Online Appendix to:

"Learning by Sharing: Monetary Policy and Common Knowledge"

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Contents of Online Appendix:

This *Online Appendix* details all proofs, derivations, and computational steps used in the paper.

Online Appendix A: A Baseline Model

This Appendix details the proofs of the Propositions in Section 3.

Proof of Proposition 1: I proceed in three steps. I first solve the representative household's problem, imposing market clearing, to derive a relationship between the real wage rate, output, and productivity in the economy. Specifically, I show that if ϕ_0 is set such that $\delta = \beta \mathbb{E}_t \left[\frac{M_t}{M_{t+1}}\right] < 1$, then the cash-in-advance constraint always binds $M_t = P_t C_t$, $W_t / L_t^\eta = M_t^s / \delta$, and the real wage rate follows a simple condition.[1] I then use this condition to derive a forward-looking expression for firms' optimal prices. Finally, I solve for the forward-solution for this expression to arrive at Proposition 1.

Step 1: The Lagrangian of the household's problem is

$$\begin{aligned} \mathcal{L} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - \frac{1}{1+\eta} L_t^{1+\eta} - \gamma_t \left(P_t C_t - M_{t-1}^d - T_t^h \right) \right. \\ &- \left. \lambda_t \left(P_t C_t + M_t^d - \int_0^1 \Pi_{it} di - W_t L_t - M_{t-1}^d - T_t^h \right) \right], \end{aligned}$$

with associated sufficient first-order conditions

$$C_t: \quad \tfrac{1}{C_t} - P_t \left(\lambda_t + \mu_t \right) = 0, \qquad \frac{1}{P_t C_t} = \lambda_t + \gamma_t \tag{A1}$$

$$L_t: \quad -L_t^\eta + \lambda_t W_t = 0, \qquad L_t^\eta = \lambda_t W_t \tag{A2}$$

$$M_t^d: \quad -\lambda_t + \beta \mathbb{E}_t \left[\lambda_{t+1} + \mu_{t+1} \right] = 0, \quad \lambda_t = \beta \mathbb{E}_t \left[\lambda_{t+1} + \mu_{t+1} \right]. \tag{A3}$$

We check that the proposed solution $M_t = P_t C_t$ and $W_t / L_t^\eta = M_t^s / \delta$ with $\delta = \beta \mathbb{E}_t \left[\frac{M_t}{M_{t+1}}\right] < 1$ satisfies the relevant first-order conditions. The combination of (A1) and (A2) shows that

$$\gamma_t = \frac{1}{P_t C_t} - \frac{L_t^\eta}{W_t} = (1 - \delta) \frac{1}{M_t^s} > 0,$$

where I have used that $\lambda_t = L_t^\eta / W_t = \delta / M_t$. This proves the first part of the first step.

To find δ, combine (A3) with $W_t / L_t^\eta = M_t^s / \delta$ to arrive at

$$\frac{\delta}{M_t^s} = \beta \mathbb{E}_t \left[\frac{\delta}{M_{t+1}^s} + \frac{1-\delta}{M_{t+1}^s} \right] = \beta \mathbb{E}_t \left[\frac{1}{M_{t+1}^s} \right].$$

This in turn shows that $\delta = \beta \mathbb{E}_t \left[\frac{M_t}{M_{t+1}}\right] = \beta \exp\left(-\phi_0 + \frac{1}{2}\phi_\theta \mathbb{V}\left[\mathbb{E}_{cbt}\theta_t\right] + \frac{1}{2}\phi_\xi \mathbb{V}\left[\mathbb{E}_{cbt}\xi_t\right]\right) > 0$, and thus that all three first-order conditions are satisfied at the candidate solution.

We now use $W_t / L_t^\eta = M_t^s / \delta$ and the binding cash-in-advance constraint to find the labor market-clearing real wage. Equating labor supply $L_t^s = \delta^{\frac{1}{\eta}} W_t^{\frac{1}{\eta}} \left(P_t Y_t\right)^{-\frac{1}{\eta}}$ with labor demand $L_t^d = \int_0^1 \frac{Y_{it}}{A_t} di = \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\rho} \frac{Y_t}{A_t} di = \frac{Y_t}{A_t}$, since all firms set the same price in equilibrium, shows that[2]

$$\frac{W_t}{A_t P_t} = \delta^{-1} \left(\frac{Y_t}{A_t} \right)^{1+\eta}. \tag{A4}$$

[1] The proof of the first step follows that of Lemma 1 in Hellwig (2005).

