Online Appendix for
"Rich Pickings? Risk, Return, and Skill in Household Wealth"

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March 2020

This online Appendix provides a detailed description of the empirical methodology used in the main text. It also reports extensive robustness checks, supplementary results on household wealth, empirical tests confirming the validity of the asset pricing models used to analyze household portfolios, and detailed comparisons with alternative studies and datasets.

The organization is the following. Section I describes the construction of wealth variables and reports summary statistics on income and wealth. Section II reports the risk and return characteristics of the pricing factors. We provide additional results on financial and pension wealth in Section III, household real estate wealth in Section IV, and private equity in Section V. Section VI discusses risk and return on total wealth and debt costs. Section VII develops estimators of the moments of individual effects. Sections VIII and IX use these results to specify a dynastic model and estimate the distribution of average returns earned over a generation. In Section X, we relate our findings to the results of Saez and Zucman (2016) by investigating the risk and return of the endowments held by US foundations. We also compare our methodology to the approach followed by Fagereng et al. (2019). Section XI discusses the relationship between our micro portfolio estimates and the dynamics of wealth inequality.

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I Measuring Household Wealth and Income

Sections *I.A* to *I.C* respectively explain the methods used to value household pension wealth, real estate wealth, and private equity. Section *I.D* analyzes the potential impact of unreported wealth items. Section *I.E* reports the thresholds used to define wealth brackets and investigates household mobility across these brackets. In Section *I.F*, we investigate household demographics, ability, and financial characteristics across wealth brackets. Section *I.G* reports the cross-section of income and Section *I.H* describes the characteristics of households in the bottom decile of the distribution of net worth.

I.A Measurement of Pension Wealth

We measure pension wealth at the individual level by implementing a variant of the procedure developed by Saez and Zucman (2016) for the US and applied to Sweden by Alstadsæter, Johannesen, and Zucman (2019). The imputation method proceeds as follows.

Our starting point is the aggregate pension wealth provided in national accounts (Statistics Sweden 2014c). We allocate 42% of this wealth to retirees and 58% to workers. The breakdown is obtained from the condition that imputed pension wealth should be roughly the same just before and just after retirement. The share allocated to workers is lower than in Saez and Zucman (2016) and Alstadsæter, Johannesen, and Zucman (2019), likely because households have contributed to funded pension schemes for a longer period of time in these other data sets compared to our sample.[1]

Among workers, we allocate pension wealth proportionately to the capitalized value of pension contributions.[2] We impute individual contributions by assuming that conditional on age and year, the proportion of earnings contributed to pensions is equal to the yearly average value

[1]Contribution rates for funded pension schemes gradually increased in Sweden from the middle of the seventies until 1996, when the compulsory funded pension scheme (PPM) was launched. By 2019, i.e. 23 years after the PPM fund was created, retirees held only 15.5% of its market value (Pensionsmyndigheten 2019).

[2]Note that this choice slightly differs from Saez and Zucman (2016) and Alstadsæter, Johannesen, and Zucman (2019), where pension wealth is allocated proportionately to current earnings.

obtained for the age group from national statistics (Statistics Sweden 2007c, Bach Calvet Sodini 2020). For each individual, we measure yearly earnings by following the methodology described in the next paragraph. Given these estimates, we compute the pension contributions of each individual in each year by multiplying yearly earnings with the estimated contribution rate. We then capitalize pension contributions made from 1950 onward, using the historical rate of return on assets held by Swedish life insurance companies as the discount rate.

We obtain individual income as follows. For the 1969 to 2007 period, the information is directly available from tax returns (Statistics Sweden 2007c). For the period prior to 1969, we lack individual tax data and impute past earnings by implementing the Bozio et al. (2017) procedure. That is, we first regress individual yearly wages on year and individual fixed effects. Second, for each gender-education-age group, we regress individual annual wage growth on the aggregate wage growth provided in national accounts (Bach Calvet Sodini 2020). We apply these regression coefficients to each individual's wage fixed effect to impute the level of wages conditional on being employed. Third, we estimate separately in each gender-education-age group how the employment rate in a given age-gender group (available in national accounts) affects the likelihood of being employed during the year. Fourth, we use these coefficients to extrapolate the probability that an individual is not employed in a given year. Fifth, we multiply the yearly extrapolated wage conditional on being employed with the yearly extrapolated probability of being employed, which provides the wage income of every individual prior to 1969.

I.B Valuation of Real Estate Wealth

Real estate prices are available from two main sources, as we explain in Section I.B of the main text. Every 3 to 7 years, tax authorities assess the tax value of every real estate property using detailed characteristics and hedonic pricing. In addition, Statistics Sweden continuously collects data on every real estate transaction in the country, which permit the construction of sales-to-tax-value multipliers for different geographic locations and property types.

The valuation of household real estate portfolios, however, is constrained along two dimensions. First, the Swedish Wealth and Income Registry (Statistics Sweden 2007a, 2007c, 2007d, 2007e) contains only limited information on the location of real estate properties. Specifically, it reports the municipality of a household's two most valuable primary residences, two most valuable secondary residences, and most valuable farmland. When a household lives in a condominium, we assume that it owns a share of the condominium and that the flat is located in the household's municipality of residence. The geographic location of rental properties is not reported.

Second, annual property returns can only be reliably estimated if a sufficient number of transactions is recorded each year. Statistics Sweden suggests that at least 25 transactions are recorded each year for an index to be meaningful. For single-family homes, this threshold is reached in almost every municipality every year; we can therefore rely on municipality-level return series for main residences. Some municipalities contain very few secondary residences, so we construct clusters of municipalities using the max-p-regions geographical clustering algorithm of Duque, Anselin, and Rey (2012). We obtain reliable return series for about 130 clusters representing on average 2.2 municipalities. For farmland, the number of transactions is only sufficient at the county level.

Overall, the price data allow us to classify real estate assets into 389 property categories, consisting of 256 primary residence types, 111 secondary residence types, 21 farmland types, and 1 type of rental property (Bach Calvet Sodini 2020).

I.C Valuation of Private Equity

The valuation of a private firm is based on its balance sheet. Following accounting regulations, financial assets (cash and stock investments with no control purpose) are recorded at market value, while other assets are recorded at historical value. For this reason, we value the firm's assets as follows.

- Financial assets are priced at book value.
- Nonfinancial assets are priced by applying a multiplier to their book values. The multiplier is a geometric average of the market-to-book ratio of all listed companies in the same two-digit sector in the same year.[3] Because the assets of an unlisted firm are not as marketable as the assets of a listed firm, we apply a 25% discount to the value obtained with listed multiples, as is common among appraisers of private firms (Koeplin, Sarin, and Shapiro 2000).

The market value of equity is obtained by subtracting financial debt from the market values of financial and nonfinancial assets. If the difference is negative, we set the market value of equity equal to zero, consistent with the assumption that shareholders have limited liability.

I.D Comparison with National Accounts and Missing Wealth Items

The Swedish Income and Wealth Registry is arguably the most detailed dataset on household wealth available for research purposes. As Appendix Table 1 shows, the wealth aggregates derived from our sample are very similar to the ones reported in national accounts, which confirms the quality of the dataset and the reasonable accuracy of the valuation methods used in the paper.

The Swedish registry, however, does not provide several wealth items. Bank account balances are unreported for a subset of households in lower brackets of the distribution of net worth. In addition, the registry does not measure consumer durables and offshore assets. We now discuss the imputation of missing bank account balances and the potential implications of other missing items.

Bank Account Balances. Section I.A of the main text defines cash as the sum of bank account balances and Swedish money market funds. Financial institutions are only required to report a household's bank account balance at year-end if the account yields more than 100 Swedish

[3] To avoid measurement error due to the infrequent trading of small stocks, we filter out listed firms whose combined book value of assets is lower than a threshold, which is set equal to 0.5% of the aggregate asset value of all listed firms.

kronor during the year (2000 to 2005 period), or if the year-end bank account balance exceeds 10,000 Swedish kronor (2006 and 2007). Calvet, Campbell, and Sodini (2007) report that bank account balances are unavailable for 2 million out of a total of 4.8 million households in 2002. We impute unreported balances by following a refinement of the methodology in Calvet, Campbell, and Sodini (2007, 2009a, 2009b) and Calvet and Sodini (2014).

The imputation relies on the subsample of individuals for which a bank account balance is observed. For every available account, we regress the log balance on the following characteristics: age and squared age of household head, household size, log real estate wealth, log household disposable income, square of log household disposable income, and log of financial assets other than bank account balances. We then use the coefficient estimates to impute unobserved bank account balances, and then compute a household's bank balance by adding up observed and imputed balances. We adjust the intercept of the imputation regression so that the aggregate value of observed and imputed bank account balances in our household panel matches the aggregate bank account balance of the household sector reported in Statistics Sweden (2014d).

Consumer Durables. The value of consumer durables is notoriously difficult to measure. In a recent assessment of Swedish national wealth, Waldenström (2016) reports that the stock of consumer durables represents 10% of total private net wealth in 2004. The US Survey of Consumer Finances (Federal Reserve Board 2007) provides similar orders of magnitude. The distribution of consumer durables in the population is an even more complex issue. The SCF reveals that the share of consumer durables in total net worth slightly declines with household wealth. However, this finding is nuanced by the fact that durables consumed by the wealthy tend to be harder to value than the durables held by the rest of the population. For instance, a Barclays Bank report uses survey evidence to show that very high net-worth households hold on average 9.6% of net worth in durables such as art, collectibles, jewelry, etc. (Mitchell 2012). It is therefore reasonable to assume that durables represent a constant share of total net wealth, so that including them in our valuation would not have a major impact on the net wealth rankings of Swedish households.

The risk and return characteristics of durables are especially challenging to measure because dividends are non-pecuniary and highly subjective for this asset class, storage and maintenance costs are substantial, and regular price listings are rarely available. The historical returns on collectibles range between the returns on safe assets and the returns on equities, with substantial risk and low correlation with equities.[4] Durables are therefore a composite asset in terms of risk-return trade-off. Taking durables into account would probably not substantially change the interpretation of the results in the paper.

Offshore Assets. Swedish residents own substantial offshore wealth, which is mostly held by the wealthiest brackets (Alstadsæter, Johannesen, and Zucman 2019). As a result, over the period 2000-2007 the share of wealth held by the top 1% would increase from 18.1% to 19.5% if one correctly accounted for ownership of offshore assets. We now examine how unobserved offshore assets potentially affect our main findings.

An obvious consequence of international tax evasion is that absolute levels of wealth are underestimated, possibly by a significant amount for households at the top. The impact of tax evasion on our main variable of interest, the household rank in the distribution of net worth, depends on the correlation between offshore wealth and domestic wealth. If both types of wealth are strongly correlated, the household rank is essentially unaffected. Otherwise, our estimates are biased downward (attenuation effect) due to an errors-in-variables problem and our paper underestimates the economic relationship between wealth and investment strategies.

Another potential complication is that hidden assets may have substantially different risk and return characteristics than declared assets. Zucman (2013) provides aggregate data on the portfolio composition of tax haven accounts held by foreigners, regardless of nationality. These accounts contain on average 24% of demand deposits, 37% of mutual funds (including money market, bond, and equity funds), and 39% of directly-held stocks and bonds.[5] In our data, the top

[4]See Spaenjers (2016) for a survey of the empirical evidence.

[5]Zucman (2013) distinguishes the proportions of directly-held stocks and bonds in his estimations, and finds a significantly

1% hold 39% of their financial portfolios (excluding derivatives and capital insurance accounts) in cash, 23% in mutual funds, and 38% in directly-held stocks and bonds. These portfolio compositions are broadly similar, so our results on portfolio risk and return among the wealthy Swedes are unlikely to be significantly affected by cross-border tax evasion.

I.E Wealth Thresholds and Wealth Dynamics

Appendix Table 2 reports the range of net wealth held by households in each bracket at the end of each year between 2000 and 2007. The median bracket ranges between 286,000 and 451,000 kronor in 2000, and between 484,000 and 771,000 kronor in 2007. To facilitate international comparison, recall that the Swedish krona traded at 0.151 US dollar at the end of 2004 (Sveriges Riksbank 2016). Using this rate of conversion, the median bracket ranges between $43,000 and $68,000 in 2000, and between $73,000 and $116,000 in 2007. Similarly, the thresholds required to enter the top 0.01% are $22.5 million in 2000 and $47.7 million in 2007.

The threshold required to be in a given bracket grows on average by 8% per year (6.4% in real terms[6]) for the median bracket and 12.7% (11.0% in real terms) per year for the top 0.01% over the sample period. These results are consistent with our key finding that the joint distribution of wealth and returns causes faster capital accumulation in wealthier groups.

Appendix Table 3 provides the transition probabilities between a household's net wealth rank in 2000 and net wealth rank in 2007, conditional on the survival of the household. As is well known from the inequality literature, the distribution of wealth is very persistent, especially among the wealthy. Despite very significant movements in asset prices between 2000 and 2007, 63% of households in the top 1% of the distribution at the beginning of our sample period are still in the top 1% bracket 7 years later. Out of the remaining 37%, 82% are still in the top 5% by the end of the sample period. Similarly, when we consider the group of households in the top

higher proportion of bondholding in offshore accounts around the world than in Sweden-based holdings. One likely reason is that the risky bond market is much less developed in Sweden than in either the US (where the corporate and mortgage-backed bond markets are deep) or emerging economies (where central government debt is a risky investment). For this reason, we choose to bundle up directly-held equities and bonds for this comparison exercise.

[6]The average annual CPI inflation rate is 1.6% over the period (Statistics Sweden 2014e).

0.01% in 2000, we obtain that 53% of them remain in the top 0.01% and 87% of them are in the top 0.1% bracket 7 years later. The strong persistence of the household rank motivates its use as our main variable of interest throughout the paper.

I.F Demographic, Financial, and IQ Characteristics Across Wealth Brackets

Appendix Table 4 reports the demographic composition of each wealth bracket. The age of the household head is a hump-shaped function of net worth. The household head is 34 years old on average in the first decile and increases to 57 in the fifth decile. The average age exhibits limited variation in the top six deciles, reaching 60 in the top 20%-2.5% and declining to 56 in the top 0.1%. These estimates reveal a division between the bottom 40% of households, which are young and hold almost none of national net wealth, and the top 60%, which are middle-aged and own almost all of national net wealth. Differences in family characteristics are more salient: richer households count more family members, from 1.9 on average in the bottom decile and 1.4 in the second decile to 2.8 for the top 0.01%, and are much more likely to be headed by a married man.

The table also reports the average cognitive ability (IQ) of household members in every bracket of net worth. Individual cognitive ability is measured for all men enlisted in the Swedish military between 1969 and 2009 (Riksarkivet 1997, Rekryteringsmyndigheten 2012). The score is standardized on a scale of 1 to 9 in such a way that its distribution follows a symmetric bell curve. A score of 5 corresponds to the median, and a score of 9 to the top 4% of the distribution.[7] When a household contains at least one member with an available score, the household's index of cognitive ability is the rounded average score of measured members. The score is available for about one third of households and is based on a single member in 95% of cases. Missing households primarily include female households and households with adult men born before 1950.

Net worth increases monotonically with ability, albeit at a slow rate. In the bottom 10% of

[7] See Lindqvist and Vestman (2011) and Dal Bó, Finan, Folke, Persson, and Rickne (2017) for more details.

the wealth distribution, average IQ is 4.5, slightly below the median. Cognitive ability reaches 5 precisely at the median of the wealth distribution. IQ keeps increasing with wealth until it reaches a plateau of about 6 in the top 10% of the distribution. To the extent that investment skill is tied to cognitive ability (Grinblatt, Keloharju, and Linnainmaa 2011), these results suggest that skill may provide part of the explanation for why the rich earn higher returns. However, the contribution of IQ is likely to be limited because a) cognitive ability is almost constant within the top 10% of the distribution, which controls more than half of aggregate wealth, and b) most wealthy households do not have an exceptional IQ.

Appendix Table 5 reports the average financial characteristics of households in each wealth bracket. For the median household, gross wealth averages 671,000 kronor ($101,000), household debt 218,000 kronor ($33,000), labor income 185,000 kronor ($28,000), and pension income 81,000 kronor ($12,000), while capital income is close to zero. By contrast, for the top 0.01% of households, gross wealth averages 801 million kronor ($121 million), household debt 20 million kronor ($3.1 million), labor income 1.1 million kronor ($161,000), pension income 1.1 million kronor ($159,000), and capital income 14.4 million kronor ($2.2 million).

The last columns of Appendix Table 5 report the percentage of households with a controlling stake (i.e. more than 5% of voting rights) in a public company, a private company, or either type of company. There are slightly more than two hundred controlling positions over listed firms among Swedish households every year. However, even among the top 0.01%, only about 11% of households control a listed firm. By contrast, most wealthy households exert their authority over a private firm. For instance, 85% of households control a private firm in the top 0.01%.

I.G Cross-Section of Household Income

In Appendix Figure 1, we illustrate the 10^{th}, 25^{th}, 50^{th}, 75^{th}, and 90^{th} percentiles of gross income in each wealth bracket. The dispersion of income is rather homogeneous across brackets contained between the median decile and the top 1% – 0.5% bracket of the distribution of net worth. As an illustration, the ratio of the 90^{th} to the 10^{th} income percentiles ranges between 4 and 5 in all these wealth brackets.

By contrast, the dispersion of income is very high at both ends of the distribution of net worth. In lower brackets, the dispersion stems from the fact that some households earn almost no income at all. Within the top 1%, income dispersion increases dramatically with net worth, and the ratio of the 90^{th} to the 10^{th} income percentiles reaches 35 for the top 0.01%. The high dispersion at the top could be in part driven by the high level of idiosyncratic risk taken in this segment of the population. A stronger factor may be that income levels are subject to large-scale optimization among the very wealthy, in particular when it comes to the realization of capital gains. For this reason, as suggested by Piketty (2014), the financial well-being of the very rich are better proxied by wealth than by annual income. This observation further justifies our focus on the impact of the concentration of wealth (rather than income) on portfolio choice.

I.H A Special Case: The First Decile of the Distribution of Net Worth

Along many dimensions considered in the paper and this online Appendix, households in the first decile of net worth differ from the rest of the population in ways that cannot be extrapolated from neighbouring deciles. In particular, they face much lower interest rates than other households in the bottom half of the distribution.

Appendix Tables 2 to 5 reveal that the first decile of net worth has the following socioeconomic characteristics. In contrast to the rest of the bottom half of the wealth distribution, households in the first decile correspond more often to families. The household size and the likelihood that its head is a married male are the highest in the bottom half, and are well in line with what is observed at the median. Households in the first decile earn on average more labor and pension income than households in slightly higher brackets of the population. Their net worth is lower than 14,000 kronor (or about 2,200 dollars), and closer analysis establishes that a large majority of households in the first decile have negative net worth. They have accumulated both more gross wealth and more debt than households in the second or the third decile. Only very few of them own shares in a private company, suggesting that we are not primarily dealing with highly indebted entrepreneurs.

The main hypothesis consistent with these summary statistics is the existence of a mismatch between the measured level of debt and the corresponding assets that this debt is meant to fund. The mismatch may be temporary because around a transaction date, the signing of a debt contract may be recorded by tax authorities before the acquisition of the corresponding asset. Indeed, households in the bottom decile in 2000 are 50% more likely to belong to the upper half of the distribution of net worth in 2007 than households in the second decile in 2000.

A complementary hypothesis is that for some households in the first decile, the low level of net worth is more persistent because it is due to substantial debt contracted to invest in assets that subsequently drop in value. In the sample period, many mortgages in Sweden are interest-only and entail high loan-to-value ratios, so a gap between outstanding debt and asset value can quickly build up as house prices fall. This scenario is consistent with the high volatility of real estate returns (13% in annual units according to Table 6 of the main text). This hypothesis is further confirmed by the fact that households in the bottom decile subsequently benefit from very large tax credits due to realized capital losses (as we show in Appendix Table 25 discussed in Section *VI.C* of this online Appendix).

The first decile is therefore an unusual mix, which includes both highly leveraged households owning depreciated assets and wealthy households incorrectly assigned to the bottom decile due to severe measurement error. For this reason, the first decile often exhibit unusual patterns in the tables presented in the main text and this online Appendix, which should be taken with caution.

II Asset Pricing Factors

Appendix Table 6 reports the historical mean and volatility of the pricing factors. The Swedish and global equity factors are obtained by taking the difference between index returns and the

Swedish T-bill. The size, value and currency factor are long-short portfolios. The return on the FASTPI real estate index is not an excess return, but a simple arithmetic return.

Column 1 reports the baseline mean and volatility used throughout the main text and this online Appendix. For equity and currency factors, these values are historical averages between 1983 and 2016, the longest period for which monthly series on global Fama-French factors are available from the AQR website (AQR Capital Management 2016). We compute FASTPI estimates on 1981 to 2014 data. The average returns are 8.7% for the Swedish stock market factor, 5.8% for the global equity factor, -0.1% for the size factor, 4.7% for the value factor, and -1.2% for the currency factor. The average return on real estate is 5.5% per year. These estimates seem reasonable by international standards. The size premium is negligible over the period. The currency premium is slightly negative, which suggests that Swedish investors earn a premium for holding the domestic currency instead of US dollars.

Column 2 reports the standard deviations of the factors. The standard deviation of the real estate index return is adjusted for positive autocorrelation, as Section III.C of the main text explains. Volatility is highest for the Swedish equity factor, with an estimated value of 21.0% in annual units. Since we observe factor returns over 34 years, the risk premia have large standard errors. For instance, the Swedish stock market has a standard error of $21.0\%/\sqrt{34} = 3.6\%$ per year. The domestic and global equity premia are significant, the value and real estate factors are strongly significant, while the size and currency premia are statistically insignificant.

In column 4, we compute the average return on the Swedish stock market and real estate over the entire postwar period, using historical data reported in Söderberg, Blöndal, and Edvinsson (2014) and Waldenström (2014a, 2014b). The risk premium is 8.6% for the Swedish index, which is remarkably close to 8.7% estimate reported in column 1. The risk premium on real estate is 5.7% over the longer sample, which is also close to the 5.5% estimate in column 1. This analysis suggests that the risk premia estimated in column 1 are closer to their long-run values than the standard errors might suggest.

Column 3 computes average factor returns over the period contemporary to the holding data (January 2001 to December 2008). The resulting values are very different from their long-term levels. Over the short 2001-2008 sample, the average return on the Swedish equity factor is -1.7% per year, and the average return on the real estate index is 8.1% per year. The Swedish real estate market experienced a boom, while the equity market suffered from two major crises, the burst of the tech bubble in 2001-2003 and the outset of the financial crisis. Therefore, computing risk premia over a long period is a key requirement for making inferences about the return distribution that are not context-specific.

In columns 5 and 6, we report the mean and standard deviation of risk premia on equity factors and real estate in the United States (French 2016, Shiller 2016). Over the period 1926-2016, the US equity premium is 7.8% on average, and the average return on US real estate is 3.9%. These values are slightly smaller but in line with the values obtained for Sweden.

III Financial and Pension Wealth

This Section provides a detailed analysis of the financial and pension wealth of households across net worth brackets. Section *III.A* documents the asset allocation of household complete financial portfolios, household fund portfolios, and Swedish pension funds. Section *III.B* studies stock market participation across wealth quantiles. In Section *III.C*, we estimate interest rates on bank account balances. Section *III.D* computes the exposures of household risky financial wealth to systematic risk factors. Finally, the welfare costs and possible origins of underdiversification are assessed in Section *III.E*.

III.A Allocation of Financial and Pension Wealth

Complete Financial Portfolio. Appendix Figure 2 plots the allocation of the complete financial portfolio. The share of cash declines rapidly with net worth. The share of risky assets (stocks and funds) correspondingly increases, which contributes to higher risk-taking at the top of the distribution. In Appendix Table 7, Panel A, we correspondingly report that the risky share increases from 10.8% for the bottom decile to 60% for the top 0.01%. Funds represent more than two thirds of risky financial wealth for households below the 95$^{\text{th}}$ percentile, while the top 0.01% allocate 79% of risky financial wealth to stocks.

Fund Portfolio. Appendix Figure 3 illustrates the composition of the fund portfolios across net wealth brackets. Pure equity funds have the largest share of the fund portfolio (which ranges between 71% and 79% across groups), while relatively "safe" funds, such as fixed income funds, have a low share (which ranges between 5% and 10% across groups). Wealthier households have a slight tendency to favor riskier funds: the combined share of pure equity funds and hedge funds increases from 73% for the median household to 81% for the top 1%. However, the shift from funds to direct stockholdings is the main channel through which the very wealthy take on more systematic risk, as the next subsections show.

Pension Funds. In Panel B of Appendix Table 7, we report the asset allocation of Swedish pension funds, which we obtain from the annual reports of the leading Swedish insurance companies (Bach Calvet Sodini 2020). The pension portfolio is evenly split between safe assets and equities. Pension funds take action against exposure to currency risk but typically do not fully hedge this risk. About 50% of the equity portfolio actively hedge currency risk.

III.B Stock Market Participation

Appendix Figure 4 illustrates patterns of stock market participation within each net wealth bracket. The participation rate steeply increases from 30% in the bottom decile to 90% in the ninth decile, and then barely goes up in the top decile.

Other patterns are salient at the top of the wealth distribution. Direct stock market participation jumps from 73% for the top 10%-5% of households to 92% for the top 0.01%. Similarly, the fraction of direct participants holding at least five different stocks increases from 39% in the top 10%-5% to 78% in the top 0.01%. These patterns confirm our findings that (i) the share of directly-held stocks is a trait of the very rich and (ii) diversification of the stock portfolio is an increasing function of wealth.

III.C Interest Rates on Bank Accounts

In Sweden, yields vary substantially across interest-bearing accounts and many accounts do not pay interest. Throughout the main text and online Appendix, we follow national accounting guidelines and treat the bank account spread, defined as the difference between a bank account's nominal interest rate and the yield on the Swedish short-term T-bill (Sveriges Riksbank 2016), as a remuneration for the services the bank provides its customers. We therefore do not treat the bank spread as a lack of investment skill on the household side. In this subsection, we investigate the validity of this assumption and its implications for our results.

In Appendix Table 8, we report: (1) the bank account spread, (2) the bank account spread weighted by the share of the bank account in financial wealth, and (3) the bank account spread weighted by the share of the bank account in total gross wealth. The nominal interest rate of a bank account is measured by the average interest income received in year t-1 and in year t (as reported in tax forms) divided by the balance of the bank account at the end of year t-1. Because the bank account may be temporarily small on December 31st, we winsorize the interest rate estimate so that it does not exceed the yield on the Swedish T-bill. This correction impacts only about 1% of observations.

The bank account spread is negative and worse than -2% across all wealth brackets. It also varies substantially along the wealth distribution. The bottom 10% pay a spread of 2.84% while the top 0.01% only pay 2.08%. These findings point to a significant level of remuneration for banking services. The lower spread paid by the wealthy seems consistent with economies of scale, originating from fixed costs in banking services that are broadly unrelated to account balances. A complementary interpretation is that the rich are more skilled at finding advantageous bank account rates than less wealthy households.

These findings have implications for our estimates of wealth returns across the wealth distribution. In the bottom decile, bank account balances represent a large component of wealth and the negative bank account spread decreases the overall return on wealth very significantly. The excess return on financial wealth falls by 2.4 percentage points (p.p.) and the excess return on total gross wealth by 1.1 p.p. when the bank account spread is included. Conversely, in the top decile of the wealth distribution, the bank account spread has a much more limited impact on the return to (financial and gross) wealth because bank accounts represent a much smaller share of wealth. Specifically, the excess return on financial wealth drops by 0.8 p.p. and the return on total wealth by 0.1 p.p. if we include the negative spread on bank accounts in the measurement of returns. Therefore, including the negative bank account spread substantially reinforces the conclusion that the rich earn higher returns, in part because the rate of return on their bank account is slightly higher but mostly because they invest their wealth away from bank accounts with negative excess returns.

In Appendix Table 9, we analyze if the lower bank spread paid by the wealthy reflects differences in investment skill or differences in the value of banking services relative to household wealth. We proxy investment skill by cognitive ability and report its impact on the measures of bank account performance considered so far. Households with the highest ability earn 0.4 p.p. more on their bank account balances than households with the lowest ability. When the bank account spread is weighted by financial wealth, this differential increases to 0.9 p.p., reflecting the fact that more able households shift their money away from low-yield bank accounts toward high-yield financial instruments. When the bank account spread is instead weighted by total gross wealth, the differential between the most and least able shrinks to 0.3 p.p. While there is a detectable impact of ability on the bank account spread, it is always smaller than 1 p.p. and, in particular, much lower than the impact of net worth. In addition, a substantial part of the effect of ability on the bank account spread disappears once we control for the impact of net worth: the differential between the most and the least able then goes down to 0.3 p.p. for the unweighted spread, 0.6 p.p. for the spread weighted by financial wealth, and 0.1 p.p. for the spread weighted by total gross wealth. A richer set of controls would likely further diminish the estimated impact of ability on yields. It is therefore unlikely that investment skill is a significant determinant of bank account rates, which validates that the bank account spread is a compensation for household consumption of banking services.

III.D Exposures to Risk Factors

III.D.1 Risk Exposures of Risky Financial Wealth

There is evidence that sophisticated households can distinguish between high and low market beta stocks, small and large stocks, or value and growth stocks, and may have a sense of the risks and returns involved (Betermier, Calvet, and Sodini 2017). Table 5 of the main text reports the expected return on risky financial wealth in each quantile. These estimates are based on portfolio exposures to five factors: the domestic stock market, the global stock market, global size, global value, and currency.

In Appendix Table 10, we report the five risk factor loadings of the risky financial portfolio. The loading on the domestic market increases with net worth, ranging from 0.61 (median household) to 0.73 (top 0.01%). The loadings on the value and size factors also strongly increase with net worth. The higher loadings on the domestic stock market, value and size factors all contribute to higher expected returns.

Interestingly, household exposures to the global equity and currency factors tend to decrease with wealth. The global stock market factor decreases very slightly with net worth, which barely impacts expected returns. The exposure to the currency factor decreases rapidly with net worth, which also has very little impact on average performance given that the currency premium is close to zero. Section *III.D.2* further investigates household exposures to these two factors.

III.D.2 Risk Exposures of Stock and Fund Holdings

Investment products offered to Swedish investors provide exposures to global risk factors (Calvet, Campbell, and Sodini 2007). At the same time, capital markets are not fully integrated and global and local market factors are both priced (Hou, Karolyi, and Kho 2011). In order to investigate if households benefit from each of these premia, we consider in the paper a global five-factor model that controls for domestic and international equity risk, as well as currency risk (Solnik 1974). Appendix Table 10 documents how the international exposure of the *risky* portfolio varies with net worth. The risky portfolio of the median household loads twice as much on the Swedish market as on the global market. Perhaps more surprisingly, the richest households load even more heavily on the local factor and bear less currency risk.

Appendix Tables 11 and 12 shed some light on this apparent puzzle by investigating the loadings of the *stock* and *fund* portfolios. For each of these portfolios, international exposures are fairly constant across wealth brackets. The striking fact is that stock and fund portfolios have very different loadings on domestic equity, global equity, and currency. Stock portfolios load much more on the local market and are not exposed to currency risk. Fund portfolios are much more exposed to global market risk than stock portfolios. Furthermore, unlike household stock portfolios, fund portfolios are also substantially exposed to currency risk. These findings reflect the fact that Swedish funds largely invest in foreign stocks but do not hedge against currency movements. As a result, by moving away from funds to stocks, wealthy households decrease their exposures to both global equity and currency risks.

III.D.3 Robustness Check: Loadings and Expected Returns under the Local CAPM

We consider the robustness of our result to an alternative asset pricing model, the local CAPM, and investigate household exposure to the domestic stock market in this context. As in the 5-factor model, the local market factor is proxied by the excess return on the SIX return index (SIXRX) from Datastream (2016).

In Appendix Table 13, we report (1) the market beta of the risky financial portfolio, (2) the market beta of the stock portfolio, (3) the market beta of the fund portfolio, (4) the expected return on the complete financial portfolio, and (5) the expected return on the risky portfolio across wealth brackets under the domestic CAPM. The market beta of the *risky* portfolio is 0.75 for the median household and substantially increases with the wealth rank, reaching 0.80 for the top 10%-5%, 0.85 for the top 1%-0.5%, and 0.87 for the top 0.1%. The market beta of the *stock* portfolio mildly declines with wealth, while the market beta of the *fund* portfolio remains almost constant. Fund portfolios are on average much less exposed to market risk than stock portfolios, consistent with the fact that many funds contain cash, bonds, and other assets. Rich households therefore achieve high market betas by shifting from funds to stocks.

The expected return associated with the domestic stock market exposure increases rapidly with net worth. The median household earns an excess return of 1.3% per year on the complete financial portfolio, while a household in the top 0.01% earns an expected excess return of 4.5% per year. These magnitudes are similar to the results of the global 5-factor model reported in the main text.

III.E Idiosyncratic Risk

Households at the top of distribution are exposed to large idiosyncratic risks. Following Calvet, Campbell, and Sodini (2007), we measure the welfare cost (or opportunity cost) of underdiversification by the return loss. By definition, the return loss is the difference between the expected return of the household's portfolio and the expected return of a diversified portfolio with the same standard deviation. In mathematical terms, the return loss is

$$RL_h = \omega_h \sigma_h (S_B - S_h),$$

where ω_h is the risky share of household h's portfolio, σ_h is the standard deviation of the risky portfolio return, S_h is the Sharpe ratio of the risky portfolio, and S_B is the Sharpe ratio of a fully diversified benchmark portfolio. The benchmark is the Swedish equity market.

Consider the relative Sharpe ratio loss:

$$RSRL_h = 1 - \frac{S_h}{S_B}.$$

The return loss can be conveniently decomposed as follows:

$$RL_h = 1_{\{\omega_h>0\}} \exp\left[\log(\mathbb{E}r_B^e) + \log(\omega_h) + \log(\beta_h) + \log\left(\frac{RSRL_h}{1-RSRL_h}\right)\right], \qquad \text{(A-1)}$$

where $1_{\{\omega_h>0\}}$ is a risky asset market participation dummy, $\mathbb{E}r_B^e$ is the expected excess return on the benchmark portfolio, and $\beta_h = \mathbb{E}r_h^e/\mathbb{E}r_B^e$ is the ratio of the expected excess return on household h's risky portfolio to the expected excess return on the benchmark portfolio.[8] Non-participants have a return loss equal zero, so that the participation decision is a major determinant of the return loss. The first term in the exponential is common to all households, the next two terms track portfolio aggressiveness, and the last term captures underdiversification.

Appendix Table 14 shows how the return loss and its four components vary with wealth, assuming that the global 5-factor model holds. The return loss from underdiversification increases monotonically with wealth (column 1), reaching 1.69% per year for the top 0.01%. This result confirms the findings of Calvet, Campbell, and Sodini (2007). A large underlying force is that, for any given level of underdiversification, richer people have larger return losses because they are taking much more systematic risk (columns 2 to 4). The underdiversification of the risky portfolio, which is captured by the relative Sharpe ratio loss (column 5), is U-shaped in financial wealth. In the top 1%, more wealth is associated with less diversification in the portfolio of risky assets.

The underdiversification of financial wealth at the top of the net worth distribution has several possible explanations: (i) wealthy households own stocks directly to save on mutual fund fees, (ii) wealthy households buy stocks in order to hold controlling stakes in listed firms, and (iii) wealthy households are skilled investors and reap risk-adjusted returns from concentrated

[8] We call this term β_h because when a local CAPM model is used to compute expected returns, the ratio $\mathbb{E}r_h^e/\mathbb{E}r_B^e$ effectively equates the market beta of household h's risky portfolio. In our regressions, in the few cases where the ratio is negative, we take its absolute value as our outcome variable.

positions. We assess the validity of these assumptions in the next subsections.

III.E.1 Mutual Fund Fees

Mutual funds manage portfolios that are an order of magnitude larger than the stock portfolio of any household, including the very rich. As a result, they are hard to beat in terms of diversification. In columns 2 and 3 of Appendix Table 15, we report the idiosyncratic share of, respectively, the stock and fund portfolios across brackets of net worth. For the median household, the idiosyncratic share of the fund portfolio is 16%, which is lower than the 36% idiosyncratic share of the stock portfolios held by the richest households. The idiosyncratic share of the fund portfolio remains between 11% and 16% in other quantiles.

By holding more stocks than funds, richer households naturally bear more idiosyncratic risk. A possible explanation is that wealthier households seek to save on mutual fund fees. Over the sample period, the typical management fee in Sweden is substantial and around 1.4% per year on average (Calvet Campbell Sodini 2007). For wealthy households, the net opportunity cost of holding stocks instead of funds can be very roughly estimated at 1.69%-1.40% = 0.29% on average per year, which is rather modest. The desire to save on mutual fund fees is therefore a plausible motive for direct stock investment, especially if it is combined with other motives.

III.E.2 Controlling Stakes

In column 1 of Appendix Table 15, we compute the idiosyncratic share of the risky portfolio when direct stockholdings representing more than 5% a company's voting rights are excluded (Euroclear 2007). The idiosyncratic share is only slightly lower as a result. For the top 0.01%, the idiosyncratic share of the risky portfolio goes from 28.7% for all direct stockholdings to 26.1% when control blocks are excluded. Therefore, corporate control is not the primary reason why rich households hold underdiversified financial portfolios.

III.E.3 Investment Skill

We have shown that wealthy households tend to hold stocks directly rather than through mutual funds. Moreover, their fund portfolios are well diversified under our 5-factor model. It is therefore unlikely that idiosyncratic risk is driven by the search for priced risks that are well recognized by the fund industry but are not captured by the five-factor model of equity returns.

Column 2 of Appendix Table 15 displays the level of diversification of stock portfolios. The stock portfolio's idiosyncratic share is slightly U-shaped, decreasing from 53% (bottom decile) to 34% (top 0.5%-0.1%) and then increasing very slightly to 36% (top 0.01%). The wealthy's propensity to hold stocks directly is therefore unlikely to be driven by stock picking. Instead, it is likely motivated by the willingness to take advantage of priced equity risk factors while saving on mutual fund fees.

In Table 5 of the main text, we measure the alpha coefficient of a household's risky portfolio, and then weigh it by the risky share to obtain the contribution of skill to the complete portfolio's performance. We find no evidence of superior investment skill in managing financial wealth.

In Appendix Table 16, we measure the alpha coefficient of the stock portfolio (columns 1 and 2) and the fund portfolio (columns 3 and 4) across wealth brackets. In contrast to Table 5 of the main text, the alpha coefficient obtained for each portfolio is not weighted. As a result, households owning very few risky assets may carry a lot of weight in the estimation. In addition, given the low stock participation rates at the bottom of the distribution of wealth, the results we obtain for bottom deciles are driven by a highly selected set of the population.

The stock and fund portfolios of the median household have a risk-adjusted performance of 0.71% and 0.04%, respectively. No wealth group earns an alpha that is significantly different from the performance reached by the median household.

IV Real Estate Wealth

This section provides a detailed analysis of household real estate portfolios. Section *IV.A* reports the allocation across property types, Section *IV.B* discusses the risk exposures of the real estate portfolio return, and Section *IV.C* compares returns with and without user costs.

IV.A Allocation of Real Estate Wealth

Appendix Figure 5 illustrates the average allocation of gross real estate wealth by property type across net worth brackets. We distinguish between residential real estate, which consists of the household's primary and secondary residence(s), and (ii) commercial real estate, which consists of rental, agricultural, and other properties. In each bracket, the average allocation is computed on the set of households that participate in real estate markets. The black line plots the fraction of households owning real estate in each bracket.

Residential real estate represents more than 90% of gross real estate wealth for households up to the 90th percentile of net worth. The share of residential real estate decreases sharply in higher brackets, dropping to 71% for the top 1%-0.5% and 59% for the top 0.01%. In all brackets, the value of the primary residence largely exceeds the value of secondary residences.

The share of commercial real estate increases with net worth, averaging 31% for the top 1% of households and 41% for the top 0.01%. These results mirror our residential estimates as the shares of residential and commercial real estate add up to unity. Since commercial properties do not provide a hedge against housing costs, the real estate portfolio is an increasing source of risk to households in higher wealth brackets.

IV.B Risk Exposures

In Appendix Table 17, we report for each wealth bracket the average value of: (1) the real estate portfolio beta relative to the FASTPI real estate index, (2) the standard deviation of the property-specific idiosyncratic return, and (3) the ratio of the property-specific idiosyncratic return variance to the sum of the property-specific and group-specific idiosyncratic return variances.

The real estate portfolio of the median household has an average beta of 0.95, and is also exposed to substantial property-specific risk. We estimate that the property-specific idiosyncratic return has a standard deviation of 7.8% per year, and accounts for 72% of the idiosyncratic variance of the real estate portfolio. These values are in line with the fact that the median household's real estate portfolio is very poorly diversified and that real estate properties are unique assets exposed to very specific risks.

Real estate portfolios have similar risk features in higher brackets. The real estate beta reaches 1.1 for the top 10% of households, which implies that real estate is slightly better compensated than in the median bracket. Because richer households tend to own more properties, the standard deviation of property-specific risk ranges between 6% and 7 % and the share of the property-specific variance between 60% and 65%, which are both slightly lower than for the median bracket. Overall, real estate is a large source of idiosyncratic risk in all brackets of net worth.

IV.C Impact of the User Cost of Real Estate Services

In the main text, we consider measures of real estate returns that include all the benefits and costs of owning a piece of property, including the benefits of homeowner occupation and depreciation costs. This comprehensive approach is useful to analyze financial well-being but seems less suited for the analysis of wealth inequality dynamics because some components of returns are not capitalized. It remains an open question whether or not non-capitalized components of returns have a major impact on the main results.

