Real GDP Interpolations Technical Note

Jakub Rybak

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Introduction

This note explains how we create monthly euro area national accounts data by interpolating the quarterly data, following Stock & Watson (2010) and Bernanke et al. (1997). The real GDP data, as well as the data for nominal GDP, its main components and GDP deflator, are published only quarterly for the euro area. However, some variables closely related to the components are published at monthly frequency. This note describes how the monthly variables, together with the quarterly data on GDP and its components, are used to estimate monthly data of real GDP.

Model

Quarterly series of components are related to their monthly unobserved values as:

$$Q_T = \frac{1}{3}(q_{3T} + q_{3T-1} + q_{3T-2})$$

Where Q_T denotes quarterly value of a given component in quarter T and q_t denotes monthly value in month t. Monthly values are treated as unobserved variables.

We can specify our system as a state-space model, with unobserved variable q_t. Then the equation above represents a measurement equation.[1] Monthly values are modelled as:

$$q_t = \alpha + z_t'\beta + u_t$$

Where z_t is a vector of monthly indicators, β is a vector of coefficients and u_t is an error term that follows AR(1) process, i.e.:

$$u_t = \rho u_{t-1} + \epsilon_t$$

[1] Note that in this case measurement equation is an identity and hence has no error term.

$$Var(\epsilon_t) = \sigma^2$$

Transition equation thus becomes:

$$\begin{pmatrix} q_t \\ q_{t-1} \\ q_{t-2} \\ u_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \rho \end{pmatrix} \begin{pmatrix} q_{t-1} \\ q_{t-2} \\ q_{t-3} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \alpha + z_t'\beta \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \epsilon_t \end{pmatrix}$$

In the Matlab code, the following vector notation is used:

Measurement equation

$$y_T = \mathbf{H}'\boldsymbol{x}_{3T}$$

Transition equation

$$\boldsymbol{x}_t = \mathbf{T}\boldsymbol{x}_{t-1} + \mathbf{Z}_t + \boldsymbol{v}_t$$

$$\mathbf{W} \equiv Var(\boldsymbol{v}_t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2 \end{pmatrix}$$

where

$$\mathbf{H} = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \\ 0 \end{pmatrix}; \boldsymbol{x}_t = \begin{pmatrix} q_t \\ q_{t-1} \\ q_{t-2} \\ u_t \end{pmatrix}; \mathbf{T} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \rho \end{pmatrix}; \mathbf{Z}_t = \begin{pmatrix} \alpha + z_t'\beta \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Note that monthly indicators may not be available for the whole sample, and hence components z_t for a given quarterly series can change. When this happens, parameters of the transition equation are allowed to change too. Parameters $\mathbf{T}$, σ^2 and β can thus be seen as time varying.

The model described above is complicated by the fact that variables are trending. To detrend them, two different approaches are applied to obtain a trend series:[2]

[2]However, when applied to EA the choice does not matter that much since the difference between nominal as well as real GDP obtained by the two methods is less than 0.7% for any given month.

1. Fitting a cubic spline to the original series
2. Using a linear trend, i.e. $Trend = a + bt$, with a and b obtained from

$$\min_{a,b}\{r_t - (a + bt)\}$$

Where r_t is a series to be detrended.

Detrended series is then obtained by dividing original observations by fitted trend values.

Hence, denoting the detrended series by $\sim$ and trends for the series by S_t for quarterly and s_t for monthly series, we get relations:

$$\tilde{Q}_T = \frac{Q_T}{S_T}$$
$$\tilde{q}_t = \frac{q_t}{s_t}$$

The model then becomes:

Measurement equation:

$$\tilde{Q}_T = \frac{1}{3S_T}\begin{pmatrix} s_{3T} & s_{3T-1} & s_{3T-2} \end{pmatrix}\begin{pmatrix} \tilde{q}_{3T} \\ \tilde{q}_{3T-1} \\ \tilde{q}_{3T-2} \end{pmatrix}$$

So now $\mathbf{H}_t = 1/3\begin{pmatrix} s_{3T}/S_T \\ s_{3T-1}/S_T \\ s_{3T-2}/S_T \\ 0 \end{pmatrix}$, which differs for every t.

