var(\varepsilon_t|I_t)$ in the case when we observe both sectors separately is given by

$$var^{\text{two sectors}}(\varepsilon_t|I_t^{\text{two sectors}}) = 1 - (1\ 1)\left(\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} \Sigma^1 & 0 \\ 0 & \Sigma^2 \end{pmatrix}\right)^{-1}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{(A-29)}$$

The corresponding formula for the case where the average is observed is

$$var^{\text{average}}(\varepsilon_t|I_t^{\text{average}}) = 1 - \frac{1}{1 + \frac{1}{4}(\Sigma^1 + \Sigma^2)} \quad \text{(A-30)}$$

Both these equations are standard Kalman filtering formulas. One can then show that the following always holds:

$$var^{\text{two sectors}}(\varepsilon_t|I_t^{\text{two sectors}}) \leq var^{\text{average}}(\varepsilon_t|I_t^{\text{average}}) \quad \text{(A-31)}$$

Furthermore, the equality is strict unless $\Sigma^1 = \Sigma^2$. The proof amounts to tedious but straightforward algebra. The result should not be surprising: you can never be worse off by using more information. Note that in our simple example one could take a weighted average of the sectors to achieve the same variance as in the case with two observables, but in practice this is not feasible because the weights would depend on the variances of the noise terms (the w terms), which are not known before estimation.

K Sectoral Impulse Responses

Sectoral impulse responses, sorted by C/Y and the prior impact to household consumption shock (which is not the same as C/Y, as it also varies with differences in overall volatility of sectoral innovations).