In this section, we construct measures of real estate returns that exclude two non-capitalized components: the benefits of homeowner occupation and depreciation costs. More specifically, we focus on the *expected capitalized return,*

$$\rho_{i,t} = \mathbb{E}_{t-1}(g_{i,t}) + rent_{i,t}, \tag{A-2}$$

where $\mathbb{E}(g_{i,t})$ is the property's expected appreciation and $rent_{i,t}$ denotes the *received rental yield*, that is the rent actually perceived if the property is rented out divided by property value. The received rental yield is

$$rent_{i,t} = (d_{i,t} - \delta_{i,t})NHO_{i,t}$$

where $d_{i,t} - \delta_{i,t}$ is the rental yield net of depreciation and maintenance costs and $NHO_{i,t}$ is a dummy variable equal to unity if i is not homeowner occupied at t. Under the no-arbitrage rental yield relationship in the main text, the received rental yield satisfies

$$rent_{i,t} = [r_{f,t} + \kappa_{i,t} - \tau_{i,t} + \gamma_i - \mathbb{E}_{t-1}(g_{i,t})]NHO_{i,t}. \tag{A-3}$$

Since the risk premium satisfies $\gamma_i = \beta_i \mathbb{E}(r^e_{RE,t})$ under the real estate CAPM, the received rental yield can be rewritten as $rent_{i,t} = [r_{f,t} + \kappa_{i,t} - \tau_{i,t} + \beta_i \mathbb{E}(r^e_{RE,t}) - \mathbb{E}_{t-1}(g_{i,t})]NHO_{i,t}$. The expected capitalized return is therefore

$$\rho_{i,t} = \begin{cases} r_{f,t} + \kappa_{i,t} - \tau_{i,t} + \beta_i \mathbb{E}(r^e_{RE,t}) & \text{if } NHO_{i,t} = 1, \\ \mathbb{E}_{t-1}(g_{i,t}) & \text{if } NHO_{i,t} = 0, \end{cases} \tag{A-4}$$

for every i and t.

The last three columns of Appendix Table 17 consider three different methods for the imputation of the capitalized return. In column 4, we apply (A-4) with the additional assumption that $\mathbb{E}_{t-1}(g_{i,t}) = \beta_i \mathbb{E}(r^e_{RE,t})$. In column 5, we also apply (A-4) and estimate $\mathbb{E}_{t-1}(g_{i,t})$ by the average price appreciation of properties in the asset class over the past thirty years, $\bar{g}_i$. Column 6 considers a purely empirical approach. We estimate $\mathbb{E}_{t-1}(g_{i,t})$ by $\bar{g}_i$ and impute the received rental yield from national accounts, as is familiar from earlier research. Specifically, Swedish national accounts report the aggregate net rental yield on residential real estate using the rental equivalence method (Statistics Sweden 2014c). If the rented property is nonresidential, we deduct its specific tax advantage (relative to owner-occupied housing) from the aggregate residential real estate net rental yield.

For the median household, the expected excess return declines to 2.4%-2.7% per year depending on the estimation method for the rental yield. Overall, all three methods deliver broadly similar estimates of expected capitalized returns along the entire wealth distribution. Because the residential share of the portfolio declines with wealth, there is a significant rise in the expected capitalized return from real estate, which reaches 4.4%-5.0% per year in the top 0.01% of net worth, about twice the expected capitalized return in the median bracket.

V Private Equity

This section provides a detailed analysis of household private equity portfolio returns. Section *V.A* explains how we measure private equity returns from firm-level financial statements (Bisnode 2014). Section *V.B* discusses the measurement and simulation of private equity risk. Section *V.C* compares aggregate returns on public and private equity and estimates the 5-factor loadings of private equity. Section *V.D* investigates the accounting performance of the private equity held by households.

V.A Measuring Private Equity Returns

We measure the historical return of every private firm j as follows. Let $E_{j,t}$ denote the observed value of firm j's equity at the end of year t (computed using the methodology in Section *I.C* of this online Appendix).

If the firm publishes a financial statement for year $t+1$, the return on a share of private equity during the year is given by:

$$R_{j,t+1} = \frac{E_{j,t+1} + NetPay_{j,t+1}}{E_{j,t}} - 1,$$

where $E_{j,t+1}$ is the value of equity at the end of year $t+1$, and $NetPay_{j,t+1}$ is the total payout (dividends and share repurchases) net of capital injections during year $t+1$. This approach can be implemented for about 93.4% of all private equity holdings and 98.9% of the aggregate market value of private equity shares in our data.

The firm may not publish financial statements for year $t+1$ because it is sold or ended through voluntary liquidation or bankruptcy. We then track if the firm files for bankruptcy in the 16 months after the last accounts are published (Bisnode 2014), a scenario that impacts 0.3% of all private equity holdings or 0.1% of the aggregate market value of private equity shares. If a bankruptcy filing is detected, we impute a return of -100% on the shares of the firm held at the end of year t. Otherwise, we proxy the return on the firm's assets by the return on the assets of comparable public firms during the year.

V.B Measuring and Simulating Private Equity Risk

In this Section, we estimate the risk profiles of household private equity portfolios in each bracket of net worth. Throughout the Section, the risk profile of a firm refers to the five factor loadings and idiosyncratic volatility of returns, which are key drivers of wealth inequality dynamics. Returns are based either on the market value of total assets or the market value of equity.

The financials of a private firm are usually available for only a limited number of years. The standard time-series regressions of (equity or total asset) returns on the factors therefore produce noisy estimates of factor loadings. For this reason, we develop an estimation procedure based on widely available firm characteristics. The approach is simulation-based and consists of three steps. First, for each private firm, we estimate the risk profile of *total assets* conditional on firm characteristics. Second, we measure the risk profile of the firm's *equity* conditional on characteristics. Third, we use the estimated risk profiles of private firms to compute the risk profiles of household private equity portfolios. We now further explain each of these steps.

V.B.1 Risk Profile of Total Assets

We estimate the risk profile of each private firm's total assets by exploiting information on publicly traded firms with similar characteristics (Bisnode 2014, FINBAS 2016), as we now explain.[9]

Risk Profiles of Publicly Traded Firms. For every *public* firm i, we regress the time series of its stock returns on the five equity factors, which produces the factor loadings and idiosyncratic volatility of firm i's equity. We then unlever the equity's risk profile to obtain the vector of factor loadings, β_i, and idiosyncratic volatility, $\sigma_{idio,i}$, of total assets. The leverage ratio we apply is the firm's average market leverage ratio over the 1998 to 2008 period.[10]

We next relate the risk profiles of public firms to widely available characteristics. Let

$$y_i = (\beta_i', \sigma_{idio,i})'$$

denote the column vector containing the five factor loadings and idiosyncratic volatility of firm i. Let X_i denote a column vector containing the following firm characteristics: the two-digit sector dummy, the tercile of the book value of assets, profitability, asset tangibility, dummies for foreign direct investments in the United States, and a constant term corresponding to the intercept. The accounting characteristics in X_i are time averages over the 1998 to 2008 period. For each component n, we estimate the regression model:

$$y_{i,n} = X_i' \gamma_n + \varepsilon_{i,n}. \tag{A-5}$$

We obtain estimates of the sensitivities of risk profiles to characteristics, $\hat{\gamma}_n$, and of the risk profile residuals, $\hat{\varepsilon}_{i,n}$, for every i and n.

Risk Profiles of Private Firms. The characteristics used in the regression (A-5) are observable

[9] We run the comparison at the asset level rather than at the equity level to avoid the biases and noise arising from differences in leverage between firms.

[10] See Berk and DeMarzo (2014) for a textbook treatment of this procedure.

for both public and private firms. We make the simplifying assumption that the risk sensitivities γ_n and the distribution of the residuals $\varepsilon_{i,n}$ are the same for both types of firms.

For each private firm j, we can sample from the distribution of firm j's risk profile conditional on the vector characteristics X_j by applying:

$$\tilde{y}_{j,n} = X_j'\hat{\gamma}_n + \hat{\varepsilon}_{j,n} \tag{A-6}$$

where $\hat{\gamma}_n$ is the regression coefficient estimated on the sample of public firms, and the residual $\hat{\varepsilon}_{j,n}$ is the residual of a randomly selected listed firm. We use (A-6) to generate M simulated profiles $\tilde{y}_j^{(m)}$, where $m = 1, \ldots, M$. In practice, we draw $M = 100$ simulated profiles for each private firm. We can view each $\tilde{y}_j^{(m)}$ as the risk profile of a pseudo-firm with the same observable characteristics as private firm j.

The methodology offers multiple benefits. The residual of the private firm, $\hat{\varepsilon}_{j,n}$, is drawn from the distribution of residuals of public firms in order to capture the heterogeneity of the risk profile $y_{j,n}$ conditional on characteristics. This random-on-unobservables imputation approach guarantees that the correlation between the types of risk exposures within each private firm j is similar to the correlation observed within each listed firm conditional on observables. This matters because there is in the data strong cross-sectional correlation between different types of risk exposures, even after controlling for observable characteristics. For example, listed firms with high idiosyncratic risk tend to have low market betas, which can occur if firm owners manage their exposure to total risk. Furthermore, the imputation approach guarantees that the heterogeneity in risk exposures across private firms will be at least as high as that which we observe among public firms.

The imputation procedure is run for non-financial assets within each firm. For financial assets held by unlisted firms, we assume that risk exposures are drawn independently from the risk exposures of listed investment vehicles that are controlled by a few individuals.[11] Because

[11] During the 2000 to 2008 period, there are five such "investmentbolag" in Sweden: Investor AB (controlled by the Wallenberg family), Investment AB Latour (controlled by the Douglas family), Ratos (controlled by the Söderberg family), Öresund (controlled by the Hagströmer and Qviberg families), and Geveko (controlled by the Bergendahl family).

these investment firms issue traded equity to the public, the risk profiles of their assets can be estimated using higher-frequency price data.

V.B.2 Risk Profile of Equity

We next estimate the risk profile of each private firm's equity. Limited liability implies that the factor loadings of equity cannot be written as closed-form functions of the factor loadings of total assets. For this reason, we use a simulation-based approach to estimate the risk profile of private equity returns conditional on firm characteristics.

Equity Simulator. Consider a firm with risk profile $y_j = (\beta_j', \sigma_{idio,j})'$, and total assets $V_{j,t}$ and equity $E_{j,t}$ at date t. Assume that firms pay no dividends and issue no new securities (debt or equity claims). We can simulate the value of total assets at $t+1$ as follows:

$$\tilde{V}_{j,t+1} = V_{j,t}(1 + r_{f,t+1} + \beta_j' f_{t+1} + \sigma_{idio,j}\, e_{j,t+1})$$

where $r_{f,t+1}$ is the risk-free rate and f_{t+1} the vector of factor realizations in $t+1$. The variable $e_{j,t+1}$ is an independent and identically distributed (i.i.d.) shock, which in practice is drawn from a log-normal distribution with unit variance. Importantly, where applicable, the vector of factor loadings β_j is the weighted average of the risk loadings of the financial and non-financial assets of firm j.

The corresponding value of the equity claim is:

$$\tilde{E}_{j,t+1} = \max\left\{\tilde{V}_{j,t+1} - [1 + (1-\theta) r^{debt}_{j,t+1}] D_{j,t}; 0\right\},$$

where $D_{j,t}$ is the book value of financial debt at the end of year t, θ is the corporate tax rate, and $r^{debt}_{j,t+1}$ is the pre-tax cost of debt.[12] We then obtain the return on equity,

$$\tilde{r}_{j,t+1} = \frac{\tilde{E}_{j,t+1}}{E_{j,t}} - 1,$$

[12]The interest rate on financial debt is imputed using the financials of firms bearing mostly financial debt (since the Swedish accounts do not separate interest payments on financial versus non-financial debt).

for every t.

Application. We use the simulator to estimate the distribution of the risk profile of equity of each private firm. We sample factors $\tilde{f}_t$ from their empirical distribution at the monthly frequency at dates $t = 1, \ldots, T$, where $T = 1,200$ months in practice. For each private firm j, we implement the following procedure.

- Consider the M pseudo-firms with risk profile $\tilde{y}_j^{(m)}$ and characteristics X_j defined in Section *V.B.1*. We assign to each pseudo-firm the actual debt, $D_{j,t}$, and debt cost, $r_{j,t+1}^{debt}$, of private firm j. Given the pseudo-factors $\tilde{f}_t$, the equity simulator allows us to draw the monthly growth rate of total assets, $V_{j,t+1}^{(m)}/V_{j,t}^{(m)}$, at dates $t = 1, \cdots, T$, for every pseudo-firm m.

- For every pseudo-firm m, we compute annual returns on total assets by compounding monthly growth rates over each year. We derive annual equity returns from the annual returns on total assets, as is explained in the simulator.

- For every private firm pseudo-firm m, we run an OLS time-series regression of annual pseudo-returns on simulated factors. This produces the pseudo-firm's equity risk profile.

- We compute the sample mean of the factor loadings of equity across pseudo-firms, which we denote by $\hat{\beta}_j^E$. We also compute the sample average of the idiosyncratic variances of the pseudo-firms, which we denote by $(\hat{\sigma}_j^E)^2$.

The idiosyncratic variance estimates, $(\hat{\sigma}_j^E)^2$, helps us gauge the level of idiosyncratic variance perceived by entrepreneurs conditional on the information available to them.[13] We use it to compute the share of idiosyncratic risk in household private equity portfolios (reported in Table 7 of the main text) and the share of idiosyncratic share in household total wealth (reported in Table 1 of the main text). In the next subsections, we use the factor loading estimates, $\hat{\beta}_j^E$, to compute the expected returns of private firms.

[13]The estimated variance $(\hat{\sigma}_j^E)^2$ does not coincide with the variance of $r_{j,t} - f_t' \hat{\beta}_j^E$. It is instead the econometrician's best estimate of the idiosyncratic variance perceived by entrepreneurs given their information sets.

V.B.3 Factor Loadings of Household Private Equity Portfolios

In Appendix Table 18, we provide the cross-sectional distribution of risk loadings on private equity portfolios in every bracket of net worth. The factor loading of an equity portfolio is the estimated loading of each private firm, $\hat{\beta}_j^E$, weighted by its share in the equity portfolio.

For private equity owners in the median bracket of net worth, the private equity portfolio has a loading of 0.72 on the local equity factor, 0.38 on the global equity factor, and -0.13 on the currency factor. These estimates are similar to the loadings of the public equity portfolio in the median bracket reported for the median bracket in Appendix Table 11. The private and public equity portfolios, however, differ in their size and value loadings. For the median household, the size beta is 0.62 for private equity but 0.02 for public equity, while the value beta is 0.44 for private equity as opposed to -0.03 for public equity.

Among richer owners of private equity, the risk exposures of the private equity portfolio are broadly similar. The exposures to local and global equity fall to, respectively, 0.65 and 0.31 among the top 0.01%. The size loading is U-shaped and the value loading is hump-shaped in net worth, while the currency loading is nearly flat. Overall, the expected return on the private equity portfolio monotonically decreases from 10.6% to 9.1 % per year between the median household and the top 0.01%, as Table 7 in the main text shows.

V.C Goodness of Fit of the Pricing Model

In Section *V.B* of this online Appendix, we assume that private firms have similar systematic exposures conditional on characteristics and do not generate abnormal risk-adjusted performance on average. We conduct a joint test of these assumptions by considering two indexes of private equity performance, which we compare to aggregate public equity.

We construct the two private equity indexes as follows. The first index, which we call the *private equity index*, tracks the performance of the aggregate private equity portfolio of Swedish households. Its return is the weighted average of the equity returns of private firms, as defined in Section *V.A* of this online Appendix. The return on the private equity index can potentially include a substantial risk-adjusted performance specific to private firms.

The second index, which we call the *matched public equity index*, tracks the aggregate value of the synthetic portfolios of public stocks that mimic the risk profile of each unlisted share. The mimicking portfolio of each private firm j is a combination of the five equity factors with the same loadings as firm j. The excess return of the synthetic portfolio is therefore $f_t'\hat{\beta}_j^E$. The return on the matched public index is the average return of synthetic portfolios, weighted by the equity value of these firms. The return on the matched public index reflects the return earned by the firm under the assumption that our asset pricing model for private equity is correct and that there is no risk-adjusted performance.

We compute the total annual return on the two private equity indexes from 2001 to 2008 and subtract the risk-free rate to obtain excess returns. We use the SIXRX index as the *unmatched public equity index*.

In the left panel of Appendix Table 19, we report the yearly excess returns on our two indices and on the Swedish public equity index, in which stocks are not reweighted to mimic the risk loadings of private firms. The returns on the private equity index are closely tracked by returns on the matched public equity index. The time series correlation coefficient between the two return series is equal to 0.97. Moreover, the right panel of Appendix Table 19 shows that our asset pricing model for private equity accurately predicts the expected return on private equity: the private equity index underperforms our matched public equity index by 0.97% per year, which is statistically insignificant. Using a specific model for private equity as we do here matters: the risk-adjusted performance of the private equity index over the unmatched public equity index is positive and equal to 6.89% per year from 2001 to 2008. The table confirms that unlisted equities have specific exposures to systematic factors and that these must be taken into account before assessing the risk-adjusted performance of private equity as an asset class.

We conclude from these results that our asset pricing model for private equity is well specified and that private equity does not deliver significant abnormal performance, which confirms that Moskowitz and Vissing-Jørgensen's (2002) findings from the US SCF also hold in our administrative dataset.

V.D Accounting Performance of Private Equity

Throughout the main text and this online Appendix, the measurement of private equity returns is based on the market value of assets, consistent with the methodology used for other wealth categories. By contrast, some recent papers relying on accounting metrics, such as Fagereng et al. (2019) and Smith et al. (2018), obtain that entrepreneurs, particularly the wealthier ones, are more skilled investors than the rest of the population, in direct conflict with the conclusions of the present study. We now show that the lack of detectable risk-adjusted performance in private equity returns reported in the main text is robust to using accounting-based measures of performance. These new results suggest that the aforementioned differences in results are not simply due to the use of market-based vs. accounting-based measures of performance.

In Appendix Table 20, we provide estimates of private equity performance based on the accounting data at our disposal. We report: (1) the return on equity (ROE), i.e. the ratio of net earnings to the book value of equity, as in Fagereng et al. (2019)[14], (2) the ratio of net earnings to the market value of equity, and (3) the ratio of net earnings to the number of workers, as in Smith et al. (2018). In all three cases, when a household owns several firms, we apportion earnings and workers according to the book value of the shares held in each firm.

The median household's private equity portfolio earns on average an ROE of 18.0% per year. This accounting-based performance measure significantly exceeds the market-based performance of the private equity class over the period 2001-2008, which is about 10.1% per year if

[14]To be precise, Fagereng et al. (2019) consider the ratio of net earnings to the tax value of equity, which includes some components registered at market value. Yet, we choose to assimilate the tax value of equity to the book value because in Norway the tax value of equity excludes intangibles, it is very correlated with the book value and it is not significantly higher or lower than the book value on average.

one adds a 3% risk-free rate to the average excess return on private equity reported in Appendix Table 19. The difference between the two metrics primarily reflects the fact that the book asset value greatly underestimates the market value of private firms: the earnings-to-equity ratio declines from 18.0% to 10.3% when we replace the book value of equity by the market value.

For each worker of her private firm, an owner with median net worth earns about 25,300 Swedish kronor per year, or about $3,800. In higher brackets, profitability per worker goes up dramatically to 330,000 Swedish kronor per year, or about $50,000. This result parallels the US estimates of Smith et al. (2018). However, the US study does not have access to data on the equity value of the investments made by entrepreneurs, even in book value terms. The US results may therefore reflect higher capital intensity and lower leverage rather than higher talent in firms owned by richer entrepreneurs. We provide evidence against the talent explanation: profitability per equity invested in the company remains basically flat across the entire wealth distribution, regardless of whether one considers the book value or market value of equity (columns 1 and 2 of Appendix Table 20).

Overall, the investigation of accounting-based performance measures does not alter our main conclusions. Private equity delivers substantial profits to their owners but these profits are well in line with the capital invested and riskiness of these ventures.

VI Total Household Wealth

This Section provides empirical evidence on household total wealth which complements the results in the main text. In Section *VI.A*, we document the asset allocation and risk profile of gross and net wealth, as well as debt costs, across brackets of net worth. Section *VI.B* verifies the robustness of our main results to alternative measures of expected returns. We investigate the impact of taxes in Section *VI.C* and the determinants of debt costs in Section *VI.D*. Section *VI.E* compares our results to the evidence from US data. Finally, we analyze the persistence of household wealth returns in Section *VI.F* and the distinction between scale-dependence and size-dependence in Section *VI.G*.

VI.A Asset Allocation and Risk Profile

The expected return on wealth is driven by (i) the allocation of total wealth to broad asset classes and (ii) the factor loadings of holdings in each asset class. The asset allocation channel is illustrated in Figure 2 and is discussed in Section II.B of the main text. In Appendix Table 21, we provide the estimates of the average asset allocation in each wealth bracket, which are used to plot Figure 2 of the main text. We now discuss the factor loading channel.

VI.A.1 Factor Loadings and Idiosyncratic Volatility of Gross Wealth

In Appendix Table 22, we report for each net worth bracket the average loading of gross wealth on the (1) local equity, (2) global equity, (3) size, (4) value, (5) currency, and (6) real estate factors. In addition, column 7 displays the idiosyncratic risk of gross wealth.

Households in the bottom 10% bear almost no risk in gross wealth. The average loadings of households in the bracket are, respectively, 0.09 for local equity, 0.14 for global equity, -0.002 for size, -0.01 for value, 0.07 for the currency factor, and 0.16 for the real estate factor. The standard deviation of idiosyncratic risk is also small and equal to 1.8% in annual units. These estimates reflect low holdings of risky assets by these households (see Figure 2 of the main text and Appendix Table 21).

The loadings on local equity, global equity, size, and value increase slowly up to the 95th percentile of net worth, and then rise rapidly in the top 5%. In the top 0.01% bracket, the local equity loading is 0.51, the global equity loading is 0.24, the size loading is 0.43, and the value loading is 0.22, which shows high exposures to these four types of systematic risk.

The loadings on the currency and real estate factors exhibit different patterns. The currency loading is slightly hump-shaped and stays close to zero throughout the distribution. The real estate beta varies strongly and is a hump-shaped function of net worth. It is equal to 0.16 for the bottom 10%, peaks at 0.59 for the top 10%-2.5% of households, and then declines to 0.18 for the top 0.01%. The increasing exposure to equity risk premia compensates the reduction in real estate risk, which explains the monotonic rise of expected returns on gross wealth along the entire wealth distribution.

Idiosyncratic volatility slowly increases to 6.3% in the top 5%-2.5% of the distribution of net worth, and then sharply rises to 26.5% in the top 0.01%. In higher brackets, the increasing idiosyncratic exposures of the financial and private equity portfolio more than offset the reduction in idiosyncratic risk of the real estate portfolio.

VI.A.2 Factor Loadings and Idiosyncratic Volatility of Net Wealth

Appendix Table 23 reports the risk profile of total *net* wealth. Column 1 to 6 display the loadings on the pricing factors and column 7 shows idiosyncratic volatility. In column 8, we compute the (negative) impact of the cost of debt on the expected return on net wealth.

Households in the second decile bear substantial systematic and idiosyncratic risk in net wealth. Idiosyncratic volatility is estimated at 3.4% per year, and the average loadings are 0.19 for local equity, 0.26 for global equity, 0.14 for the currency factor, and 0.28 for the real estate factor. Despite substantial exposure to priced risk, the expected return on net wealth remains low in the first decile because debt costs reduce the net expected return by 3.6%, as column 8 shows.

Risk exposures vary strongly with net worth. The loadings on the local and global equity are U-shaped in net worth and bottom out in the top 70%-95% bracket. The loadings on size and value are negligible outside the top 2.5% of households, and the loading on the currency factor monotonically declines with wealth. The exposure of net wealth to the real estate factor is a hump-shaped function of net worth, rising to 0.73 in the top 50%-30% and then declining monotonically to 0.22 among the top 0.01%. The patterns obtained for the value, size, currency, and real estate loadings are in line with the patterns obtained for gross wealth. Local and global equity loadings, however, exhibit sharply different properties depending on the type of wealth we consider.

Idiosyncratic risk remains flat between 6% and 8% between the 20th and 97.5th percentiles and then increases rapidly. It reaches 27% for the top 0.01% of households. This pattern is very consistent with gross wealth.

Column 8 reports the negative impact of debt costs on the expected return of net wealth. The cost of debt declines monotonically with net worth, ranging from 3.6% in the second decile to 0.1% in the top 1% of the distribution. Thus, debt has an important impact on expected returns. In the bottom 20%, interest rates are high and more than offset the positive effect of leverage on expected returns. As a result, the expected return on net wealth is much more in line with the expected return on gross wealth than factor loadings alone might suggest.

VI.B Robustness Checks on Expected Returns

In Appendix Table 24, we verify that our results on expected wealth returns are robust to alternative measurement methods. Columns 1 and 2 of Panel A shows the baseline estimates of expected returns on gross and net wealth across wealth brackets, which are also reported in Tables 1 and 2 of the main text.

Columns 3 and 4 of Appendix Table 24, Panel A, investigate the potential effects of look-ahead bias. Instead of using average risk premia estimated for the entire period 1981-2016, we use the average risk premia from 1981 up until the time household holdings are measured. The resulting measures of expected returns are noisier because they are evaluated over a shorter period, but do not include any information on risk premia measured after year t. The table shows that the level and distribution of expected returns are very similar to baseline estimates. Therefore, none of the results in the paper are driven by look-ahead bias.

Columns 5 and 6 of Appendix Table 24, Panel A, document the impact of pension wealth imputation. In the main text, we impute funded pension wealth from information on past labor income and current pension income, as well as a simple rule for allocation of aggregate funded pension wealth between current workers and retirees. As Section *I.A* of this online Appendix explains, the imputation strategy in the main text differs along two key dimensions from the method suggested by Saez and Zucman (2016) and applied to Sweden by Alstadsæter, Johannesen, and Zucman (2019). First, we use the entire individual labor income series, rather than current labor income, to impute the pension wealth of current workers. Second, for the period 2000-2007, we allocate 42% of funded pension wealth to current retirees and 58% to current workers, as compared to 60% and 40%, respectively, in the aforementioned papers. Columns 5 and 6 display the expected returns resulting from the imputation method in Saez and Zucman (2016) and Alstadsæter, Johannesen, and Zucman (2019). Expected returns are almost identical to our baseline estimates for the top 80% of households. Differences are larger but remain modest for the bottom 20%, where pension wealth is a large component of wealth.

In columns 1 and 2 of Appendix Table 24, Panel B, we report the expected return on gross and net wealth when pension assets are excluded from the definition of household wealth. Expected returns are substantially lower in the bottom half of the population because non-pension wealth is held primarily in low-yielding cash. By contrast, for households in the top half of the distribution, expected returns are slightly higher when pension wealth is excluded because non-pension wealth is held primarily in high-yielding risky assets. The relationship between expected returns and wealth remains the same under both methods. Thus, accounting for pension wealth is important only if we focus on the differences in expected returns between the bottom and top halves of the wealth distribution.

In columns 3 and 4 of Appendix Table 24, Panel B, we analyze the sensitivity of our results to private equity. Measuring the value and returns on private equity requires strong assumptions, as explained in Section V of this online Appendix. For this reason, we now measure expected returns and wealth ranks under the assumption that private equity does not belong to household wealth. This would be the case if one assumes that private equity wealth is in fact only human capital wealth. Unsurprisingly, excluding private equity reduces the gap in expected returns between the median and the top of the wealth distribution of wealth. The expected return spread between the top 40%-50% and the top 0.01% is 3.1% for gross wealth and 3.0% for net returns, compared to 4.4% and 3.8%, respectively, in the baseline case. However, the expected return spread remains very substantial. Excluding private equity therefore has no impact on our key result that expected returns are positively correlated to net worth.

VI.C Taxes

VI.C.1 Impact of Different Forms of Taxation on Capital Income

The main text documents a hump-shaped relationship between the capital tax rate (i.e., the ratio of all taxes on capital divided by wealth) and the household net worth rank. We now investigate the origins of this pattern.

Capital taxes can be subdivided into personal taxes on capital income (including realized capital gains), personal taxes on the capital stock, and the corporate tax paid by companies held by the household. Furthermore, households receive a tax credit for mortgage interest. Starting in fiscal year 2007, taxes on the capital stock are either capped (in case of the property tax) or entirely eliminated (in the case of the wealth tax), and taxes on dividends from private firms are also cut. We therefore consider two different subperiods: 2001-2006 and 2007-2008.

Appendix Table 25 displays the following statistics across the distribution of net worth. For the 2001-2006 subperiod, we report the personal capital tax, defined as the sum of personal taxes on capital income and the capital stock, scaled by gross wealth (column 1 of Panel A) or net wealth (column 2), and the personal capital income tax scaled by gross wealth (column 3) or net wealth (column 4). Columns 5 to 8 of Panel A report the same tax ratios for the 2007-2008 subperiod. Finally, Panel B shows the average corporate tax rate over the full 2001-2008 period. We do not consider the mortgage interest rate deduction in the calculation of tax rates expressed as a fraction of gross wealth (columns 1, 3, 5, and 7 of Panel A) but we consider it for tax rates relative to net wealth (columns 2, 4, 6, and 8 of Panel A).

Consider a household in the median wealth bracket. Between 2001 and 2006, the median household pays capital taxes amounting to 0.7% of gross wealth, but receives a net subsidy amounting to 0.2% of net wealth once the mortgage interest tax credit is taken into account. These estimates are primarily driven by the taxation of capital income and not by the taxation of the capital stock. For instance, the yearly tax on capital income represents +0.6% of gross wealth. Starting in 2007, taxation of the capital stock is greatly reduced but the reform barely affects the overall capital tax rate of the median household. Finally, over the entire period, the corporate tax paid by the median household is equal to 0.5% of gross wealth and 0.6% of net wealth, primarily because half of pension wealth is invested in equities and therefore subject to the corporate tax.

We next consider the taxation of capital in higher brackets. Over the full sample period, the capital income tax is a hump-shaped function of gross wealth, which increases from 0.6% for the median household to 1.2% for the top 1%-0.1% and then declines to 0.7% (2001-2006 period) and 0.6% (2007-2008 period) for the top 0.01%. The increase from the median to the top 1% primarily stems from the fact that the dividend yield on owner-occupied housing, which represents a large source of returns for households in the middle of the wealth distribution, is not taxed. The decline of the capital income tax rate within the top 1% is due to the fact that very rich households tend to retain corporate income in their private companies and thus defer personal taxation. This effect is partly compensated by the higher corporate tax exposures of richer households: the corporate tax rate monotonically rises from 0.5% of gross wealth for the median household to 1.4% for the top 0.01%.

The personal tax on the capital stock is a substantial and hump-shaped fraction of gross wealth in the top half of the wealth distribution over the 2001-2006 period. Specifically, the personal tax of the capital stock increases from 0.1% of gross wealth for the median household to 0.4% for the top 0.5%-0.1% and then declines to 0.2% for the top 0.01%.[15] The decline in higher brackets is due to the fact that most of private equity is exempt from the wealth tax. This tax is almost entirely eliminated starting in 2007.

[15]We obtain these estimates by subtracting column 3 from column 1 of Panel A.

Overall, this analysis shows that the hump-shaped relationship between the capital tax rate and wealth stems primarily from the perimeter of taxable capital income, which is itself a hump-shaped proportion of wealth. These important empirical patterns are attributed to several key features of the tax system, including the absence of taxation of owner-occupied housing dividends and the deferred taxation of capital gains.

VI.C.2 Cross-Sectional Dispersion of the Capital Tax Rate

As Table 8 of the main text shows, the taxation of capital increases the dispersion of returns on *gross* wealth but has the opposite effect on the returns on *net* wealth. We now further explain these regularities.

In Appendix Table 26, we report the cross-sectional standard deviation of a household's capital income tax scaled either by gross wealth (column 1) or net wealth (column 2), excluding corporate taxes from consideration. These ratios exhibit very strong dispersion across households in a given year, with a standard deviation of 4.0% for gross wealth and 6.1% for net wealth. We estimate the variance decomposition formula:

$$Var(\tau_{h,t}) = \mathbb{E}[Var(\tau_{h,t}|W_{h,t})] + Var[\mathbb{E}(\tau_{h,t}|W_{h,t})],$$

where $\tau_{h,t}$ denote the capital tax rate and $W_{h,t}$ the wealth of household h in year t. The within share, $\mathbb{E}[Var(\tau_{h,t}|W_{h,t})]/Var(\tau_{h,t})$, is estimated at 91% when we focus on gross wealth and 97% when we focus on net wealth. The dispersion is therefore almost entirely due to heterogeneity within wealth brackets.

The post-tax return is given by

$$r^{post}_{h,t} = r^{pre}_{h,t} - \tau_{h,t},$$

where $r^{pre}_{h,t}$ denotes the pre-tax return and $\tau_{h,t}$ is the capital tax rate.[16] The cross-sectional vari-

[16]The wealth accumulation equation, $W_{h,t} = (1 + r_{h,t})W_{h,t-1} - \tau_{h,t}W_{h,t-1}$, implies that the post-tax return is $r_{h,t} - \tau_{h,t}$.

ances of post- and pre-tax returns therefore differ by

$$Var(r_{h,t}^{post}) - Var(r_{h,t}^{pre}) = Var(\tau_{h,t}) - 2Cov(r_{h,t}^{pre}; \tau_{h,t}).$$

Post-tax returns are more dispersed than pre-tax returns if the correlation between the capital tax rate and the pre-tax return on wealth is lower than $0.5[Var(\tau_{h,t})/Var(r_{h,t}^{pre})]^{1/2}$. This condition is satisfied for gross wealth but not for net wealth, as the estimates in Table 8 of the main text imply.

In columns 3 to 6 of Appendix Table 26, we investigate the mechanisms driving the heterogeneity of capital income tax rates within wealth brackets. In principle, if the taxation of returns were progressive, capital income taxes could play an insurance role and mitigate the dispersion of pre-tax returns. For this reason, we compute the marginal impact on a household's capital tax rate of a 1 percentage point increase in the historical wealth return. The tax rate and historical return refer to gross wealth in column 3 and to net wealth in column 4.

Our main finding is that at the yearly frequency, a household's historical wealth return has very little effect on capital taxes. Across the population, a one percentage-point increase in historical returns induces a 0.02-0.03 percentage point increase in the capital income tax rate, whether one considers gross wealth or net wealth. Within wealth brackets, the sensitivity of taxes to historical returns is also very small, except when one considers net wealth among indebted fractiles at the bottom of the wealth distribution.[17]

The weak relationship between taxes and returns at the annual frequency may be due to the fact that capital gains need not be realized in the year when they are they generated. For this reason, we now investigate the connection between long-term returns and capital income taxes. Given data constraints, we proxy long-term returns by expected returns, which are measured with good accuracy. In columns 5 and 6, we indeed find that the capital tax rates are more sensitive

[17] In the second decile of the wealth distribution, a one percentage-point increase in the historical return on net wealth induces a 0.22 percentage-point increase in the tax rate. This effect is likely driven by the mortgage interest deduction: households with low historical returns are those that pay very high interest rates and hence obtain a large tax credit from the Swedish tax authority. The insurance effect of mortgage deductions explains on its own why after-tax returns on net wealth are less dispersed than pre-tax net wealth returns, even though gross wealth returns are substantially more dispersed before taxes than after taxes.

to expected returns than to annual historical returns. In the population, a one percentage-point increase in expected returns is associated on average with a 0.31 percentage-point increase in the capital tax rate when we focus on gross wealth and 0.19 percentage-point when we focus on net wealth. These estimates are partly due to a composition effect, since the rich earn high expected returns and face a progressive capital tax system. For this reason, it is important to measure capital tax sensitivities within wealth groups.

The sensitivity of taxes to expected returns is generally negative within most wealth brackets. These results sharply contrast with the strong positive link between taxes and expected returns at the population level. Hence, the positive correlation of taxes and returns at the population level stems from a positive correlation between capital tax rates and returns between wealth brackets, which overcomes the negative sensitivity measured within wealth brackets.[18] Both effects, however, have important implications for the wealth inequality dynamics. The tax system tends to both mitigate inequality between brackets and increase inequality within brackets, which tends to foster mobility.

The sensitivity of taxes to expected returns (conditional on wealth) tends to vary substantially across wealth brackets. The relationship is U-shaped, whether one considers gross wealth and net wealth. Since the sensitivity is generally negative, households from the middle of the wealth distribution receive the largest tax subsidies on wealth returns. A possible explanation is that in this part of the distribution, the taxation of capital operates primarily through the capital income tax, while the tax on the wealth stock only plays a marginal role. Assets providing high expected returns, such as equity and real estate, usually generate returns in the form of untaxed housing dividends and capital gains that are realized and taxed rather infrequently, which in-

[18] This relationship can be understood by decomposing the sensitivity of taxes to returns in the population as a function of the sensitivity between and within wealth brackets. Specifically, let $\beta_{within}(W_{h,t}) = Cov(r_{h,t}, \tau_{h,t}|W_{h,t})/Var(r_{h,t}|W_{h,t})$ denote the sensitivity conditional on wealth, and let $\beta_{between} = Cov[\mathbb{E}(r_{h,t}|W_{h,t}), \mathbb{E}(\tau_{h,t}|W_{h,t})]/Var[\mathbb{E}(r_{h,t}|W_{h,t})]$ denote the sensitivity between wealth brackets. The population sensitivity of taxes to returns, $\beta = Cov(r_{h,t}, \tau_{h,t})/Var(r_{h,t})$ satisfies

$$\beta = \mathbb{E}\left[\beta_{within}\frac{Var(r_{h,t}|W_{h,t})}{Var(r_{h,t})}\right] + \beta_{between}\frac{Var[\mathbb{E}(r_{h,t}|W_{h,t})]}{Var(r_{h,t})},$$

as the law of total variance and the law of total covariance imply.

duces a negative sensitivity of taxes to expected returns at the yearly frequency. Consistent with this explanation, the sensitivity becomes less negative once taxes on the stock of wealth become more important, as is the case for households in the top 5%.

Overall, this analysis suggests that the tax system has a contrasted role on the heterogeneity of wealth returns and the dynamics of inequality. Besides the progressivity of the tax system, the absence of taxation of owner-occupied housing dividends, the deferred taxation of capital gains, and the mortgage interest tax credit seem to be of primary importance.

VI.D Cost of Debt

In Appendix Table 27, we investigate the determinants of debt costs on household liabilities. Column 1 reproduces the benchmark results in Table 2 of the main text, which show a very strong negative effect of net worth on the cost of debt. For instance, the cost of debt is 3.4% lower for households in the top 0.01% than for the median household. The remaining columns of Appendix Table 27 consider additional explanatory variables: dummies for deciles of the debt level (column 2), the debt coverage, defined as the log of one plus the real estate asset-to-debt ratio (column 3), and all explanatory variables together (column 4).

A high debt level is associated with a low interest rate spread (column 2), which is consistent with economies of scale or a demand effect. By contrast, higher debt coverage is associated with a lower debt cost (column 3), as risk management implies. In both columns, the negative relationship between debt costs and net worth is somewhat weaker than in column 1, which suggests that the debt level and the debt coverage partly explain the average variation of debt costs across wealth brackets.

When all covariates are simultaneously included, the regression coefficient on debt level and coverage remain very similar and the coefficients on net worth are close to zero in most brackets. Overall, the results suggest that a mix of economies of scale and lower default risk can almost entirely explain the low household debt spread of the wealthy.

VI.E Comparison with US Households

Sweden differs from the US along many economic, cultural, and institutional dimensions. Unfortunately, we cannot fully replicate our results on US households for lack of comprehensive data.

In Appendix Table 28, we use the US Survey of Consumer Finances (SCF, Federal Reserve Board 2007), to provide rough estimates of the gross and net expected returns earned by US households. We compare these results to the expected returns of Swedish households.

Columns 1 and 2 report expected returns estimated on US data. Wealth ranks are estimated using sampling weights provided in the SCF. Household holdings are reported at the level of three asset classes: public and private equity, real estate, and safe financial assets. We impute household wealth returns by assuming that the expected return earned by every household on each asset class is the expected return of the asset class's index. We use the following indexes: the value-weighted CRSP index for equity, the Case-Shiller index for real estate (Shiller 2016), and the US one-month T-bill for safe financial assets (French 2016). The expected return of each index is proxied by its average return over the 1981 to 2016 period. We also use the household debt cost reported in the SCF.

The median US household earns a risk premium on gross wealth of 3.7% and a risk premium on net wealth of 3.8%. Like in Sweden, the risk premium increases strongly with net wealth, reaching 5.3% for gross wealth and 5.4% for net wealth in the top 1%-0.5% bracket, and 6.6% for gross and net wealth alike in the top 0.01% bracket.

In columns 3 and 4, we report the expected excess returns on the gross and net wealth of Swedish households computed with the same imputation method as the one used for US households. That is, we consider household holdings aggregated at the asset class level and assume that a household's portfolio of assets in a class earns the expected excess return of the corresponding index (SIX equity index (Datastream 2016), FASTPI housing index (Statistics Sweden

2014b), or Swedish 1-month T-bill (Sveriges Riksbank 2016)). We apply the average risk premia corresponding to each index over the 1981-2016 period. For the computation of net wealth returns, we use the same measure of debt cost as in the rest of the paper.

Across most of the distribution of net worth, the expected excess return on gross wealth is remarkably similar in both countries. For instance, the expected excess return on gross wealth is 2.1% in the US and 2.4% in Sweden on average for a household in the bottom decile; 3.7% in the US and 3.8% in Sweden for the top 40%-50%; 5.3% in the US and 5.6% in Sweden for the top 1%-0.5%. For the top 0.01%, however, a discrepancy arises: the expected excess return on gross wealth is 6.6% in the US and 7.5% in Sweden. The difference is possibly a reflection of differences in the household data. The top 400 families are excluded from the SCF but not from the Swedish series. Furthermore, the response rate at the top is very low in the SCF but is close to 100% in the Swedish registry.