Transition equation[3] (for detrended variables):

$$\begin{pmatrix} \tilde{q}_t \\ \tilde{q}_{t-1} \\ \tilde{q}_{t-2} \\ u_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \rho \end{pmatrix}\begin{pmatrix} \tilde{q}_{t-1} \\ \tilde{q}_{t-2} \\ \tilde{q}_{t-3} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \alpha + \tilde{z}_t'\beta \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \epsilon_t \end{pmatrix}$$

Two approaches

The bottom-up approach: First, we interpolate the GDP deflator and the GDP components: private final consumption, government consumption, gross fixed capital formation, change in business inventories, net exports of goods and services. Second, we add up the monthly GDP components to obtain monthly

[3]Note that in the original Stock and Watson's methodology file there is a mistake in this equation, as the transition equation includes non-detrended z_t

nominal GDP. Third, we divide the monthly nominal GDP by the monthly GDP deflator to obtain the monthly real GDP

Direct approach: We directly interpolate real GDP using unemployment and industrial production as monthly indicators.

Estimation

Parameters in the transition equation are estimated using Gaussian maximum likelihood method, with starting values for $\beta : \begin{pmatrix} 0 & 1 & 1 \end{pmatrix}'$ where 0 corresponds to the constant, and ones to the monthly indicators (their number varies between quarterly series). Starting values for ρ and σ are 0.2 and 0.3 respectively.[4]

To see that the starting value of 0.3 is reasonable for σ, note that

$$\sigma = \sqrt{(1-\rho^2)Var(u_t)} = \sqrt{(1-\rho^2)Var(\tilde{q}_t - \alpha - \tilde{z}_t'\beta)}$$

where α, β, ρ are estimated jointly with σ and variables with $\sim$ are divided by trends, so their values should be around 1. All variables are either in levels, or expressed as an index number or % balances.

Data

All data, apart from HICP are obtained from Haver database and can be found in the file "Quarterly", which is easily refreshable. HICP series is obtained from the ECBs Statistical Data Warehouse (SDW) (series code: SDW ICP.M.U2.Y.000000.3.ERX).

The quarterly and monthly series used are:

[4] An alternative is to compute the starting value of the standard deviation from the model, using starting values for other coefficients, i.e. $\sigma^2 = \widehat{Var(u_t)}(1-\rho^2_{start})$ where $\widehat{Var(u_t)} = \hat{\sigma}^2; \hat{\sigma} = st.dev(q_t - x_t'\beta^{start})$. This is slightly different from the way Stock and Watson compute them, as they use only approximation to the equation above. However, results do not change significantly when either of these methods is applied.

Quarterly	Monthly
Private final consumption	○ Imports of consumer goods (from 1999) ○ Retail trade
Government final consumption	
Gross fixed capital formation	○ Construction output (from 1995)
Change in business inventories and acquisitions less disposable values	○ Stocks of finished products (manufacturing industry), percent balance ○ Volume of Stocks (retail), percent balance
Net exports of goods and services	○ Trade balances with RoW in goods (from 1990) ○ Volume of export order books, % balance, manufacturing industry ○ Manufacturing new orders (non-EA nondomestic market) (from 2003)
GDP Deflator	○ Domestic PPI (industry excluding construction) (from 1981) ○ HICP
Real GDP (direct approach)	○ Unemployment rate ○ Industrial production index

References

Bernanke, B. S., Gertler, M. & Watson, M. (1997), 'Systemic monetary policy and the effects of oil price shocks', *Brookings Papers on Economic Activity* .

Stock, J. H. & Watson, M. W. (2010), 'Distribution of quarterly values of GDP/GDI across months within the quarter', *Research Memorandum* .