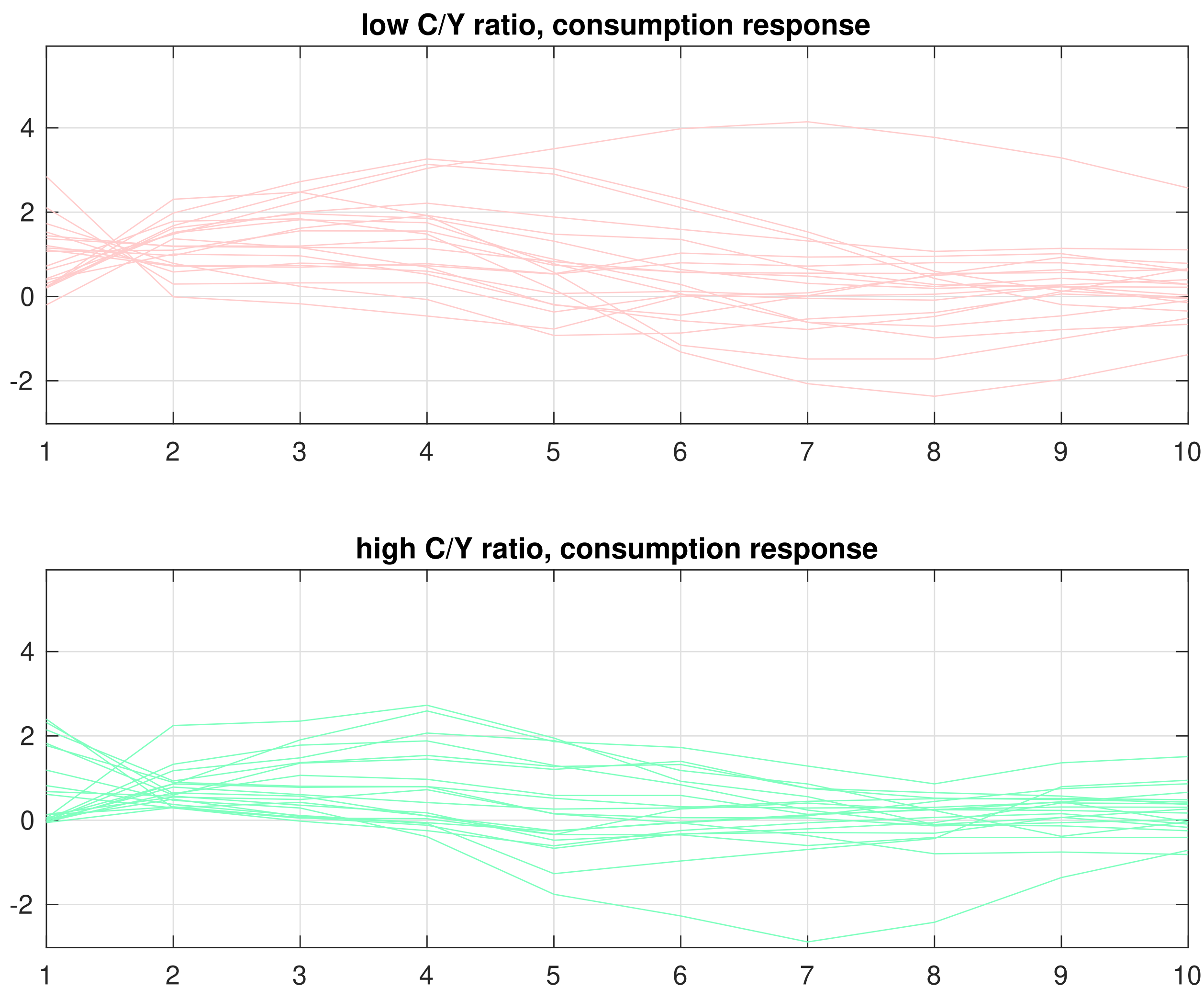

Figure A-9: Sectoral IRFs, high C/Y vs. low C/Y

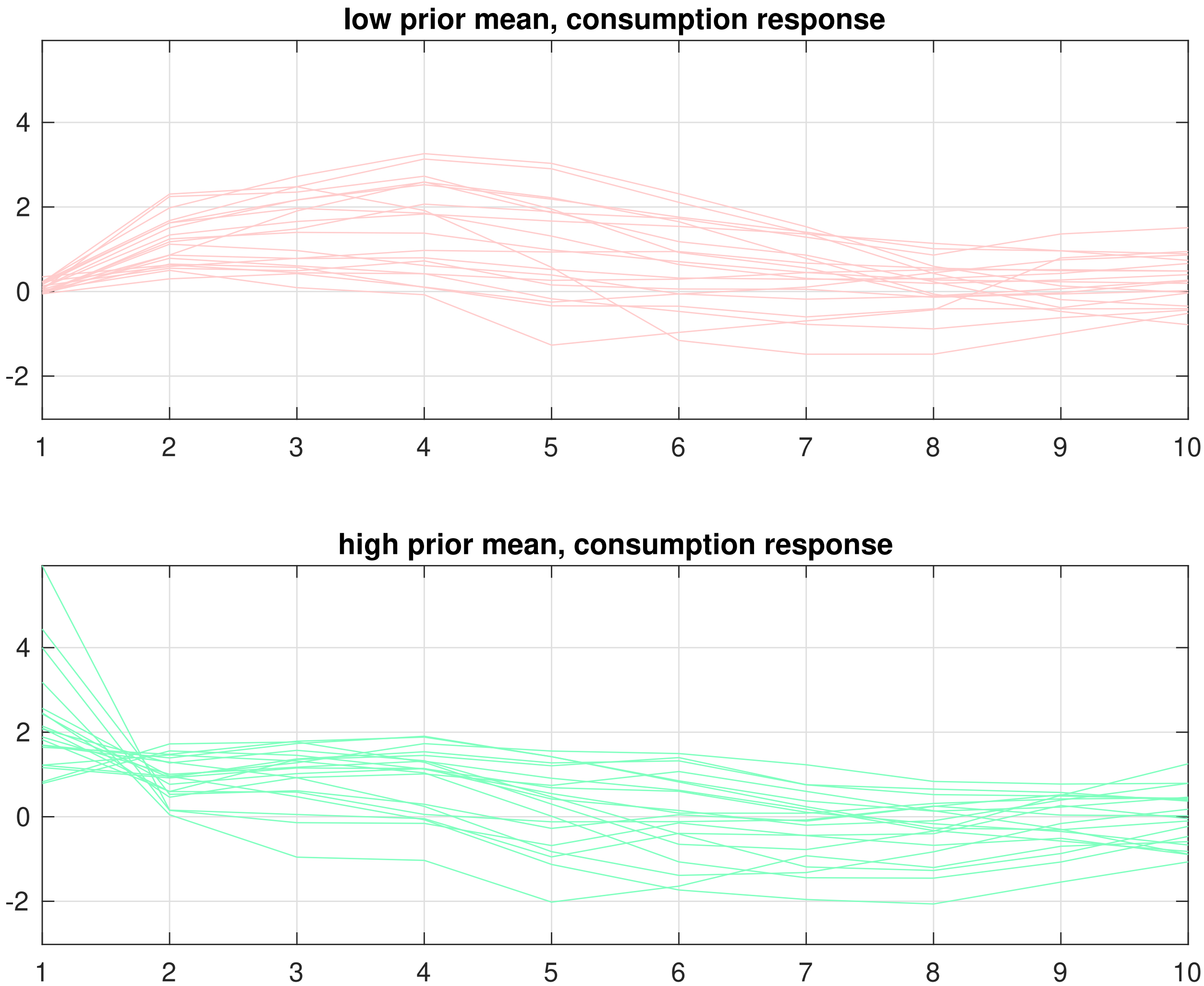

Figure A-10: Sectoral IRFs, high prior mean vs. low prior mean

L Impulse Responses to Other Economic Shocks

Note that the responses to the household consumption