The expected excess return on net wealth exhibits more pronounced differences between the two countries. In the bottom 70% of the wealth distribution, expected returns are at least one percentage point lower in the US than in Sweden, and the gap is even more pronounced for the bottom 20%-30% bracket (the first decile for which net worth is positive in the US). The main reason is that household debt costs are significantly higher in the US than in Sweden, which has multiple origins. Swedish households cannot walk away from their loans due to the absence of a personal bankruptcy procedure and the impossibility of nonrecourse loans. By contrast, US households have access to bankruptcy procedures in all states and in some states can also contract nonrecourse loans, which limit liabilities to property values and exempting a significant part of their properties from repossession by lenders. It is therefore justified that interest rate spreads on household debt are higher in the United States. Furthermore, the possibility of filing for personal bankruptcy implies that the expected return on household net wealth is underestimated for US households. Indeed, interest payments in case of a bad shock are far lower than what is suggested by the interest rate prior to default, which we use here to compute expected returns. This suggests that a careful estimation of default probabilities is required for a thorough international comparison of household wealth returns.

In columns 5 and 6, we report our baseline results on expected returns in Sweden using the full data at our disposal. A comparison with the two previous columns, where we limit the data to what is typically available in surveys, reveals that the differences in expected returns with and without full data are smaller than 0.25 percentage points in the bottom 97.5% of the distribution. However, using data on security-level holdings leads to significantly larger expected returns within the top 1%. The imputation method decreases the expected return on gross wealth from 6.2% to 5.6% for the top 1%-0.5%, and from 7.9% to 7.5% for the top 0.01%.

Overall, because we have access to the full population of rich households and exhaustive data on their holdings, the expected gross return we measure in our data is about one percentage point higher than in the SCF for the top 1%-0.5% and one and a half percentage points higher for the top 0.01%. The difference in the expected excess return on gross wealth between the median household and the top 0.01% goes from 2.9 percentage points in the SCF to 4.4 percentage points using our data. Therefore, access to better data largely amplifies the role played by return heterogeneity in the dynamics of wealth inequality. It also makes it possible to measure the dispersion of returns within wealth brackets, an important metric that cannot be observed using current surveys.

VI.F Time Persistence of Household Wealth Returns

We now study the time persistence of household wealth returns. This investigation is motivated by the intuition that the cross-sectional heterogeneity of household wealth returns reported in the main text and this online Appendix has a strong impact on inequality dynamics because differences in household annual returns tend to persist over time.

VI.F.1 Permanent Effects

We consider historical household wealth returns, $r_{h,t}$, over the 2001 to 2008 period, as defined in the main text. We measure the explanatory power of fixed effects as follows. First, we compute the adjusted coefficient of determination, R_t^2, of a panel regression with year fixed effects:

$$r_{h,t} = \alpha + \lambda_t + \varepsilon_{h,t}.$$

Second, we compute the adjusted coefficient of determination, $R_{h,t}^2$, of a panel regression with both household and year fixed effects:

$$r_{h,t} = \alpha + \lambda_t + \mu_h + \varepsilon_{h,t}.$$

The marginal explanatory power of household fixed effects is then given by

$$\rho = R_{h,t}^2 - R_t^2, \tag{A-7}$$

as in Fagereng et al. (2019). This simple method permits the use of historical returns. In Section VIII, we develop a more advanced model of return persistence, which allows us to disentangle the respective roles of risk loadings, factor returns, and underdiversification.

In Appendix Table 29, we report (1) the share explained by time fixed effects, R_t^2, (2) the share explained by household and year fixed effects, $R_{h,t}^2$, (3) the incremental share explained by household fixed effects, ρ, and (4) the sample standard deviation for a large number of return outcomes. The rows report the results for a variety of return measures, which provide useful information on the possible sources of return persistence.

Baseline Results. In the first two rows of Appendix Table 29, we consider the historical excess return on gross and net wealth, respectively, measured using the baseline approach outlined in the main text. Year fixed effects account for 44% of the variance of household wealth returns, which shows that household returns on gross wealth vary substantially over time. This finding is consistent with the joint exposure of household portfolios to general market conditions.

Permanent household characteristics only have a weak impact and account for only 5% of the cross-sectional variance of gross wealth returns.

Permanent household characteristics have a stronger impact on net wealth returns. Year fixed effects account for 29% of the return variance. The marginal explanatory power of household fixed effects reaches 12%, which is higher than the 5% estimate obtained for gross wealth. We attribute these results to heterogeneity in leverage, which creates higher dispersion of household returns conditional on overall market returns and thereby weakens the role of year effects. Because interest spreads on household debt are very persistent, household fixed effects have higher explanatory power.

Determinants of the Persistence of Wealth Returns. The other rows of the table provide other clues on the drivers of persistence in household wealth returns. Expected wealth returns are strongly explained by household fixed effects, which are driven by the loadings of wealth on priced factors. The marginal explanatory power of household fixed effects is 75% for gross wealth and 51% for net wealth, which emphasizes the importance of factor loadings.

By contrast, investment skill, as measured by the difference between the historical portfolio return and the return predicted by our asset pricing model given factor returns, is marginally explained by household fixed effects. This result implies that in the long run, annual differences in risk-adjusted performance across households will cancel out. Tax rates are highly persistent. Household fixed effects explain about 40% of tax rates, whether one considers gross wealth or net wealth.

Overall, the simple panel regressions of household historical returns considered in this section reveal that persistence in household risk exposures is the main driver of persistence in household wealth returns. We further investigate this important channel in Section VIII.

VI.F.2 Transitory Effects

The fixed effects approach introduced in Section *VI.F.1* captures the effect of constant characteristics on households wealth returns. However, there may be additional persistence if there is substantial serial correlation in the time-varying component of returns. We assess the presence of serial correlation by implementing the following test suggested by Wooldridge (2002).

Consider the following data-generating process for household returns:

$$r_{h,t} = \lambda_t + \mu_h + \varepsilon_{h,t}. \tag{A-8}$$

We wish to test the null hypothesis that the idiosyncratic components $\varepsilon_{h,t}$ are serially uncorrelated. Take first differences of equation (A-8):

$$\Delta r_{h,t} = a_t + \Delta\varepsilon_{h,t}.$$

Under the null, the correlation between $\Delta\varepsilon_{h,t}$ and $\Delta\varepsilon_{h,t}$ satisfies

$$Corr(\Delta\varepsilon_{h,t}, \Delta\varepsilon_{h,t-1}) = -0.5.$$

The coefficient of a regression of $\Delta r_{h,t}$ on $\Delta r_{h,t-1}$ with time fixed-effects should not be significantly different from -0.5.

We assess significance as follows.[19]

1. For each year t, run the OLS regression of $\Delta r_{h,t}$ on $\Delta r_{h,t-1}$, which produces the linear coefficient $\widehat{\theta_t}$.
2. Compute the sample mean $\overline{\widehat{\theta_t}}$ and sample standard deviation $\widehat{\sigma_{\widehat{\theta}}}$ of the linear coefficients $\{\widehat{\theta_t}; t = 1 \dots T\}$.
3. Compute the statistic $\sqrt{T}(\widehat{\theta_t} + 0.5)/\widehat{\sigma_{\widehat{\theta}}}$, which has a Student distribution with $T-1$ degrees

[19]In the absence of substantial within-year correlation in returns, the t-test associated with the coefficient using household-level clustering would be appropriate. However, a panel of household wealth returns contains substantial within-year correlation, due to the interaction of market-wide returns with heterogeneous household risk exposures.

of freedom under the null.

The third step is based on the fact that each $\widehat{\theta_t}$ is estimated on large number of households observed at date t and is therefore approximately normal.

In Appendix Table 30, we display estimates of $\overline{\widehat{\theta_t}}$ and $\widehat{\sigma_{\widehat{\theta}}}$ for the returns on gross and net wealth corresponding to various brackets of net worth. The linear coefficient ranges between -0.39 and -0.52 and never significantly differs from -0.5. The table shows that there is very little serial autocorrelation in returns left once household fixed effects are taken into account.

VI.G Identifying Scale Dependence

In Table 9 of the main text, we use twin regressions to disentangle scale- and type-dependence in wealth returns. The twin regressions address the concern that unobserved heterogeneity drives the relationship between wealth and returns. However, twin regressions do not control for reverse causality, that is the possibility that returns drive wealth. Furthermore, twin regressions rely on substantial identification assumptions. We now address these issues.

VI.G.1 Reverse Causality

In Appendix Table 31, we show how the expected return of household h in years t, $t+1$, $t+2$, and $t+4$ depends on its rank in the wealth distribution at the end of year t. The rank at t has a long-lasting effect on expected returns. For gross wealth, the difference in expected returns between the top 0.01% and the median bracket is 4.4% in year t, 4.4% in year $t+1$, 4.5% in year $t+2$, and 4.2% in year $t+4$. The results are similar for net wealth: the difference between the top 0.01% and the median bracket is 3.8% in year t, 4.6% in year $t+1$, 4.4% in year $t+2$, and 3.8% in year $t+4$. These findings suggest that reverse causality, i.e. high expected returns causing a subsequent increase in net worth, is not a major concern in our setting.

VI.G.2 Robustness of Twin Regressions

In Appendix Table 32, we verify the robustness of the twin regressions reported in Table 9 of the main text. As in any twin study, the main identification condition is that there is no residual unobserved heterogeneity between twins that correlates with both the regressor and the regressand, that is household wealth and expected return in our case. We test this assumption by considering three types of tests.

Twins of Same Gender. In columns 1 and 2 of Appendix Table 32, we check whether the twin fixed-effect estimates change if we only consider twins of the same gender, instead of considering all twin pairs as we do in the main text. Monozygotic twins represent 30.5% of all twins in Sweden and same-gender twins represent 64.5% of all twins (Calvet and Sodini 2014), so by Bayes' rule the likelihood of dealing with identical twins is 47% in a sample of same-gender twins. When we regress expected returns on wealth and twin pair-year fixed effects in this restricted sample, the point estimates of the wealth coefficients are very similar to our baseline results (columns 5 and 6 of Table 9 in the main text) and the adjusted R^2 barely increases (it is equal to 0.55 for all twins and 0.57 for twins of the same gender). Therefore, unobserved genetic heterogeneity is unlikely to drive our results.

Heterogeneity Between Twin Siblings. A second test of the identification condition is to consider individual variables potentially influencing the asset allocation of each twin and test if a) such variables affect asset allocation even after the inclusion of twin pair-year fixed effects, and b) the coefficients on wealth effects differ significantly once we include such variables as controls. This test is inspired by Sandewall, Cesarini, and Johannesson (2014), who propose an analogous procedure in the context of schooling regressions. We consider two sources of individual variation, IQ and an elicited measure of risk aversion.

IQ is known from earlier studies to have a strong correlation with financial sophistication (Grinblatt, Keloharju, and Linnainmaa 2011). We focus on the sample of twins for whom such a measure is available. We report the OLS regression of the expected gross wealth return on own wealth rank and twin pair-year fixed effects (column 3 of Appendix Table 32) and on the combination of these explanatory variables with own IQ (column 4). The IQ measure does not significantly impact expected returns in the presence of twin pair-year fixed effects (t-stat = 1.41). In addition, the IQ measure does not influence the wealth rank coefficients, as a close comparison of columns 3 and 4 reveals.

Similarly, in columns 5 and 6 of Appendix Table 32, we consider a measure of risk aversion elicited through a survey (Karolinska Institutet 2002).[20] The risk aversion measure has a slightly significant effect on the expected gross wealth return even when twin pair-year fixed effects are included (t-stat = 1.85). However, including such a variable as control does not affect the size of the wealth coefficients in any detectable way. These findings demonstrate that twin pair-year fixed effects have the ability to pick up variation in important individual variables that co-vary with individual wealth and expected wealth returns. Unexplained heterogeneity between twins is therefore unlikely to explain our results.

Measuring Wealth at Household vs. Individual Level. Throughout the main text and the online Appendix, we investigate wealth and returns at the household level, consistent with financial theory and most of the empirical household finance literature (see, e.g., Campbell 2006; Guiso, Haliassos, and Jappelli 2002). However, analyzing finances at the household level may result in a loss of accuracy when studying twins. In columns 9 and 10 of Appendix Table 32, we rerun our baseline regression using individual wealth to measure the rank in the distribution and individual asset allocation to measure expected returns. We find a very strong relationship between wealth and return under this alternative specification, suggesting that our methodological choice to consider household wealth rather than individual wealth has no impact on our results.[21]

[20] One imperfection of this variable is that unlike IQ, risk aversion is measured in 2000, i.e. almost contemporaneously to our wealth measurement.

[21] The only apparent discrepancy is the low coefficient on the top 0.01% dummy for expected net wealth returns, which stems from large estimation error (the standard error is 3.8 percentage points).

VII Estimating the Moments of Individual Effects

This Section provides background material on the estimation of the moments of error components models. We consider balanced panels in Section *VII.A* and unbalanced panels in Section *VII.B*.

VII.A Balanced Panel

Consider the two-way random effect model:

$$y_{h,t} = \lambda_t + \mu_h + u_{h,t}, \tag{A-9}$$

where $h = 1, ..., H$ and $t = 1, ..., T$. The time effects λ_t are independent and identically distributed (i.i.d.) with zero population mean, $\mathbb{E}(\lambda_t) = 0$, and variance $Var(\lambda_t) = \sigma^2_\lambda$. The individual effects μ_h are i.i.d. with variance $\sigma^2_\mu = Var(\mu_h)$. The stochastic disturbance $u_{h,t}$ are i.i.d with zero population mean, $\mathbb{E}(u_{h,t}) = 0$, and variance $\sigma^2_u = Var(u_{h,t})$. The random variables λ_t, μ_h, and $u_{h,t}$ are mutually independent and are independent across h and t.

Consistent with the literature (e.g., Eisenhart 1947), the estimator of each individual effect, μ_h, is the time-series average:

$$\hat{\mu}_h = \frac{1}{T}\sum_{t=1}^{T} y_{h,t}. \tag{A-10}$$

The estimator of each time effect λ_t is the period's cross-sectional mean deviation of individual observations from estimated individual effects:

$$\hat{\lambda}_t = \frac{1}{H}\sum_{h=1}^{H}(y_{h,t} - \hat{\mu}_h). \tag{A-11}$$

Let

$$\bar{\hat{\mu}} = \frac{1}{H}\sum_{h=1}^{H}\hat{\mu}_h \qquad \text{and} \qquad \overline{(\hat{\mu})^2} = \frac{1}{H}\sum_{h=1}^{H}(\hat{\mu}_h)^2.$$

Proposition 1 *The random variables*

$$\begin{aligned}
\widehat{\sigma_u^2} &= \frac{1}{(H-1)(T-1)}\sum_{h=1}^{H}\sum_{t=1}^{T}(y_{h,t}-\hat{\mu}_h-\hat{\lambda}_t)^2, \\
\widehat{\sigma_\lambda^2} &= \frac{1}{T-1}\sum_{t=1}^{T}(\hat{\lambda}_t)^2-\frac{1}{H}\widehat{\sigma_u^2}, \\
\widehat{\sigma_\mu^2} &= \frac{1}{H-1}\sum_{h=1}^{H}\left(\hat{\mu}_h-\bar{\hat{\mu}}\right)^2-\frac{\widehat{\sigma_u^2}}{T}
\end{aligned}$$

are unbiased estimators of σ_u^2, σ_λ^2, *and* σ_μ^2, *respectively. Furthermore,*

$$\hat{M}_2=\overline{(\hat{\mu})^2}-\frac{1}{T}(\widehat{\sigma_\lambda^2}+\widehat{\sigma_u^2})$$

is a consistent and unbiased estimator of $\mathbb{E}(\mu_h^2)$.

Proof. The results are established in Eisenhart (1947). For completeness, we provide here a direct proof. It follows from equations (A-9) to (A-11) that

$$y_{h,t}-\hat{\mu}_h-\hat{\lambda}_t=v_{h,t}-\frac{1}{T}\sum_{s=1}^{T}v_{h,s}, \tag{A-12}$$

where

$$v_{h,t}=u_{h,t}-\frac{1}{H}\sum_{k=1}^{H}u_{k,t}.$$

Since the random variables $v_{h,t}$ are uncorrelated across t, we infer from (A-12) that

$$\mathbb{E}\left[(y_{h,t}-\hat{\mu}_h-\hat{\lambda}_t)^2\right]=(1-T^{-1})Var(v_{h,t})=(1-T^{-1})(1-H^{-1})\sigma_u^2.$$

Hence, $\widehat{\sigma_u^2}$ is an unbiased estimator of σ_u^2.

Each time effect estimator can be written as

$$\hat{\lambda}_t=\lambda_t-\frac{1}{T}\sum_{s=1}^{T}\lambda_s+\frac{1}{H}\sum_{h=1}^{H}u_{h,t}-\frac{1}{T}\sum_{s=1}^{T}\left(\frac{1}{H}\sum_{h=1}^{H}u_{h,s}\right).$$

Hence $Var(\hat{\lambda}_t)=(1-T^{-1})\left(\sigma_\lambda^2+H^{-1}\sigma_u^2\right)$, and $\widehat{\sigma_\lambda^2}$ is an unbiased estimator of σ_λ^2.

The estimator of each individual effect satisfies

$$\hat{\mu}_h^2=\mu_h^2+\frac{2\mu_h}{T}\sum_{t=1}^{T}(\lambda_t+u_{h,t})+\frac{1}{T^2}\left[\sum_{t=1}^{T}(\lambda_t+u_{h,t})\right]^2,$$

which implies

$$\mathbb{E}(\hat{\mu}_h^2)=\mathbb{E}(\mu_h^2)+\frac{1}{T}(\sigma_\lambda^2+\sigma_u^2).$$

Hence, $\hat{M}_2$ is an unbiased estimator of $\mathbb{E}(\mu_h^2)$.

It is useful to note that

$$\hat{\mu}_h = g_h + \bar{\lambda}, \tag{A-13}$$

where $\bar{\lambda} = T^{-1}\sum_{t=1}^{T}\lambda_t$ and

$$g_h = \mu_h + \frac{1}{T}\sum_{t=1}^{T} u_{h,t}. \tag{A-14}$$

We infer that

$$\hat{\mu}_h - \bar{\hat{\mu}} = g_h - H^{-1}\sum_{k=1}^{H} g_k \tag{A-15}$$

and therefore

$$\mathbb{E}[(\hat{\mu}_h - \bar{\hat{\mu}})^2] = \left(1 - \frac{1}{H}\right) Var(g_h) = \left(1 - \frac{1}{H}\right)\left(\sigma_\mu^2 + \frac{1}{T}\sigma_u^2\right).$$

Hence $\widehat{\sigma_\mu^2}$ is an unbiased estimator of σ_μ^2. ■

The next step is to estimate the variance of μ_h^2. We make the additional assumption that $u_{h,t}$ has a Gaussian distribution.

Proposition 2 *The sample variance of squared individual estimates,*

$$\widehat{V}_1 = \frac{1}{H-1}\sum_{h=1}^{H}\left[(\hat{\mu}_h)^2 - \overline{(\hat{\mu})^2}\right]^2,$$

satisfies

$$\mathbb{E}(\widehat{V}_1) = Var(\mu_h^2) + \frac{4}{T}\left[\sigma_u^2\,\mathbb{E}(\mu_h^2) + \sigma_\lambda^2\sigma_\mu^2\right] + \frac{2\sigma_u^2}{T^2}(\sigma_u^2 + 2\sigma_\lambda^2).$$

Proof. By (A-13), the estimator of each individual effect satisfies

$$(\hat{\mu}_h)^2 = g_h^2 + 2\bar{\lambda}g_h + \bar{\lambda}^2, \tag{A-16}$$

and therefore

$$\overline{(\hat{\mu})^2} = \frac{1}{H}\sum_{k=1}^{H} g_k^2 + 2\bar{\lambda}\left(\frac{1}{H}\sum_{k=1}^{H} g_k\right) + \bar{\lambda}^2.$$

We infer that

$$(\hat{\mu}_h)^2 - \overline{(\hat{\mu}_h)^2} = g_h^2 - \frac{1}{H}\sum_{k=1}^{H} g_k^2 + 2\bar{\lambda}\left(g_h - \frac{1}{H}\sum_{k=1}^{H} g_k\right).$$

Hence

$$\begin{aligned}
\mathbb{E}\left\{\left[(\hat{\mu}_h)^2 - \overline{(\hat{\mu}_h)^2}\right]^2\right\} &= \mathbb{E}\left[\left(g_h^2 - \frac{1}{H}\sum_{k=1}^{H} g_k^2\right)^2\right] + 4Var(\bar{\lambda})\,\mathbb{E}\left[\left(g_h - \frac{1}{H}\sum_{k=1}^{H} g_k\right)^2\right] \\
&= \left(1 - H^{-1}\right) Var(g_h^2) + 4Var(\bar{\lambda})\left(1 - H^{-1}\right) Var(g_h).
\end{aligned}$$

The sample variance of $(\hat{\mu}_h)^2$ therefore satisfies

$$\mathbb{E}(\widehat{V}_1) = Var(g_h^2) + \frac{4\sigma_\lambda^2}{T} Var(g_h). \tag{A-17}$$

Equation (A-14) implies

$$g_h^2 = \mu_h^2 + \frac{2\mu_h}{T}\sum_{t=1}^{T} u_{h,t} + \frac{1}{T^2}\left(\sum_{t=1}^{T} u_{h,t}\right)^2.$$

The variance of g_h^2 can therefore be decomposed as follows:

$$\begin{aligned} Var(g_h^2) \quad = \quad & Var(\mu_h^2) + Var\left(\frac{2\mu_h}{T}\sum_{t=1}^{T} u_{h,t}\right) + Var\left[\frac{1}{T^2}\left(\sum_{t=1}^{T} u_{h,t}\right)^2\right] \\ & +2Cov\left[\mu_h^2; \frac{2\mu_h}{T}\sum_{t=1}^{T} u_{h,t} + \frac{1}{T^2}\left(\sum_{t=1}^{T} u_{h,t}\right)^2\right] \\ & +2Cov\left[\frac{2\mu_h}{T}\sum_{t=1}^{T} u_{h,t}; \frac{1}{T^2}\left(\sum_{t=1}^{T} u_{h,t}\right)^2\right]. \end{aligned} \tag{A-18}$$

We now compute separately the four terms on the right-hand side involving the error terms $u_{h,t}$..

1. We note that
$$Var\left(\frac{2\mu_h}{T}\sum_{t=1}^{T} u_{h,t}\right) = \mathbb{E}\left[\frac{4\mu_h^2}{T^2}\left(\sum_{t=1}^{T} u_{h,t}\right)^2\right] = \frac{4\mathbb{E}(\mu_h^2)}{T^2} Var\left(\sum_{t=1}^{T} u_{h,t}\right)$$
and therefore
$$Var\left(\frac{2\mu_h}{T}\sum_{t=1}^{T} u_{h,t}\right) = \frac{4\sigma_u^2}{T}\mathbb{E}(\mu_h^2).$$

2. Since $\sum_{t=1}^{T} u_{h,t}$ is $\mathcal{N}(0, \sigma_u^2 T)$, the random variable $Z = (\sigma_u\sqrt{T})^{-1}\sum_{t=1}^{T} u_{h,t}$ is $\mathcal{N}(0,1)$ and therefore
$$Var\left[\frac{1}{T^2}\left(\sum_{t=1}^{T} u_{h,t}\right)^2\right] = \frac{1}{T^4} Var(\sigma_u^2 T Z^2) = \frac{\sigma_u^4}{T^2} Var(Z^2) = \frac{2\sigma_u^4}{T^2}.$$

3. We note that
$$Cov\left[\mu_h^2; \frac{2\mu_h}{T}\sum_{t=1}^{T} u_{h,t} + \frac{1}{T^2}\left(\sum_{t=1}^{T} u_{h,t}\right)^2\right] = \mathbb{E}\left(\mu_h^2 \frac{2\mu_h}{T}\sum_{t=1}^{T} u_{h,t}\right) = 0.$$

4. Since the random variable $u_{h,t}\left(\sum_{s=1}^{T} u_{h,s}\right)^2$ is symmetric around 0, we have
$$Cov\left[\frac{2\mu_h}{T}\sum_{t=1}^{T} u_{h,t}; \frac{1}{T^2}\left(\sum_{t=1}^{T} u_{h,t}\right)^2\right] = \frac{2\mathbb{E}(\mu_h)}{T^3}\sum_{t=1}^{T}\mathbb{E}\left[u_{h,t}\left(\sum_{s=1}^{T} u_{h,s}\right)^2\right] = 0.$$

We plug these results into (A-18) and obtain:

$$Var(g_h^2) = Var(\mu_h^2) + \frac{4\sigma_u^2}{T}\mathbb{E}(\mu_h^2) + \frac{2\sigma_u^4}{T^2}.$$

We conclude from (A-17) that the Proposition holds. ■

The Proposition implies that

$$Var(\mu_h^2) = \mathbb{E}(\widehat{V}_1) - \frac{4}{T}\left[\sigma_u^2 \mathbb{E}(\mu_h^2) + \sigma_\lambda^2 \sigma_\mu^2\right] - \frac{2\sigma_u^2}{T^2}(\sigma_u^2 + 2\sigma_\lambda^2).$$

We estimate $Var(\mu_h^2)$ by

$$\widehat{V}_2 = \widehat{V}_1 - \frac{4}{T}\left(\hat{M}_2 \widehat{\sigma_u^2} + \widehat{\sigma_\lambda^2}\,\widehat{\sigma_\mu^2}\right) - \frac{2}{T^2}\,\widehat{\sigma_u^2}\left(\widehat{\sigma_u^2} + 2\widehat{\sigma_\lambda^2}\right). \tag{A-19}$$

We also show the following result.

Proposition 3 *The sample covariance of* $(\hat{\mu}_h)^2$ *and* $\hat{\mu}_h$ *satisfies*

$$\mathbb{E}\left\{\frac{1}{H-1}\sum_{h=1}^{H}(\hat{\mu}_h)^2\left[\hat{\mu}_h - \overline{(\hat{\mu})}\right]\right\} = Cov(\mu_h;\mu_h^2) + \frac{2}{T}\sigma_u^2\mathbb{E}(\mu_h).$$

Proof. We infer from (A-15) and (A-16) that

$$(\hat{\mu}_h)^2\left[\hat{\mu}_h - \overline{(\hat{\mu})}\right] = \left(g_h^2 + 2\bar{\lambda}g_h + \bar{\lambda}^2\right)\left(g_h - H^{-1}\sum_{k=1}^{H} g_k\right)$$

and therefore

$$\frac{1}{H-1}\sum_{h=1}^{H}(\hat{\mu}_h)^2\left[\hat{\mu}_h - \overline{(\hat{\mu})}\right] = \frac{1}{H-1}\sum_{h=1}^{H}\left(g_h^2 + 2\bar{\lambda}g_h\right)\left(g_h - H^{-1}\sum_{k=1}^{H} g_k\right).$$

Hence

$$\begin{aligned}
\mathbb{E}\left\{\frac{1}{H-1}\sum_{h=1}^{H}(\hat{\mu}_h)^2\left[\hat{\mu}_h - \overline{(\hat{\mu})}\right]\right\} &= \frac{1}{H-1}\sum_{h=1}^{H}\left[\left(1-\frac{1}{H}\right)\mathbb{E}(g_h^3) - \mathbb{E}(g_h^2)\sum_{k\neq h}\mathbb{E}(g_k)\right] \\
&= \mathbb{E}(g_h^3) - \mathbb{E}(g_h^2)\,\mathbb{E}(g_h).
\end{aligned}$$

Recall that $\mathbb{E}(g_h) = \mathbb{E}(\mu_h)$, $\mathbb{E}(g_h^2) = \mathbb{E}(\mu_h^2) + \sigma_u^2/T$, and let $\bar{u}_h = H^{-1}\sum_{h=1}^{H} u_{h,t}$. We note that

$$\begin{aligned}
\mathbb{E}\left(g_h^3\right) &= \mathbb{E}\left[\mu_h^3 + 3\mu_h^2\bar{u}_h + 3\mu_h(\bar{u}_h)^2 + (\bar{u}_h)^3\right] \\
&= \mathbb{E}(\mu_h^3) + 3\mathbb{E}(\mu_h)\frac{\sigma_u^2}{T} \\
&= Cov(\mu_h;\mu_h^2) + \mathbb{E}(\mu_h)\left[\mathbb{E}(\mu_h^2) + 3\frac{\sigma_u^2}{T}\right],
\end{aligned}$$

and therefore

$$\mathbb{E}\left\{\frac{1}{H-1}\sum_{h=1}^{H}(\hat{\mu}_h)^2\left[\hat{\mu}_h - \overline{(\hat{\mu})}\right]\right\} = Cov(\mu_h;\mu_h^2) + \mathbb{E}(\mu_h)\left[\mathbb{E}(\mu_h^2) + 3\frac{\sigma_u^2}{T}\right] - \mathbb{E}(\mu_h)\left[\mathbb{E}(\mu_h^2) + \frac{\sigma_u^2}{T}\right],$$

We conclude that the proposition holds. ∎

We therefore estimate $Cov(\mu_h;\mu_h^2)$ by

$$\frac{1}{H-1}\sum_{h=1}^{H}(\hat{\mu}_h)^2\left[\hat{\mu}_h-\overline{(\hat{\mu})}\right]-\frac{2}{T}\bar{\hat{\mu}}\widehat{\sigma_u^2}.$$

These results form the basis of the empirical application.

VII.B Unbalanced Panel

We now explain how to extend the previous results to the case of an unbalanced panel. Let T_h denote the number of periods when household h is observed, H_t the number of households observed at t, and $\mathcal{H}_t$ the set of individuals observed at t. We assume that $y_{h,t}$ is given by (A-9). The estimator of the individual effect is

$$\hat{\mu}_h=\frac{1}{T_h}\sum_{t\in\mathcal{T}_h}y_{h,t}=\mu_h+\frac{1}{T_h}\sum_{t\in\mathcal{T}_h}(\lambda_t+u_{h,t})$$

for every h. The estimator of the time effect is

$$\hat{\lambda}_t=\frac{1}{H_t}\sum_{k\in\mathcal{H}_t}(y_{kt}-\hat{\mu}_k) \tag{A-20}$$

for every period t.

Proposition 4 *The fixed effect estimators satisfy:*

$$\mathbb{E}\left[\frac{1}{T-1}\sum_{t=1}^{T}(\hat{\lambda}_t)^2\right]=A_\lambda\sigma_\lambda^2+A_u\sigma_u^2, \tag{A-21}$$

$$\mathbb{E}\left[\frac{1}{T-1}\sum_{t=1}^{T}\frac{1}{H_t-1}\sum_{h\in\mathcal{H}_t}(y_{h,t}-\hat{\lambda}_t-\hat{\mu}_h)^2\right]=B_\lambda\sigma_\lambda^2+B_u\sigma_u^2,$$

where

$$\begin{aligned}
A_\lambda &= \frac{1}{T-1}\sum_{t=1}^{T}\left[\left(1-\frac{1}{H_t}\sum_{k\in\mathcal{H}_t}\frac{1}{T_k}\right)^2+\frac{1}{H_t^2}\sum_{s\neq t}\left(\sum_{k\in\mathcal{H}_t\cap\mathcal{H}_s}\frac{1}{T_k}\right)^2\right],\\
A_u &= \frac{1}{T-1}\sum_{t=1}^{T}\frac{1}{H_t^2}\sum_{k\in\mathcal{H}_t}\left(1-\frac{1}{T_k}\right),\\
B_\lambda &= \frac{1}{T-1}\sum_{t=1}^{T}\frac{1}{H_t-1}\sum_{h\in\mathcal{H}_t}b_{\lambda,h,t},\\
B_u &= \frac{1}{T-1}\sum_{t=1}^{T}\left(1-\frac{1}{H_t}\sum_{h\in\mathcal{H}_t}\frac{1}{T_h}\right),
\end{aligned}$$

and

$$\begin{aligned}
b_{\lambda,h,t} &= 1-\frac{1}{T_h}+\left(1-\frac{1}{H_t}\sum_{k\in\mathcal{H}_t}\frac{1}{T_k}\right)^2+\frac{1}{H_t^2}\sum_{s\neq t}\left(\sum_{k\in\mathcal{H}_t\cap\mathcal{H}_s}\frac{1}{T_k}\right)^2\\
&\quad -2\left(1-\frac{1}{T_h}\right)\left(1-\frac{1}{H_t}\sum_{k\in\mathcal{H}_t}\frac{1}{T_k}\right)-\frac{2}{T_hH_t}\sum_{s\in\mathcal{T}_h\setminus\{t\}}\left(\sum_{k\in\mathcal{H}_t\cap\mathcal{H}_s}\frac{1}{T_k}\right)
\end{aligned}$$

for every h and t.

Proof. By (A-20), the estimator of each time effect satisfies

$$\begin{aligned}
\hat\lambda_t &= \frac{1}{H_t}\sum_{k\in\mathcal{H}_t}\left[\lambda_t+\mu_k+u_{k,t}-\mu_k-\frac{1}{T_k}\sum_{s\in\mathcal{T}_k}(\lambda_s+u_{k,s})\right]\\
&= \lambda_t-\frac{1}{H_t}\sum_{k\in\mathcal{H}_t}\frac{1}{T_k}\sum_{s\in\mathcal{T}_k}\lambda_s+\frac{1}{H_t}\sum_{k\in\mathcal{H}_t}\left(u_{k,t}-\frac{1}{T_k}\sum_{s\in\mathcal{T}_k}u_{k,s}\right),
\end{aligned}$$

and therefore

$$\hat\lambda_t = \lambda_t\left(1-\frac{1}{H_t}\sum_{k\in\mathcal{H}_t}\frac{1}{T_k}\right)-\frac{1}{H_t}\sum_{\substack{1\le s\le T\\ s\neq t}}\lambda_s\sum_{k\in\mathcal{H}_t\cap\mathcal{H}_s}\frac{1}{T_k}+\frac{1}{H_t}\sum_{k\in\mathcal{H}_t}\left(u_{kt}-\frac{1}{T_k}\sum_{s\in\mathcal{T}_k}u_{ks}\right).$$

We infer that

$$Var(\hat\lambda_t) = \left[\left(1-\frac{1}{H_t}\sum_{k\in\mathcal{H}_t}\frac{1}{T_k}\right)^2+\frac{1}{H_t^2}\sum_{s\neq t}\left(\sum_{k\in\mathcal{H}_t\cap\mathcal{H}_s}\frac{1}{T_k}\right)^2\right]\sigma_\lambda^2+\frac{1}{H_t^2}\sum_{k\in\mathcal{H}_t}\left(1-\frac{1}{T_k}\right)\sigma_u^2 \qquad \text{(A-22)}$$

for every t. Hence equation (A-21) holds.

We note that

$$\mathbb{E}\left[(y_{h,t}-\hat\lambda_t-\hat\mu_h)^2\right]=Var(y_{ht}-\hat\mu_h)+Var(\hat\lambda_t)-2Cov(y_{ht}-\hat\mu_h;\hat\lambda_t).$$

We compute separately each of these three terms.

1. We note that
$$y_{h,t}-\hat{\mu}_h=\lambda_t+u_{h,t}-\frac{1}{T_h}\sum_{s\in\mathcal{T}_h}(\lambda_s+u_{h,s}).$$
Hence
$$Var(y_{h,t}-\hat{\mu}_h)=\left(1-\frac{1}{T_h}\right)(\sigma_\lambda^2+\sigma_u^2).$$

2. The term $Var(\hat{\lambda}_t)$ is given by (A-22).

3. We observe that
$$y_{h,t}-\hat{\mu}_h=\left(1-\frac{1}{T_h}\right)\lambda_t-\frac{1}{T_h}\sum_{\substack{s\in\mathcal{T}_h\\ s\neq t}}\lambda_s+u_{h,t}-\frac{1}{T_h}\sum_{s\in\mathcal{T}_h}u_{h,s}.$$
Hence $Cov(y_{hnt}-\hat{\mu}_h;\hat{\lambda}_t)$ is given by
$$\left[\left(1-\frac{1}{T_h}\right)\left(1-\frac{1}{H_t}\sum_{k\in\mathcal{H}_t}\frac{1}{T_k}\right)+\frac{1}{T_hH_t}\sum_{\substack{s\in\mathcal{T}_h\\ s\neq t}}\left(\sum_{k\in\mathcal{H}_t\cap\mathcal{H}_s}\frac{1}{T_k}\right)\right]\sigma_\lambda^2+\frac{1}{H_t}\left(1-\frac{1}{T_h}\right)\sigma_u^2.$$
for every h and t.

The mean squared error $\mathbb{E}\left[(y_{h,t}-\hat{\lambda}_t-\hat{\mu}_h)^2\right]$ is therefore equal to

$$\left(1-\frac{1}{T_h}\right)(\sigma_\lambda^2+\sigma_u^2)+\left[\left(1-\frac{1}{H_t}\sum_{k\in\mathcal{H}_t}\frac{1}{T_k}\right)^2+\frac{1}{H_t^2}\sum_{s\neq t}\left(\sum_{k\in\mathcal{H}_t\cap\mathcal{H}_s}\frac{1}{T_k}\right)^2\right]\sigma_\lambda^2+\frac{1}{H_t^2}\sum_{k\in\mathcal{H}_t}\left(1-\frac{1}{T_k}\right)\sigma_u^2$$
$$-\frac{2}{H_t}\left(1-\frac{1}{T_h}\right)\sigma_u^2-2\left[\left(1-\frac{1}{T_h}\right)\left(1-\frac{1}{H_t}\sum_{k\in\mathcal{H}_t}\frac{1}{T_k}\right)+\frac{1}{T_hH_t}\sum_{\substack{s\in\mathcal{T}_h\\ s\neq t}}\left(\sum_{k\in\mathcal{H}_t\cap\mathcal{H}_s}\frac{1}{T_k}\right)\right]\sigma_\lambda^2,$$

or equivalently

$$\mathbb{E}\left[(y_{h,t}-\hat{\lambda}_t-\hat{\mu}_h)^2\right]=\left[\left(1-\frac{1}{T_h}\right)\left(1-\frac{1}{H_t}\right)+\frac{1}{H_t^2}\sum_{k\in\mathcal{H}_t}\left(\frac{1}{T_h}-\frac{1}{T_k}\right)\right]\sigma_u^2+b_{\lambda,h,t}\sigma_\lambda^2,$$

Hence
$$\mathbb{E}\left[\sum_{h\in\mathcal{H}_t}(y_{ht}-\hat{\lambda}_t-\hat{\mu}_h)^2\right]=\sum_{h\in\mathcal{H}_t}\left(1-\frac{1}{T_h}\right)\left(1-\frac{1}{H_t}\right)\sigma_u^2+\sum_{h\in\mathcal{H}_t}b_{\lambda,h,t}\,\sigma_\lambda^2,$$
and we conclude that the proposition holds. ■

If the panel is balanced, the coefficients satisfy $A_\lambda=1$, $A_u=H^{-1}$, $B_\lambda=0$, $B_u=1$ and the results of the proposition are consistent with Proposition 1.

Similarly, we consider

$$\begin{aligned}
\check{\lambda}_t &= \frac{1}{H_t} \sum_{k \in \mathcal{H}_t} y_{k,t}, \\
\check{\mu}_h &= \frac{1}{T_h} \sum_{t \in \mathcal{T}_h} (y_{h,t} - \check{\lambda}_t).
\end{aligned}$$

Let H the total number of households in the panel. By a simple change of notation, the proposition implies that

$$\mathbb{E}\left[\frac{1}{H-1}\sum_h (\hat{\mu}_h)^2\right] = C_\mu \sigma_\mu^2 + C_u \sigma_u^2,$$

$$\mathbb{E}\left[\frac{1}{H-1}\sum_h \frac{1}{T_h-1}\sum_{t \in \mathcal{T}_h} (y_{h,t} - \check{\lambda}_t - \check{\mu}_h)^2\right] = D_\mu \sigma_\mu^2 + D_u \sigma_u^2,$$

where

$$\begin{aligned}
C_\mu &= \frac{1}{H-1}\sum_h \left[\left(1 - \frac{1}{T_h}\sum_{t \in \mathcal{T}_h} \frac{1}{H_t}\right)^2 + \frac{1}{T_h^2}\sum_{k \neq h}\left(\sum_{t \in \mathcal{T}_h \cap \mathcal{T}_k} \frac{1}{H_t}\right)^2\right], \\
C_u &= \frac{1}{H-1}\sum_h \frac{1}{T_h^2}\sum_{t \in \mathcal{T}_h}\left(1 - \frac{1}{H_t}\right), \\
D_\mu &= \frac{1}{H-1}\sum_h \frac{1}{T_h-1}\sum_{t \in \mathcal{T}_h} d_{\mu,h,t}, \\
D_u &= \frac{1}{H-1}\sum_h \left(1 - \frac{1}{T_h}\sum_{t \in \mathcal{T}_h} \frac{1}{H_t}\right),
\end{aligned}$$

and

$$\begin{aligned}
d_{\mu,h,t} &= 1 - \frac{1}{T_h} + \left(1 - \frac{1}{H_t}\sum_{k \in \mathcal{H}_t} \frac{1}{T_k}\right)^2 + \frac{1}{H_t^2}\sum_{s \neq t}\left(\sum_{k \in \mathcal{H}_t \cap \mathcal{H}_s} \frac{1}{T_k}\right)^2 \\
&\quad -2\left(1 - \frac{1}{T_h}\right)\left(1 - \frac{1}{H_t}\sum_{k \in \mathcal{H}_t} \frac{1}{T_k}\right) - \frac{2}{T_h H_t}\sum_{s \in \mathcal{T}_h \setminus \{t\}}\left(\sum_{k \in \mathcal{H}_t \cap \mathcal{H}_s} \frac{1}{T_k}\right)
\end{aligned}$$

for every h and t.

We can estimate σ_λ^2 and σ_u^2 by the solution to the system of equations:

$$A_\lambda \widehat{\sigma_\lambda^2} + A_u \widehat{\sigma_u^2} = \frac{1}{T-1}\sum_{t=1}^{T}(\hat{\lambda}_t)^2 \tag{A-23}$$

$$B_\lambda \widehat{\sigma_\lambda^2} + B_u \widehat{\sigma_u^2} = \frac{1}{T-1}\sum_{t=1}^{T}\frac{1}{H_t-1}\sum_{h\in\mathcal{H}_t}(y_{h,t}-\hat{\lambda}_t-\hat{\mu}_h)^2. \tag{A-24}$$

The estimators $\widehat{\sigma_\lambda^2}$ and $\widehat{\sigma_u^2}$ are unbiased, as Proposition 4 implies. Furthermore, the solution to

$$C_\mu \widehat{\sigma_\mu^2} + C_u \widehat{\sigma_u^2} = \frac{1}{H-1}\sum_{h}(\hat{\mu}_h)^2 \tag{A-25}$$

is an unbiased estimator of σ_μ^2.

For computational convenience, we can simplify the estimation method when the number of households observed each year is very large, which is the case for an administrative dataset of Swedish residents. When $\min_{1\leq t\leq T} H_t \to \infty$, the variance of $\hat{\lambda}_t$ in equation (A-22) only depends on σ_λ^2, and the variance of the time effect is consistently estimated by

$$\widehat{\sigma_\lambda^2} = \frac{1}{A_\lambda(T-1)}\sum_{t=1}^{T}(\hat{\lambda}_t)^2.$$

Given $\widehat{\sigma_\lambda^2}$, the solution to (A-24), denoted by $\widehat{\sigma_u^2}$, provides a consistent estimator of σ_u^2. The estimation of σ_μ^2 is simplified by the observations that as $\min_{1\leq t\leq T} H_t \to \infty$, the coefficient C_μ converges to unity and the coefficient C_u is approximately equal to the harmonic mean of the household number of observations,

$$T^* = \left(\frac{1}{H}\sum_{h}\frac{1}{T_h}\right)^{-1}.$$

We infer from (A-25) that

$$\widehat{\sigma_\mu^2} = \frac{1}{H-1}\sum_{h}(\hat{\mu}_h)^2 - \frac{\widehat{\sigma_u^2}}{T^*} \tag{A-26}$$

consistently estimates the variance of the individual effect. For higher moments, we apply the estimation methodology developed in Propositions 2 and 3, where T is replaced with the effective number of time periods, T^*.

VIII Cross-Sectional Distribution of Average Returns over a Generation: Theory

In Section IV.D of the main text, we report measures of household return heterogeneity and provides calibration parameters for the theoretical model of Benhabib, Bisin, and Luo (2019). The present section develops the specification underlying these results, as well as the implied population moments.

VIII.A Specification

The model of Benhabib, Bisin, and Luo (2019) uses as a key ingredient the wealth return earned by generation G of dynasty h, which is denoted by r_h^G in annual units. An important feature of their approach is that r_h^G is drawn only once for each generation of a dynasty, whose investment life spans $T_g = 36$ years.

The empirical equivalent of r_h^G is the geometric average household return:

$$r_h^G = \left[\prod_{t=1}^{T_g}(1+r_{h,t})\right]^{\frac{1}{T_g}} - 1. \tag{A-27}$$

Benhabib, Bisin, and Luo (2019) calibrate their model using US data and find that the standard deviation of r_h^G should be 2.69% in annual units in order to fit the observed distribution of wealth. We cannot measure r_h^G directly because the effective duration of a household is typically much shorter than 36 years and the Swedish panel provides a household's wealth return for up to eight years. For these reasons, we develop a model of excess returns that allows us to estimate the mean and standard deviation of a generation's return over the investment horizon T_g using a shorter household portfolio panel of length T, where $T \leq T_g$. The framework controls for heterogeneity in factor loadings and heterogeneity in risk-adjusted returns.

We consider a panel of dynasties indexed by h observed at dates $t = 1, ..., T$, where $T \leq T_g$. Let $r_{h,t}$ denote the wealth return earned by dynasty h in year t. Throughout this section, we consider both cross-sectional means and time averages. For every panel of random variables $x_{h,t}$, we denote by $\mathbb{E}^*(x_{h,t})$ the cross-sectional average of $x_{h,t}$ at t, and by $\mathbb{E}(x_{h,t})$ the time-series unconditional average of $x_{h,t}$. In particular, $\mathbb{E}[\mathbb{E}^*(x_{h,t})]$ denotes the time-series average of the cross-sectional mean of $x_{h,t}$. We use similar notation for the variance and covariance operators. For every h, we denote by $\mathbb{E}_h(x_{h,t})$ the expectation of $x_{h,t}$ under the stationary distribution of $\{x_{h,t}\}_t$.

In order to distinguish between long-run and transitory effects in portfolio composition, we specify the vector of factor loadings of dynasty h at date t by

$$\beta_{h,t} = \beta_h + \gamma_t + \delta_{h,t}, \tag{A-28}$$

where β_h is the long-run level of factor loadings, γ_t is a vector of time effects, and $\delta_{h,t}$ is an idiosyncratic term. We assume that the vectors β_h, γ_t and $\delta_{h,t}$ are mutually independent and are serially independent across h and t. In addition, we assume that

$$\mathbb{E}^*(\beta_h) = \beta_0 \quad \text{and} \quad \mathbb{E}(\gamma_t) = 0.$$

The idiosyncratic yearly component, $\delta_{h,t}$, has mean zero for every h and t.

The return of dynasty h at date t is given by

$$r_{h,t} = \beta'_{h,t-1} f_t + \varepsilon_{h,t}. \tag{A-29}$$

Consistent with the empirical evidence, we assume that the risk-adjusted return, $\varepsilon_{h,t}$, has mean zero for every h and t. Furthermore, the risk-adjusted return $\varepsilon_{h,t}$ is serially uncorrelated:

$$Cov_h(\varepsilon_{h,t};\varepsilon_{h,s}) = 0 \tag{A-30}$$

for every h and $t \neq s$. We also make the simplifying assumption that the risk-adjusted return $\varepsilon_{h,t}$ is independent of the vector of factors f_t and the components of $\beta_{h,t-1}$ at all leads or lags.

We emphasize that our analysis does not require us to make any assumptions on the serial correlation of $\varepsilon^2_{h,t}$. We will in fact show that time-series persistence in portfolio underdiversification, as captured by $Cov_h(\varepsilon^2_{h,t};\varepsilon^2_{h,s})$, plays an important role in practice.

Under the chosen specification of factor loadings, the expected return of dynasty h in year t,

$$\mu^{\exp}_{h,t} = \beta'_{h,t-1}\mathbb{E}(f_t),$$

has a classic panel structure of the type considered in Section VII of this online Appendix. Specifically, the expected return of dynasty h at t satisfies

$$\mu^{\exp}_{h,t} = \lambda_t + \mu_h + u_{h,t}, \tag{A-31}$$

where μ_h is a household fixed effect:

$$\mu_h = \beta'_h\mathbb{E}(f_t), \tag{A-32}$$

λ_t is a time effect driven by average loadings at t across the population:

$$\lambda_t = \gamma'_{t-1}\mathbb{E}(f_t), \tag{A-33}$$

and the residual $u_{h,t}$ is driven by the yearly deviation of $\beta_{h,t-1} - \mathbb{E}^*(\beta_{h,t-1})$ from the long-run mean:

$$u_{h,t} = \delta'_{h,t-1}\mathbb{E}(f_t). \tag{A-34}$$

It follows from the chosen specification of factor loadings that the components λ_t, μ_h, and $u_{h,t}$ are mutually independent. We also assume that $u_{h,t}$ is i.i.d. Gaussian. The variances of λ_t, μ_h, and $u_{h,t}$ are denoted by σ^2_λ, σ^2_μ, and σ^2_u, respectively.

The return process is driven by both the expected value of the factors, $\mathbb{E}(f_t)$, and their unexpected realizations:

$$\tilde{f}_t = f_t - \mathbb{E}(f_t).$$

The return of dynasty h in year t, as defined in equation (A-29), can be decomposed as the sum of three components:

$$r_{h,t} = \mu_{h,t}^{exp} + \mu_{h,t}^{dev} + \varepsilon_{h,t},$$

where $\mu_{h,t}^{exp} = \beta_{h,t-1}' \mathbb{E}(f_t)$ is the expected return given the dynasty's loadings at the beginning of the year,

$$\mu_{h,t}^{dev} = \beta_{h,t-1}' \tilde{f}_t,$$

is the portfolio return due to deviations of factor returns from their long-term means, and $\varepsilon_{h,t} = r_{h,t} - \beta_{h,t-1}' f_t$ is the return due to portfolio underdiversification. All three components can be measured with good accuracy on portfolio holdings and return data.

Since expected returns follow the panel structure (A-29), the excess return of dynasty h at date t can therefore be written as

$$r_{h,t} = \lambda_t + \mu_h + u_{h,t} + \mu_{h,t}^{dev} + \varepsilon_{h,t}. \tag{A-35}$$

The household fixed effect μ_h captures the long-run impact of the factor loadings on expected returns. The time fixed effect λ_t quantifies the impact of deviations of average household loadings at year t from their long-run means. The terms $u_{h,t}$ and $\mu_{h,t}^{dev}$ capture the impact stemming from short-run individual deviations of loadings and asset returns, respectively, from their long-run means. The term $\varepsilon_{h,t}$ is the portfolio return due to underdiversification.

It is convenient to denote the sum of individual innovations at date t by

$$\eta_{h,t} = u_{h,t} + \mu_{h,t}^{dev} + \varepsilon_{h,t}. \tag{A-36}$$

The return process can then be written as

$$r_{h,t} = \lambda_t + \mu_h + \eta_{h,t}, \tag{A-37}$$

for every h and t.

VIII.B Population Moments

We now express the key population moments of the distribution of (unobserved) average returns over a generation as a function of the population moments of (observed) yearly returns.

Consider the average performance of a dynasty's portfolio over a generation. The arithmetic average return earned by dynasty h is:

$$\bar{r}_h = \frac{1}{T_g} \sum_{t=1}^{T_g} r_{h,t}, \tag{A-38}$$

where $T_g = 36$ years in the calculation of Benhabib, Bisin, and Luo. The properties of the average arithmetic return over a generation, $\bar{r}_h$, follow from the structure of yearly returns in equation (A-35).

Proposition 5 (Arithmetic Mean of Yearly Returns) *The first moment of the arithmetic mean return over T_g years is given by*

$$\mathbb{E}[\mathbb{E}^*(\bar{r}_h)] = \mathbb{E}^*(\mu_h) = \beta_0' \mathbb{E}(f_t). \tag{A-39}$$

The unconditional mean of the cross-sectional variance is:

$$\mathbb{E}\left[Var^*\left(\bar{r}_h\right)\right] = \sigma_\mu^2 + \frac{\sigma_u^2 + \sigma_{dev}^2 + \sigma_\varepsilon^2}{T_g}, \tag{A-40}$$

where $\sigma_{dev}^2 = \mathbb{E}[Var^*(\mu_{h,t}^{dev})] = \mathbb{E}\left\{\tilde{f}_t'\left[Var^*(\beta_h) + Var^*(\delta_{h,t-1})\right]\tilde{f}_t\right\}.$

Proof. Equations (A-35) and (A-37) imply that

$$\bar{r}_h = \mu_h + \bar{\lambda} + \frac{1}{T_g} \sum_{t=1}^{T_g} (u_{h,t} + \mu_{h,t}^{dev} + \varepsilon_{h,t}) = \mu_h + \bar{\lambda} + \frac{1}{T_g} \sum_{t=1}^{T_g} \eta_{h,t}, \tag{A-41}$$

where

$$\bar{\lambda} = \frac{1}{T_g} \sum_{t=1}^{T_g} \lambda_t. \tag{A-42}$$

The component $\mu_{h,t}^{dev}$ satisfies

$$\begin{aligned}
\mathbb{E}^*(\mu_{h,t}^{dev}) &= (\beta_0+\gamma_{t-1})'\tilde{f}_t, \\
Var^*(\mu_{h,t}^{dev}) &= \tilde{f}_t'\left[Var^*(\beta_h)+Var^*(\delta_{h,t-1})\right]\tilde{f}_t, \\
\mathbb{E}^*[(\mu_{h,t}^{dev})^2] &= [(\beta_0+\gamma_{t-1})'\tilde{f}_t]^2+\tilde{f}_t'\left[Var^*(\beta_h)+Var^*(\delta_{h,t-1})\right]\tilde{f}_t.
\end{aligned}$$

The covariances of individual yearly innovations are given by

$$\begin{aligned}
Cov^*(\mu_h,u_{h,t}) &= 0, \\
Cov^*(\mu_h,\mu_{h,t}^{dev}) &= \mathbb{E}(f_t)'Var^*(\beta_h)\,\tilde{f}_t, \\
Cov^*(\mu_h,\varepsilon_{h,t}) &= 0, \\
Cov^*(u_{h,t};\mu_{h,t}^{dev}) &= \mathbb{E}(f_t)'Var^*(\delta_{h,t-1})\,\tilde{f}_t, \\
Cov^*(u_{h,t},\varepsilon_{h,t}) &= 0, \\
Cov^*(\mu_{h,t}^{dev},\varepsilon_{h,t}) &= 0.
\end{aligned}$$

The sum of yearly individual innovations therefore satisfy

$$\begin{aligned}
\mathbb{E}^*(\eta_{h,t}) &= (\beta_0+\gamma_{t-1})'\tilde{f}_t, \\
Var^*(\eta_{h,t}) &= \sigma_u^2+Var^*(\mu_{h,t}^{dev})+\sigma_\varepsilon^2+2\mathbb{E}(f_t)'Var^*(\delta_{h,t-1})\,\tilde{f}_t, \\
\mathbb{E}[Var^*(\eta_{h,t})] &= \sigma_u^2+\sigma_{dev}^2+\sigma_\varepsilon^2,
\end{aligned}$$

for every h and t.

The cross-sectional mean return is

$$\mathbb{E}^*(\bar{r}_h)=\mathbb{E}^*(\mu_h)+\bar{\lambda}+\frac{1}{T_g}\sum_{t=1}^{T_g}(\beta_0+\gamma_{t-1})'\tilde{f}_t. \tag{A-43}$$

The cross-sectional variance of the mean return is

$$\begin{aligned}
Var^*(\bar{r}_h) &= \sigma_\mu^2+\frac{1}{T_g^2}\sum_{t=1}^{T_g}Var^*(\eta_{h,t})+\frac{2}{T_g}\sum_{t=1}^{T_g}Cov^*(\mu_{h,t};\eta_{h,t}) \\
&= \sigma_\mu^2+\frac{1}{T_g^2}\sum_{t=1}^{T_g}Var^*(\eta_{h,t})+\frac{2}{T_g}\sum_{t=1}^{T_g}\mathbb{E}(f_t)'Var^*(\beta_h)\,\tilde{f}_t.
\end{aligned}$$

The unconditional mean of $Var^*(\bar{r}_h)$ is therefore given by (A-40). ■

The variance is driven by (i) the cross-sectional variance of the long-run expected return, (ii) the variance of short-run deviations in individual loadings divided by T_g, and (iii) the variance of risk-adjusted returns divided by T_g.

Let $\overline{r_h^2}=T_g^{-1}\sum_{t=1}^{T_g}r_{h,t}^2$ denote the average squared return earned by a generation. The cross-sectional moments of $\overline{r_h^2}$ are provided by the following proposition.

Proposition 6 (Arithmetic Mean of Squared Yearly Returns) *The unconditional mean of the cross-sectional average squared return is given by:*

$$\mathbb{E}\left[\mathbb{E}^*\left(\overline{r_h^2}\right)\right] = \sigma_\lambda^2 + \mathbb{E}^*(\mu_h^2) + \mathbb{E}\left\{[\mathbb{E}^*(\mu_{h,t}^{dev})]^2\right\} + \sigma_u^2 + \sigma_{dev}^2 + \sigma_\varepsilon^2. \qquad \text{(A-44)}$$

The unconditional mean of the cross-sectional variance of the average squared return satisfies

$$\mathbb{E}\left[Var^*(\overline{r_h^2})\right] = \mathbb{E}\left[Var^*\left(\frac{1}{T_g}\sum_{t=1}^{T_g}\eta_{h,t}^2\right)\right] + Var^*(\mu_h^2) + 2\mathbb{E}[Cov^*(\mu_h^2;\eta_{h,t}^2)] + \frac{A}{T_g}, \qquad \text{(A-45)}$$

where

$$\begin{aligned} A \;=\;& 4\sigma_\lambda^2\sigma_\mu^2 + 4\mathbb{E}[\lambda_t^2\, Var^*(\eta_{h,t})] + 4\mathbb{E}^*(\mu_h^2)\,(\sigma_u^2+\sigma_\varepsilon^2) + 4\mathbb{E}[Var^*(\mu_h\mu_{h,t}^{dev})] \\ & +4\mathbb{E}\left[Cov^*\left(\eta_{h,t}^2;\mu_h\eta_{h,t}\right)\right] + 4\mathbb{E}\left[\lambda_t Cov^*\left(\eta_{h,t}^2;\eta_{h,t}\right)\right] + 8\mathbb{E}\left[\lambda_t Cov^*(\mu_h\eta_{h,t};\eta_{h,t})\right]. \end{aligned}$$

The unconditional cross-sectional covariance is

$$\mathbb{E}\left[Cov^*(\overline{r_h};\overline{r_h^2})\right] = Cov^*(\mu_h;\mu_h^2) + \mathbb{E}\left[Cov^*(\mu_h;\eta_{h,t}^2)\right] + \frac{B}{T_g}, \qquad \text{(A-46)}$$

where $B = \mathbb{E}[Cov^*(\eta_{h,t};\eta_{h,t}^2)] + 2\mathbb{E}[Cov^*(\eta_{h,t};\mu_h\eta_{h,t})] + 2\mathbb{E}[\lambda_t\, Var^*(\eta_{h,t})]$.

Proof. We begin by developing a few useful results. The cross-sectional second moment of $\eta_{h,t}^2$ is given by

$$\mathbb{E}^*(\eta_{h,t}^2) = [\mathbb{E}^*(\mu_{h,t}^{dev})]^2 + Var^*(\eta_{h,t}),$$

so that

$$\mathbb{E}\left[\mathbb{E}^*(\eta_{h,t}^2)\right] = \mathbb{E}\left\{[\mathbb{E}^*(\mu_{h,t}^{dev})]^2\right\} + \sigma_u^2 + \sigma_{dev}^2 + \sigma_\varepsilon^2. \qquad \text{(A-47)}$$

Since $\mu_h = \beta_h'\,\mathbb{E}(f_t)$ and $\eta_{h,t} = \beta_{h,t-1}'\,\tilde{f}_t + u_{h,t} + \varepsilon_{h,t}$, the cross-sectional mean of $\mu_{h,t}\eta_{h,t}$ is given by

$$\mathbb{E}^*(\mu_h\eta_{h,t}) = \mathbb{E}(f_t)'\,\mathbb{E}^*(\beta_h\beta_{h,t-1}')\tilde{f}_t,$$

and therefore

$$\mathbb{E}[\mathbb{E}^*(\mu_h\eta_{h,t})] = 0. \qquad \text{(A-48)}$$

for every t.

By (A-37), the squared portfolio return can be written as

$$r_{h,t}^2 \;=\; \lambda_t^2 + \mu_h^2 + \eta_{h,t}^2 + 2\lambda_t\,\mu_h + 2\mu_h\,\eta_{h,t} + 2\lambda_t\,\eta_{h,t},$$

and the average squared yearly return as

$$\overline{r_h^2} \quad = \quad \frac{1}{T_g}\sum_{t=1}^{T_g}\lambda_t^2+\mu_h^2+\frac{1}{T_g}\sum_{t=1}^{T_g}\eta_{h,t}^2+2\bar{\lambda}\mu_h+\frac{2}{T_g}\sum_{t=1}^{T_g}\mu_h\,\eta_{h,t}+\frac{2}{T_g}\sum_{t=1}^{T_g}\lambda_t\,\eta_{h,t} \tag{A-49}$$

for every dynasty h.

The cross-sectional mean of the average squared return is therefore given by

$$\mathbb{E}^*\left(\overline{r_h^2}\right)=\frac{1}{T_g}\sum_{t=1}^{T_g}\lambda_t^2+\mathbb{E}^*(\mu_h^2)+\mathbb{E}^*\left(\frac{1}{T_g}\sum_{t=1}^{T_g}\eta_{h,t}^2\right)+2\bar{\lambda}\mathbb{E}^*(\mu_h)+\frac{2}{T_g}\sum_{t=1}^{T_g}\mathbb{E}^*(\mu_h\,\eta_{h,t})+\frac{2}{T_g}\sum_{t=1}^{T_g}\lambda_t(\beta_0+\gamma_{t-1})'\tilde{f}_t.$$

By (A-47) and (A-48), the unconditional mean of $\mathbb{E}^*\left(\overline{r_h^2}\right)$ satisfies equation (A-44).

The specification of the average squared return in equation (A-49) implies that its cross-sectional variance, $Var^*(\overline{r_h^2})$, is equal to

$$\begin{aligned}
&Var^*(\mu_h^2)+Var^*\left(\tfrac{1}{T_g}\textstyle\sum_{t=1}^{T_g}\eta_{h,t}^2\right)+4(\bar{\lambda})^2Var^*(\mu_h)+\tfrac{4}{T_g^2}Var^*\left(\textstyle\sum_{t=1}^{T_g}\mu_h\,\eta_{h,t}\right)+\tfrac{4}{T_g^2}Var^*\left(\textstyle\sum_{t=1}^{T_g}\lambda_t\,\eta_{h,t}\right)\\
&+\tfrac{2}{T_g}\textstyle\sum_{t=1}^{T_g}Cov^*(\mu_h^2;\eta_{h,t}^2)+4\bar{\lambda}Cov^*[(\mu_h)^2,\mu_h]+\tfrac{4}{T_g}\textstyle\sum_{t=1}^{T_g}Cov^*(\mu_h^2;\mu_h\,\eta_{h,t})+\tfrac{4}{T_g}\textstyle\sum_{t=1}^{T_g}\lambda_t\,Cov^*(\mu_h^2;\eta_{h,t})\\
&+4\bar{\lambda}Cov^*\left(\tfrac{1}{T_g}\textstyle\sum_{t=1}^{T_g}\eta_{h,t}^2;\mu_h\right)+2Cov^*\left(\tfrac{1}{T_g}\textstyle\sum_{s=1}^{T_g}\eta_{h,s}^2;\tfrac{2}{T_g}\textstyle\sum_{t=1}^{T_g}\mu_h\,\eta_{h,t}\right)+\tfrac{4}{T_g}\textstyle\sum_{t=1}^{T_g}\lambda_tCov^*\left(\tfrac{1}{T_g}\textstyle\sum_{s=1}^{T_g}\eta_{h,s}^2;\eta_{h,t}\right)\\
&+8\bar{\lambda}Cov^*\left(\mu_h;\tfrac{1}{T_g}\textstyle\sum_{t=1}^{T_g}\mu_h\eta_{h,t}\right)+\tfrac{8\bar{\lambda}}{T_g}\textstyle\sum_{t=1}^{T_g}\lambda_t\,Cov^*(\mu_h,\eta_{h,t})+\tfrac{8}{T_g^2}\textstyle\sum_{t=1}^{T_g}\lambda_tCov^*(\textstyle\sum_{s=1}^{T_g}\mu_h\eta_{h,s};\eta_{h,t}).
\end{aligned}$$

We successively examine the unconditional expectation of each of these 15 terms.

1. The variance of the individual effect,

$$\mathbb{E}[Var^*(\mu_h^2)]=Var^*(\mu_h^2), \tag{A-50}$$

   can be easily estimated from Proposition 1 of this online Appendix.

2. The addend $\mathbb{E}[Cov^*(\eta_{h,t}^2;\eta_{h,s}^2)]$ is equal to a positive constant for all $t\neq s$. Hence

$$\mathbb{E}\left[Var^*\left(\frac{1}{T_g}\sum_{t=1}^{T_g}\eta_{h,t}^2\right)\right] \tag{A-51}$$

   converges to a positive constant as T_g goes to infinity.

3. We note that

$$\mathbb{E}[4\,(\bar{\lambda})^2Var^*(\mu_h)]=\frac{4}{T_g}\sigma_\lambda^2\,\sigma_\mu^2 \tag{A-52}$$

   for every h.

4. The cross-sectional variance of $\sum_{t=1}^{T_g}\mu_h\,\eta_{h,t}$ can be rewritten as

$$Var^*\left(\sum_{t=1}^{T_g}\mu_h\,\eta_{h,t}\right)=\sum_{t=1}^{T_g}Var^*(\mu_h\,\eta_{h,t})+\sum_{s\neq t}Cov^*(\mu_h\,\eta_{h,s};\mu_h\,\eta_{h,t}).$$

For every $s \neq t$, the cross-sectional covariance of $\mu_h \eta_{h,s}$ and $\mu_h \eta_{h,t}$ simplifies as follows:

$$\begin{aligned} Cov^*(\mu_h \eta_{h,s}; \mu_h \eta_{h,t}) &= Cov^*[\mu_h (u_{h,s} + \mu_{h,s}^{dev} + \varepsilon_{h,s}); \mu_h (u_{h,t} + \mu_{h,t}^{dev} + \varepsilon_{h,t})] \\ &= Cov^*(\mu_h \mu_{h,s}^{dev}; \mu_h \mu_{h,t}^{dev}) \\ &= \tilde{f}_s' Cov^*(\mu_h \beta_{h,s-1}; \mu_h \beta_{h,t-1}) \tilde{f}_t, \end{aligned}$$

and therefore

$$Cov^*(\mu_h \eta_{h,s}; \mu_h \eta_{h,t}) = \tilde{f}_s' Var^*(\mu_h \beta_h) \tilde{f}_t.$$

Hence

$$\mathbb{E}[Cov^*(\mu_h \eta_{h,t}; \mu_h \eta_{h,s})] = \mathbb{E}(\tilde{f}_s)' Var^*(\mu_h \beta_h) \mathbb{E}(\tilde{f}_t) = 0.$$

for every $t \neq s$. We infer that

$$\mathbb{E}\left[\frac{4}{T_g^2} Var^*\left(\sum_{t=1}^{T_g} \mu_h \eta_{h,t}\right)\right] = \frac{4}{T_g} \mathbb{E}\left[Var^*(\mu_h \eta_{h,t})\right].$$

Since $\mu_h \eta_{h,t} = \mu_h (u_{h,t} + \varepsilon_{h,t}) + \mu_h \mu_{h,t}^{dev}$, the cross-sectional variance of $\mu_h \eta_{h,t}$ is given by

$$\begin{aligned} Var^*(\mu_h \eta_{h,t}) &= Var^*[\mu_h (u_{h,t} + \varepsilon_{h,t})] + Var^*(\mu_h \mu_{h,t}^{dev}) + 2Cov^*[\mu_h (u_{h,t} + \varepsilon_{h,t}); \mu_h \mu_{h,t}^{dev}] \\ &= \mathbb{E}^*(\mu_h^2)(\sigma_u^2 + \sigma_\varepsilon^2) + Var^*(\mu_h \mu_{h,t}^{dev}) + 2Cov^*(\mu_h u_{h,t}; \mu_h \beta_{h,t-1}') \tilde{f}_t. \end{aligned}$$

Hence

$$\mathbb{E}\left[\frac{4}{T_g^2} Var^*\left(\sum_{t=1}^{T_g} \mu_h \eta_{h,t}\right)\right] = \frac{4}{T_g}\left\{\mathbb{E}^*(\mu_h^2)(\sigma_u^2 + \sigma_\varepsilon^2) + \mathbb{E}[Var^*(\mu_h \mu_{h,t}^{dev})]\right\} \tag{A-53}$$

for every h and t.

5. The cross-sectional variance of $\sum_{t=1}^{T_g} \lambda_t \eta_{h,t}$ satisfies

$$Var^*\left(\sum_{t=1}^{T_g} \lambda_t \eta_{h,t}\right) = \sum_{t=1}^{T_g} \lambda_t^2 Var^*(\eta_{h,t}) + \sum_{s \neq t} \lambda_s \lambda_t Cov^*(\eta_{h,s}; \eta_{h,t}).$$

We note that for every $s \neq t$, the cross-sectional covariance of $\eta_{h,s}$ and $\eta_{h,t}$ simplifies to $Cov^*(\eta_{h,s}; \eta_{h,t}) = \tilde{f}_s' Var^*(\beta_h) \tilde{f}_t$ and therefore

$$\mathbb{E}[\lambda_s \lambda_t Cov^*(\eta_{h,s}; \eta_{h,t})] = \mathbb{E}(\tilde{f}_s)' \mathbb{E}[\lambda_s \lambda_t Var^*(\beta_h)] \mathbb{E}(\tilde{f}_t).$$

Hence

$$\mathbb{E}\left[\frac{4}{T_g^2} Var^*\left(\sum_{t=1}^{T_g} \lambda_t \eta_{h,t}\right)\right] = \frac{4}{T_g} \mathbb{E}[\lambda_t^2 Var^*(\eta_{h,t})]. \tag{A-54}$$

for every h.

6. We observe that

$$\mathbb{E}\left[\frac{2}{T_g} \sum_{t=1}^{T_g} Cov^*(\mu_h^2; \eta_{h,t}^2)\right] = 2\mathbb{E}[Cov^*(\mu_h^2; \eta_{h,t}^2)] \tag{A-55}$$

is a positive constant that does not vary with T_g.

7. We observe that

$$\mathbb{E}[4\bar{\lambda} Cov^*(\mu_h^2, \mu_h)] = 0. \tag{A-56}$$

8. The cross-sectional covariance of μ_h^2 and $\mu_h \eta_{h,t}$ satisfies

$$Cov^*(\mu_h^2;\mu_h \eta_{h,t}) = Cov^*(\mu_h^2;\mu_h \mu_{h,t}^{dev}) = \tilde{f}_t' Cov^*(\mu_h^2;\mu_h \beta_{h,t-1}).$$

   Its unconditional mean is therefore equal to zero:

$$\mathbb{E}[Cov^*(\mu_h^2;\mu_h \eta_{h,t})] = \mathbb{E}(\tilde{f}_t)' \mathbb{E}[Cov^*(\mu_h^2;\mu_h \beta_{h,t-1})] = 0. \tag{A-57}$$

9. Similarly, the cross-sectional covariance of μ_h^2 and $\eta_{h,t}$ is

$$Cov^*(\mu_h^2;\eta_{h,t}) = \tilde{f}_t' Cov^*(\mu_h^2;\beta_{h,t-1}).$$

   Its unconditional mean is therefore equal to zero:

$$\mathbb{E}[\lambda_t Cov^*(\mu_h^2;\eta_{h,t})] = \mathbb{E}(\tilde{f}_t)' \mathbb{E}[\lambda_t Cov^*(\mu_h^2;\beta_{h,t-1})] = 0. \tag{A-58}$$

10. We observe that for every $s \neq t$,

$$\mathbb{E}[\lambda_s Cov^*(\eta_{h,t}^2;\mu_h)] = \mathbb{E}(\lambda_s)\, \mathbb{E}[Cov^*(\eta_{h,t}^2;\mu_h)] = 0.$$

   Hence

$$\mathbb{E}\left[4\bar{\lambda} Cov^*\left(\frac{1}{T_g}\sum_{t=1}^{T_g}\eta_{h,t}^2;\mu_h\right)\right] = \frac{4}{T_g}\mathbb{E}\left[\lambda_t Cov^*\left(\eta_{h,t}^2;\mu_h\right)\right].$$

   The definition of $\eta_{h,t}$ provided in equation (A-36) implies that

$$\begin{aligned} Cov^*\left(\eta_{h,t}^2;\mu_h\right) &= Cov^*\left[(\mu_{h,t}^{dev})^2;\mu_h\right] + 2Cov^*\left[\mu_{h,t}^{dev}(\varepsilon_{h,t}+u_{h,t});\mu_h\right] + Cov^*\left[(\varepsilon_{h,t}+u_{h,t})^2;\mu_h\right] \\ &= Cov^*\left[(\mu_{h,t}^{dev})^2;\mu_h\right] + 2Cov^*(\mu_{h,t}^{dev} u_{h,t};\mu_h), \end{aligned}$$

   so that

$$\begin{aligned} \mathbb{E}\left[\lambda_t Cov^*\left(\eta_{h,t}^2;\mu_h\right)\right] &= \mathbb{E}\left\{\lambda_t Cov^*\left[(\mu_{h,t}^{dev})^2;\mu_h\right]\right\} + 2\mathbb{E}\left[\tilde{f}_t' Cov^*(\lambda_t u_{h,t}\beta_{h,t-1};\mu_h)\right] \\ &= \mathbb{E}\left\{\lambda_t Cov^*\left[(\mu_{h,t}^{dev})^2;\mu_h\right]\right\}. \end{aligned}$$

   Since the cross-sectional covariance of $(\mu_{h,t}^{dev})^2$ and μ_h reduces to

$$\begin{aligned} Cov^*\left[(\mu_{h,t}^{dev})^2;\mu_h\right] &= Cov^*\left[(\beta_h'\tilde{f}_t)^2 + 2(\beta_h'\tilde{f}_t)(\gamma_{t-1}+\delta_{h,t-1})'\tilde{f}_t + ((\gamma_{t-1}+\delta_{h,t-1})'\tilde{f}_t)^2;\mu_h\right] \\ &= Cov^*\left[(\beta_h'\tilde{f}_t)^2;\mu_h\right], \end{aligned}$$

   we infer that $\mathbb{E}\left[\lambda_t Cov^*\left(\eta_{h,t}^2;\mu_h\right)\right] = \mathbb{E}\left\{\lambda_t Cov^*\left[(\beta_h'\tilde{f}_t)^2;\mu_h\right]\right\} = 0$, and therefore

$$\mathbb{E}\left[4\bar{\lambda} Cov^*\left(\frac{1}{T_g}\sum_{t=1}^{T_g}\eta_{h,t}^2;\mu_h\right)\right] = 0. \tag{A-59}$$

11. For every $s \neq t$, the cross-sectional covariance of $\eta_{h,s}^2$ and $\mu_h \eta_{h,t}$ satisfies

$$Cov^*(\eta_{h,s}^2;\mu_h \eta_{h,t}) = Cov^*(\eta_{h,s}^2;\mu_h \mu_{h,t}^{dev}) = \tilde{f}_t' Cov^*(\eta_{h,s}^2;\mu_h \beta_{h,t-1}),$$

and therefore has zero unconditional mean:

$$\mathbb{E}[Cov^*(\eta_{h,s}^2;\mu_h \eta_{h,t})] = \mathbb{E}(\tilde{f}_t)'\mathbb{E}[Cov^*(\eta_{h,s}^2;\mu_h \beta_{h,t-1})] = 0.$$

Hence

$$\mathbb{E}\left[2Cov^*\left(\frac{1}{T_g}\sum_{s=1}^{T_g}\eta_{h,s}^2;\frac{2}{T_g}\sum_{t=1}^{T_g}\mu_h \eta_{h,t}\right)\right] = \frac{4}{T_g}\mathbb{E}\left[Cov^*\left(\eta_{h,t}^2;\mu_h\eta_{h,t}\right)\right] \tag{A-60}$$

for every h and t.

12. Since

$$\mathbb{E}[\lambda_t Cov^*\left(\eta_{h,s}^2;\eta_{h,t}\right)] = \mathbb{E}[\tilde{f}_t' Cov^*\left(\eta_{h,s}^2;\lambda_t \beta_{h,t-1}\right)] = 0,$$

whenever $t \neq s$, we infer that

$$\mathbb{E}\left[\frac{4}{T_g}\sum_{t=1}^{T_g}\lambda_t Cov^*\left(\frac{1}{T_g}\sum_{t=1}^{T_g}\eta_{h,s}^2;\eta_{h,t}\right)\right] = \frac{4}{T_g}\mathbb{E}\left[\lambda_t Cov^*\left(\eta_{h,t}^2;\eta_{h,t}\right)\right] \tag{A-61}$$

for every h and t.

13. Since $Cov^*(\mu_h;\mu_h \eta_{h,t}) = Cov^*(\mu_h;\mu_h \mu_{h,t}^{dev})$, we infer that

$$\bar{\lambda} Cov^*(\mu_h;\mu_h \eta_{h,t}) = \tilde{f}_t' \bar{\lambda} Cov^*\left(\mu_h;\mu_h \beta_{h,t-1}\right),$$

and therefore

$$\mathbb{E}[\bar{\lambda} Cov^*\left(\mu_h;\mu_h\eta_{h,t}\right)] = \mathbb{E}(\tilde{f}_t)'\,\mathbb{E}[\bar{\lambda} Cov^*\left(\mu_h;\mu_h \beta_{h,t-1}\right)] = 0. \tag{A-62}$$

14. Similarly, we note that $Cov^*(\mu_h;\eta_{h,t}) = Cov^*(\mu_h;\mu_{h,t}^{dev})$ and therefore

$$\mathbb{E}[\bar{\lambda}\lambda_t\, Cov^*(\mu_h;\eta_{h,t})] = \mathbb{E}(\tilde{f}_t)'\,\mathbb{E}[\bar{\lambda}\lambda_t\, Cov^*(\mu_h;\beta_{h,t-1})] = 0. \tag{A-63}$$

15. For every $s \neq t$, the cross-sectional covariance of $\mu_h \eta_{h,s}$ and $\eta_{h,t}$ satisfies

$$Cov^*(\mu_h \eta_{h,s};\eta_{h,t}) = Cov^*(\mu_h \eta_{h,s};\mu_{h,t}^{dev}) = \tilde{f}_t' Cov^*(\mu_h \eta_{h,s};\beta_{h,t-1}),$$

and therefore

$$\mathbb{E}[Cov^*(\mu_h \eta_{h,s};\eta_{h,t})] = \mathbb{E}(\tilde{f}_t)'\mathbb{E}[Cov^*(\mu_h \eta_{h,s};\beta_{h,t-1})] = 0.$$

Hence

$$\mathbb{E}\left[\frac{8}{T_g^2}\sum_{t=1}^{T_g}\lambda_t Cov^*\left(\sum_{s=1}^{T_g}\mu_h\eta_{h,s};\eta_{h,t}\right)\right] = \frac{8}{T_g}\mathbb{E}\left[\lambda_t Cov^*(\mu_h\eta_{h,t};\eta_{h,t})\right] \tag{A-64}$$

for every h and t.

We have now computed the unconditional mean of all 15 components of $Var^*(\overline{r_h^2})$. By equations (A-50) to (A-64), the unconditional cross-sectional variance, $\mathbb{E}\left[Var^*(\overline{r_h^2})\right]$, is equal to

$$\begin{aligned}
&Var^*(\mu_h^2)+\mathbb{E}\left[Var^*\left(\tfrac{1}{T_g}\textstyle\sum_{t=1}^{T_g}\eta_{h,t}^2\right)\right]+\tfrac{4}{T_g}\sigma_\lambda^2\sigma_\mu^2+\tfrac{4}{T_g}\left\{\mathbb{E}^*(\mu_h^2)\,(\sigma_u^2+\sigma_\varepsilon^2)+\mathbb{E}[Var^*(\mu_h\mu_{h,t}^{dev})]\right\}\\
&+\tfrac{4}{T_g}\mathbb{E}[\lambda_t^2\,Var^*(\eta_{h,t})]+2\,\mathbb{E}[Cov^*(\mu_h^2;\eta_{h,t}^2)]+\tfrac{4}{T_g}\mathbb{E}[Cov^*(\eta_{h,t}^2;\mu_h\eta_{h,t})]\\
&+\tfrac{4}{T_g}\mathbb{E}\left[\lambda_t Cov^*\left(\eta_{h,t}^2;\eta_{h,t}\right)\right]+\tfrac{8}{T_g}\mathbb{E}\left[\lambda_t\,Cov^*(\mu_h\eta_{h,t};\eta_{h,t})\right],
\end{aligned}$$

which implies that equation (A-45) holds.