shock and the monetary shock are in the main text (Figures 3 and 8).

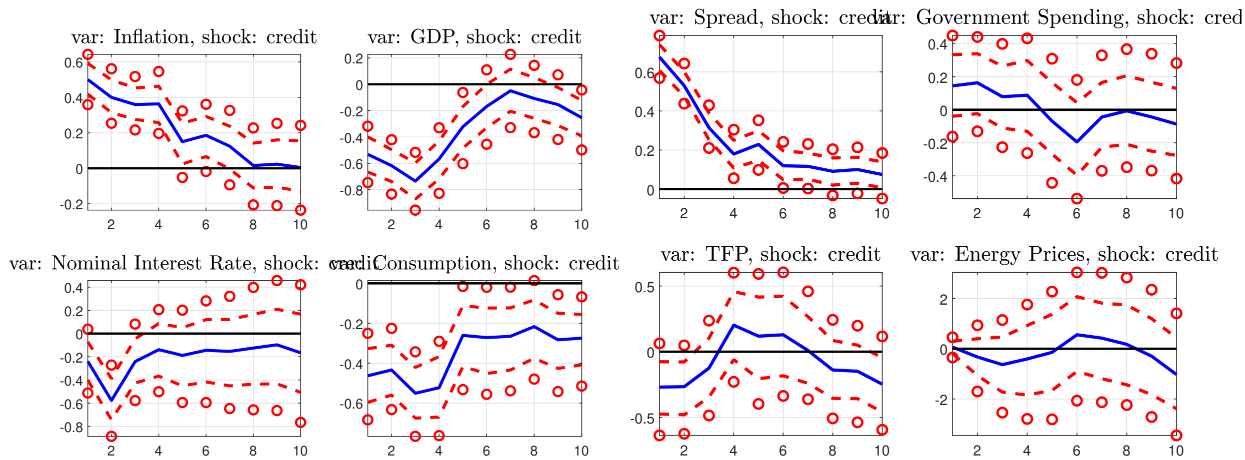

Figure A-12: Responses to Credit Shock. Dashed lines are 16th and 84th Posterior Percentile Bands, Dots are 5th and 95th Posterior Percentiles. The x-axis Shows Time in Quarters.