The cross-sectional covariance of $\overline{r_h}$ and $\overline{r_h^2}$ can be written as:

$$Cov^*\left(\overline{r_h};\overline{r_h^2}\right)=Cov^*\left(\mu_h;\overline{r_h^2}\right)+\frac{1}{T_g}\sum_{s=1}^{T_g}Cov^*\left(\eta_{h,s};\overline{r_h^2}\right). \tag{A-65}$$

The cross-sectional average of μ_h and $\overline{r_h^2}$ satisfies

$$\begin{aligned}
Cov^*\left(\mu_h,\overline{r_h^2}\right) &= Cov^*\left(\mu_h;\mu_h^2+\frac{1}{T_g}\sum_{t=1}^{T_g}\eta_{h,t}^2+2\,\bar{\lambda}\mu_h+\frac{2}{T_g}\sum_{t=1}^{T_g}\mu_h\,\eta_{h,t}+\frac{2}{T_g}\sum_{t=1}^{T_g}\lambda_t\,\eta_{h,t}\right)\\
&= Cov^*(\mu_h;\mu_h^2)+\frac{1}{T_g}\sum_{t=1}^{T_g}Cov^*(\mu_h;\eta_{h,t}^2)+2\,\bar{\lambda}\sigma_\mu^2\\
&\quad+\frac{2}{T_g}\sum_{t=1}^{T_g}Cov^*(\mu_h;\mu_h\,\eta_{h,t})+\frac{2}{T_g}\sum_{t=1}^{T_g}\lambda_t Cov^*(\mu_h;\eta_{h,t}).
\end{aligned}$$

and therefore

$$\mathbb{E}\left[Cov^*\left(\mu_h,\overline{r_h^2}\right)\right]=Cov^*(\mu_h;\mu_h^2)+\mathbb{E}\left[Cov^*(\mu_h;\eta_{h,t}^2)\right]. \tag{A-66}$$

Similarly, the cross-sectional average of $\eta_{h,s}$ and $\overline{r_h^2}$ is given by

$$\begin{aligned}
Cov^*\left(\eta_{h,s};\overline{r_h^2}\right) &= Cov^*(\eta_{h,s};\mu_h^2)+\frac{1}{T_g}\sum_{t=1}^{T_g}Cov^*(\eta_{h,s};\eta_{h,t}^2)+2\,\bar{\lambda}Cov^*(\eta_{h,s};\mu_h)\\
&\quad+\frac{2}{T_g}\sum_{t=1}^{T_g}Cov^*(\eta_{h,s};\mu_h\,\eta_{h,t})+\frac{2}{T_g}\sum_{t=1}^{T_g}\lambda_t Cov^*(\eta_{h,s};\eta_{h,t}).
\end{aligned}$$

We observe that

$$\begin{aligned}
\mathbb{E}[Cov^*(\eta_{h,s};\mu_h^2)] &= \mathbb{E}(\tilde{f}_s)'\mathbb{E}[Cov^*(\beta_{h,s-1};\mu_h^2)]=0,\\
\mathbb{E}[\bar{\lambda}Cov^*(\eta_{h,s};\mu_h)] &= \mathbb{E}(\tilde{f}_s)'\mathbb{E}[\bar{\lambda}Cov^*(\beta_{h,s-1};\mu_h)]=0.
\end{aligned}$$

Moreover, whenever $t\neq s$, we have

$$\begin{aligned}
\mathbb{E}[Cov^*(\eta_{h,s};\eta_{h,t}^2)] &= \mathbb{E}(\tilde{f}_s)'\mathbb{E}[Cov^*(\beta_{h,s-1};\eta_{h,t}^2)]=0,\\
\mathbb{E}[Cov^*(\eta_{h,s};\mu_h\,\eta_{h,t})] &= \mathbb{E}(\tilde{f}_s)'\mathbb{E}[Cov^*(\beta_{h,s-1};\mu_h\,\eta_{h,t})]=0,\\
\mathbb{E}[\lambda_t Cov^*(\eta_{h,s};\eta_{h,t})] &= \mathbb{E}(\tilde{f}_s)'\mathbb{E}[\lambda_t Cov^*(\beta_{h,s-1};\eta_{h,t})]=0.
\end{aligned}$$

Hence

$$\mathbb{E}Cov^*\left(\eta_{h,s};\overline{r_h^2}\right)=\frac{B}{T_g}. \tag{A-67}$$

We infer from equations (A-65) to (A-67) that (A-46) holds. ■

The geometric average return earned by dynasty h, which is defined in equation (A-27), satisfies:

$$\ln(1+r_h^G)=\frac{1}{T_g}\sum_{t=1}^{T_g}\ln(1+r_{h,t}).$$

Each log yearly return $r_{h,t}$ can be approximated by the second-order Taylor expansion:

$$\ln(1+r_{h,t})\approx r_{h,t}-r_{h,t}^2/2.$$

The dynasty's geometric average return is therefore given by

$$\ln(1+r_h^G)\approx\overline{r_h}-\frac{1}{2}\overline{r_h^2},$$

By equation (A-39), the unconditional mean log return,

$$\mu_G=\mathbb{E}\left\{\mathbb{E}^*[\ln(1+r_h^G)]\right\}, \tag{A-68}$$

satisfies

$$\mu_G\approx\mathbb{E}^*(\mu_h)-\frac{1}{2}\mathbb{E}\left[\mathbb{E}^*\left(\overline{r_h^2}\right)\right], \tag{A-69}$$

where $\mathbb{E}\left[\mathbb{E}^*\left(\overline{r_h^2}\right)\right]$ is provided by equation (A-44).

Similarly, we note that

$$Var^*\left[\ln(1+r_h^G)\right]\approx Var^*\left(\bar{r}_h-\overline{r_h^2}/2\right)=Var^*\left(\bar{r}_h\right)+\frac{1}{4}Var^*(\overline{r_h^2})-Cov^*\left(\bar{r}_h;\overline{r_h^2}\right).$$

The unconditional mean of the cross-sectional variance of a dynasty's log return,

$$\sigma_G^2=\mathbb{E}\left\{Var^*\left[\ln(1+r_h^G)\right]\right\},$$

therefore satisfies

$$\sigma_G^2 = \mathbb{E}[Var^*(\bar{r}_h)] + \frac{1}{4}\mathbb{E}[Var^*(\overline{r_h^2})] - \mathbb{E}\left[Cov^*(\bar{r}_h; \overline{r_h^2})\right], \tag{A-70}$$

where $\mathbb{E}[Var^*(\bar{r}_h)]$, $\mathbb{E}[Var^*(\overline{r_h^2})]$, and $\mathbb{E}\left[Cov^*(\bar{r}_h; \overline{r_h^2})\right]$ are provided by equations (A-40), (A-45), and (A-46), respectively.

By the Central Limit Theorem, $1 + r_h^G$ is approximately lognormal. Its variance is therefore:

$$Var(r_h^G) \approx \left[\exp(\sigma_G^2) - 1\right] \exp\left(2\mu_G + \sigma_G^2\right). \tag{A-71}$$

The key population moments of (unobserved) average returns over a generation are now known functions of the population moments of (observable) yearly returns. These results are the basis of the estimation procedure.

IX Cross-Sectional Distribution of Average Returns over a Generation: Estimation

This Section develops the estimation procedure and provides empirical estimates of the model of dynastic wealth returns developed in Section VIII.

IX.A Definition of Estimators

We use the population moments of Section *VIII.B* to estimate the model of wealth return over a generation defined in Section *VIII.A*.

IX.A.1 Baseline Estimator

We develop an estimator of the key moments of average returns over a generation, called estimator #1, in which factor risk premia are taken as given. This conditional procedure is consistent with the approach considered in the main text and earlier sections of this online Appendix, and is therefore viewed as the baseline method. It proceeds as follows.

- We estimate the vector of factor loadings, $\beta_{h,t}$, of each household, as is explained in the main text. Since the estimation of asset loadings is based on long time series, we henceforth neglect the corresponding estimation error.
- We compute the panel of expected returns, $\{\mu_{h,t}^{\exp}\}_{h,t}$, by interacting household loadings with risk premia.
- We estimate the decomposition $\beta_{h,t} = \beta_h + \gamma_t + \delta_{h,t}$, and impute the individual fixed effect μ_h and the time fixed effect λ_t.
- We estimate the second moments σ_μ^2, σ_λ^2, σ_u^2, $Var(\mu_h^2)$, and $\mathbb{E}(\mu_h^2)$, by following the procedure outlined in Section VII of this online Appendix.
- We estimate σ_ε^2 by the sample variance of the risk-adjusted return $\varepsilon_{h,t} = r_{h,t} - \beta_{h,t-1}' f_t$.
- We estimate the cross-sectional mean and variance of long-run performance, μ_G and σ_G^2, by computing the finite-sample equivalents of equations (A-39), (A-40), (A-69), and (A-70).

The cross-sectional variance of the distribution of average returns over a generation, $Var(r_h^G)$, is obtained by plugging the estimates of μ_G and σ_G^2 into equation (A-71).

The baseline procedure focuses on the dispersion of pre-tax returns. In the main text, we also consider pre-tax systematic returns as well post-tax returns. Pre-tax systematic returns are obtained by multiplying household factor loadings with historical factor returns, setting the risk-adjusted return, $\varepsilon_{h,t}$, equal to zero. The Swedish panel provides the capital taxes paid by each household. We obtain household h's post tax return in year t by deducting the household's capital tax-to-wealth ratio from the household's expected pre-tax return. The estimation then follows the same steps as in the baseline case.

IX.A.2 Unconditional Estimator

We consider a variant of the baseline method, called estimator #2, in which factor risk premia are estimated and therefore a source of estimation noise. By equations (A-32) to (A-34), the cross-sectional second moments are quadratic functions of expected risk premia:

$$\begin{aligned}
\mathbb{E}(\mu_h^2) &= \mathbb{E}(f_t)'\,\mathbb{E}^*(\beta_h\beta_h')\,\mathbb{E}(f_t), \\
\sigma_\mu^2 &= \mathbb{E}(f_t)'\,Var^*(\beta_h)\,\mathbb{E}(f_t), \\
\sigma_\lambda^2 &= \mathbb{E}(f_t)'Var^*(\gamma_{t-1})\,\mathbb{E}(f_t), \\
\sigma_u^2 &= \mathbb{E}(f_t)'\,Var^*(\delta_{h,t-1})\,\mathbb{E}(f_t).
\end{aligned}$$

We estimate $\mathbb{E}(f_t)$ by the sample mean of factor realizations, $\overline{f}$, over a period of T_{prem} years. We verify that

$$\mathbb{E}(\overline{f}\,\overline{f}') = \mathbb{E}(f_t)\mathbb{E}(f_t)' + \frac{Var(f_t)}{T_{prem}}.$$

Let $\hat{V}_f$ denote the (time series) sample-covariance matrix of the factors. It is also convenient to denote by $\hat{M}_{2,\beta}$ the sample equivalent of $\mathbb{E}(\beta_h\,\beta_h')$, and by $\hat{V}_\beta$, $\hat{V}_\gamma$, and $\hat{V}_\delta$, respectively, the sample variance-covariance matrix of the estimated individual effect, $\hat{\beta}_h$, time effect, $\hat{\gamma}_t$, and residual $\hat{\delta}_{h,t}$. We estimate the cross-sectional second moments by

$$\widehat{M_2} = \overline{f}'\,\hat{M}_{2,\beta}\,\overline{f} - \mathrm{tr}\left(\hat{M}_{2,\beta}\,\hat{V}_f\right)/T_{prem}, \tag{A-72}$$

$$\widehat{\sigma_\mu^2} = \overline{f}\,\hat{V}_\beta\,\overline{f} - \mathrm{tr}\left(\hat{V}_\beta\,\hat{V}_f\right)/T_{prem}, \tag{A-73}$$

$$\widehat{\sigma_\lambda^2} = \overline{f}'\,\hat{V}_\gamma\,\overline{f} - \mathrm{tr}(\hat{V}_\gamma\hat{V}_f)/T_{prem}, \tag{A-74}$$

$$\widehat{\sigma_u^2} = \overline{f}'\,\hat{V}_\delta\,\overline{f} - \mathrm{tr}\left(\hat{V}_\delta\,\hat{V}_f\right)/T_{prem}. \tag{A-75}$$

Risk premium estimation error may also bias the estimation of the higher-order moments of the average moment over a generation. However, by the Central Limit Theorem, the vector $\overline{f_t}$ is approximately Gaussian. For this reason, estimator #2 focuses on estimation error in the first two moments of risk premia.

IX.A.3 Benchmark Estimators

In addition to estimators #1 and #2, we consider the following benchmark estimators of the cross-sectional variance of the geometric average return over a generation, $Var^*(r_h^G)$.

- Estimator #3 is the cross-sectional variance of the average arithmetic return, $\bar{r}_h = \sum_t r_{h,t}/T$, based on equation (A-40) and the baseline estimators of σ_μ^2, σ_u^2, σ_{dev}^2 and σ_ε^2.

- Estimator #4 is the variant of estimator #3 that controls for risk premium estimation error.

- Estimator #5 is the cross-sectional variance of the average arithmetic return, $Var^*(\bar{r}_h) = \sigma_\mu^2 + \sigma_u^2/T$, under the two-way fixed effect model of returns: $r_{h,t} = \lambda_t + \mu_h + u_{h,t}$, estimated using the methodology outlined in Section VII of this online Appendix.

- Estimator #6 is the cross-sectional variance of the average arithmetic return of households over the sample period, $\bar{r}_h = \sum_t r_{h,t}/T$, as in Fagereng et al. (2019).

IX.B Monte Carlo Simulations

We assess the performance of estimators of the average return over a generation by Monte Carlo simulations. The data-generating process is the model laid out in Section *VIII.A* of this online Appendix, with the parameters provided below. Consistent with previous notation, dynasties are indexed by h and years by t. All numerical values are approximately set to their levels in the Swedish population.

- The population comprises 500 dynasties observed over 36 years.

- The risk-free rate is 3% in annual units.

- Household portfolio returns follow a one-factor market model: $r^e_{h,t} = \beta_{h,t-1}\, r^e_{m,t} + \varepsilon_{h,t}$.

- The excess return on the market factor $r^e_{m,t}$ has a normal distribution with mean 0.08 and standard deviation 0.2.

- The loading of dynasty h in year t, $\beta_{h,t}$, is the sum of (i) an individual effect, β_h, drawn from a normal distribution with mean 0.438 and standard deviation 0.2, (ii) a time effect, γ_t, drawn from a centered normal distribution with standard deviation 0.031, and (iii) an idiosyncratic component, $\delta_{h,t}$, drawn from a centered normal with standard deviation 0.125.

- The risk-adjusted return is specified by the stochastic variance model:

$$\varepsilon_{h,t} = \sigma_{\varepsilon,h,t}\, z_{h,t},$$

  where $z_{h,t}$ has a standard normal distribution, $\sigma_{\varepsilon,h,t} = \max(-0.03 + 0.15\,\beta_{h,t} + \nu_{h,t}; 0)$, and $\nu_{h,t}$ has a centered normal distribution with standard deviation 0.046.

Under this specification, the population cross-sectional standard deviation of the geometric average return, $[Var^*(r_h^G)]^{1/2}$ is equal to 2.04% in annual units, while the cross-sectional standard deviation of the arithmetic average return, $[Var^*(\bar{r}_h)]^{1/2}$, is 2.34%.

In Appendix Figure 6, we report boxplots of Monte Carlo simulations of the estimators defined in Section *IX.C*. Specifically, we simulate 10,000 different panels of 500 households and for each simulated panel we compute the estimators of $[Var^*(r_h^G)]^{1/2}$ defined above. The distribution of each estimator is illustrated in a boxplot. In each boxplot, the solid red line shows the target population standard deviation of the *geometric* average return, $[Var^*(r_h^G)]^{1/2}$. The dashed red line illustrates the population standard deviation of the *arithmetic* average return, $[Var^*(\bar{r}_h)]^{1/2}$, as a benchmark.

Panel A illustrates the distribution of estimators #1, #3 and #5 applied to simulated unbalanced panels of 8 years (first three boxplots from the left).[22] The panel also plots estimator #6 applied to simulated unbalanced panels of 11 years (fourth boxplot), as in the Norwegian data of Fagereng et al. (2019), and to balanced panels of 36 years (fifth boxplot), the length of a generation in the Benhabib, Bisin, and Luo (2019) model. By construction, estimator #6 coincides with estimator #5 on a panel of 36 years.

Panel B illustrates estimators #1 to #4 (in this order from the left of the panel) based on 8 years of household data. Estimators #1 and #3 are conditional on risk premia, while estimators #2 and #4 also control for risk premium estimation error. The risk premia are estimated on a panel of 33 years, consistent with the Swedish data.

[22] Each simulated unbalanced panel is obtained by simulating a balanced panel of 8 years and then dropping each simulated household-year observation with probability 14%.

Our baseline estimator (estimator #1 illustrated in the leftmost boxplot of Panels A and B) is the most precise and least biased of all the estimators considered in this Section. It remains accurate even when risk premia are estimated on a relatively short sample of 33 years (estimator #2 illustrated in the second boxplot of Panel B). Estimator #2 has of course a higher variance than estimator #1 since it incorporates risk premium estimation error.

Estimator #3 (second boxplot of Panel A and third boxplot of Panel B) and estimator #4 (fourth boxplot of Panel B) exhibit strong upward bias and are clustered around the population cross-sectional standard deviation of the *arithmetic* average return. These properties follow naturally from the definitions of these estimators. Estimator #3 estimates the population standard deviation of the *geometric* average return, $Var^*(r_h^G)$, by the sample standard deviation of the *arithmetic* average return, which generates upward bias. Estimator #3, however, has a low standard deviation because it hinges on an asset pricing model. Estimator #4 exhibits the same bias than its conditional version, but is much noisier due to risk premium estimation error.

Estimator #5 (third boxplot of Panel A) is very noisy. This property results from the fact that this estimator does not rely on an asset pricing model and is therefore noisier than the other estimators.

Finally, estimator #6 (fourth and fifth boxplots of Panel A) exhibits both a strong upward bias and high variability. The performance is especially poor when we apply it to a household panel of 11 years, as in Fagereng et al. (2019). The explanation for this poor performance is that estimator #6 treats the average of idiosyncratic shocks over 11 years as equally dispersed as the average of idiosyncratic shocks over 36 years. In fact, the former exceeds the latter by a factor of $\sqrt{36/11} = 1.8$, which explains the large upward bias in Fagereng et al. (2019). Estimator #6 remains biased and dispersed even on a longer sample of 36 years (the full investment lifespan of a generation). Overall, the Monte Carlo simulations illustrate the finite-sample accuracy of the baseline approach developed in this paper, which provide good estimates of the cross-sectional dispersion of generational returns on a panel of 8 years.

IX.C Empirical Results

We now apply the estimation methodology developed in earlier sections to the 2000-2007 panel of Swedish households.

IX.C.1 Dispersion of the Average Return over a Generation: Comparison of Five Estimators

In Appendix Table 33, we report empirical estimates of the cross-sectional standard deviation of the geometric average return over a generation, $[Var^*(r_h^G)]^{1/2}$. Estimators #1 to #3 deliver similar results for the entire population. For gross wealth, the standard deviation of the average return over a generation, $[Var^*(r_h^G)]^{1/2}$, is about 2.2% under all three methods. For net wealth, the differences between the various methods are slightly larger but the orders of magnitude are similar. The standard deviation of the average return on net wealth over a generation is 7.8% per year using our baseline method (estimator #1), 7.7% using the variant of the baseline approach that adjusts for risk premium estimation error (estimator #2), and 5.8% using an asset-pricing-based arithmetic average (estimator #3). We obtain higher estimates, equal to 2.8% for gross wealth and 8.2% for net wealth, when we use a purely econometric decomposition of returns (estimator #4).

Estimators #1 and #2 produce similar estimates of the cross-sectional dispersion within wealth fractiles of the average return over generation, for both gross and net wealth. Like estimators #1 and #2, estimator #3 and #4 capture that the cross-sectional dispersion of the average return on net wealth over a generation is U-shaped in initial net worth. However, estimator #4 provides noisier estimates, consistent with the Monte Carlo simulations of Section *IX.B* and as one expects from a purely econometric approach that does not rely on an asset pricing model.

The pure fixed-effects approach (estimator #5) delivers much higher estimates of dispersion than other estimators. The standard deviation of the average return over a generation obtained under this approach exceeds our baseline estimate by a factor of 2.1 for gross wealth and 1.4 for net wealth in the entire population. This large gap subsists within all wealth brackets. These results show that estimator #5 exhibits a very large upward bias, as the Monte Carlo simulations of Section *IX.B* confirm.

IX.C.2 Cross-Sectional Moments of the Average Logarithmic Return

In Appendix Table 34, we report the cross-sectional mean and standard deviation of the average return over a generation. Columns 1 and 2 show empirical estimates of the mean logarithmic return, $\mu_G = \mathbb{E}\left\{\mathbb{E}^*[\ln(1+r_h^G)]\right\}$, on gross and net wealth. The mean logarithmic return is a hump-shaped function of a dynasty's initial wealth, which parallels the findings in Tables 1 and 2 of the main text. By (A-27), the mean logarithmic return, μ_G, is independent of a generation's investment lifespan, T_g.

Columns 3 and 4 of Appendix Table 34 report the cross-sectional standard deviation of the average logarithmic return, $\sigma_G = [\mathbb{E}\left\{Var^*\left[\ln(1+r_h^G)\right]\right\}]^{1/2}$, for a generation with an investment lifespan of 36 years horizon, while columns 5 and 6 consider an infinite investment life span. The cross-sectional standard deviation σ_G is very substantial for an infinitely-lived generation and about 25% smaller than the value obtained for a generation with a 36-year investment lifespan. These results suggest that the investment lifespan of generation needs to be quite long for the effect of idiosyncratic shocks to be fully washed out.

Finally, the estimates in the table can be used to compute the cross-sectional mean and standard deviation of a generation's average geometric return over an investment period M other than 1 year. Indeed, since the distribution of $\ln(1+r_h^G)$ is approximately normal $\mathcal{N}(\mu_G, \sigma_G^2)$, the average generational return over an investment period of M periods, $(1+r_h^G)^M$, is approximately lognormal with known moments.[23]

[23] That is, the log return over M periods, $\ln[(1+r_h^G)^M]$, is approximately normal with mean $M\mu_G$ and variance $M^2\sigma_G^2$.

X Comparison with Alternative Studies

We now compare our results to the findings from other datasets. Section *X.A* analyzes the returns on the endowments of US foundations considered by Saez and Zucman (2016). Section *X.B* compares our results to the Norwegian study of Fagereng et al. (2019).

X.A Evidence From US Foundations

As Section I of the main text explains, we measure expected returns under the assumption that households are rewarded for their exposures to the factors but do not earn abnormal risk-adjusted returns. We test this assumption in our data and do not find significant evidence against it. It may be, however, that rich Swedish households do not have as much access to high-performing assets as rich investors in the United States. For this reason, we now consider US foundations (Saez and Zucman 2016), one of the few US examples where historical returns are observable at the micro level.

The Internal Revenue Service (IRS) provides detailed balance sheet and profit and loss data on US foundations from 1985 onward (Internal Revenue Service 2013). The IRS dataset contains all foundations with more than \$10 million in total assets (valued at fair market prices). Smaller foundations are sampled with probability 0.1. As Saez and Zucman (2016) explain, the IRS foundation dataset is unique because it provides (i) the market value of total assets at the beginning and the end of each fiscal year as well as the breakdown of holdings into broad asset classes, (ii) dividends and realized capital gains during the fiscal year, and (iii) operating costs and benefits. The IRS data permit the broadest possible measurement of capital income and its decomposition into dividends, interest payments, and realized and unrealized capital gains. Cash balances in non-interest bearing accounts permit us to impute interest payments in the form of banking services. As in Fagereng et al. (2019), we measure the return on wealth by

$$r_{j,t+1} = \frac{Y_{j,t+1}}{(W_{j,t} + W_{j,t+1})/2},$$

where $W_{j,t}$ denotes the total net assets held by foundation j at the end of fiscal year t, and $Y_{j,t+1}$ is the capital income perceived during fiscal year $t+1$. This method is designed to synchronize the flow of capital income and the corresponding capital stock. In order to avoid outliers, we remove foundations with less than \$1,000 in net assets at the beginning of the year and winsorize returns at the 1% level.

We rank foundations by net worth at the beginning of each fiscal year. We use the same wealth thresholds as Saez and Zucman (2016), that is we partition the population of US foundations into seven groups indexed by g. We denote by $r_{g,t}$ the average return earned during year t by each foundation belonging to group g at the beginning of year t. For each group g, we model returns by the following CAPM equation:

$$r_{g,t} = \alpha_g + \beta_g MKT_t + u_{g,t}, \tag{A-76}$$

where MKT_t is the US stock market excess return in fiscal year t,[24] and $u_{g,t}$ is an uncorrelated residual.

In Appendix Table 35, we estimate the pricing equation (A-76) for each foundation group using the 28 annual observations between 1986 and 2013.[25] We report the results for three different definitions of returns: the total return (Panel A); the total return minus unrealized capital gains (Panel B), as in Fagereng et al. (2019); and the dividend yield (Panel C). Column 5 of Panel A shows that when one considers the most exhaustive measure of returns, the CAPM accounts for a large share of the variation in returns across foundation groups, with an R^2 coefficient ranging from 0.65 to 0.94. Crucially, columns 3 and 4 of Panel A shows that no group substantially outperforms the returns predicted by the factor loadings, either statistically or economically.

Why did many past studies (Piketty 2014; Fagereng et al. 2019; Cao and Luo 2017) argue that richer investors have access to more skilled investments? Appendix Figure 7 provides a first

[24]The US risk-free rate and the equity market return are obtained from Kenneth French's data library. Market returns are matched to each foundation's fiscal year using information on the month of the fiscal year-end.

[25]It is precisely because we have few data points that we do not include other factors such as value and size and focus on a simple CAPM model.

answer by comparing the average return observed from 1986 to 2013 with the expected return predicted by the CAPM. We replicate Saez and Zucman's (2016) evidence that average returns increase with foundation net worth. However, we show that the relationship between average returns and wealth (dotted line) is closely mimicked by the relationship between expected returns and wealth (the solid line). In other words, foundation returns increase with net worth only because they are more exposed to systematic and compensated equity risks (Panel A of Appendix Table 35), which is fully consistent with our analysis of Swedish households.

A second reason why the existing literature may wrongly suggest that richer investors exhibit investment skill is that some of this research (in particular, Cao and Luo (2017) for US households) relies on a measure of returns that does not take into account unrealized capital gains on assets other than the primary residence. The foundation data allow us to analyze if such measures of returns, which are unconventional in finance, are consistent with the CAPM, an asset pricing model whose validity is confirmed by Panel A. As Panel B shows, returns that exclude capital gains exhibit strong deviations from the CAPM. The R^2 coefficient drops very substantially and the CAPM alpha is very significantly positive for any level of wealth. This analysis casts doubts about the financial relevance of using partial measures of returns. Furthermore, using these measures can be quite misleading. The table shows that the alpha coefficient increases significantly with net worth while the beta is a low and flat function of wealth. One could wrongly conclude from these findings that historical returns do not follow a simple asset pricing model and/or investors have substantial skill, which would contradict a very large body of the empirical finance literature. When returns only include dividends and interest rate payments (Panel C), the CAPM α becomes a decreasing function of wealth and the market return loses all explanatory power for the returns of foundations. Panels B and C of Table 35 therefore illustrate the perils of working with flawed measures of returns.

In Appendix Table 36, we show that another key flaw of computing returns without unrealized capital gains is that it vastly understates the heterogeneity of wealth returns across foundations. In the general population of foundations, the cross-sectional standard deviation of returns is

14.7% if we use total returns, 8.3% if we exclude unrealized capital gains, and 2.3% only if we only consider dividend yields. Consistent with the evidence reported for Swedish households, we find that return heterogeneity is very high among foundations with very low net worth (18.3% for foundations with less than $100,000 in assets). An important difference is that the standard deviation of returns reaches a plateau of about 11-12% for foundations with at least $10 million in net worth.

Quite strikingly, the heterogeneity among the richest foundations seems much lower if we instead use partial measures of returns, such as dividend yields or returns without unrealized capital gains. Among foundations with a net worth in excess of $5 billion, the historical standard deviation is 11.7% for total returns, 3.3% for returns excluding unrealized capital gains, and 1.1% for the dividend yield.[26] Since the main text shows that the variation in return heterogeneity across the wealth distribution is a key determinant of wealth inequality dynamics, this last result suggests that omitting unrealized capital gains results in vastly underestimating the contribution of return heterogeneity to the dynamics of top wealth shares.

X.B Comparison with Fagereng et al.'s (2019) Study of Norwegian Residents

Fagereng et al. (2019) use Norwegian registry data to explore the cross-sectional properties of individual wealth returns. Their return measure diverges from our baseline approach along two key dimensions: (i) the return on a bank account is assumed to be purely monetary and excludes non-pecuniary services, and (ii) the return on private equity is based on an accounting measure. In a previous version of their work and a contribution of Cao and Luo (2017), unrealized capital gains are excluded from the return measure. By contrast, our baseline approach includes both nonpecuniary banking services (consistent with national accounting practices) and market-based private equity returns.

[26] These estimates probably reflect the fact that very rich foundations have more predictable consumption needs in relation to their wealth and are therefore more able to smooth their asset withdrawals.

X.B.1 Mean Return

In Appendix Table 37, we investigate the impact of the return measurement methodology on the historical average return across brackets of net worth. The table reports statistics on the excess returns of gross and net wealth that exclude unrealized capital gains and non-pecuniary banking services, as in Cao and Luo (2017) (columns 1 and 2 of Panel A), exclude non-pecuniary banking services and use accounting-based private equity returns, as in Fagereng et al. (2019) (columns 3 and 4 of Panel A), include non-pecuniary banking services and accounting-based private equity returns (columns 5 and 6 of Panel A), exclude non-pecuniary banking services and use market-based private equity returns (columns 1 and 2 of Panel B), or include non-pecuniary banking services and market-based private equity returns, which is the baseline approach followed throughout the paper and in the rest of this online Appendix (columns 3 and 4 of Panel B).

The mean return generally increases with net worth under all methods. However, there are strong differences in the level of returns and in the steepness of the relationship between wealth and returns depending on the return measurement methodology.

Median Household. When non-pecuniary banking services and unrealized capital gains are excluded, as in Cao and Luo (2017), the median household earns on average an excess return of 0.7% on gross wealth and -0.4% on net wealth (columns 1 and 2 of Panel A). If we include unrealized capital gains and use accounting-based private equity returns, as in Fagereng et al. (2019), the median household earns an average return of 1% on gross wealth and 2.2% on net wealth (columns 3 and 4 of Panel A). The results are almost similar if we use market-based private equity returns while continuing to exclude non-pecuniary banking services (columns 1 and 2 of Panel B). These results establish that unrealized gains are a substantial component of household wealth returns.

If we now include non-pecuniary banking services, the average return earned by the median household is unsurprisingly higher and equal to 1.3% for gross wealth and 2.5% for net wealth, regardless of the method used to compute private equity returns (columns 5 and 6 of Panel A and columns 3 and 4 of Panel B). For the median household, accounting for unrealized capital gains is essential and including banking services makes a significant difference, while the method used to compute private equity returns is essentially irrelevant.

Top Half of the Wealth Distribution. Between the median household and households in the top 0.01%, the variation of expected returns along the wealth distribution is sensitive to the method used to compute returns. If non-pecuniary banking services and unrealized capital gains are excluded, as in Cao and Luo (2017), the expected return on wealth is a humped-shaped function of net worth, whether one considers gross or net wealth. For gross wealth, the expected excess return increases from 0.7% for the median household to 1.9% for the top 5%-0.5% and then declines to 0.1% for the top 0.01% (column 1 of Panel A). For net wealth, the excess return increases from -0.4% for the median household to 1.9% for the top 2.5%-0.5% and then declines to 0.0% for the top 0.01% (column 2 of Panel A).

The mean return on wealth monotonically earned by households at the top of the wealth distribution increases very substantially once unrealized capital gains are taken into account. With accounting-based equity returns, as in Fagereng et al. (2019), the return differential between the median bracket and the top 0.01% is on average 5.4 p.p. for gross wealth and 4.9 p.p. for net wealth on average (columns 3 and 4 of Panel A). If we also include non-pecuniary banking services, then the average return differential between median bracket and the top 0.01% is equal to 5.1 p.p. for gross wealth and 4.6 p.p. for net wealth (columns 5 and 6 of Panel A).

The relationship between mean returns and wealth remains positive but becomes less steep when one considers market-based private equity returns. The gap between the median bracket and the top 0.01% is 1.2 p.p. for gross wealth and 0.4 p.p. for net wealth in the absence of non-pecuniary banking services (columns 1 and 2 of Panel B), and 0.9 p.p. for gross wealth and 0.0 p.p. for net wealth under our baseline approach (columns 3 and 4 of Panel B).

Overall, the average return differential between the wealthy and the median bracket is about 5 p.p. higher if one chooses the method of Fagereng et al. (2019) rather than our methodology. This is primarily due to the effect of using an accounting-based rather than market-based return on equity. Indeed, as Section *V.D* of this online Appendix shows, the accounting-based return on private equity is structurally higher than the market-based return, most likely because the book value of assets systematically underestimates the value of intangible assets.[27]

Bottom Half of the Wealth Distribution. Between households in the lowest decile where returns can be measured (bottom 10% for gross wealth returns, bottom 10%-20% for net wealth returns) and the median household, the return differential is always positive, and it is equal on average to 11.0 p.p. for gross wealth and 8.4 p.p. for net wealth when non-pecuniary banking services and unrealized capital gains are excluded (columns 1 and 2 of Panel A), as in Cao and Luo (2017). Using the methodology in Fagereng et al. (2019), in which unrealized capital gains are included, the return on private equity is on an accounting basis, and non-pecuniary banking services are excluded, the gap becomes 2.0% for gross wealth and 9.3% for net wealth (columns 3 and 4 of Panel A). If we now include non-pecuniary services from banks, then the average return differential between the bottom bracket and the median is equal to 1.2% for gross wealth and 7.5% for net wealth. If instead we exclude non-pecuniary services but compute returns to private equity on a market basis, then this return differential is 2.0% for gross wealth and 9.2% for net wealth. Finally, if we use our preferred measure of returns, with non-pecuniary banking services and private equity returns measured on a market basis, then the average gap between the median and the bottom bracket is 1.2% for gross wealth and 6.9% for net wealth. Overall, the average return differential between the poor and the middle class is between 33% and 66% larger if one chooses the method of Fagereng et al. (2019) rather than our methodology. This is exclusively due to the effect of excluding non-pecuniary banking services from the measure of

[27] In the most recent version of their work, Fagereng et al. (2019) confirm our results and provide evidence (in Figure 7 of their working paper) that the gap in expected net wealth returns between the top percentile and the median of the wealth distribution is significantly smaller once returns on private equity are measured on a market basis rather than on an accounting basis.

returns to wealth.[28]

X.B.2 Cross-Sectional Dispersion

In Appendix Table 38, we report the cross-sectional standard deviation of returns estimated under the definitions of returns introduced in Section *X.B.1*. The organization of the panels is the same as in Appendix Table 37.

Full Population. The cross-sectional standard deviation of returns, measured on an annual basis, is large across all types of return measurement. In the entire population, the dispersion of returns is equal to 8.6% for gross wealth and 11.4% for net terms when non-pecuniary banking services and unrealized capital gains are excluded, as in Cao and Luo (2017) (columns 1 and 2 of Appendix Table 38, Panel A). With unrealized capital gains and accounting-based private equity returns, as in Fagereng et al. (2019), then the dispersion of returns within the entire population is 7.0% for gross wealth and 14.7% for net wealth (columns 3 and 4 of Panel A). The estimates are very similar if we also include non-pecuniary banking services.

The measured dispersion increases very substantially when one consider market-based measures of private equity returns. In the absence of non-pecuniary banking services, the dispersion of returns rises substantially to 8.6% for gross wealth and 16.3% for net wealth (columns 1 and 2 of Panel B). The results are very similar under our baseline approach, which includes non-pecuniary banking services: the standard deviation is then 9.2% for gross returns and 18.6% for net wealth.

Overall, the methodology used in Fagereng et al. (2019) generates full-population dispersion estimates that are about 25% lower than our baseline estimates, primarily due to their use of accounting-based private equity returns that are much smoother than market-based private equity returns.

[28] In the most recent version of their work, Fagereng et al. (2019) confirm our results and provide evidence (in Figure 2 of their working paper) that the spread in financial wealth returns earned by the top percentile and the bottom decile goes from 2 percentage points to 0.3 percentage points once one assumes that all safe assets earn the risk-free rate of return.

Variation Across Wealth Brackets. Across all types of return measurement, the cross-sectional standard deviation of returns is not constant along the wealth distribution. In the case of returns measured without unrealized capital gains, the dispersion is particularly high in the bottom of the distribution, at 17.9% for gross wealth and 22.3% for net wealth. The explanation is that realized capital gains are measured using the purchase price as the base, so that gains and losses are often disproportionately high relative to the low level of wealth just at the beginning of the year. This effect fades out quickly and in the top 70% of the distribution the dispersion remains in a range between 5 and 10% in both gross and net terms.

Using the methodology from Fagereng et al. (2019), the dispersion of gross wealth returns remains in a narrow 6%-7.5% range up until the top 2.5% of the distribution of net worth. It then quickly rises up to a level of 18.8% within the top 0.01%. The dispersion of net wealth returns is U-shaped in net worth, reaching a minimum of 7.6% for the top 20%-10% of households. This pattern is due to the fact that poorer households are much more levered, so that even small differences in gross returns are largely amplified at the bottom of the distribution.

Under our baseline methodology, the dispersion of returns exhibits a similar shape but with a much bigger increase in return heterogeneity at the top of the distribution. The dispersion of gross wealth returns remains flat at around 7-9% up to the 90th percentile of the distribution of households, and then continuously rises with net worth until it reaches a level of 35.8% at the very top. The larger dispersion measured under our baseline methodology is entirely driven by the fact that a) we use market-based rather than accounting-based private equity returns and b) private equity is far more prevalent among the wealthy.

X.B.3 Persistence

In Appendix Table 39, we investigate how the measurement of household fixed effects is impacted by the measurement of private equity returns and non-pecuniary banking services. The table has a similar structure to Appendix Table 29. That is, we consider a variety of return measures and for each measure, we report (1) the share explained by time fixed effects, R_t^2, (2) the share explained by household and year fixed effects, $R_{h,t}^2$, (3) the incremental share explained by household fixed effects, ρ, and (4) the sample standard deviation for a large number of return outcomes.

We consider the persistence of household returns on gross and net wealth when unrealized capital gains and non-pecuniary banking services are excluded from the analysis, as in Cao and Luo (2017). Year fixed effects have much smaller explanatory power ($R_t^2 = 20\%$ for gross wealth and 17% for net wealth) than under the baseline approach. This finding suggests that overall market movements have a weaker impact on household returns when unrealized capital gains are not factored in. By contrast, realized wealth returns contain a much larger household-specific persistent component than total wealth returns, and the marginal explanatory power ρ reaches more than 28% for both gross and net wealth. Household realizations of gains and losses are largely independent from overall market movements and are more likely driven by heterogeneous preferences and liquidity needs than by innate skills.

In the next rows, we consider measures of wealth returns that include accounting-based private equity returns but exclude non-pecuniary banking services, as in Fagereng et al. (2019). The marginal explanatory power of household fixed effects, ρ, is substantially higher than under our baseline approach and reaches 9.6% for gross wealth (compared to 4.7% under the baseline approach) and 17% for net wealth (compared to 12.3% under the baseline approach).[29]

The differences between our baseline results and the results of the Fagereng et al. method are driven mostly by differences in the measurement of private equity returns but also significantly by differences in the treatment of bank account returns, as the next rows of the table show. When we measure the returns on gross and net wealth, respectively, using accounting-based private equity returns (as in Fagereng et al.) but including non-pecuniary banking services. The marginal explanatory power of household fixed effects goes down from 9.6% to 7.9% for gross wealth

[29]Reassuringly, Fagereng et al. (2019) report R^2 estimates for net wealth returns that are similar to the estimates we obtain when we use their methodology. In Table 5 of their latest working paper, the reported R^2 is 0.33 in Norway (compared to 0.32 in our Swedish dataset) without individual fixed effects and 0.50 (0.49 in Sweden) in the presence of individual fixed effects.

and from 17.0% to 16.2% for net wealth. This means the treatment by Fagereng et al. (2019) of banking services explains (9.6-7.9)/(9.6-4.7) = 35% of the difference in marginal explanatory power of household fixed effects between our approach and their approach for gross wealth and (17.0-16.2)/(17.0-12.3) = 17% for net wealth.

We confirm this result by considering market-based private equity returns but exclude non-pecuniary banking services. Using this comparison, the treatment by Fagereng et al. (2019) of banking services explains (6.1-4.7)/(9.6-4.7) = 29% of the difference in marginal explanatory power of household fixed effects between our approach and their approach for gross wealth terms and (13-12.3)/(17-12.3) = 15% of the difference in net terms. Thus the method used to compute private equity returns explains between 68% and 84% of the difference in marginal explanatory power of household fixed effects between our two approaches.

We zero in on private equity and compare the magnitude of fixed effects in market- and accounting-based returns of household private equity portfolios. The market-based return has a strong time component and a weak household component, suggesting weak persistence at the household level. By contrast, the accounting-based return has a weak time component and a strong household component.[30] This confirms a long-established fact in the accounting literature that accruals earnings (the numerator of Fagereng et al.'s measure) and the book value of equity (the denominator) of public and private companies are heavily smoothed in comparison with the underlying economic cash flows (Burgstahler et al. 2006). This also provides further evidence that the measured persistence of household wealth returns is sensitive to the method used to compute private equity returns.

Yields on banking account balances exhibit substantial persistence. The adjusted coefficient of determination increases from $R_t^2 = 32\%$ to $R_{h,t}^2 = 64\%$ once household fixed effects are included. In the latter case, we investigate earlier in the appendix whether investor sophistication affects the bank account interest rate and find that there is only a small correlation, which sug-

[30]In Table 6 of the latest version of their working paper, Fagereng et al. (2019) also find a very weak time component and a strong individual component with an accounting-based measure of returns.

gests that differences in bank account rates mainly reflect differences in preferences.

In conclusion, the methodology used in Fagereng et al. (2019) generates a distribution of returns that exhibits much greater variation between wealth brackets and slightly stronger persistence at the household level compared to our baseline method. The wider heterogeneity in Fagereng et al. (2019) originates from their choice to use accounting-based private equity returns and to exclude non-pecuniary banking services from the analysis. The numbers we obtain in order to quantify such heterogeneity are very similar to theirs when we make the same methodological choices. However, our analysis shows that a large part of the return heterogeneity one estimates using those assumptions is driven by heterogeneity in accounting methods and banking preferences rather than skill.

XI Risk Exposures and Wealth Inequality

This section complements the results on wealth inequality dynamics reported in the main text. Section *XI.A* estimates a decomposition of the growth rate of top wealth shares in the spirit of Saez and Zucman (2016). Section *XI.B* relates our results to the variance decomposition of wealth inequality dynamics proposed by Campbell (2016).

XI.A A Decomposition of Wealth Inequality

In Section IV of the main text, we assess the contribution of wealth returns to the dynamics of top wealth inequality in Sweden between 2000 and 2007. We highlight the role of household wealth allocations by comparing the historical variation in inequality with the variation that would take place if net saving flows out of labor income were strictly proportional to initial wealth. Saez and Zucman (2016) propose a decomposition of wealth accumulation across different wealth groups that also aims at quantifying the relative contributions of theoretically important drivers of wealth inequality. We now show how our results fit with their framework.