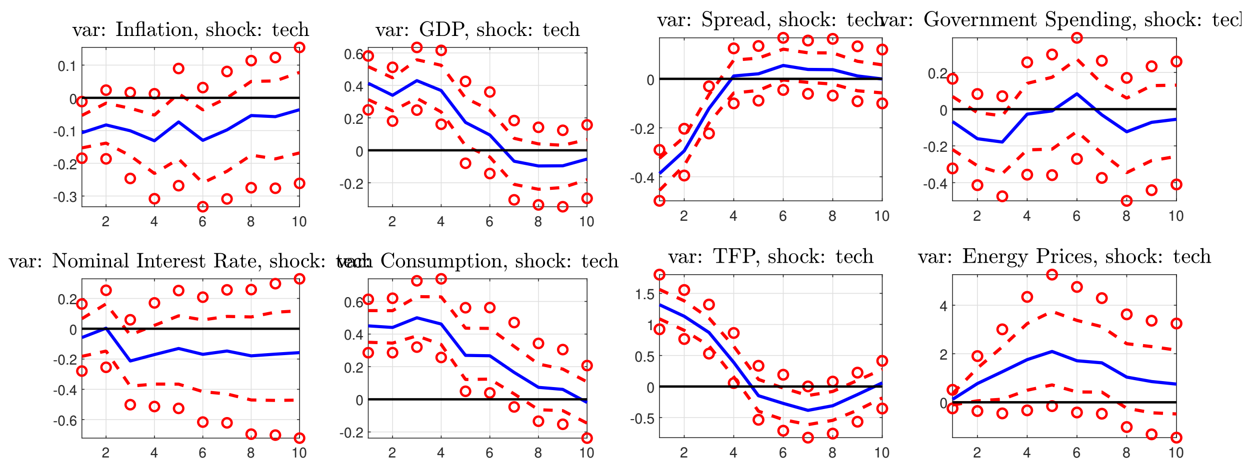

Figure A-11: Responses to Technology Shock. Dashed lines are 16th and 84th Posterior Percentile Bands, Dots are 5th and 95th Posterior Percentiles. The x-axis Shows Time in Quarters.

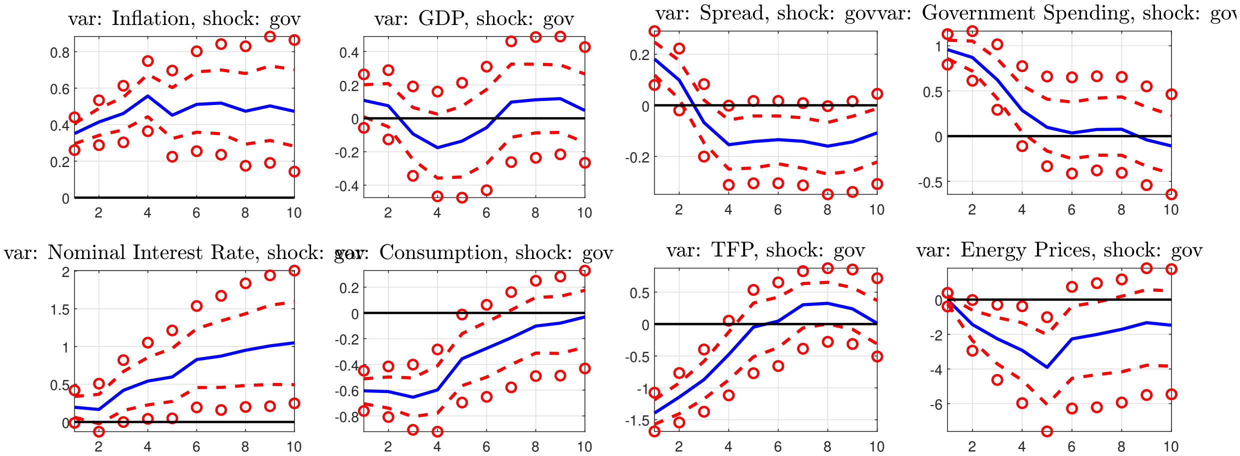

Figure A-13: Responses to Government Spending Shock. Dashed lines are 16th and 84th Posterior Percentile Bands, Dots are 5th and 95th Posterior Percentiles. The x-axis Shows Time in Quarters.