Saez and Zucman (2016) consider the following accounting identity:

$$W_{t+1}^{p} = (1 + r_{t+1}^{p})\ (W_t^p + S_{t+1}^{p}), \tag{A-77}$$

where W_t^p is the aggregate net wealth held by households in quantile p of net worth at the end of year t, r_{t+1}^{p} is the year $t+1$ value-weighted wealth return earned by households in quantile p at the end of year t, and S_{t+1}^{p} is the level of saving flows such that equation (A-77) holds. The imputed saving flow S_{t+1}^{p} does not necessarily reflect actual saving because households in quantile p at the end of year t are not necessarily in the same quantile at the end of year $t+1$. For this reason, Saez and Zucman call S_{t+1}^{p} the "synthetic" saving flow.

Let W_t denote the aggregate stock of net wealth at the end of year t, and let r_{t+1} denote the value-weighted return on aggregate net wealth. We infer from (A-77) that the growth rate of the share of quantile p satisfies:

$$\frac{W_{t+1}^{p}/W_{t+1}}{W_t^p/W_t} = \frac{1 + (S_{t+1}^{p}/W_t^p)}{1 + (S_{t+1}/W_t)}\ \frac{1 + r_{t+1}^{p}}{1 + r_{t+1}}. \tag{A-78}$$

This equation decomposes the growth rate of wealth shares into a differential "synthetic" saving effect, $[1 + (S_{t+1}^{p}/W_t^p)]/[1 + (S_{t+1}/W_t)]$, and a differential return effect, $(1 + r_{t+1}^{p})/(1 + r_{t+1})$.

In columns 1 to 3 of Appendix Table 40, we consider five fractiles of net wealth in Sweden and for each quantile p we estimate (1) the growth rate of its wealth share, $(W_{t+1}^{p}/W_{t+1})/(W_t^p/W_t)$, (2) the differential saving effect, and (3) the differential return effect in equation (A-78).

Wealthier groups have larger synthetic saving rates. For instance, synthetic saving explains 48% of the increase in the top 0.01% share over the 2000 to 2007 period, while the differential return effect explains the other 52%. These results are in line with Saez and Zucman (2016), who show that the positive relationship between synthetic saving flows and net worth plays a major role in explaining US wealth inequality dynamics. However, the oft-made interpretation that higher synthetic saving implies higher household-level saving rates in top brackets could be invalid if there is substantial wealth mobility.

To illustrate this possibility, assume that a household's net saving flow is a constant proportion of initial net wealth, as in Section IV of the main text. In columns 5 and 6 of Appendix Table 40, we estimate the growth rate of the top wealth share, $(W^p_{t+1}/W_{t+1})/(W^p_t/W_t) - 1$, and the corresponding differential saving effect predicted by the combination of constant saving rates and the portfolio allocations observed for each household at the beginning of the year. Strikingly, even though individual saving rates do not differ by construction, we find a very strong positive relationship between the synthetic saving rate of each wealth quantile and its rank in the distribution of wealth. In particular, the relative contribution of synthetic saving flows to top wealth inequality is very similar in the simulated data (42%) and in the real data (48%). This analysis illustrates that, to a very large extent, the contribution of synthetic saving flows to wealth inequality reflects the wide heterogeneity of returns within each wealth group, in particular within top brackets.

As a further illustration of this mechanism, assume that the saving rate is not only constant but equal to zero for all households. In the top quantile, all households exiting the group between t and $t+1$ earn a bad return during the year, which diminishes the group-level return to wealth r^p_{t+1}. They are replaced by households from lower fractiles who have earned a good return during the year. The new entrants do not impact the wealth return of households in the top quantile at t, r^p_{t+1}, but tend to increase the total wealth of the top quantile at the end of the year, so that $W^p_{t+1} > W^p_t \, r^p_{t+1}$. As a result, the imputed synthetic saving flow of the rich, S^p_{t+1}, is positive, even though by construction all household saving flows are equal to zero.

To sum up, Section IV of the main text documents that heterogeneity in individual saving rates has a second-order effect on wealth inequality dynamics. As the analysis of the present section shows, this result is fully compatible with Saez and Zucman's (2016) findings that a large proportion of the recent increase in US wealth inequality is associated with higher synthetic saving rates in top brackets.

XI.B Decomposing the Variance of Financial Wealth

Campbell (2016) considers a synthetic measure of inequality, the variance of log financial wealth, and proposes a parsimonious dynamic model of wealth inequality that allows for significant diversity in investment strategies. For simplicity, the model abstracts away from active saving. Let $Var^*(x_{h,t})$ denote the cross-sectional variance of a variable $x_{h,t}$ at date t. On average over time, the change in the cross-sectional variance of log financial wealth is governed by the following law of motion:

$$\begin{aligned}\mathbb{E}[Var^*(f_{h,t+1}) - Var^*(f_{h,t})] &= \mathbb{E}[Var^*(\mathbb{E}_t r_{h,t+1})] + \mathbb{E}[Var^*(\tilde{r}_{h,t+1})] \\ &\quad + 2\mathbb{E}[Cov^*(\mathbb{E}_t r_{h,t+1}; f_{h,t})],\end{aligned} \tag{A-79}$$

where $f_{h,t}$ is the logarithm of the financial wealth held by household h at the beginning of year t, $\mathbb{E}_t r_{h,t+1}$ is the annual log return expected by the household at the end of year t, and $\tilde{r}_{h,t+1} = r_{h,t+1} - \mathbb{E}_t r_{h,t+1}$ is the return innovation in year $t+1$. All returns are measured before taxes.

In Appendix Table 41, we estimate the variance decomposition (A-79) in our data. In line with Campbell's (2016) focus on financial assets, we consider the dynamics of inequality in financial wealth, including riskless assets. The sample contains all Swedish households with strictly positive financial wealth. We assume that expected returns on household portfolios are entirely driven by exposures to priced factors and that all household portfolio alphas are equal to zero. In order to verify the robustness of our results, we use both the global 5-factor model and the local CAPM. The risk premia are the historical annual returns on each of these factors during the period 1983 to 2016. We compute historical returns under the assumption that households choose their holdings on December 31st and rebalance monthly to keep security weights constant.

Between December 1999 and December 2007, the variance of log financial wealth increases on average by 0.0389 a year. When we estimate expected returns by using the global 5-factor model, the three terms on the right-hand side of (A-79) sum up to about 0.0366, which illustrates that overall return heterogeneity tends to amplify inequality. The positive relationship between risk loadings and wealth, as measured by the covariance term $2\mathbb{E}[Cov^*(\mathbb{E}_t r_{h,t+1}; f_{h,t})]$ in (A-79), is the dominant channel and alone contributes to about 72.5% of the predicted increase in the cross-sectional wealth variance. The impact of return innovations emphasized by Benhabib, Bisin, and Zhu (2011), which is measured here by the term $\mathbb{E}[Var^*(\tilde{r}_{h,t+1})]$, is another important contributor to inequality that accounts for about 26.5% of the total effect of change in the wealth variance. By contrast, the diversity of expected returns per se, as measured by the term $\mathbb{E}[Var^*(\mathbb{E}_t r_{h,t+1})]$, explain less than 1% of the overall effect of returns.

Campbell (2016) estimates the variance decomposition (A-79) on Indian data and finds similar orders of magnitude for the impact of returns on inequality, but with a much higher contribution of the return innovation variance, $\mathbb{E}[Var^*(\tilde{r}_{h,t+1})]$. One reason for this gap may be that Indian households have virtually no access to mutual funds, which makes it harder for them to diversify their portfolios and induces a high cross-sectional variance of the return innovation, $\mathbb{E}[Var^*(\tilde{r}_{h,t+1})]$. Another likely reason for the discrepancy is that Campbell (2016) only considers returns to stock wealth, so that the positive link between wealth on risk-taking plays no role in his results, while this link is the main mechanism through which wealthier Swedish households obtain higher returns on their complete financial portfolios.

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Appendix Table 1
Aggregate Wealth Statistics

This table reports the aggregate holdings of Swedish household wealth on 31 December 2004 obtained from (1) the aggregation of the micro data used in the paper; and (2) the national accounts produced by Statistics Sweden (2014c, 2014d) for all assets except private equity, and the estimate of aggregate private equity from Waldenström (2016). All holdings are expressed in billion dollars using the exchange rate on 31 December 2004, when the Swedish kronor traded at 0.151 US dollar.

	Aggregate Holdings (in Billion Dollars)	
	Micro Data (1)	National Accounts (2)
Financial wealth:		
Deposits	85.3	87.3
Stocks	61.3	67.4
Funds	63.4	57.4
Capital insurance	17.7	17.9
Other	11.6	17.0
Total financial wealth	239.3	247.0
Funded pension wealth	220.6	223.8
Real estate	568.2	603.5
Private equity	122.6	104.2
Total gross wealth	1,150.7	1,178.5
Debt	218.4	216.0
Total net wealth	932.2	962.5
Number of households	4,755,879	N/A

Appendix Table 2
Net Wealth Thresholds

This table reports the minimum and maximum levels of net wealth in each bracket of the net wealth distribution in Sweden over the period 2000-2007. All amounts are expressed in thousands of Swedish kronor. On 31 December 2004, 1 Swedish krona traded at 0.151 US dollars.

Panel A: Years 2000 to 2003

	Net wealth brackets (thousand Swedish kronor)							
	2000		2001		2002		2003	
	Min	Max	Min	Max	Min	Max	Min	Max
Wealth group								
P0-P10	-	5	-	8	-	14	-	3
P10-P20	5	40	8	44	14	48	3	39
P20-P30	40	149	44	150	48	148	39	144
P30-P40	149	286	150	286	148	280	144	303
P40-P50	286	451	286	449	280	440	303	476
P50-P60	451	684	449	683	441	672	476	731
P60-P70	684	1,012	683	1,010	672	991	731	1,082
P70-P80	1,012	1,496	1,010	1,494	991	1,466	1,082	1,611
P80-P90	1,496	2,395	1,494	2,396	1,466	2,347	1,611	2,607
P90-P95	2,395	3,480	2,396	3,465	2,347	3,366	2,607	3,768
P95-P97.5	3,480	4,889	3,465	4,828	3,366	4,643	3,768	5,234
P97.5-P99	4,889	7,678	4,828	7,451	4,643	7,074	5,234	8,039
P99-P99.5	7,678	11,214	7,451	10,705	7,074	9,983	8,039	11,571
P99.5-P99.9	11,214	30,595	10,706	28,989	9,984	27,054	11,572	32,762
P99.9-P99.99	30,602	148,723	28,991	143,907	27,057	128,322	32,764	165,399
Top 0.01%	148,731	-	144,343	-	128,501	-	165,409	-

Panel B: Years 2004 to 2007

	Net wealth brackets (thousand Swedish kronor)							
	2004		2005		2006		2007	
	Min	Max	Min	Max	Min	Max	Min	Max
Wealth group								
P0-P10	-	10	-	14	-	0	-	0
P10-P20	10	48	14	60	0	53	0	57
P20-P30	48	167	60	202	53	214	57	234
P30-P40	167	349	202	413	214	445	234	484
P40-P50	349	546	413	640	445	703	484	771
P50-P60	546	831	640	968	703	1,073	771	1,180
P60-P70	831	1,225	968	1,419	1,073	1,580	1,180	1,733
P70-P80	1,225	1,812	1,419	2,097	1,580	2,337	1,733	2,557
P80-P90	1,812	2,918	2,097	3,385	2,337	3,786	2,557	4,133
P90-P95	2,918	4,224	3,385	4,940	3,786	5,523	4,133	6,021
P95-P97.5	4,224	5,895	4,940	6,949	5,523	7,787	6,021	8,427
P97.5-P99	5,895	9,212	6,949	11,087	7,787	12,482	8,427	13,212
P99-P99.5	9,212	13,478	11,087	16,581	12,483	18,730	13,213	19,174
P99.5-P99.9	13,479	40,408	16,582	50,697	18,731	59,354	19,175	56,822
P99.9-P99.99	40,411	209,447	50,697	271,690	59,365	335,423	56,825	315,551
Top 0.01%	209,523	-	271,840	-	335,792	-	315,604	-

Appendix Table 3
Mobility of Net Wealth from 2000 to 2007

This table reports the transition probabilities between a household's net wealth rank in 2000 and its net wealth rank in 2007, conditional on the household being observed at both dates. One should read the table as follows: among households belonging to the top 0.01% of the net wealth distribution in 2000 and still in existence in 2007, 1.7% are in the bottom 90% of the distribution in 2007, 0.6% are in the top 10%-5%, 2.2% are in the top 5%-1%, 9.1% are in the top 1%-0.1%, 33.7% are in the top 0.1%-0.01%, and 52.8% stay in the top 0.01%.

		Wealth rank in 2007					
		Bottom 90%	Top 10%-5%	Top 5%-1%	Top 1%-0.1%	Top 0.1%-0.01%	Top 0.01%
Wealth rank in 2000	Bottom 90%	95.6%	3.3%	1.0%	0.1%	0.0%	0.0%
	Top 10%-5%	33.5%	43.8%	21.6%	1.0%	0.0%	0.0%
	Top 5%-1%	10.0%	21.2%	60.6%	8.0%	0.2%	0.0%
	Top 1%-0.1%	3.2%	3.7%	32.9%	56.1%	3.9%	0.1%
	Top 0.1%-0.01%	1.8%	1.2%	5.0%	39.3%	49.5%	3.3%
	Top 0.01%	1.7%	0.6%	2.2%	9.1%	33.7%	52.8%

Appendix Table 4
Demographic Characteristics

This table reports average demographic characteristics of households in different brackets of the net wealth distribution in Sweden over the period 2000-2007. Gender, age, marital status and education refer to the household head. The cognitive ability score, or IQ, is standardized on a scale going from 1 to 9. The household's cognitive ability is the average score of men in each household who were enlisted in Swedish Armed Forces during the 1969 to 2009 period.

	Average demographic characteristics						
	Gender	Age	High school degree	Higher education	IQ	Family size	Marriage dummy
Wealth group							
P0-P10	0.622	34.33	0.794	0.226	4.50	1.890	0.219
P10-P20	0.496	31.35	0.811	0.318	4.76	1.426	0.114
P20-P30	0.532	38.28	0.835	0.311	4.84	1.746	0.152
P30-P40	0.482	50.70	0.694	0.238	4.89	1.716	0.171
P40-P50	0.475	57.18	0.630	0.208	5.02	1.691	0.187
P50-P60	0.537	56.59	0.663	0.244	5.16	1.879	0.297
P60-P70	0.561	57.67	0.670	0.264	5.32	2.001	0.412
P70-P80	0.582	58.92	0.684	0.291	5.52	2.084	0.515
P80-P90	0.605	59.95	0.724	0.337	5.72	2.165	0.609
P90-P95	0.631	60.32	0.777	0.404	5.89	2.263	0.681
P95-P97.5	0.659	60.06	0.812	0.463	5.98	2.359	0.719
P97.5-P99	0.682	59.35	0.833	0.501	5.99	2.459	0.734
P99-P99.5	0.709	58.24	0.841	0.506	5.98	2.549	0.731
P99.5-P99.9	0.747	56.52	0.846	0.492	6.03	2.617	0.718
P99.9-P99.99	0.795	55.51	0.857	0.487	6.14	2.661	0.727
Top 0.01%	0.832	55.74	0.892	0.559	6.33	2.756	0.755
Number of observations	38,025,055	38,025,055	37,097,079	37,097,079	12,469,299	38,025,055	38,025,055

Appendix Table 5
Financial Characteristics

This table reports average financial characteristics of households in different brackets of the net wealth distribution in Sweden over the period 2000-2007. The controlling block dummy variable is set equal to unity if the household owns at least 5% of the voting rights of a company that is either listed, private, or any of these two categories. Monetary amounts are expressed in thousands of Swedish kronor. On 31 December 2004, 1 Swedish krona traded at 0.151 US dollars.

	Average financial characteristics (thousand Swedish kronor)					Owning at least 1 controlling block		
	Gross wealth	Household debt	Labor income	Pension Income	Capital Income	In a listed firm	In a private firm	In any firm
Wealth group								
P0-P10	230	394	204	41	-11	0.0%	1.0%	1.0%
P10-P20	66	44	116	24	-2	0.0%	0.2%	0.2%
P20-P30	239	135	198	38	-5	0.0%	0.7%	0.7%
P30-P40	465	197	198	57	-6	0.0%	1.2%	1.2%
P40-P50	671	218	185	81	-6	0.0%	1.5%	1.5%
P50-P60	986	285	220	94	-6	0.0%	2.2%	2.2%
P60-P70	1,375	329	242	110	-3	0.0%	3.2%	3.2%
P70-P80	1,898	360	264	128	3	0.0%	4.6%	4.6%
P80-P90	2,745	390	294	151	16	0.0%	7.3%	7.3%
P90-P95	4,025	444	339	177	43	0.0%	12.2%	12.2%
P95-P97.5	5,620	532	392	202	84	0.0%	19.1%	19.1%
P97.5-P99	8,122	703	452	227	166	0.0%	30.1%	30.1%
P99-P99.5	12,349	1,015	518	262	335	0.1%	44.9%	44.9%
P99.5-P99.9	23,195	1,815	604	321	827	0.3%	61.7%	61.7%
P99.9-P99.9(	83,607	5,897	747	452	2,763	1.6%	77.2%	77.5%
Top 0.01%	801,015	20,462	1,069	1,051	14,422	11.1%	84.8%	86.7%
Number of observations	38,025,055	38,025,055	38,025,055	38,025,055	38,025,055	38,025,055	38,025,055	38,025,055

Appendix Table 6
Historical Mean and Volatility of Asset Factors

This table reports the risk and return characteristics of the systematic factors used throughout our analysis and provide a comparison with their US counterparts. The global equity and Fama-French factors are obtained from AQR Capital Management (2016), the Swedish equity market index is from Datastream (2016), Sveriges Riksbank (2016), and Waldenström (2014a, 2014b), the currency factor is from Datastream (2016) and Sveriges Riksbank (2016), the Swedish real estate index is from Söderberg, Blöndal, and Edvinsson (2014) and Statistics Sweden (2014b), the US Fama-French factors are from French (2016), and the US real estate index is from Shiller (2016). Columns 1 and 2 report the mean and standard deviation of Swedish factors, computed over the period July 1983-April 2016 for equity factors and over the period 1981-2014 period for the Swedish real estate index. We also report the arithmetic average of risk premia over (3) the sample period of the Swedish household holding data (December 31, 2000 to December 31, 2008) and (4) the entire postwar era (1950 to 2016 for equity factors, 1950 to 2014 for the real estate index). In columns 5 and 6, we report the risk and return characteristics over the 1926 to 2016 period for asset benchmarks relevant to US investors. The size and value factor rows in columns 1, 2 and 4 refer to global Fama-French factors, while those in columns 5 and 6 refer to the US Fama-French factors. All characteristics are expressed in annual units. The standard deviation of returns on the national real estate index is computed by multiplying the standard deviation of the three-year moving average of returns by the square root of three. The Swedish equity market return is expressed in Swedish kronor in excess of the Swedish 1-month Treasury bill. The global equity market return and the US equity market return are expressed in US dollars in excess of the US 1-month Treasury bill.

	Moments of pricing factors					
	Baseline values		Alternative benchmarks			
	Sweden		Sweden		United States	
	1983-2016	1983-2016	1950-2016	2001-2008	1926-2016	1926-2016
	Mean	Standard deviation	Mean	Mean	Mean	Standard deviation
	(1)	(2)	(3)	(4)	(5)	(6)
Equity factors:						
National market index	8.7%	21.0%	8.6%	-1.7%	7.8%	18.6%
Global market index	5.8%	15.5%	-	-1.2%	-	-
Size factor	-0.1%	6.8%	-	2.1%	2.6%	11.1%
Value factor	4.7%	7.7%	-	9.4%	4.8%	12.1%
Currency factor	-1.2%	11.1%	-	0.6%	-	-
Real estate factor:						
FASTPI national index return	5.5%	8.9%	5.7%	8.1%	3.9%	9.2%

Appendix Table 7
Asset Allocation of Liquid Financial Wealth and Funded Pension Wealth

Panel A reports the average share of financial wealth invested in risky financial assets in different brackets of net worth in Sweden over the period 2000-2007. Panel B reports the average share of various asset classes in the portfolio of Swedish life insurance companies from December 31st, 2000 to December 31st, 2007. The numbers are drawn from the annual reports of the Swedish FSA, the AP7 public pension fund, and four life insurance companies: Alecta, AMF, Skandia, and SEB-Gamla Liv. We assume that 50% of the foreign equity portfolio is not hedged against currency risk, as is reported in the annual reports of the largest Swedish life insurers over the sample period.

Panel A: Share of Risky Assets in Complete Financial Portfolio

Wealth group	
P0-P10	10.8%
P10-P20	9.9%
P20-P30	21.0%
P30-P40	18.5%
P40-P50	19.7%
P50-P60	25.3%
P60-P70	29.5%
P70-P80	34.6%
P80-P90	41.0%
P90-P95	47.6%
P95-P97.5	52.4%
P97.5-P99	56.0%
P99-P99.5	58.3%
P99.5-P99.9	58.5%
P99.9-P99.99	57.4%
Top 0.01%	60.0%

Panel B: Share of Funded Pension Investments (2000-2007)

Safe assets (cash, bills, bonds, etc.)	46.0%
Swedish equity	17.7%
Foreign equity	33.1%
not currency-hedged	16.6%
Real estate	3.2%

Appendix Table 8
Performance of Bank Accounts
Effect of Net Worth

This table reports the average performance of household bank accounts in different net worth brackets in Sweden over the period 2000-2007. We consider (1) the spread between the return on the bank account and the return on the Swedish 1-month Treasury bill, (2) the spread between the return on the bank account and the return on the Swedish 1-month Treasury bill weighted by the bank account balance-to-total *financial* wealth ratio, and (3) the spread between the return on the bank account and the return on the Swedish 1-month Treasury bill weighted by bank account balance-to-total *gross* wealth ratio.

	Contribution (in % per year) of bank account performance to return on:		
	Bank account	Financial wealth	Gross wealth
	(1)	(2)	(3)
Wealth group			
P0-P10	-2.84	-2.40	-1.10
P10-P20	-2.78	-2.40	-1.64
P20-P30	-2.65	-1.98	-0.61
P30-P40	-2.66	-2.05	-0.36
P40-P50	-2.60	-1.93	-0.29
P50-P60	-2.51	-1.67	-0.25
P60-P70	-2.44	-1.49	-0.22
P70-P80	-2.37	-1.30	-0.20
P80-P90	-2.29	-1.09	-0.18
P90-P95	-2.22	-0.92	-0.17
P95-P97.5	-2.17	-0.82	-0.16
P97.5-P99	-2.15	-0.77	-0.15
P99-P99.5	-2.14	-0.75	-0.14
P99.5-P99.9	-2.14	-0.75	-0.13
P99.9-P99.99	-2.13	-0.79	-0.10
Top 0.01%	-2.08	-0.78	-0.06

Appendix Table 9
Performance of Bank Accounts
Effect of Cognitive Ability

This table reports estimates of the impact of cognitive ability on the performance of household bank accounts in Sweden over the period 2000-2007. In columns 1 to 3, we report the unconditional marginal effect of possessing a particular IQ score relative to having the median IQ score. In columns 4 to 6, we report the marginal effect of possessing a particular IQ score relative to having the median IQ score after controlling for a set of fifteen dummies corresponding to the brackets of the net wealth distribution used in the rest of the paper. We consider as performance outcomes: the spread between the return on the bank account and the return on the Swedish 1-month Treasury bill (columns 1 and 4), the spread between the return on the bank account and the return on the Swedish 1-month Treasury bill weighted by the ratio of bank account balance-to-total *financial* wealth ratio (columns 2 and 5), and the spread between the return on the bank account and the return on the Swedish 1-month Treasury bill weighted by the bank account balance-to-total *gross* wealth ratio (columns 3 and 6). The IQ score is standardized on a 1 to 9 scale and is available for male members of the household enlisted in Swedish Armed Forces between 1969 and 2009.

	Unconditional effect of IQ			Effect of IQ conditional on net worth		
	Contribution (in % per year) of bank account performance to return on:			Contribution (in % per year) of bank account performance to return on:		
	Bank account	Financial wealth	Gross wealth	Bank account	Financial wealth	Gross wealth
	(1)	(2)	(3)	(4)	(5)	(6)
IQ level						
1	-0.167	-0.527	-0.272	-0.147	-0.371	-0.115
2	-0.131	-0.389	-0.146	-0.116	-0.273	-0.043
3	-0.092	-0.256	-0.087	-0.082	-0.182	-0.025
4	-0.051	-0.137	-0.043	-0.046	-0.101	-0.015
5	REF.	REF.	REF.	REF.	REF.	REF.
6	0.053	0.126	0.029	0.046	0.074	-0.006
7	0.107	0.234	0.048	0.096	0.146	-0.008
8	0.154	0.326	0.065	0.137	0.203	-0.013
9	0.211	0.406	0.067	0.190	0.256	-0.023

Appendix Table 10
International Fama-French Portfolio Loadings
Risky Financial Portfolio

This table reports the average loadings of household risky financial portfolios in different brackets of the net wealth distribution in Sweden over the period 2000-2007. The loadings are computed on: (1) the Swedish stock market, (2) the global stock market, (3) the global size factor, (4) the global value factor, and (5) the currency factor. The Swedish market factor is the excess return on the SIXRX index relative to the Swedish 1-month Treasury bill. The global Fama and French factors are obtained from AQR Capital Management (2016). The currency factor consists of monthly returns on the carry trade in which the investor is long the US 1-month Treasury bill and short the Swedish 1-month Treasury bill. All factor loadings are estimated by a multivariate regression at the asset level.

	Factor loadings of household risky financial wealth				
	Local equity	Global equity	Global size	Global value	Currency
	(1)	(2)	(3)	(4)	(5)
Wealth group					
P0-P10	0.600	0.326	-0.099	-0.195	0.250
P10-P20	0.598	0.311	-0.139	-0.201	0.253
P20-P30	0.609	0.309	-0.135	-0.169	0.238
P30-P40	0.612	0.315	-0.126	-0.130	0.216
P40-P50	0.608	0.303	-0.132	-0.100	0.194
P50-P60	0.612	0.298	-0.132	-0.092	0.187
P60-P70	0.618	0.294	-0.132	-0.081	0.179
P70-P80	0.627	0.287	-0.131	-0.072	0.173
P80-P90	0.643	0.285	-0.121	-0.056	0.169
P90-P95	0.668	0.287	-0.103	-0.036	0.167
P95-P97.5	0.687	0.291	-0.080	-0.015	0.167
P97.5-P99	0.703	0.295	-0.051	0.005	0.167
P99-P99.5	0.718	0.295	-0.019	0.007	0.169
P99.5-P99.9	0.728	0.296	0.018	-0.005	0.170
P99.9-P99.99	0.727	0.296	0.068	-0.006	0.156
Top 0.01%	0.726	0.288	0.109	0.028	0.137

Appendix Table 11
International Fama-French Portfolio Loadings
Stock Portfolio

This table reports the average loadings of household stock portfolios in different brackets of the net wealth distribution in Sweden over the period 2000-2007. The loadings are computed on (1) the Swedish stock market, (2) the global stock market, (3) the global size factor, (4) the global value factor, and (5) the currency factor. The Swedish market factor is the excess return on the SIXRX index relative to the Swedish 1-month Treasury bill. The global Fama and French factors are obtained from AQR Capital Management (2016). The currency factor consists of monthly returns on the carry trade in which the investor is long the US 1-month Treasury bill and short the Swedish 1-month Treasury bill. All factor loadings are estimated by a multivariate regression at the asset level.

	Factor loadings of household stock portfolio				
	Local equity	Global equity	Global size	Global value	Currency
	(1)	(2)	(3)	(4)	(5)
Wealth group					
P0-P10	0.896	0.296	0.177	-0.348	0.049
P10-P20	0.894	0.284	0.118	-0.385	0.053
P20-P30	0.888	0.308	0.053	-0.265	0.039
P30-P40	0.869	0.338	0.042	-0.134	0.005
P40-P50	0.854	0.359	0.019	-0.030	-0.032
P50-P60	0.850	0.361	-0.003	0.002	-0.039
P60-P70	0.844	0.364	-0.027	0.038	-0.045
P70-P80	0.839	0.363	-0.049	0.064	-0.044
P80-P90	0.836	0.360	-0.056	0.103	-0.032
P90-P95	0.838	0.352	-0.041	0.140	-0.006
P95-P97.5	0.842	0.346	-0.015	0.167	0.018
P97.5-P99	0.844	0.337	0.018	0.173	0.042
P99-P99.5	0.848	0.318	0.049	0.137	0.067
P99.5-P99.9	0.855	0.294	0.083	0.069	0.088
P99.9-P99.99	0.845	0.281	0.128	0.035	0.092
Top 0.01%	0.817	0.273	0.150	0.094	0.081

Appendix Table 12
International Fama-French Portfolio Loadings
Fund Portfolio

This table reports the average of household fund portfolio loadings in different brackets of the net wealth distribution in Sweden over the period 2000-2007. The loadings are computed on (1) the Swedish stock market, (2) the global stock market, (3) the global size factor, (4) the global value factor, and (5) the currency factor. The Swedish market factor is the excess return on the SIXRX index relative to the Swedish 1-month Treasury bill. The global Fama and French factors are obtained from AQR Capital Management (2016). The currency factor consists of monthly returns on the carry trade in which the investor is long the US 1-month Treasury bill and short the Swedish 1-month Treasury bill. All factor loadings are estimated by a multivariate regression at the asset level.

	Factor loadings of household fund portfolio				
	Local equity	Global equity	Global size	Global value	Currency
	(1)	(2)	(3)	(4)	(5)
Wealth group					
P0-P10	0.519	0.339	-0.169	-0.161	0.304
P10-P20	0.536	0.320	-0.186	-0.170	0.296
P20-P30	0.535	0.316	-0.181	-0.157	0.288
P30-P40	0.530	0.313	-0.178	-0.150	0.283
P40-P50	0.527	0.293	-0.182	-0.143	0.268
P50-P60	0.527	0.287	-0.179	-0.139	0.262
P60-P70	0.529	0.281	-0.176	-0.137	0.256
P70-P80	0.534	0.276	-0.173	-0.134	0.249
P80-P90	0.543	0.274	-0.167	-0.133	0.246
P90-P95	0.554	0.280	-0.158	-0.133	0.246
P95-P97.5	0.559	0.288	-0.149	-0.133	0.249
P97.5-P99	0.557	0.299	-0.140	-0.133	0.255
P99-P99.5	0.548	0.314	-0.128	-0.135	0.262
P99.5-P99.9	0.529	0.334	-0.112	-0.134	0.273
P99.9-P99.99	0.502	0.353	-0.089	-0.119	0.279
Top 0.01%	0.479	0.368	-0.046	-0.108	0.284

Appendix Table 13
CAPM Beta and Excess Return of Financial Wealth

This table reports the average local CAPM beta and the corresponding expected return of household portfolios in different brackets of the net wealth distribution in Sweden over the period 2000-2007. The local CAPM beta is computed with respect to the Swedish stock market factor. Expected excess returns on the risky portfolio are computed by multiplying the risk loadings in column 1 with the historical mean annual arithmetic excess return on the SIXRX index over the 1983 to 2016 period. Expected excess returns on the complete financial portfolio are computed by multiplying the expected excess return on the risky portfolio by the financial risky share. Excess returns are measured pre-tax and relative to the yield on the Swedish 1-month Treasury bill.

	Market beta			Expected return	
	Risky portfolio (1)	Stock portfolio (2)	Fund portfolio (3)	Risky portfolio (4)	Complete portfolio (5)
Wealth group					
P0-P10	0.762	1.069	0.681	6.60%	0.74%
P10-P20	0.751	1.060	0.689	6.50%	0.65%
P20-P30	0.759	1.049	0.686	6.57%	1.40%
P30-P40	0.761	1.032	0.679	6.59%	1.23%
P40-P50	0.748	1.014	0.665	6.48%	1.28%
P50-P60	0.749	1.006	0.663	6.48%	1.64%
P60-P70	0.751	0.996	0.662	6.50%	1.92%
P70-P80	0.756	0.987	0.664	6.55%	2.27%
P80-P90	0.771	0.980	0.672	6.68%	2.75%
P90-P95	0.796	0.978	0.687	6.89%	3.31%
P95-P97.5	0.817	0.980	0.696	7.07%	3.75%
P97.5-P99	0.836	0.981	0.699	7.24%	4.12%
P99-P99.5	0.853	0.983	0.699	7.39%	4.38%
P99.5-P99.9	0.868	0.989	0.690	7.51%	4.45%
P99.9-P99.99	0.869	0.980	0.673	7.53%	4.35%
Top 0.01%	0.865	0.944	0.658	7.49%	4.50%

Appendix Table 14
Return Loss from Underdiversification

This table reports the average of the return loss from underdiversification (column 1) and its components (columns 2 to 5) in different brackets of the net wealth distribution in Sweden over the period 2000-2007. A household's return loss from underdiversification is the loss in expected return implied by not investing in a portfolio consisting of a well-diversified equity benchmark and the risk-free asset with a risky share chosen so that the portfolio the same standard deviation as the household's portfolio. The SIXRX Swedish stockmarket index is used as the benchmark. The return loss can be decomposed into a stock market participation dummy (column 2) multiplied by the exponential of the sum of three terms: the log risky share of the financial portfolio, the log beta of the risky portfolio and a nonlinear and increasing transformation of the relative Sharpe ratio loss. Columns 1 and 2 are based on the entire sample of Swedish households, and columns 3 to 5 on stock market participants.

			Components of the return loss in the risky portfolio		
	Return loss on complete portfolio	Stock market participation	Risky share $\log(\omega_h)$	Market beta $\log\lvert\beta_h\rvert$	Diversification loss $\log\lvert RSRL_h/(1-RSRL_h)\rvert$
	(1)	(2)	(3)	(4)	(5)
Wealth group					
P0-P10	0.0038	0.3559	-1.8773	-0.5389	-1.1602
P10-P20	0.0030	0.3154	-1.7500	-0.5674	-1.2579
P20-P30	0.0058	0.5229	-1.3889	-0.5110	-1.2967
P30-P40	0.0049	0.4758	-1.4581	-0.4812	-1.3115
P40-P50	0.0047	0.5118	-1.4521	-0.4956	-1.3473
P50-P60	0.0056	0.6276	-1.3744	-0.4818	-1.3660
P60-P70	0.0060	0.7051	-1.3202	-0.4599	-1.4037
P70-P80	0.0065	0.7849	-1.2335	-0.4313	-1.4647
P80-P90	0.0070	0.8592	-1.1169	-0.3778	-1.5516
P90-P95	0.0077	0.9154	-0.9917	-0.3072	-1.6481
P95-P97.5	0.0082	0.9440	-0.8970	-0.2526	-1.7119
P97.5-P99	0.0087	0.9580	-0.8324	-0.2104	-1.7368
P99-P99.5	0.0096	0.9626	-0.7940	-0.1886	-1.6874
P99.5-P99.9	0.0113	0.9603	-0.8172	-0.1862	-1.5741
P99.9-P99.99	0.0134	0.9609	-0.8879	-0.1932	-1.4499
Top 0.01%	0.0169	0.9704	-0.9131	-0.1876	-1.4718

Appendix Table 15
Diversification of Financial Wealth

This table reports the average household idiosyncratic share in different brackets of the net wealth distribution in Sweden over the period 2000-2007. The idiosyncratic share is the ratio of idiosyncratic portfolio variance to total portfolio variance. We report the results for (1) the risky portfolio without asset holdings providing control over more than 5% of voting rights of the issuing company (*controlling blocks*), (2) the stock portfolio, (3) the mutual fund portfolio, and (4) the risky portfolio. The total variance of a portfolio is computed using the historical variance-covariance matrix of all individual securities weighted by household loadings of each factor. The systematic variance of a portfolio is computed by using the historical variance-covariance matrix of the factors weighted by household loadings of each factor; the idiosyncratic variance is obtained by subtracting the systematic variance from the total variance.

	Share of idiosyncratic risk			
	Risky portfolio without controlling blocks	Stock portfolio	Mutual fund portfolio	Risky portfolio
	(1)	(2)	(3)	(4)
Wealth group				
P0-P10	23.6%	53.1%	16.3%	23.7%
P10-P20	22.5%	54.3%	16.5%	22.5%
P20-P30	21.6%	51.6%	15.1%	21.6%
P30-P40	22.5%	50.4%	15.3%	22.6%
P40-P50	23.6%	50.1%	16.4%	23.6%
P50-P60	23.3%	49.4%	16.1%	23.3%
P60-P70	22.8%	48.4%	15.6%	22.9%
P70-P80	22.0%	47.1%	14.9%	22.1%
P80-P90	20.8%	45.0%	13.5%	20.8%
P90-P95	19.7%	42.2%	12.0%	19.7%
P95-P97.5	19.3%	39.4%	11.3%	19.3%
P97.5-P99	19.6%	36.7%	11.1%	19.6%
P99-P99.5	20.5%	34.7%	11.3%	20.6%
P99.5-P99.9	22.3%	34.3%	12.2%	22.4%
P99.9-P99.99	24.7%	34.5%	13.9%	25.1%
Top 0.01%	26.1%	36.0%	15.3%	28.7%

Appendix Table 16
Risk-Adjusted Performance of Risky Financial Assets
Effect of Net Worth

This table reports the average household portfolio alpha in different brackets of the net wealth distribution in Sweden over the period 2000-2007. We compute the risk-adjusted performance of the stock portfolio over 12 months (columns 1 and 2), and the risk-adjusted performance of the fund portfolio over 12 months (columns 3 and 4). The calculations are based on the 5-factor global asset pricing model described in the main text. For each regression, we report the alpha coefficient within each wealth bracket as well as the statistical significance of the difference between the bracket's alpha and the alpha coefficient in the median bracket of net worth (P40-P50). Alphas are computed monthly and are expressed in natural annual units. We assume that households rebalance their portfolios monthly to keep security weights constant during each calendar year. Monthly alphas are winsorized at the 1% level and standard errors are clustered at the calendar month level.

	Stock portfolio alpha		Fund portfolio alpha	
	Estimate	p-value vs. median	Estimate	p-value vs. median
	(1)	(2)	(3)	(4)
Wealth group				
P0-P10	2.75	0.27	0.18	0.43
P10-P20	3.63	0.12	0.07	0.86
P20-P30	2.65	0.11	0.03	0.86
P30-P40	1.50	0.13	0.06	0.87
P40-P50	0.71	REF.	0.04	REF.
P50-P60	0.81	0.69	0.03	0.61
P60-P70	0.67	0.94	0.03	0.68
P70-P80	0.54	0.81	0.02	0.67
P80-P90	0.29	0.65	0.01	0.75
P90-P95	-0.14	0.49	0.03	0.88
P95-P97.5	-0.44	0.45	0.04	0.98
P97.5-P99	-0.58	0.48	0.03	0.96
P99-P99.5	-0.24	0.64	0.04	0.98
P99.5-P99.9	0.69	0.99	0.11	0.80
P99.9-P99.99	1.64	0.70	0.22	0.63
Top 0.01%	2.25	0.57	0.66	0.17

Appendix Table 17
Risk and Return of Real Estate Wealth
Additional Characteristics

This table reports the average characteristics of the household real estate wealth return in different brackets of the net wealth distribution in Sweden over the period 2000-2007. We compute: (1) the loading of the real estate portfolio return on the Swedish real estate index, (2) the standard deviation of the real estate portfolio return due to property-specific risk, (3) the ratio of the property-specific variance to total idiosyncratic portfolio variance, (4) the yearly expected excess return on household real estate wealth net of the imputed rental yield for owner-occupied residences, estimated with a user cost of real estate that assumes no predictability in risk-adjusted capital gain returns, (5) the yearly expected excess return on household real estate wealth net of the imputed rental yield, estimated with a user cost of real estate that assumes predictable risk-adjusted capital gains, and (6) the yearly expected excess return on household real estate wealth net of the imputed rental yield for owner-occupied residences, where the rental yield and expected capital gains returns are equal to their long-term average. Property-specific risk refers to risk uncorrelated to the Swedish real estate index and uncorrelated to the risk of other properties in the same locality and property type. Expected returns are measured pre-tax and in excess of the yield on the Swedish 1-month Treasury bill.

	Measures of real estate risk			Excess return excluding owner-occupation benefit (% per year)		
	Real estate beta	Property-specific return: Standard deviation (% per year)	Property-specific return: Share of idiosyncratic risk (%)	User cost without return predictabiliy	User cost with return predictabiliy	Model-free
	(1)	(2)	(3)	(4)	(5)	(6)
Wealth group						
P0-P10	0.92	7.93	71.6	2.26	2.38	2.54
P10-P20	0.87	8.11	73.8	1.89	1.99	2.11
P20-P30	0.90	8.03	73.6	2.05	2.17	2.30
P30-P40	0.93	7.92	72.8	2.26	2.39	2.52
P40-P50	0.95	7.82	71.9	2.38	2.51	2.66
P50-P60	0.97	7.72	71.2	2.49	2.63	2.80
P60-P70	0.98	7.57	70.2	2.61	2.77	2.96
P70-P80	1.00	7.37	68.9	2.76	2.94	3.17
P80-P90	1.04	7.00	66.7	3.06	3.25	3.54
P90-P95	1.07	6.60	63.7	3.40	3.60	3.96
P95-P97.5	1.09	6.36	61.5	3.63	3.86	4.27
P97.5-P99	1.09	6.30	60.1	3.81	4.07	4.52
P99-P99.5	1.09	6.38	60.4	3.91	4.18	4.61
P99.5-P99.9	1.09	6.60	62.6	3.96	4.26	4.59
P99.9-P99.99	1.08	7.00	65.3	4.16	4.48	4.76
Top 0.01%	1.12	6.78	63.3	4.38	4.75	5.04

Appendix Table 18
International Fama-French Portfolio Loadings
Private Equity Portfolio

This table reports the average household private equity portfolio loadings in different brackets of the net wealth distribution in Sweden over the period 2000-2007. The loadings are computed on (1) the Swedish stock market, (2) the global stock market, (3) the global size factor, (4) the global value factor, and (5) the currency factor. The Swedish market factor is the excess return on the SIXRX index relative to the Swedish 1-month Treasury bill. The global Fama and French factors are obtained from AQR Capital Management (2016). The currency factor consists of monthly returns on the carry trade in which the investor is long the US 1-month Treasury bill and short the Swedish 1-month Treasury bill.