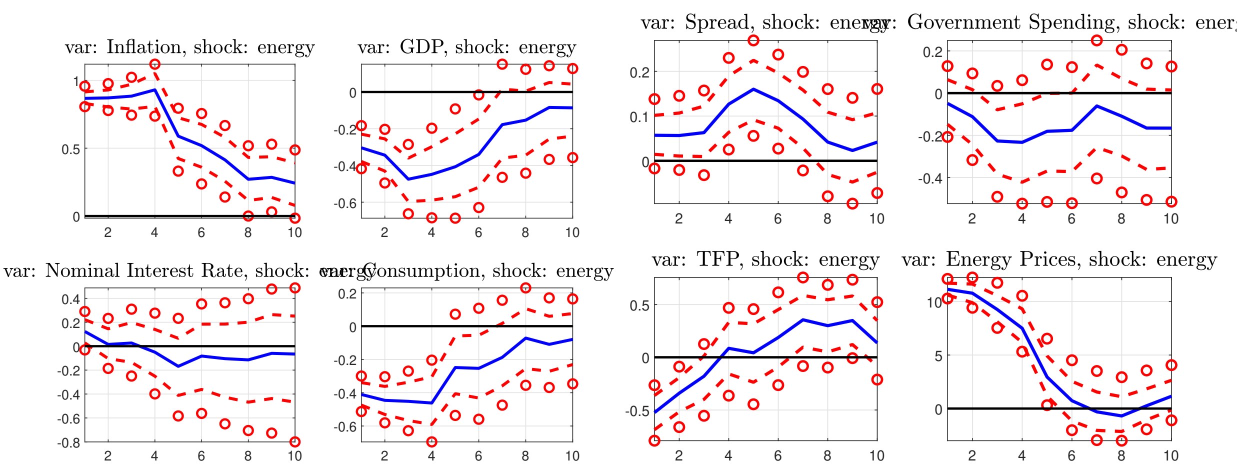

Figure A-14: Responses to Energy Price Shock. Dashed lines are 16th and 84th Posterior Percentile Bands, Dots are 5th and 95th Posterior Percentiles. The x-axis Shows Time in Quarters.

L.1 Sentiment Shock

We can examine whether the sentiment series is a good IV for the consumption shock, by estimating impulse responses to a consumer "sentiment" shock using the series for consumer sentiment as an IV (figure A-15). In particular, to estimate the IRFs, sentiment is ordered first in the VAR(4) and identification of the sentiment shock is achieved via Cholesky decomposition. We use the Canova and Ferroni (2021) toolbox to implement Minnesota priors with estimated hyperparameters (Giannone et al., 2015) and otherwise use standard prior settings as implemented by Canova and Ferroni (2021).

We find that they look similar to the IRFs for the consumption shock in some but not all instances. In particular, it is also associated with increased TFP and stable inflation, indicating that consumer sentiment also captures the response of household expectations to productivity news.