	Factor loadings of household private equity portfolio				
	Local equity	Global equity	Global size	Global value	Currency
	(1)	(2)	(3)	(4)	(5)
Wealth group					
P0-P10	0.878	0.436	0.748	0.372	-0.143
P10-P20	0.869	0.417	0.763	0.282	-0.136
P20-P30	0.756	0.388	0.646	0.334	-0.127
P30-P40	0.732	0.383	0.629	0.377	-0.126
P40-P50	0.722	0.382	0.625	0.437	-0.131
P50-P60	0.695	0.379	0.593	0.467	-0.125
P60-P70	0.683	0.368	0.580	0.470	-0.122
P70-P80	0.663	0.359	0.557	0.469	-0.116
P80-P90	0.646	0.342	0.539	0.444	-0.106
P90-P95	0.632	0.329	0.525	0.421	-0.096
P95-P97.5	0.629	0.322	0.522	0.409	-0.090
P97.5-P99	0.625	0.317	0.520	0.407	-0.085
P99-P99.5	0.617	0.315	0.525	0.410	-0.085
P99.5-P99.9	0.609	0.319	0.550	0.424	-0.094
P99.9-P99.99	0.608	0.326	0.594	0.462	-0.107
Top 0.01%	0.647	0.308	0.616	0.352	-0.089

Appendix Table 19
Historical Returns on Public and Private Equity

This table reports the historical returns and sample statistics of: (1) the portfolio of all private equity shares held by Swedish households at year-end, (2) a matched portfolio of public firms with the same systematic and idiosyncratic risk exposures and the same leverage as private firms, and (3) the SIXRX Swedish public equity index. We also report (4) the difference in historical excess returns between the private equity index and the matched public index, and (5) the difference in historical excess return between the private equity index and the SIXRX Swedish public equity index. The historical return on the matched portfolio is equal to the risk loadings of the value-weighted private equity holdings displayed in Appendix Table 18 multiplied by the historical realization of the corresponding factor returns. The equity returns of each private firm are winsorized at the 0.01% level.

	Excess return			Private equity risk-adjusted performance	
	Private equity index	Matched public index	Unmatched public index	Matched public index	Unmatched public index
	(1)	(2)	(3)	(4)	(5)
Year					
2001	1.69%	-5.58%	-18.90%	7.27%	20.59%
2002	-21.38%	-22.15%	-40.01%	0.76%	18.63%
2003	50.63%	63.05%	30.99%	-12.42%	19.64%
2004	32.94%	29.13%	18.57%	3.81%	14.37%
2005	35.18%	25.42%	34.57%	9.76%	0.61%
2006	27.29%	27.17%	25.83%	0.12%	1.46%
2007	-15.77%	-6.62%	-6.05%	-9.15%	-9.72%
2008	-53.63%	-45.71%	-43.14%	-7.92%	-10.49%
Sample statistics					
Mean	7.12%	8.09%	0.23%	-0.97%	6.89%
Standard error	12.54%	12.20%	11.20%	2.85%	4.61%
Standard deviation	35.48%	34.52%	31.68%		
Sharpe ratio	0.20	0.23	0.01		
Correlation of private and public index		0.97	0.93		

Appendix Table 20
Accounting Performance of Private Equity

This table reports the average accounting performance of household private equity portfolios in different brackets of the net wealth distribution in Sweden over the period 2000-2007. We report: (1) the ratio of annual earnings (after corporate taxes) to the book value of equity, (2) the ratio of annual earnings (after corporate taxes) to the market value of equity, (3) the ratio of annual earnings (in thousands of Swedish kronor, after corporate taxes) to the number of workers working for owned companies.

	Return on on equity (1)	Ratio of earnings to market value of equity (2)	Earnings per worker in thousand kronor (3)
Wealth group			
P0-P10	19.60%	11.51%	31.6
P10-P20	19.81%	11.44%	19.1
P20-P30	17.54%	10.43%	17.0
P30-P40	17.77%	10.21%	22.0
P40-P50	17.95%	10.33%	25.3
P50-P60	18.55%	10.48%	28.8
P60-P70	18.66%	10.60%	31.9
P70-P80	18.46%	10.60%	35.8
P80-P90	18.23%	10.57%	42.4
P90-P95	17.92%	10.51%	53.3
P95-P97.5	18.20%	10.65%	68.4
P97.5-P99	17.90%	10.34%	91.2
P99-P99.5	17.95%	9.99%	124.8
P99.5-P99.9	18.53%	9.60%	180.6
P99.9-P99.99	18.62%	8.81%	266.6
Top 0.01%	17.80%	8.51%	329.6

Appendix Table 21
Allocation of Gross Wealth

This table reports the average share of gross wealth held by Swedish households in different brackets of the net wealth distribution in Sweden over the period 2000-2007. The share invested in each asset class is reported for: (1) cash, defined as the sum of bank account balances and money market funds, (2) risky financial assets, (3) pension wealth, (4) residential real estate, (5) commercial real estate, and (6) private equity. In (7), we display the leverage ratio, defined as the ratio of total household debt to gross wealth.

	Share of gross wealth (%)						Debt-to-gross wealth ratio (%)
	Cash	Risky financial wealth	Pension wealth	Residential real estate	Commercial real estate	Private equity	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Wealth group							
P0-P10	39.71	4.94	38.37	15.77	0.97	0.24	> 100
P10-P20	58.63	8.02	28.18	4.97	0.15	0.06	20.02
P20-P30	26.15	14.93	40.25	17.82	0.57	0.28	25.89
P30-P40	15.46	8.78	48.05	26.34	0.98	0.40	23.24
P40-P50	13.15	7.02	49.19	28.91	1.30	0.42	19.07
P50-P60	12.02	8.22	39.16	37.94	2.08	0.59	19.34
P60-P70	11.23	9.20	32.88	42.84	3.05	0.79	17.40
P70-P80	10.79	10.64	27.97	45.01	4.48	1.11	14.63
P80-P90	10.11	12.76	23.26	45.12	6.95	1.79	11.47
P90-P95	9.26	15.29	18.92	42.98	10.32	3.23	9.11
P95-P97.5	8.57	17.48	15.49	39.17	13.75	5.55	7.89
P97.5-P99	7.94	19.51	11.96	33.34	16.99	10.25	7.15
P99-P99.5	7.33	21.48	8.35	26.03	18.24	18.57	6.66
P99.5-P99.9	6.78	21.63	4.97	17.20	17.40	32.02	6.26
P99.9-P99.99	5.08	18.27	1.68	6.91	18.79	49.27	5.76
Top 0.01%	2.94	18.25	0.41	2.04	14.44	61.93	3.90

Appendix Table 22
Factor Loadings
Gross Wealth

This table reports the average loadings of household gross wealth in different brackets of the net wealth distribution in Sweden over the period 2000-2007. The loadings are computed relative to (1) the Swedish stock market, (2) the global stock market, (3) the global size factor, (4) the global value factor, (5) the currency factor, and (6) the Swedish real estate market. In (7), we report the idiosyncratic volatility of the return on gross wealth. In (8), we report the expected yearly arithmetic pre-tax return on household gross wealth in excess of the yield on the Swedish 1-month Treasury bill. The Swedish equity market factor is the excess return on the SIXRX index. The global Fama and French factors are obtained from AQR Capital Management (2016). The currency factor consists of monthly returns on the carry trade in which the investor is long the US 1-month Treasury bill and short the Swedish 1-month Treasury bill. The Swedish real estate market factor is the return on the FASTPI real estate index. All factor loadings are estimated by a multivariate regression at the asset level.

	Factor loadings of household gross wealth						Idiosyncratic
	Local equity	Global equity	Global size	Global value	Currency	Swedish real estate	return volatility
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Wealth group							
P0-P10	0.092	0.136	-0.002	-0.010	0.072	0.156	1.8%
P10-P20	0.090	0.110	-0.010	-0.014	0.062	0.051	0.8%
P20-P30	0.155	0.173	-0.017	-0.022	0.099	0.180	2.5%
P30-P40	0.136	0.183	-0.007	-0.010	0.097	0.277	3.0%
P40-P50	0.127	0.181	-0.005	-0.006	0.094	0.315	3.1%
P50-P60	0.115	0.150	-0.005	-0.004	0.078	0.418	4.0%
P60-P70	0.110	0.131	-0.005	-0.002	0.069	0.483	4.5%
P70-P80	0.112	0.119	-0.005	0.001	0.062	0.526	4.8%
P80-P90	0.122	0.111	-0.003	0.005	0.057	0.567	5.1%
P90-P95	0.143	0.109	0.005	0.014	0.053	0.593	5.6%
P95-P97.5	0.171	0.112	0.020	0.029	0.050	0.593	6.3%
P97.5-P99	0.212	0.123	0.051	0.056	0.043	0.563	7.8%
P99-P99.5	0.273	0.145	0.105	0.100	0.032	0.493	10.4%
P99.5-P99.9	0.346	0.182	0.198	0.167	0.009	0.385	14.7%
P99.9-P99.99	0.413	0.223	0.331	0.254	-0.033	0.283	21.2%
Top 0.01%	0.515	0.240	0.434	0.224	-0.044	0.177	26.5%

Appendix Table 23
Factor Loadings and Debt Costs
Net Wealth

This table reports the average loadings of household net wealth in different brackets of the net wealth distribution in Sweden over the period 2000-2007. The loadings are computed relative to (1) the Swedish stock market, (2) the global stock market, (3) the global size factor, (4) the global value factor, (5) the currency factor, and (6) the Swedish real estate market. In (7), we report the idiosyncratic volatility of the return on net wealth. In (8), we report the level of the interest rate spread weighted by the debt-to-net-wealth ratio for each household. The Swedish equity market factor is the excess return on the SIXRX index. The global Fama and French factors are obtained from AQR Capital Management (2016). The currency factor consists of monthly returns on the carry trade in which the investor is long the US 1-month Treasury bill and short the Swedish 1-month Treasury bill. The Swedish real estate market factor is the return on the FASTPI real estate index. All factor loadings are estimated by a multivariate regression at the asset level.

	Factor loadings of household net wealth						Idiosyncratic	Impact of
	Local equity	Global equity	Global size	Global value	Currency	Swedish real estate	return volatility	debt on net wealth return
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Wealth group								
P10-P20	0.188	0.257	-0.013	-0.024	0.139	0.278	3.36%	3.58%
P20-P30	0.232	0.278	-0.018	-0.030	0.154	0.734	8.03%	3.06%
P30-P40	0.180	0.241	-0.006	-0.014	0.128	0.779	7.59%	1.91%
P40-P50	0.156	0.217	-0.004	-0.007	0.113	0.666	6.16%	1.19%
P50-P60	0.141	0.181	-0.004	-0.005	0.094	0.726	6.58%	0.96%
P60-P70	0.132	0.156	-0.004	-0.002	0.081	0.725	6.48%	0.71%
P70-P80	0.131	0.139	-0.003	0.001	0.071	0.708	6.29%	0.50%
P80-P90	0.138	0.126	0.000	0.007	0.063	0.697	6.20%	0.34%
P90-P95	0.157	0.120	0.008	0.016	0.057	0.689	6.46%	0.24%
P95-P97.5	0.185	0.122	0.024	0.032	0.053	0.671	7.17%	0.19%
P97.5-P99	0.228	0.132	0.058	0.061	0.045	0.631	8.71%	0.17%
P99-P99.5	0.291	0.155	0.116	0.107	0.033	0.554	11.39%	0.15%
P99.5-P99.9	0.366	0.193	0.211	0.177	0.009	0.442	15.89%	0.13%
P99.9-P99.99	0.432	0.233	0.347	0.264	-0.035	0.342	22.50%	0.11%
Top 0.01%	0.530	0.246	0.445	0.230	-0.043	0.218	27.48%	0.06%

Appendix Table 24
Expected Return on Total Wealth: Robustness Checks

This table reports the average expected return on household gross and net wealth in different brackets of the net wealth distribution in Sweden over the period 2000-2007, using alternative return computations and/or definitions of wealth. In columns 1 and 2 of Panel A, we compute expected returns with our baseline methodology. In columns 3 and 4 of Panel A, we compute expected returns using the average risk premia observed up until the year when wealth holdings are measured. In columns 5 and 6 of Panel A, we compute expected returns using the pension imputation method suggested by Saez and Zucman (2016). In columns 1 and 2 of Panel B, we report expected returns on wealth other than pension wealth. In columns 3 and 4 of Panel B, we report expected returns on wealth other than private equity wealth. Excess returns are measured pre-tax and relative to the yield on the Swedish 1-month Treasury bill.

Panel A: Baseline, Look-Ahead Bias, and Pension Imputation

	Expected excess return (% per year)					
	Baseline methodology		Factor returns measured without look-ahead bias		Saez-Zucman pension imputation method	
	Gross wealth (1)	Net wealth (2)	Gross wealth (3)	Net wealth (4)	Gross wealth (5)	Net wealth (6)
Wealth group						
P0-P10	2.23	-	2.13	-	2.08	-
P10-P20	1.53	0.43	1.46	0.28	2.69	1.95
P20-P30	3.01	3.81	2.87	3.62	3.16	3.65
P30-P40	3.44	4.64	3.30	4.50	3.49	4.71
P40-P50	3.56	4.52	3.44	4.40	3.65	4.66
P50-P60	3.81	4.74	3.73	4.66	3.82	4.73
P60-P70	4.00	4.81	3.94	4.75	4.04	4.87
P70-P80	4.19	4.85	4.15	4.81	4.22	4.90
P80-P90	4.49	5.01	4.47	4.99	4.49	5.03
P90-P95	4.86	5.29	4.86	5.30	4.85	5.30
P95-P97.5	5.21	5.61	5.24	5.64	5.21	5.62
P97.5-P99	5.64	6.04	5.71	6.12	5.66	6.07
P99-P99.5	6.18	6.61	6.31	6.74	6.22	6.65
P99.5-P99.9	6.85	7.32	7.05	7.52	6.89	7.36
P99.9-P99.99	7.65	8.15	7.92	8.43	7.66	8.17
Top 0.01%	7.92	8.30	8.06	8.44	7.92	8.30

Appendix Table 24 – *Continued*
Expected Return on Total Wealth: Robustness Checks

Panel B: Without Pension Savings or Private Equity

	Expected excess return (% per year)			
	Excluding pensions		Excluding private equity	
	Gross wealth	Net wealth	Gross wealth	Net wealth
	(1)	(2)	(3)	(4)
Wealth group				
P0-P10	1.59	-	2.25	-
P10-P20	0.67	-	1.50	0.27
P20-P30	0.70	-0.87	2.97	3.73
P30-P40	1.84	2.62	3.40	4.62
P40-P50	2.98	4.89	3.52	4.49
P50-P60	3.71	5.56	3.76	4.70
P60-P70	4.15	5.61	3.94	4.76
P70-P80	4.46	5.59	4.11	4.78
P80-P90	4.83	5.65	4.37	4.89
P90-P95	5.21	5.82	4.67	5.08
P95-P97.5	5.54	6.06	4.91	5.29
P97.5-P99	5.94	6.42	5.16	5.51
P99-P99.5	6.43	6.92	5.42	5.79
P99.5-P99.9	7.04	7.54	5.73	6.18
P99.9-P99.99	7.73	8.24	6.16	6.88
Top 0.01%	7.94	8.32	6.61	7.49

Appendix Table 25
Capital Taxes
Decomposition by Time Period and Type of Tax

This table reports the average characteristics of capital taxes paid by households in different brackets of the net wealth distribution in Sweden over the period 2000-2007. Personal taxes on gross wealth include capital income taxes, taxes on net capital gains, property taxes, and the wealth tax (prior to 2006). Taxes on net wealth include taxes on gross wealth minus mortgage interest deductions. In columns 1 and 2 of Panel A, we consider the 2001 to 2006 period and report the personal capital taxes (wealth and property taxes included) paid during the year expressed as a proportion of the wealth held at the beginning of the year. For the same period, columns 3 and 4 of Panel A report the personal capital taxes net of wealth and property taxes paid during the year expressed as a proportion of the wealth held at the beginning of the year. In columns 5 and 6 of Panel A, we consider years 2007 and 2008 and compute personal capital taxes (wealth and property taxes included) paid during the year expressed as a proportion the wealth held at the beginning of the year. For the same period, columns 7 and 8 of Panel A report personal capital taxes net of wealth and property taxes expressed as a proportion of the wealth held at the beginning of the year. In columns 1 and 2 of Panel B, we compute the corporate taxes paid by household portfolio companies during the year expressed as a proportion of household wealth at the beginning of the year. Tax rates are winsorized at the 0.1% level.

Panel A: Subperiods 2001-2006 and 2007-2008

	2001 to 2006				2007 and 2008			
	Personal tax rate		Personal tax rate excluding taxes on capital stock		Personal tax rate		Personal tax rate excluding taxes on capital stock	
	% of gross wealth	% of net wealth	% of gross wealth	% of net wealth	% of gross wealth	% of net wealth	% of gross wealth	% of net wealth
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Wealth group								
P0-P10	-3.74	-	-3.95	-	-6.51	-	-6.66	-
P10-P20	-0.52	-3.74	-0.60	-3.94	-0.77	-6.73	-0.82	-6.87
P20-P30	0.10	-2.58	0.00	-2.94	0.14	-2.74	0.10	-2.90
P30-P40	0.38	-1.22	0.28	-1.55	0.36	-1.31	0.31	-1.45
P40-P50	0.74	-0.21	0.63	-0.47	0.66	-0.28	0.61	-0.39
P50-P60	0.78	0.07	0.63	-0.21	0.68	-0.03	0.62	-0.14
P60-P70	0.83	0.36	0.66	0.09	0.73	0.27	0.67	0.17
P70-P80	0.91	0.62	0.71	0.35	0.81	0.52	0.74	0.43
P80-P90	1.02	0.86	0.80	0.59	0.92	0.76	0.85	0.67
P90-P95	1.15	1.07	0.89	0.78	1.03	0.95	0.96	0.87
P95-P97.5	1.29	1.25	0.97	0.90	1.12	1.08	1.05	1.00
P97.5-P99	1.44	1.44	1.05	1.02	1.17	1.15	1.11	1.09
P99-P99.5	1.57	1.60	1.13	1.13	1.19	1.20	1.14	1.15
P99.5-P99.9	1.62	1.67	1.19	1.21	1.20	1.23	1.16	1.18
P99.9-P99.99	1.37	1.45	1.03	1.08	0.95	0.97	0.92	0.93
Top 0.01%	0.99	1.01	0.74	0.74	0.65	0.65	0.63	0.63

Appendix Table 25 – *Continued*

Capital Taxes

Decomposition by Time Period and Type of Tax

Panel B: Full Period 2001-2008

	Full period	
	Corporate tax rate	
	% of gross wealth	% of net wealth
	(1)	(2)
Wealth group		
P0-P10	0.41	-
P10-P20	0.35	0.73
P20-P30	0.55	0.84
P30-P40	0.54	0.71
P40-P50	0.53	0.64
P50-P60	0.46	0.56
P60-P70	0.42	0.51
P70-P80	0.41	0.48
P80-P90	0.42	0.48
P90-P95	0.47	0.52
P95-P97.5	0.54	0.60
P97.5-P99	0.68	0.75
P99-P99.5	0.90	0.98
P99.5-P99.9	1.18	1.27
P99.9-P99.99	1.37	1.46
Top 0.01%	1.45	1.50

Appendix Table 26
Dispersion of Tax Rates

This table reports various moments of the tax rate (amount of capital taxes paid annually over initial wealth) in the Swedish population both across the entire population and in different brackets of the net wealth distribution in Sweden over the 2000-2007 period. In columns 1 and 2, we report the standard deviation of the tax rate defined either as gross taxes on gross wealth or as taxes net of subsidies over net worth. In columns 3 and 4, we report the slope coefficient of a simple OLS regression of the tax rate on the household's historical return. In columns 5 and 6, we report the slope coefficient of a simple OLS regression of the tax rate on the household's expected return. Household returns are winsorized at the 0.01% level and tax rates are winsorized at the 0.1% level. All returns are in excess of the rate of return on the Swedish 1-month Treasury bill.

	Standard deviation of tax rate		Marginal impact historical return on tax rate		Marginal impact expected return on tax rate	
	% of gross wealth	% of net wealth	% of gross wealth	% of net wealth	% of gross wealth	% of net wealth
	(1)	(2)	(3)	(4)	(5)	(6)
Full population	3.99	6.09	0.019	0.028	0.307	0.193
Share of within-group dispersion	0.915	0.975				
Wealth group						
P0-P10	9.54	-	0.096	-	0.944	-
P10-P20	4.93	23.16	0.003	0.218	0.023	0.570
P20-P30	2.80	14.22	-0.004	0.134	-0.005	0.572
P30-P40	1.97	7.55	-0.012	-0.029	-0.110	-0.047
P40-P50	1.80	4.85	-0.025	-0.061	-0.224	-0.292
P50-P60	1.78	3.53	-0.026	-0.065	-0.233	-0.348
P60-P70	1.72	2.99	-0.021	-0.050	-0.209	-0.333
P70-P80	1.68	2.55	-0.015	-0.035	-0.173	-0.301
P80-P90	1.67	2.27	-0.008	-0.022	-0.129	-0.256
P90-P95	1.70	2.09	-0.003	-0.010	-0.084	-0.192
P95-P97.5	1.78	2.07	0.000	-0.002	-0.052	-0.121
P97.5-P99	1.96	2.15	0.003	0.001	-0.029	-0.070
P99-P99.5	2.28	2.38	0.004	0.004	-0.021	-0.034
P99.5-P99.9	2.69	2.79	0.003	0.005	-0.043	-0.016
P99.9-P99.99	2.76	3.21	0.003	0.004	-0.032	-0.024
Top 0.01%	2.04	3.20	0.004	0.003	-0.006	-0.010

Appendix Table 27
Determinants of Household Debt Cost

This table reports the estimates from a regression of the cost of debt (in excess of the risk-free rate) on net worth bracket dummies, debt deciles, the log debt coverage, and year fixed effects. The debt coverage is the ratio of gross real estate wealth to outstanding debt. The debt deciles are dummies for the rank in the distribution of the absolute level of debt outstanding. The coverage ratio is capped below at 15% of debt outstanding. The sample comprises all households with a debt level representing at least 1% or more of total assets.

	Dependent variable: debt cost in excess of risk-free rate			
	(1)	(2)	(3)	(4)
Debt coverage in logs			-0.59	-0.43
Debt deciles				
P0-P10		6.13		5.87
P10-P20		2.75		2.60
P20-P30		1.56		1.49
P30-P40		0.71		0.69
P40-P50		REF.		REF.
P50-P60		-0.46		-0.47
P60-P70		-0.70		-0.74
P70-P80		-0.83		-0.91
P80-P90		-0.97		-1.10
P90-P100		-1.22		-1.45
Wealth group				
P0-P10	-0.93	-1.34	-1.83	-1.99
P10-P20	2.51	-0.30	1.60	-0.88
P20-P30	1.40	0.05	0.79	-0.36
P30-P40	0.51	0.09	0.24	-0.10
P40-P50	REF.	REF.	REF.	REF.
P50-P60	-0.90	-0.42	-0.61	-0.22
P60-P70	-1.47	-0.73	-0.95	-0.36
P70-P80	-1.84	-0.94	-1.13	-0.44
P80-P90	-2.09	-1.07	-1.19	-0.44
P90-P95	-2.25	-1.12	-1.21	-0.38
P95-P97.5	-2.34	-1.08	-1.24	-0.28
P97.5-P99	-2.40	-0.99	-1.29	-0.17
P99-P99.5	-2.48	-0.91	-1.44	-0.12
P99.5-P99.9	-2.61	-0.89	-1.71	-0.16
P99.9-P99.99	-2.92	-1.02	-2.19	-0.37
Top 0.01%	-3.40	-1.46	-2.84	-0.90
Adjusted R^2	0.083	0.216	0.095	0.222

Appendix Table 28
Expected Wealth Returns of US and Swedish Households

This table reports the average expected excess return on household wealth in different brackets of the net wealth distribution in the US and Sweden. In columns 1 to 2, we provide the expected return using data from the 1998, 2001, 2004 and 2007 waves of the US Survey of Consumer Finances (Federal Reserve Board 2007). We assume that risky pension wealth, financial wealth and private equity wealth earn the same expected return as US aggregate equity over the period 1981-2016. We also assume that real estate wealth earns the same expected returns as the US aggregate real estate market over the period 1981-2016, while all other assets earn the risk-free rate. In columns 3 and 4, we provide the expected return using the Swedish micro data as if only the level of detail of the SCF was available. We assume that risky financial wealth earns the same expected return as Swedish pension wealth invested in equities, that real estate wealth earns the same expected return as the Swedish aggregate real estate market over the period 1981-2014, that private equity wealth earns the same expected return as the Swedish equity market portfolio, while all other assets earn the risk-free rate. In columns 5 and 6, we provide the expected return using the same Swedish micro data and the same method as in Tables 1 and 2 of the main text. In all three cases, we measure the cost of debt as is reported either in the SCF or in the Swedish administrative data where applicable. Expected returns are expressed in excess of the rate of the return on either the US 1-month Treasury bill (columns 1 and 2) or the Swedish 1-month Treasury bill (columns 3 to 6).

	Expected return (% per year)					
	United States		Sweden			
	Survey of Consumer Finances		Imputed data		Administrative data	
	Gross wealth (1)	Net wealth (2)	Gross wealth (3)	Net wealth (4)	Gross wealth (5)	Net wealth (6)
Wealth group						
P0-P10	2.08	-	2.43	-	2.23	-
P10-P20	1.08	-	1.64	0.82	1.53	0.43
P20-P30	2.19	-0.38	3.28	4.65	3.01	3.81
P30-P40	3.57	3.13	3.69	5.24	3.44	4.64
P40-P50	3.68	3.82	3.79	4.91	3.56	4.52
P50-P60	3.78	4.02	4.06	5.09	3.81	4.74
P60-P70	3.72	3.90	4.24	5.07	4.00	4.81
P70-P80	3.88	4.16	4.37	5.03	4.19	4.85
P80-P90	4.15	4.37	4.54	5.04	4.49	5.01
P90-P95	4.39	4.57	4.74	5.14	4.86	5.29
P95-P97.5	4.83	5.03	4.95	5.31	5.21	5.61
P97.5-P99	5.08	5.23	5.24	5.59	5.64	6.04
P99-P99.5	5.33	5.43	5.64	6.01	6.18	6.61
P99.5-P99.9	5.76	5.86	6.18	6.56	6.85	7.32
P99.9-P99.99	5.98	6.04	6.88	7.27	7.65	8.15
Top 0.01%	6.60	6.62	7.45	7.74	7.92	8.30

Appendix Table 29
Explanatory Power of Household Fixed Effects
Components of Wealth Returns

This table reports statistics on the explanatory power of year and household fixed effects for various measures of returns. We consider the historical excess return on gross and net wealth, the expected return on gross and net wealth, the risk-adjusted return on gross and net wealth, and the ratio of capital taxes over either gross or net wealth. All quantities are computer over the period 2001-2008. We display: (1) the adjusted R^2 of a regression with year fixed effects, (2) the adjusted R^2 of a regression with both household and year, (3) the marginal explanatory power of household fixed effects, defined as the difference between the first two columns, and (4) the sample standard deviation.

	Explanatory power of year fixed effects (1)	Explanatory power of year and household fixed effects (2)	Marginal explanatory power of household fixed effects (3)	Sample standard deviation (4)
Historical excess return				
Gross wealth	0.438	0.486	0.047	11.31%
Net wealth	0.293	0.415	0.122	19.17%
Expected excess return				
Gross wealth	0.016	0.768	0.752	1.82%
Net wealth	0.008	0.521	0.512	5.36%
Risk-adjusted performance				
Gross wealth	0.003	0.005	0.002	6.11%
Net wealth	0.002	0.047	0.045	10.70%
Capital tax rate				
Gross wealth	0.002	0.397	0.395	3.98%
Net wealth	0.002	0.419	0.417	6.09%

Appendix Table 30
Serial Correlation of Returns to Wealth

This table reports the Wooldridge (2002) correlation statistic designed identify the presence of residual serial correlation in wealth returns once household fixed effects are accounted for. The statistic is the intra-household correlation between the first difference in returns in a given year and the first difference in returns from the previous year. In the absence of serial correlation, the statistic should not be significantly different from -0.5. In order to account for the effect of serial correlation across households within a given year, we use a two-step procedure: first, we compute the correlation statistic separately for each year over the period 2003-2008; then we compute and report below the average correlation statistic across these six years and the standard error of this average. We report the statistic separately for historical returns on gross and net wealth. The statistics within a given wealth bracket are computed conditional on the wealth rank upon entry in the panel.

	Serial correlation of first difference in historical returns			
	Gross wealth		Net wealth	
	Correlation coefficient	Standard error	Correlation coefficient	Standard error
	(1)	(2)	(3)	(4)
Wealth group				
Full population	-0.461	0.066	-0.440	0.071
P0-P10	-0.462	0.058	-	-
P10-P20	-0.439	0.079	-0.394	0.074
P20-P30	-0.451	0.075	-0.432	0.087
P30-P40	-0.455	0.070	-0.447	0.073
P40-P50	-0.464	0.073	-0.465	0.065
P50-P60	-0.458	0.068	-0.457	0.060
P60-P70	-0.468	0.067	-0.469	0.060
P70-P80	-0.469	0.059	-0.475	0.055
P80-P90	-0.462	0.066	-0.473	0.062
P90-P95	-0.473	0.061	-0.488	0.057
P95-P97.5	-0.472	0.061	-0.480	0.058
P97.5-P99	-0.493	0.053	-0.504	0.048
P99-P99.5	-0.502	0.057	-0.511	0.051
P99.5-P99.9	-0.508	0.046	-0.503	0.048
P99.9-P99.99	-0.512	0.056	-0.506	0.070
Top 0.01%	-0.524	0.072	-0.514	0.079

Appendix Table 31
The Persistence of Expected Returns Along the Wealth Distribution

This table reports the average expected excess wealth return in year *t+n* for households in different brackets of the net wealth distribution at *t*, where $n \in \{0, 1, 2, 4\}$. Columns 1 and 2 display the expected returns on gross and net wealth computed in the year the wealth rank is measured. Columns 3 and 4 display the expected returns on gross and net wealth 1 year after the wealth rank is measured. Column 5 and 6 display the expected returns on gross and net wealth 2 years after the wealth rank is measured. Columns 7 and 8 display the expected returns on gross and net wealth 4 years after the wealth rank is measured. Excess returns are computed relative to the yield on the Swedish 1-month Treasury bill.

	Expected return (% per year)							
	Year t		Year t+1		Year t+2		Year t+4	
	Gross wealth	Net wealth	Gross wealth	Net wealth	Gross wealth	Net wealth	Gross wealth	Net wealth
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Wealth group at end of year t								
P0-P10	2.23	-	3.51	-	3.50	-	3.65	-
P10-P20	1.53	0.43	2.51	4.06	2.72	4.15	3.17	3.88
P20-P30	3.01	3.81	1.76	3.15	1.99	3.82	2.57	4.47
P30-P40	3.44	4.64	3.06	1.11	3.13	1.72	3.40	2.30
P40-P50	3.56	4.52	3.40	3.24	3.39	3.42	3.56	3.50
P50-P60	3.81	4.74	3.74	4.00	3.71	4.05	3.82	3.81
P60-P70	4.00	4.81	3.92	4.37	3.89	4.45	3.98	4.12
P70-P80	4.19	4.85	4.11	4.50	4.07	4.58	4.13	4.23
P80-P90	4.49	5.01	4.41	4.57	4.36	4.67	4.41	4.28
P90-P95	4.86	5.29	4.79	4.75	4.75	4.85	4.78	4.44
P95-P97.5	5.21	5.61	5.14	5.04	5.10	5.15	5.14	4.74
P97.5-P99	5.64	6.04	5.57	5.36	5.53	5.48	5.54	5.06
P99-P99.5	6.18	6.61	6.11	5.79	6.04	5.91	6.04	5.47
P99.5-P99.9	6.85	7.32	6.75	6.36	6.67	6.46	6.63	5.99
P99.9-P99.99	7.65	8.15	7.50	7.05	7.37	7.14	7.30	6.62
Top 0.01%	7.92	8.30	7.85	7.84	7.88	7.87	7.76	7.31

Appendix Table 32
Identifying Scale Effects Using Twin Fixed Effects: Robustness Checks

This table reports robustness checks of the twin regressions reported in the main text, estimated on Swedish households with a twin over the period 2000-2007. In each column, we regress the expected wealth return on wealth rank and twin pair-year fixed effects. In columns 1 and 2, we focus on the subsample of twins of same gender and we compute wealth rank at the household level. In columns 3 and 4, we focus on the subsample of household pairs for which the IQ of each twin is available, the dependent variable is the expected return on household gross wealth, and we exclude or include IQ from the set of controls. In columns 5 and 6, we focus on the subsample of household pairs for which the elicited measure of risk aversion is available for each twin, the dependent variable is the expected return on gross wealth, and we exclude or include risk aversion from the set of controls. In columns 7 and 8, the dependent variable is the expected return on a twin's individual wealth and the rank is computed from the distribution of individual net wealth. In columns 4 and 6, we display the t-stat for the coefficient on IQ and risk aversion, respectively. The risk aversion measure is elicited through a survey administered to a large sample of twins, standardized to have zero mean and unit variance.

	Expected Return							
	Twins of same gender		IQ		Risk aversion		Individual data	
	Gross wealth	Net wealth	Without IQ as Control	With IQ as Control	Without risk aversion as control	With risk aversion as control	Gross wealth	Net wealth
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Twin pair-year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Wealth group								
P0-P10	-1.292	-	-1.327	-1.326	-0.215	-0.219	-1.209	-
P10-P20	-1.829	-4.830	-1.728	-1.728	-0.636	-0.639	-1.793	-5.265
P20-P30	-0.890	-1.634	-0.827	-0.826	-0.434	-0.434	-0.947	-1.459
P30-P40	-0.292	-0.222	-0.317	-0.316	-0.238	-0.239	-0.393	-0.222
P40-P50	REF.	REF.	REF.	REF.	REF.	REF.	REF.	REF.
P50-P60	0.238	0.021	0.281	0.280	0.188	0.188	0.245	0.123
P60-P70	0.487	0.164	0.532	0.531	0.461	0.460	0.493	0.199
P70-P80	0.685	0.178	0.772	0.771	0.723	0.721	0.776	0.335
P80-P90	0.988	0.380	1.037	1.035	1.051	1.047	1.106	0.566
P90-P95	1.312	0.598	1.264	1.260	1.443	1.437	1.498	0.820
P95-P97.5	1.553	0.672	1.417	1.415	1.711	1.702	1.821	1.145
P97.5-P99	1.917	1.056	1.827	1.823	2.054	2.048	2.198	1.491
P99-P99.5	2.365	1.256	2.242	2.240	2.773	2.767	2.755	2.011
P99.5-P99.9	3.025	1.953	2.570	2.567	3.047	3.044	3.269	2.472
P99.9-P99.99	3.779	2.387	3.565	3.559	2.261	2.246	3.704	2.724
Top 0.01%	4.287	4.115	2.784	2.795	3.776	3.802	4.261	1.588
t-stat of control variable				1.41		-1.85		
Adjusted R^2	0.571	0.246	0.587	0.587	0.483	0.483	0.566	0.203
Number of twin pairs per year	27,314	23,373	7,378	7,378	3,187	3,187	41,672	35,009

Appendix Table 33
Cross-Sectional Standard Deviation
of the Geometric Average Return Earned over a Generation

This table reports the cross-sectional standard deviation of the geometric average yearly household wealth return over a 36-year period. The estimators defined this online Appendix are applied to the entire population and different net worth brackets. Households are assigned to a wealth bracket the year they enter the 2000-2007 panel. In columns 1 and 2 of Panel A, we report the results of our baseline estimator (estimator #1), which relies on a second-order Taylor expansion of the logarithmic return and the assumption that returns behave according to a known asset pricing model. In columns 3 and 4 of Panel A, we show the version of the baseline estimator that controls for risk premium estimation error (estimator #2). In columns 5 and 6 of Panel A, we report the standard deviation of the arithmetic average return, under the assumption that returns behave according to a known asset pricing model (estimator #3). In columns 1 and 2 of Panel B, we report the cross-sectional standard deviation of the arithmetic average return, without assuming that returns behave according to a specific asset pricing model (estimator #4). In columns 3 and 4 of Panel B, we report the cross-sectional dispersion of household fixed effects estimated in our 8-year sample (estimator #5).

Panel A: Estimators #1 to 3

	Cross-sectional standard deviation of geometric average yearly return over 36-year period					
	Estimator #1 (Baseline methodology)		Estimator #2 (Estimation error in risk premia)		Estimator #3 (Arithmetic average, with asset pricing model)	
	Gross wealth (1)	Net wealth (2)	Gross wealth (3)	Net wealth (4)	Gross wealth (5)	Net wealth (6)
Wealth group						
Full population	2.19	7.83	2.12	7.73	2.18	5.76
P0-P10	2.19	-	2.13	-	2.13	-
P10-P20	1.75	11.85	1.71	11.81	1.86	9.04
P20-P30	1.97	8.39	1.90	8.19	1.99	6.98
P30-P40	1.91	6.54	1.82	6.32	1.85	5.42
P40-P50	1.78	5.21	1.69	5.00	1.75	4.35
P50-P60	1.85	4.63	1.76	4.44	1.81	3.94
P60-P70	1.90	4.55	1.82	4.40	1.85	3.56
P70-P80	1.92	4.28	1.84	4.17	1.90	3.24
P80-P90	2.00	4.39	1.94	4.30	1.96	3.03
P90-P95	2.23	4.84	2.18	4.78	2.11	3.00
P95-P97.5	2.59	5.07	2.54	5.01	2.37	3.16
P97.5-P99	3.21	6.47	3.16	6.43	2.81	3.69
P99-P99.5	4.35	9.08	4.32	9.06	3.45	4.46
P99.5-P99.9	5.53	11.45	5.50	11.45	4.30	5.56
P99.9-P99.99	7.43	16.71	7.43	16.90	5.48	7.09
Top 0.01%	7.88	12.05	8.04	12.70	6.52	7.84

Appendix Table 33 – *Continued*
Cross-Sectional Standard Deviation of the Geometric Average Return Earned over a Generation

Panel B: Estimators #4 and 5

	Cross-sectional standard deviation of geometric average yearly return over 36-year period			
	Estimator #4 (Arithmetic average, no asset pricing model)		Estimator #5 (Fixed effects, no asset pricing model)	
	Gross wealth	Net wealth	Gross wealth	Net wealth
	(1)	(2)	(3)	(4)
Wealth group				
Full population	2.82	8.23	4.70	11.23
P0-P10	2.68	-	4.82	-
P10-P20	2.17	13.01	4.18	18.26
P20-P30	3.02	8.75	5.12	13.42
P30-P40	3.00	7.23	4.80	10.73
P40-P50	2.90	5.93	4.44	8.69
P50-P60	2.84	5.35	4.34	7.86
P60-P70	2.65	4.68	4.16	7.01
P70-P80	2.49	3.99	4.05	6.27
P80-P90	2.35	3.68	4.05	5.94
P90-P95	2.22	3.26	4.31	5.91
P95-P97.5	2.29	3.33	4.86	6.40
P97.5-P99	2.61	3.69	5.92	7.71
P99-P99.5	2.61	3.84	7.36	9.51
P99.5-P99.9	2.76	4.19	9.35	12.03
P99.9-P99.99	2.95	3.94	12.10	15.51
Top 0.01%	0.93	6.96	13.90	17.63

Appendix Table 34
Cross-Sectional Moments of the Average Logarithmic Return over a Generation

This table reports the cross-sectional mean and the cross-sectional standard deviation of the average logarithmic wealth return earned over a generation. The estimation is conducted on the entire population and on different brackets of the distribution of net worth. A household is assigned to a bracket upon entering our panel. Columns 1 and 2 report the mean logarithmic return on gross and net wealth. Columns 3 and 4 report the cross-sectional standard deviation of the average logarithmic return over an investment lifespan of 36 years, while columns 5 and 6 consider an infinite investment lifespan. The estimation relies on the baseline estimator, which relies on a second-order Taylor expansion of the logarithmic return and the assumption that returns behave according to a known asset pricing model.