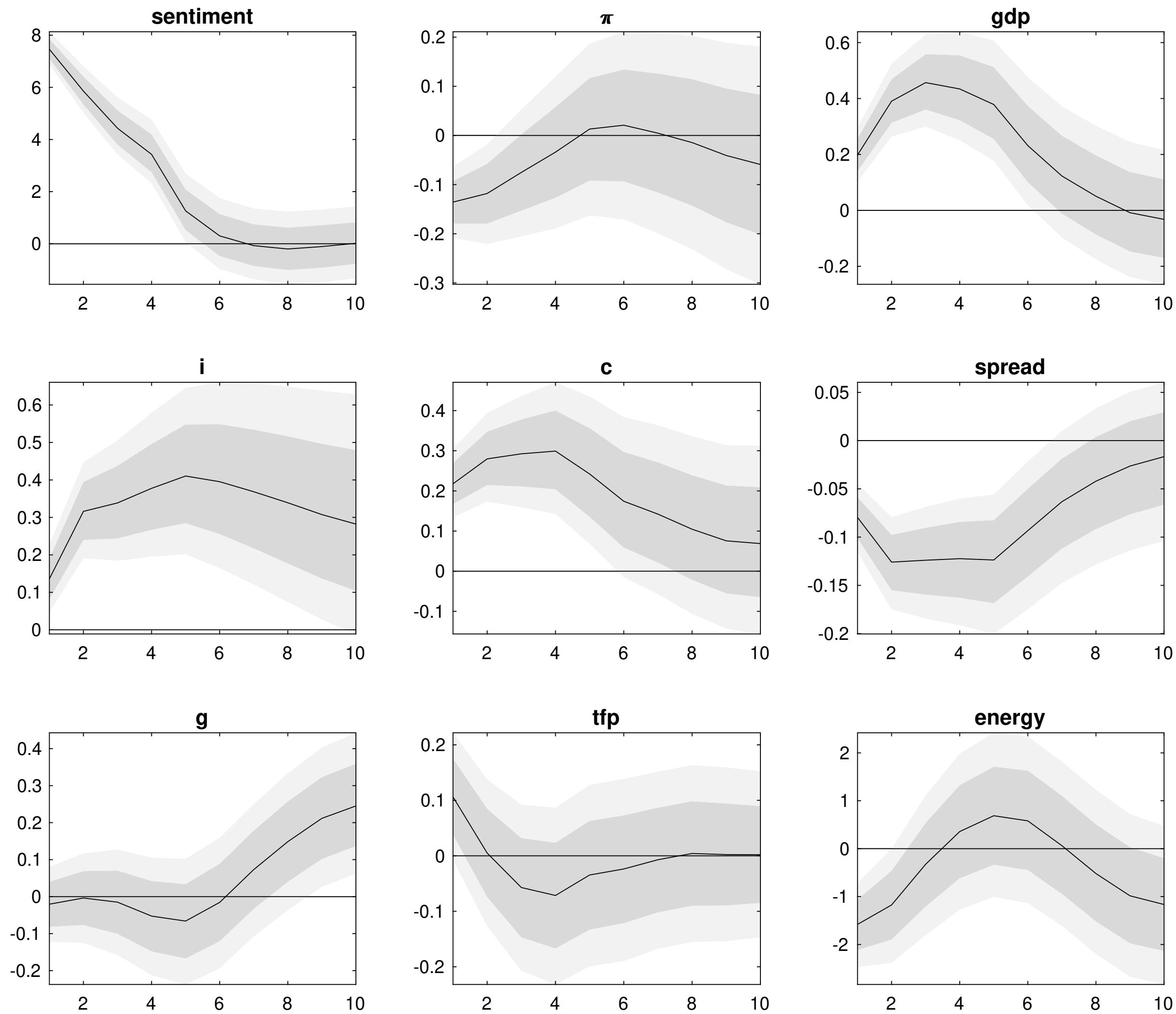

Figure A-15: Impulse response to a one-standard deviation sentiment shock. Black line is the posterior median, error bands represent 68% (darker area) and 90% posterior probability.

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M Further Robustness checks

To economize on space, we focus in our robustness checks on the importance/variance decomposition (for business cycle frequencies) of the consumption shock for aggregate variables. Relative to the main text, we also show the 5th and 95th percentiles of this variance decomposition. Therefore, we start by showing the results for our benchmark case. Throughout all these specifications, the household consumption shock remains a key driver of economic activity.

M.1 Benchmark

Inflation	29.4	34.6	42.4
GDP	28.7	37.3	42.3
Nominal Interest Rate	29.6	36.3	43.4
Consumption	36.6	41.7	44.7
Spread	5.6	8.9	10.5
Government Spending	8.2	26.8	35.7
TFP	5.1	9.8	16.6
Energy Prices	7.2	10.4	13.1

Table A-4: Variance decomposition across business cycle frequencies, consumption shock. Benchmark specification.

M.2 Aggregates only identification

To show the marginal gain from using sectoral data for identification of shocks, we show the variance decomposition when only informative priors on the effect of aggregate shocks are used.

	5th Percentile	Mean	95th Percentile
Inflation	6.7	13.5	17.4
GDP	12.2	15.6	29.7
Nominal Interest Rate	13.1	15.4	17.1
Consumption	34.5	42.2	47.4
Spread	5.7	9.0	11.8
Government Spending	13.1	23.7	44.1
TFP	3.2	5.5	13.4
Energy Prices	6.8	10.9	13.5

Table A-5: Variance decomposition across business cycle frequencies, consumption shock, only aggregate identification restrictions.

M.3 Larger Prior Variance on Impact of Consumption Shock

Next, we increase the prior standard deviation for the impact of the consumption shock on aggregate consumption equal to $1/2 \times abs\left(E\left[D_c\right]\right)$, where D_c is the prior mean of the impact of the household shock on aggregate consumption.[9]

	5th Percentile	Mean	95th Percentile
Inflation	24.6	31.2	38.4
GDP	22.0	30.9	37.6
Nominal Interest Rate	25.1	33.5	43.9
Consumption	35.5	41.3	45.8
Spread	15.1	20.3	27.0
Government Spending	7.6	22.6	30.4
TFP	6.4	10.3	13.9
Energy Prices	3.8	7.3	11.2

Table A-6: Variance decomposition across business cycle frequencies, consumption shock. Larger prior variance.

M.4 Shorter Sample

To assess whether or not our results are driven by the Great Recession, we re-estimate the model ending our sample in 2004:Q3.

	5th Percentile	Mean	95th Percentile
Inflation	21.4	26.5	41.3
GDP	21.4	25.4	29.2
Nominal Interest Rate	21.1	31.5	45.1
Consumption	31.7	34.3	37.8
Spread	9.4	15.6	19.9
Government Spending	12.8	20.5	27.1
TFP	5.4	12.4	17.6
Energy Prices	7.9	14.7	18.5

Table A-7: Variance decomposition across business cycle frequencies, consumption shock. Shorter sample.

[9] For our benchmark, we use $0.1 \times abs\left(E\left[D_c\right]\right)$. The prior standard deviation for the aggregate impact of the other aggregate shocks is set in the same fashion.

M.5 Fewer Lags

We now reduce the number of lags L and L^X to 4 from our benchmark specification of 6.

	5th Percentile	Mean	95th Percentile
Inflation	12.1	18.6	34.4
GDP	34.6	39.5	41.8
Nominal Interest Rate	12.9	19.3	34.6
Consumption	37.3	39.0	39.7
Spread	13.8	15.1	18.6
Government Spending	6.8	12.4	17.9
TFP	7.2	10.3	20.4
Energy Prices	3.6	9.1	20.3

Table A-8: Variance decomposition across business cycle frequencies, consumption shock. Fewer lags.

M.6 Investment specific technology shock

In this robustness check we modify our benchmark specification in two ways:

1. We add year-over-year growth in investment to our set of aggregate observables. As a measure of investment we use Real Gross Private Domestic Investment (FRED mnemonic GPDIC1).

2. We also identify an investment shock. This shock moves aggregate investment positively on impact (the prior is set in the same fashion as for our consumption shock, for example). At the sectoral level, it decreases inflation while increasing quantities. These effects are stronger the higher the investment intensity for a sector is, which we measure as the ratio between the value of goods produced in the sector that go towards gross capital formation and its total gross output.

As displayed in Table A-9, our consumption shock still remains the main driver of business cycle fluctutations.

	5th Percentile	Mean	95th Percentile
Inflation	14.5	23.6	38.4
GDP	12.4	17.0	19.9
Nominal Interest Rate	15.0	26.0	41.2
Consumption	25.8	30.4	34.6
Spread	12.1	17.6	28.6
Government Spending	8.1	12.7	15.3
TFP	5.5	8.7	12.0
Energy Prices	6.6	8.9	13.6
Investment	8.0	12.3	20.4

Table A-9: Variance decomposition across business cycle frequencies, consumption shock. Specification with investment-specific technology shocks .

M.7 Sample starting in 1985

To assess whether or not our results are driven by the Great Inflation, we re-estimate the model starting our sample in 1985:Q1.

	5th Percentile	Mean	95th Percentile
Inflation	7.6	14.2	25.2
GDP	23.2	24.7	27.9
Nominal Interest Rate	8.2	15.8	21.2
Consumption	27.1	30.2	33.1
Spread	4.8	8.5	11.0
Government Spending	10.4	12.7	17.1
TFP	8.0	10.6	12.9
Energy Prices	6.3	9.9	11.7

Table A-10: Variance decomposition across business cycle frequencies, consumption shock. Sample starting in 1985.

M.8 Comparison of Main Business Cycle Shock in Angeletos et al. (2020)

Our results suggest that consumption shocks are one of several important shocks, rather than a single main business cycle shock. We tested this by regressing the main business cycle shock from Angeletos et al. (2020) on the various shocks we identify. We found that this main shock has a small correlation with the consumption shock and can be better understood as a combination of various shocks, with the coefficients shown in Table A-11. This supports the view that multiple shocks play a significant role in business cycles, and the consumption shock plays a prominent but not dominant role.

	tech	credit	demand	gov	energy	monetary	investment
β_i	0.3	-0.1	0.0	0.1	-0.1	-0.1	0.2

Table A-11: Regression coefficients: $ACD_t = \sum \beta_i shock_t^i + v_t$