	Cross-sectional moments of the average logarithmic return (% per year)					
	Mean		Standard deviation			
			36-year investment lifespan		Infinite investment lifespan	
	Gross wealth (1)	Net wealth (2)	Gross wealth (3)	Net wealth (4)	Gross wealth (5)	Net wealth (6)
Wealth group						
Full population	4.33	2.89	2.07	7.49	1.48	6.31
P0-P10	3.52	-	2.09	-	1.55	-
P10-P20	3.06	-3.37	1.68	12.01	1.10	10.52
P20-P30	3.98	1.04	1.87	8.17	1.12	6.51
P30-P40	4.27	3.04	1.81	6.25	1.13	4.78
P40-P50	4.41	3.89	1.69	4.94	1.04	3.64
P50-P60	4.61	4.37	1.74	4.37	1.11	3.05
P60-P70	4.78	4.70	1.79	4.28	1.21	3.14
P70-P80	4.93	4.91	1.80	4.02	1.21	2.97
P80-P90	5.13	5.11	1.88	4.11	1.28	3.03
P90-P95	5.31	5.23	2.09	4.52	1.44	3.47
P95-P97.5	5.37	5.22	2.42	4.74	1.69	3.53
P97.5-P99	5.23	4.83	3.00	6.07	2.20	4.42
P99-P99.5	4.74	3.91	4.09	8.59	3.24	6.55
P99.5-P99.9	3.77	2.24	5.25	10.98	4.23	8.34
P99.9-P99.99	1.76	-1.24	7.21	16.57	6.07	12.72
Top 0.01%	-0.92	-3.55	7.99	12.87	6.62	8.85

Appendix Table 35
Tests of Asset Pricing Models on US Foundation Return Data

This table reports OLS regressions of the excess net wealth returns earned by US foundations on a US public equity index over the period 1986-2013 for various groups of US foundations sorted by net worth. We run regressions for three measures of returns: the total return, which includes realized and unrealized capital gains, dividends, and imputed interest from cash holdings (Panel A); a return measure that includes realized capital gains, dividends, and interest but excludes unrealized capital gains (Panel B); and a return measure that includes dividends and interest but excludes all forms of capital gains (Panel C). For each return measure and US foundation group, we report (1) the estimate and (2) standard error of the beta coefficient (slope), (3) the estimate and (4) standard error of the alpha coefficient in percentage points (intercept), and (5) the R^2 coefficient. One should read the table as follows: among foundations with more than 5 billion US dollars and using total returns as a return measure, the market beta is equal to 0.551 and is statistically significant at all conventional levels; the alpha coefficient is equal to -0.24% per year and is statistically insignificant. Wealth group thresholds are determined according to the level of net wealth at the beginning of each year expressed in 2010 US dollars.

Panel A: Total Return

	CAPM beta		CAPM alpha (%)		R^2
	Estimate	Standard error	Estimate	Standard error	
	(1)	(2)	(3)	(4)	(5)
Wealth group					
Below \$100k	0.200	(0.029)	0.21	(0.51)	0.65
\$100k to \$1m	0.391	(0.023)	-0.22	(0.40)	0.92
\$1m to \$10m	0.454	(0.025)	-0.08	(0.43)	0.93
\$10m to \$100m	0.466	(0.023)	0.25	(0.39)	0.94
\$100m to \$500m	0.537	(0.026)	0.07	(0.45)	0.94
\$500m to \$5bn	0.543	(0.039)	0.54	(0.69)	0.88
Above \$5bn	0.551	(0.071)	-0.24	(1.25)	0.70

Panel B: Return without Unrealized Capital Gains

	CAPM beta		CAPM alpha (%)		R^2
	Estimate	Standard error	Estimate	Standard error	
	(1)	(2)	(3)	(4)	(5)
Wealth group					
Below \$100k	0.049	(0.013)	1.42	(0.23)	0.35
\$100k to \$1m	0.052	(0.017)	1.96	(0.30)	0.26
\$1m to \$10m	0.061	(0.018)	2.41	(0.32)	0.29
\$10m to \$100m	0.065	(0.021)	2.75	(0.37)	0.26
\$100m to \$500m	0.084	(0.024)	2.92	(0.41)	0.33
\$500m to \$5bn	0.069	(0.026)	3.03	(0.45)	0.22
Above \$5bn	0.042	(0.025)	2.38	(0.44)	0.10

Appendix Table 35 – *Continued*
Tests of Asset Pricing Models on US Foundation Return Data

Panel C: Return without Capital Gains

	CAPM beta		CAPM alpha (%)		R^2
	Estimate	Standard error	Estimate	Standard error	
	(1)	(2)	(3)	(4)	(5)
Wealth group					
Below $100k	0.008	(0.010)	0.45	(0.17)	0.03
$100k to $1m	0.009	(0.015)	0.44	(0.26)	0.01
$1m to $10m	0.010	(0.018)	0.25	(0.32)	0.01
$10m to $100m	0.012	(0.019)	-0.08	(0.33)	0.01
$100m to $500m	0.009	(0.019)	-0.32	(0.33)	0.01
$500m to $5bn	0.008	(0.019)	-0.78	(0.34)	0.01
Above $5bn	0.006	(0.022)	-1.10	(0.39)	0.00

Appendix Table 36
Heterogeneity of Historical Returns Across US Foundations

This table reports the cross-sectional standard deviation of the historical return on net wealth held by US foundations. The results are computed over the period 1986-2013 for various wealth groups and for the entire population of US foundations. We consider three measures of returns: (1) the total return, which includes realized and unrealized capital gains, dividends, and imputed interest from cash holdings; (2) a return measure that includes realized capital gains, dividends, and interest but excludes unrealized capital gains; and (3) a return measure that includes dividends and interest but excludes all forms of capital gains. Wealth group thresholds are determined according to the level of net wealth at the beginning of each year expressed in 2010 US dollars.

	Cross-sectional standard deviation of annual wealth return		
	Total return (1)	Return without unrealized capital gains (2)	Return without capital gains (3)
Wealth group			
Entire population	14.67%	8.31%	2.28%
Below $100k	18.30%	9.64%	2.70%
$100k to $1m	13.79%	8.19%	2.26%
$1m to $10m	12.45%	7.37%	1.93%
$10m to $100m	11.42%	6.60%	1.74%
$100m to $500m	11.32%	6.32%	1.57%
$500m to $5bn	11.49%	6.15%	1.34%
Above $5bn	11.67%	3.33%	1.07%

Appendix Table 37
Return Statistics With Alternative Return Measurements
Mean Returns

This table reports the mean of historical excess returns on total wealth in different brackets of the net wealth distribution in Sweden over the period 2000-2007, using five distinct measurement methods for historical returns. In columns 1 and 2 of Panel A, historical returns are the sum of the dividend yield, excluding non-pecuniary banking services, and the realized capital gain return, excluding unrealized capital gains. In columns 3 and 4 of Panel A, historical returns are the sum of the dividend yield, excluding non-pecuniary services, the total capital gain return for all but private equity holdings, and the accounting return on equity for private equity holdings. In columns 5 and 6 of Panel A, historical returns are the sum of the dividend yield, including non-pecuniary services, the total capital gain return for all but private equity holdings, and the accounting return on equity for private equity holdings. In columns 1 and 2 of Panel B, historical returns are the sum of the dividend yield, excluding non-pecuniary services, and the total capital gain return for all types of holdings. In columns 3 and 4 of Panel B, historical returns are the sum of the dividend yield, including non-pecuniary banking services, and the total capital gain return for all types of holdings. All returns are in excess of the rate of return on the Swedish 1-month Treasury bill.

Panel A: Without Unrealized Gains and Accounting-Based Private Equity Returns

	Historical mean excess return (% per year)					
	Without unrealized capital gains		Computed using accounting measure of private equity returns			
			Without banking services		With banking services	
	Gross wealth (1)	Net wealth (2)	Gross wealth (3)	Net wealth (4)	Gross wealth (5)	Net wealth (6)
Wealth group						
P0-P10	-10.29	-	-0.98	-	0.11	-
P10-P20	-3.02	-8.87	-2.50	-7.06	-0.94	-4.95
P20-P30	-0.59	-4.51	-0.47	-0.18	0.14	0.65
P30-P40	0.28	-1.76	0.69	2.19	1.05	2.66
P40-P50	0.74	-0.43	1.04	2.17	1.33	2.53
P50-P60	0.97	0.16	1.97	3.05	2.22	3.35
P60-P70	1.27	0.76	2.55	3.43	2.76	3.68
P70-P80	1.50	1.22	2.93	3.63	3.13	3.85
P80-P90	1.69	1.56	3.26	3.79	3.45	4.00
P90-P95	1.82	1.76	3.49	3.92	3.66	4.10
P95-P97.5	1.87	1.84	3.61	4.01	3.77	4.18
P97.5-P99	1.88	1.87	3.75	4.16	3.90	4.33
P99-P99.5	1.87	1.89	3.91	4.40	4.06	4.56
P99.5-P99.9	1.76	1.81	4.31	4.95	4.45	5.10
P99.9-P99.99	1.12	1.18	5.47	6.34	5.59	6.48
Top 0.01%	0.01	0.00	6.40	7.02	6.47	7.10

Appendix Table 37 – *Continued*
Return Statistics With Alternative Return Measurements
Mean Returns

Panel B: Market-Based Private Equity Returns

	Historical mean excess return (% per year)			
	Computed using market measure of private equity returns			
	Without banking services		With banking services	
	Gross wealth	Net wealth	Gross wealth	Net wealth
	(1)	(2)	(3)	(4)
Wealth group				
P0-P10	-0.95	-	0.13	-
P10-P20	-2.50	-7.05	-0.95	-4.33
P20-P30	-0.44	-0.15	0.17	0.67
P30-P40	0.71	2.20	1.07	2.67
P40-P50	1.06	2.19	1.35	2.55
P50-P60	2.00	3.09	2.25	3.39
P60-P70	2.56	3.44	2.78	3.71
P70-P80	2.95	3.64	3.15	3.88
P80-P90	3.25	3.78	3.44	3.98
P90-P95	3.41	3.84	3.59	4.02
P95-P97.5	3.44	3.83	3.60	4.01
P97.5-P99	3.36	3.76	3.51	3.92
P99-P99.5	3.08	3.53	3.21	3.67
P99.5-P99.9	2.96	3.49	3.05	3.59
P99.9-P99.99	2.93	3.56	2.96	3.59
Top 0.01%	2.27	2.59	2.21	2.53

Appendix Table 38
Return Statistics With Alternative Return Measurements
Cross-Sectional Standard Deviation of Returns

This table reports the cross-sectional standard deviation of historical excess returns on total wealth in different brackets of the net wealth distribution in Sweden over the period 2000-2007, using five distinct measurement methods for historical returns. In columns 1 and 2 of Panel A, historical returns are the sum of the dividend yield, excluding non-pecuniary banking services, and the realized capital gain return, excluding unrealized capital gains. In columns 3 and 4 of Panel A, historical returns are the sum of the dividend yield, excluding non-pecuniary services, the total capital gain return for all but private equity holdings, and the accounting return on equity for private equity holdings. In columns 5 and 6 of Panel A, historical returns are the sum of the dividend yield, including non-pecuniary services, the total capital gain return for all but private equity holdings, and the accounting return on equity for private equity holdings. In columns 1 and 2 of Panel B, historical returns are the sum of the dividend yield, excluding non-pecuniary services, and the total capital gain return for all types of holdings. In columns 3 and 4 of Panel B, historical returns are the sum of the dividend yield, including non-pecuniary banking services, and the total capital gain return for all types of holdings. All returns are in excess of the rate of return on the Swedish 1-month Treasury bill.

Panel A: Without Unrealized Gains and Accounting-Based Private Equity Returns

	Cross-sectional standard deviation of household annual wealth return (% per year)					
	Without unrealized capital gains		Computed using accounting measure of private equity returns			
			Without banking services		With banking services	
	Gross wealth	Net wealth	Gross wealth	Net wealth	Gross wealth	Net wealth
	(1)	(2)	(3)	(4)	(5)	(6)
Wealth group						
Full population	8.60	11.41	6.96	14.66	6.81	14.49
P0-P10	17.95	-	6.92	-	6.73	-
P10-P20	8.50	22.27	6.19	26.77	6.16	26.56
P20-P30	7.00	15.06	7.50	20.16	7.43	20.11
P30-P40	5.77	9.98	6.88	14.48	6.81	14.47
P40-P50	5.21	7.63	6.35	10.98	6.29	10.96
P50-P60	5.35	7.11	6.20	9.66	6.14	9.63
P60-P70	5.42	6.71	5.92	8.55	5.87	8.52
P70-P80	5.30	6.32	5.79	7.74	5.75	7.71
P80-P90	5.15	5.98	6.00	7.58	5.96	7.56
P90-P95	5.10	5.83	6.55	8.01	6.53	8.00
P95-P97.5	5.20	5.95	7.40	8.92	7.39	8.91
P97.5-P99	5.63	6.43	8.68	10.67	8.67	10.67
P99-P99.5	6.62	7.63	10.37	13.28	10.37	13.28
P99.5-P99.9	7.91	9.09	13.04	17.35	13.03	17.35
P99.9-P99.99	8.41	9.52	16.22	22.29	16.21	22.28
Top 0.01%	6.85	7.37	18.77	21.16	18.76	21.15

Appendix Table 38 – *Continued*

Return Statistics With Alternative Return Measurements

Cross-Sectional Standard Deviation of Returns

Panel B: Market-Based Private Equity Returns

	Cross-sectional standard deviation of household annual wealth return (% per year)			
	Computed using market measure of private equity returns			
	Without banking services		With banking services	
	Gross wealth (1)	Net wealth (2)	Gross wealth (3)	Net wealth (4)
Wealth group				
Full population	8.59	16.26	9.19	18.58
P0-P10	7.78	-	8.13	-
P10-P20	6.46	25.98	6.63	27.62
P20-P30	8.29	20.92	8.78	24.57
P30-P40	7.83	16.11	8.59	19.66
P40-P50	7.31	12.79	8.13	15.34
P50-P60	7.37	11.84	8.35	14.00
P60-P70	7.32	10.94	8.36	12.70
P70-P80	7.49	10.57	8.50	11.98
P80-P90	8.14	10.83	9.04	11.93
P90-P95	9.55	12.12	10.30	12.96
P95-P97.5	11.61	14.49	12.21	15.15
P97.5-P99	14.85	18.09	15.31	18.62
P99-P99.5	19.43	23.40	19.76	23.81
P99.5-P99.9	25.21	29.49	25.44	29.81
P99.9-P99.99	32.20	36.67	32.43	37.01
Top 0.01%	35.64	37.89	35.79	38.14

Appendix Table 39
The Explanatory Power of Household Fixed Effects for Returns to Wealth Using Alternative Return Measures

This table reports statistics on the explanatory power of year and household fixed effects for various measures of returns. We consider: the historical return, excluding non-pecuniary banking services and unrealized capital gains, on gross and net wealth over the period 2001-2008; the historical return, excluding non-pecuniary services, including the total capital gain return for all but private equity holdings, and using the accounting return on equity for returns on private equity holdings, on gross and net wealth over the period 2001-2008; the historical return, including non-pecuniary services, the total capital gain return for all but private equity holdings, and using the accounting return on equity for private equity holdings, on gross and net wealth over the period 2001-2008; the historical return, excluding non-pecuniary services, and including the total capital gain return for all types of holdings, on gross and net wealth; the historical return, including all types of capital gains and non-pecuniary banking services, on gross and net wealth over the period 2001-2008; the interest yield on bank account balances in excess of the risk-free rate; the return on private equity holdings measured either as the sum of the dividend yield and the total capital gain or the accounting return on equity over the period 2001-2008. We display the adjusted R^2 of a regression with year fixed effects (column 1), and with year fixed effects plus household fixed effects (column 2). In column 3, we report the marginal explanatory power of household fixed effects, defined as the difference between column 2 and column 1 The sample standard deviation is displayed in column 4.

	Explanatory power of year fixed effects (1)	Explanatory power of year and household fixed effects (2)	Marginal explanatory power of household fixed effects (3)	Sample standard deviation (4)
Historical excess return on wealth				
- Excluding unrealized capital gains				
Gross wealth	0.197	0.484	0.287	9.6%
Net wealth	0.172	0.454	0.282	12.5%
- Accounting-based private equity returns, excluding banking services				
Gross wealth	0.516	0.612	0.096	10.0%
Net wealth	0.317	0.487	0.170	17.8%
- Accounting-based private equity returns, with banking services				
Gross wealth	0.525	0.604	0.079	9.9%
Net wealth	0.320	0.482	0.162	17.6%
- Market-based private equity returns, without banking services				
Gross wealth	0.433	0.494	0.061	11.4%
Net wealth	0.291	0.421	0.130	19.3%
- Market-based private equity returns, with banking services				
Gross wealth	0.438	0.486	0.047	11.3%
Net wealth	0.293	0.415	0.122	19.2%
Historical excess return on wealth components				
- Yield on bank account balances	0.317	0.638	0.321	1.1%
- Private equity return				
Market measure	0.171	0.194	0.023	74.3%
Accounting measure	0.013	0.177	0.163	51.1%

Appendix Table 40
Historical Returns, Saving, and Inequality Dynamics

This table reports estimates of the Saez and Zucman (2016) decomposition of the wealth inequality dynamics applied to our panel of Swedish households over the period 2000-2007. For each group of the net wealth distribution, we compute (1) the historical annual growth of the group's share of net wealth, (2) the synthetic saving flow required to match this change divided by the initial stock of wealth held the group, (3) the historical nominal return on net wealth held by households in the group, (4) the group's historical initial share, (5) the annual growth of the group's wealth share predicted by our model, and (6) the differential synthetic saving effect implied by our model. The returns reported in column 3 are computed using available data on the historical return of each group member during the year. The results in column 5 and 6 are based on our model of wealth accumulation explained in Section IV of the main text, which assumes that households simply capitalize returns on beginning-of-the-year asset holdings and have zero net saving out of labor income throughout the year. One should read the table as follows: from 2000 to 2007, according to the data, the group of households in the top 0.01% of the net wealth distribution increased its share of net wealth by 6.12% a year; 48% (2.95/6.12) of this effect is due to a higher synthetic saving rate within the group and 52% is due to a higher return to wealth. If households in the group had zero net saving, the group would have increased its share of net wealth by 5.20% a year, out of which 42% (2.17/5.20) would be attributed to a higher synthetic saving rate within the group compared to the full household population.

	Historical values				Model predictions	
	Annual growth of wealth share	Differential synthetic saving effect	Differential return effect	Initial wealth share	Annual growth of wealth share	Differential synthetic saving effect
	(1)	(2)	(3)	(4)	(5)	(6)
Wealth group						
Bottom 90%	-0.42%	-0.18%	-0.24%	47.56%	-0.59%	-0.35%
P90-P99	0.43%	0.75%	-0.25%	31.32%	0.58%	0.89%
P99-P99.9	1.78%	1.27%	0.43%	10.56%	2.20%	1.69%
P99.9-P99.99	3.97%	1.74%	2.06%	5.07%	3.98%	1.75%
Top 0.01%	6.12%	2.95%	2.99%	5.48%	5.20%	2.17%

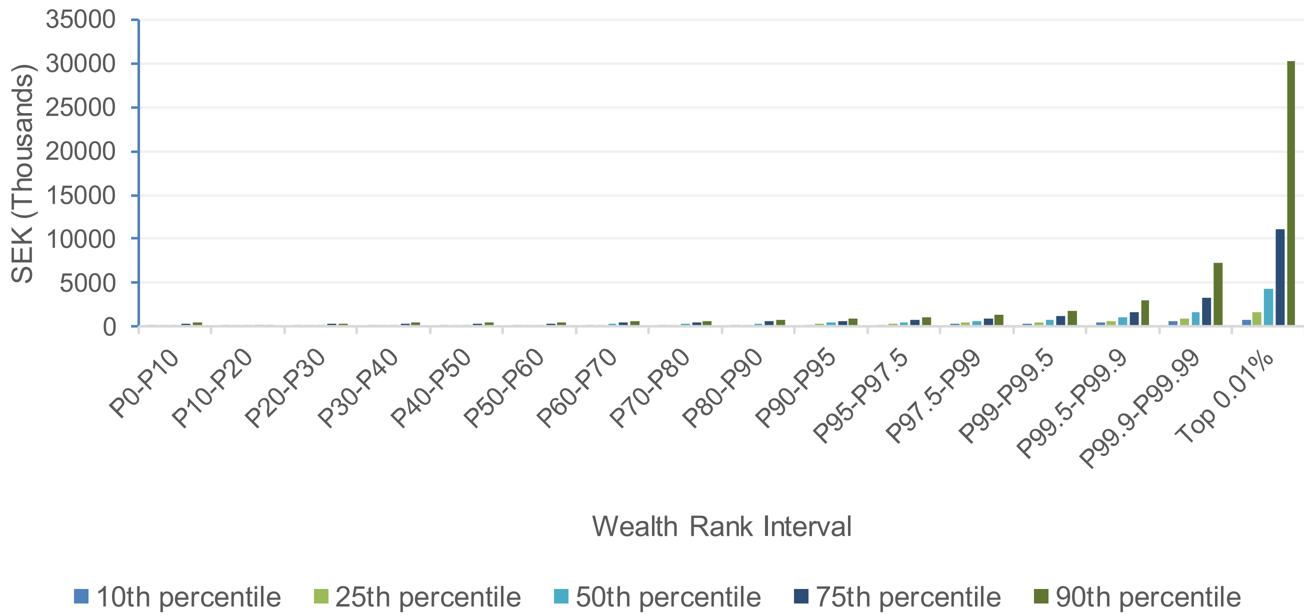
Appendix Table 41
Financial Portfolio Heterogeneity and Inequality Dynamics

This table reports estimates the Campbell (2016) decomposition of the 1-year change in the cross-sectional variance of log financial wealth applied to Swedish households over the sample period 2000-2007. We report (1) the cross-sectional variance of the expected complete portfolio return, (2) the cross-sectional variance of the return innovation, (3) the covariance of expected return and log financial wealth multiplied by a factor of two, (4) the 1-year change in the cross-sectional variance of log financial wealth predicted by the model, and (5) the historical average 1-year change in log financial wealth over the sample period. We compute estimates of these moments in each sample year for the entire cross-section of households with positive financial wealth; we then provide here the time-series average of these cross-sectional moments. We assume households rebalance their portfolio to keep security weights constant in each of the twelve months following December 31st of each year. Column 4 is the sum of columns 1 to 3. We present the results using two asset pricing models: the local CAPM and the global 5-factor model described in the main text, and assume in each case that household portfolio alphas are equal to zero.

	Decomposition of the 1-year change in the cross-sectional variance of log financial wealth				
	Cross-sectional moments			Predicted yearly change in variance of financial wealth	Average yearly change in variance of financial wealth
	Variance of expected return $Var^*(E_t r_{h,t+1})$	Variance of return innovation $Var^*(r_{h,t+1} - E_t r_{h,t+1})$	Covariance of financial wealth and expected return $2Cov^*[E_t r_{h,t+1}; \log(W_{h,t+1})]$		
	(1)	(2)	(3)	(4)	(5)
Asset pricing model					
Local CAPM	0.0003	0.0099	0.0264	0.0366	0.0389
Global 5 factors	0.0003	0.0098	0.0275	0.0377	0.0389

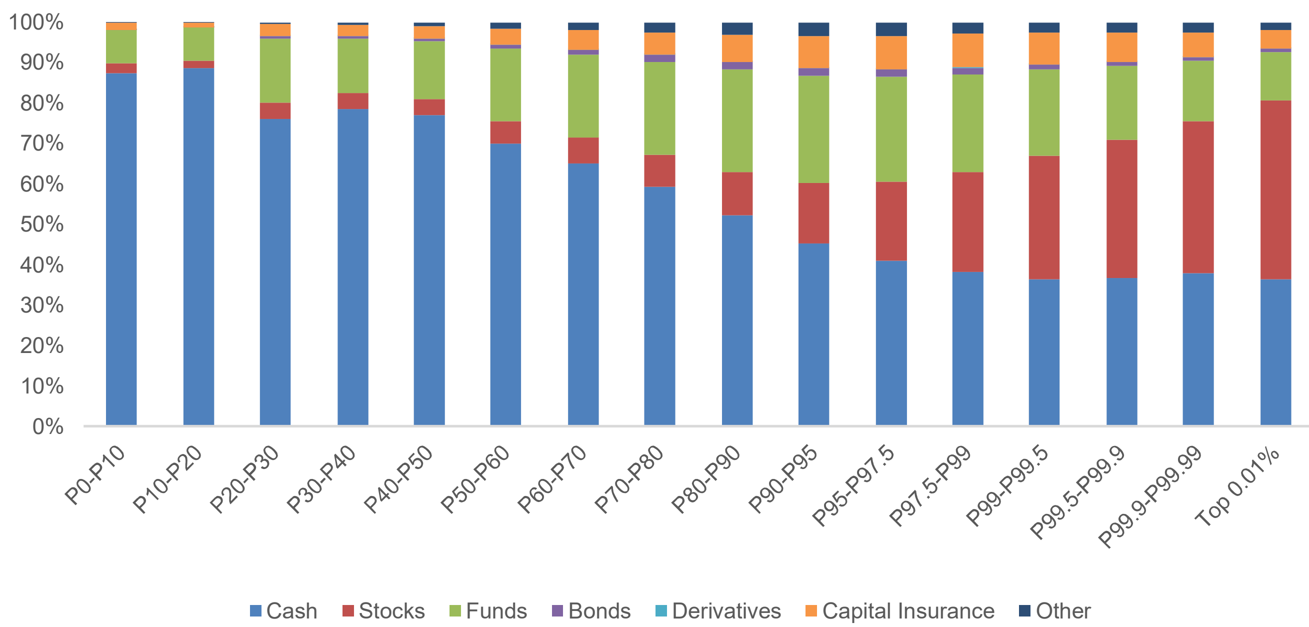
Appendix Figure 1
Gross Income

This figure illustrates the 10th, 25th , 50th, 75th, and 90th percentiles of gross income in different brackets of the net wealth distribution in Sweden over the period 2000-2007. Gross income is the sum of labor income before income tax and capital income, and is expressed in thousands of Swedish kronor. On 31 December 2004, 1 Swedish krona traded at 0.151 US dollar. One should read the graph as follows: in the top 0.01% of the net wealth distribution, a household with a median level of gross income earns 4,400,000 Swedish kronor (about $660,000) a year, while a household in the 90th percentile of the gross income distribution earns 30,300,000 Swedish kronor (about $4,600,000) a year.



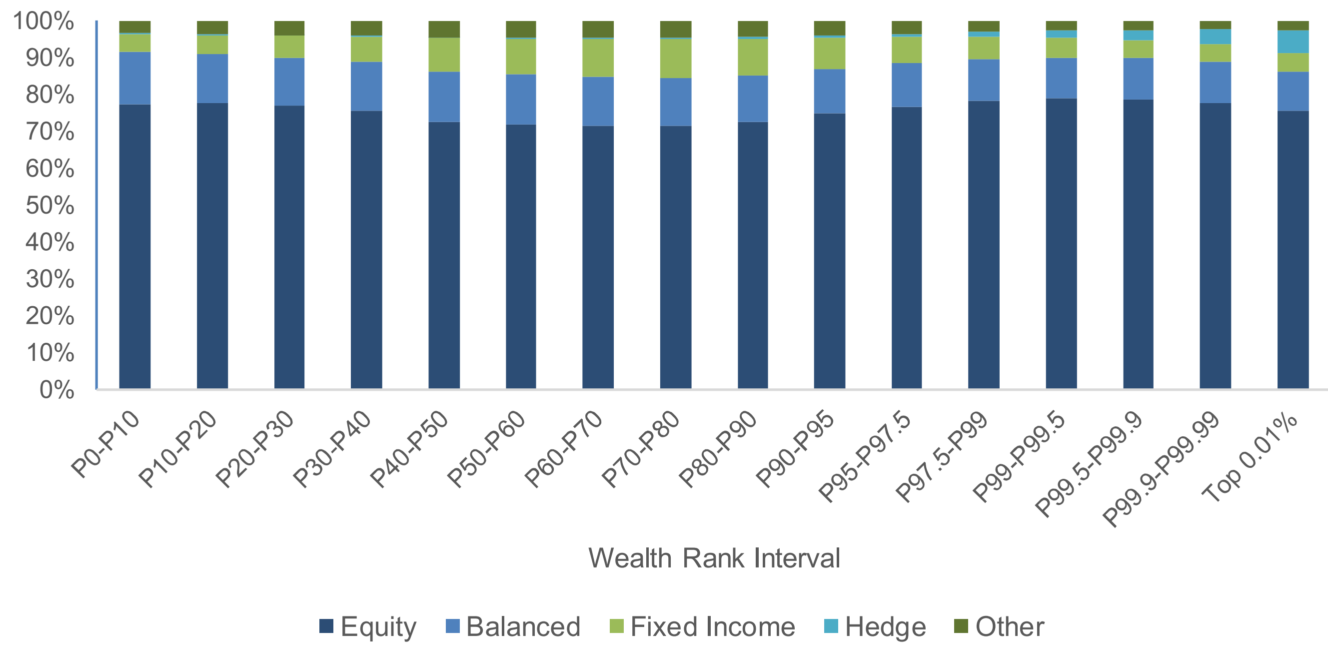
Appendix Figure 2
Allocation of Financial Wealth

This figure illustrates the average asset allocation of the financial wealth held by Swedish households in different brackets of the net wealth distribution in Sweden over the 2000-2007 period. We consider cash (bank account balances and money market funds), directly-held stocks, funds (mutual funds other than money-market funds), bonds, derivatives, capital insurance, and other assets. Capital insurance accounts are tax-favored savings accounts whose proceeds can be invested either in stocks, mutual funds or in riskless assets. One should read the graph as follows: a household in the top 0.01% allocates on average 36.5% of its financial portfolio to cash, 44.2% to stocks, 11.8% to funds, 0.8% to bonds, 0.1% to derivatives, 4.5% to capital insurance, and 2.1% to other investment vehicles.



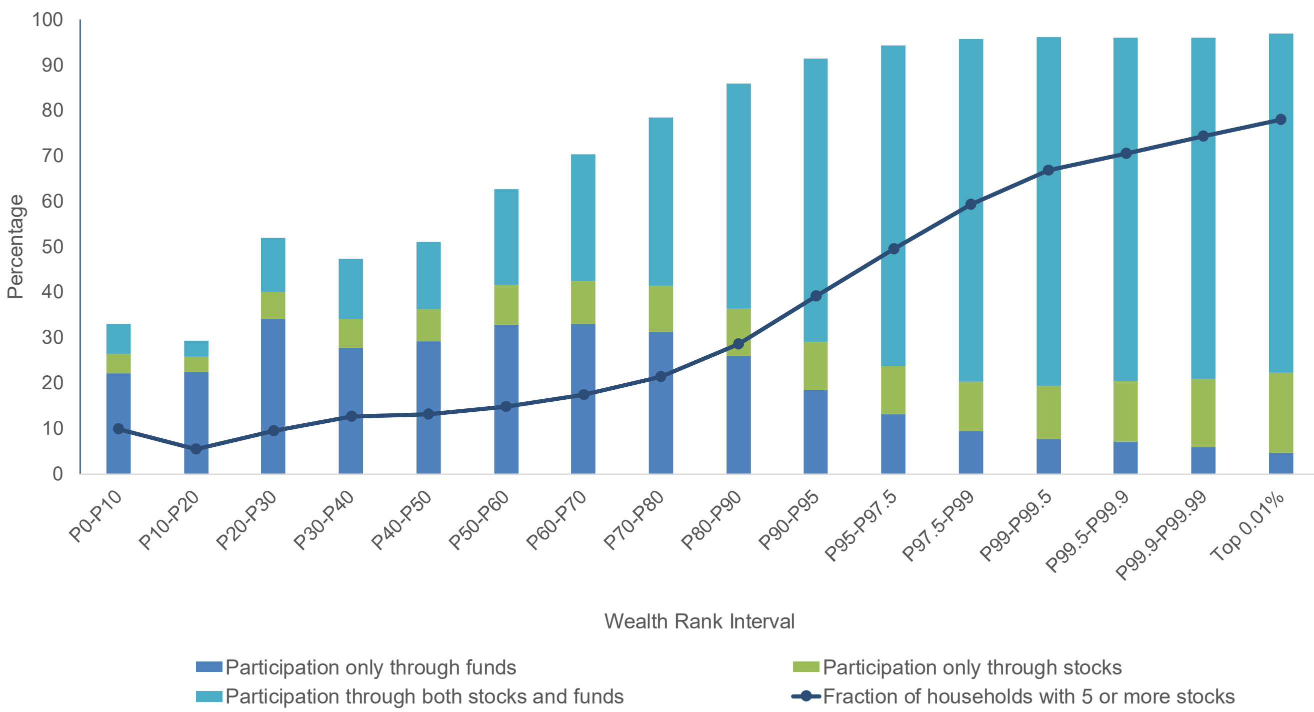
Appendix Figure 3
Allocation of Fund Portfolio

This figure illustrates the average allocation of the fund portfolio to pure equity funds, balanced funds, pure fixed-income funds, hedge funds, and other funds in different brackets of the net wealth distribution in Sweden between 2000 and 2007. The reported allocations are averages over the period 2000-2007. One should read the graph as follows: a household in the top 0.01% of the net wealth distribution allocates on average 75% of its fund portfolio to equity funds, 11% to balanced funds, 5% to fixed income funds, 6% to hedge funds, and 2% to other fund types.



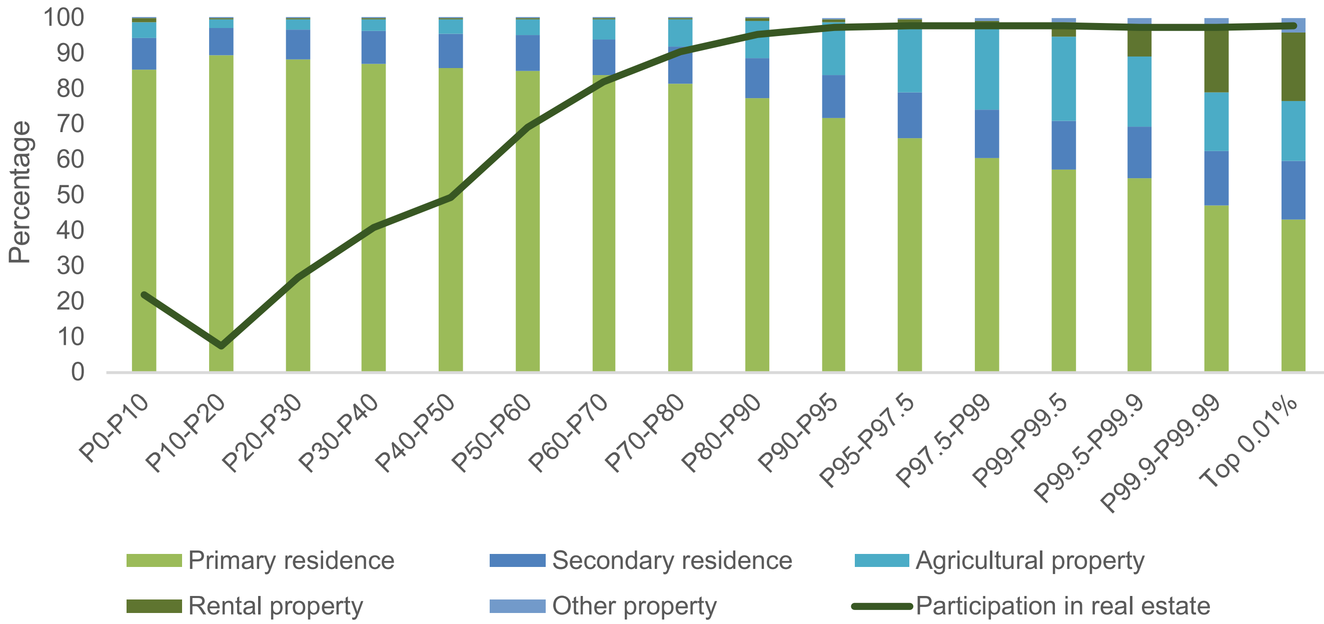
Appendix Figure 4
Stock Market Participation

This figure illustrates the average rates of participation in funds, stocks, and risky assets (bar chart), and the fraction of households owning directly at least 5 stocks (black line) in different brackets of the net wealth distribution in Sweden over the period 2000-2007. Stocks refer to directly-held stocks, and funds refer to mutual funds other than money-market funds. The fraction of households owning at least 5 stocks is measured conditional on directly holding stocks. One should read the graph as follows: in the top 0.01% of the net wealth distribution, 74.7% of households own both stocks and funds, 17.6% own only stocks, 4.7% own only funds, and 78% of direct stockholders own at least 5 different stocks.



Appendix Figure 5
Allocation of Real Estate Wealth

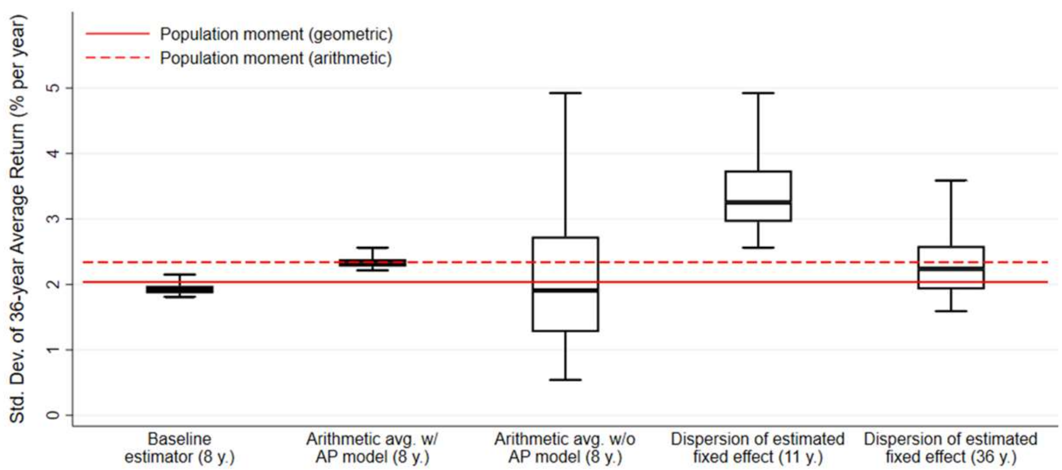
This figure illustrates the average asset allocation of the real estate wealth held by households in different brackets of the net wealth distribution in Sweden over the period 2000-2007. The black line plots the rate of participation rate in the real estate market. We consider the following five property classes: primary residence, secondary residence, agricultural property, rental property, and other property. The latter category mainly includes foreign housing and the industrial properties of sole proprietors. One should read the graph as follows: a household the top 0.01% allocates on average 43.1% of real estate wealth to its primary residence, 16.2% to secondary residence(s), 16.9% to agricultural property, 19.5% to rental property, and 4.2% to other property.



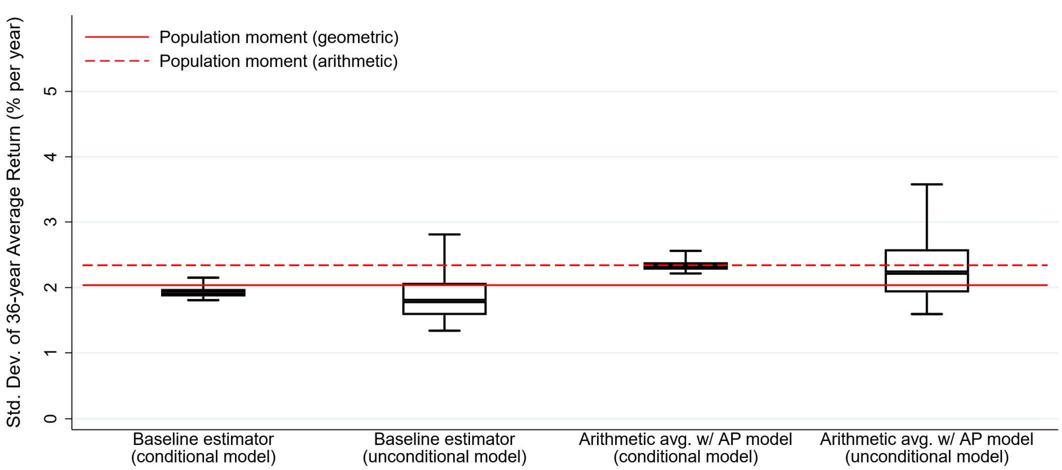
Appendix Figure 6
Estimators of the Cross-Sectional Standard Deviation of the Geometric Average Return over a Generation

This figure illustrates the distribution of several estimators of the cross-sectional standard deviation of the geometric average yearly return over a generation (36 years). We simulate 10,000 panels of the returns obtained by 500 dynasties over 36 years according to the data generating process specified in section IX.B of this online Appendix. We apply the following method to each panel. Estimator #1 relies on a second-order Taylor expansion of the logarithmic return and the assumption that returns behave according to a known asset pricing model, estimated on 8 years of household wealth returns. Estimator #2 is a variant of estimator #1 that controls for risk premium estimation error. Estimator #3 is the cross-sectional standard deviation of the arithmetic average return, under the assumption that returns behave according to a known asset pricing model, estimated on 8 years of household wealth returns. Estimator #4 is variant of estimator #3 that controls for risk premium estimation error. Estimator #5 is the standard deviation of the arithmetic average return, which assumes a two-way fixed-effect model of household returns but does not rely on a specific asset pricing model, estimated using 8 years of data. Estimator #6 considers the cross-sectional dispersion of households' sample average return. Panel A reports, from left to right, estimators #1, #3, and #5 applied to 8-year household samples, and estimator #6 applied to household samples of 11 years (as in Fagereng et al. (2019)) and 36 years. Panel B reports estimators #1 to #4 applied to household samples of 8 years and pricing factor samples of 33 years. The solid red line shows the target population cross-sectional standard deviation of the geometric average return. The dashed red line shows the population cross-sectional standard deviation of the arithmetic average return as a benchmark.

Panel A: Estimators With and Without Asset Pricing Models



Panel B: Estimators Conditional and Unconditional on Risk Premia



Appendix Figure 7
Mean Excess Return Earned by US Foundations

This figure illustrates the relationship between the mean yearly arithmetic return on net wealth and the level of net wealth in the population of US foundations over the period 1986-2013. Returns are measured pre-tax and in excess of the yield on the US 1-month Treasury bill. The dotted line plots mean returns using the equal-weighted average of historical returns within each wealth group over the period 1986-2013. The solid line plots mean returns using the expected return derived from the CAPM betas for each wealth group and the historical average of the market portfolio's annual arithmetic return over the period 1986-2013. The CAPM beta for each wealth group is computed using a regression of each wealth group's yearly average return on the realization of the return on the US equity market portfolio. Wealth group thresholds are determined according to the level of net wealth in 2010 US dollars.