[2] I here set C_t equal to Y_t (see the last section of this appendix). This is because the real resource cost of inflation is of second-order, $Y_t = C_t + \mathcal{O}(2)$, and I subsequently linearize all resultant expressions.

A log-linear approximation of (A4) then completes the first step and shows that

$$w_t - p_t - a_t = (1+\eta)(y_t - a_t). \tag{A5}$$

Step 2: The demand for firm $i \in [0, 1]$ goods is

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\rho} Y_t.$$

The representative firm's problem is therefore

$$\max_{P_{it}} \mathbb{E}_{f0} \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{P_t C_t}\right) \left[(1+T_t^s) P_{it} Y_{it} - W_t L_{it} - \frac{\psi}{2}\left(\frac{P_{it}}{P_{it-1}} - 1\right) P_t Y_t\right] =$$

$$\max_{P_{ijt}} \mathbb{E}_{f0} \sum_{t=0}^{\infty} \beta^t \left[(1+T_t^s)\left(\frac{P_{it}}{P_t}\right)^{1-\rho} - \left(\frac{W_t}{A_t P_t}\right)\left(\frac{P_{it}}{P_t}\right)^{-\rho} - \frac{\psi}{2}\left(\frac{P_{it}}{P_{it-1}} - 1\right)\right].$$

where $1 + T_t^s$ has mean $\frac{\rho}{\rho-1}$. The sufficient first-order condition to this problem is

$$\mathbb{E}_{ft}\left[(1+T_t^s)(1-\rho)\left(\frac{P_{it}}{P_t}\right)^{-\rho}\frac{1}{P_t} + \rho\left(\frac{W_t}{A_t P_t}\right)\left(\frac{P_{it}}{P_t}\right)^{-\rho}\frac{1}{P_{it}}\right] =$$
$$\mathbb{E}_{ft}\left[\psi\left(\frac{P_{it}}{P_{it-1}} - 1\right)\frac{1}{P_{it-1}} - \beta\psi\left(\frac{P_{it+1}}{P_{it}} - 1\right)\frac{P_{it+1}}{P_{it}^2}\right]. \tag{A6}$$

Since firm prices are symmetric ($P_{it} = P_t$), this first-order condition reduces to

$$\mathbb{E}_{ft}\left[-\frac{1}{\mathcal{M}_t} + \left(\frac{W_t}{A_t P_t}\right) - \frac{\psi}{\rho}\left(\frac{P_t}{P_{t-1}} - 1\right)\left(\frac{P_t}{P_{t-1}}\right) + \beta\frac{\psi}{\rho}\left(\frac{P_{t+1}}{P_t} - 1\right)\left(\frac{P_{t+1}}{P_t}\right)\right] = 0. \tag{A7}$$

Equation (A7) represents the *New Keynesian Phillips Curve* for our economy.

A straightforward log-linearization of (A7) then shows that

$$p_t = \gamma_{-1} p_{t-1} + \gamma_0 \mathbb{E}_{ft}\left[w_t - p_t - a_t\right] + \gamma_1 \mathbb{E}_{ft}\left[p_{t+1}\right] + \gamma_2 \mathbb{E}_{ft}\left[\mu_t\right], \tag{A8}$$

where $\gamma_{-1} = \frac{1}{1+\beta}$, $\gamma_1 = \frac{\beta}{1+\beta}$, $\gamma_2 = \frac{\rho}{\psi(1+\beta)}$, $\gamma_0 = \frac{\rho \underline{W^{ss}}}{\psi(1+\beta)}$, and $\underline{W^{ss}}$ denotes the steady state of $\frac{W_t}{P_t A_t}$.

We can now use (A5) and that $y_t = m_t - p_t$ to re-write (A8) as

$$p_t = \lambda_{-1} p_{t-1} + \lambda_0 \mathbb{E}_{ft}\left[m_t - a_t\right] + \lambda_1 \mathbb{E}_{ft}\left[p_{t+1}\right] + \lambda_2 \mathbb{E}_{ft}\left[\mu_t\right], \tag{A9}$$

where $\lambda_2 = \left[\underline{W^{ss}}(1+\eta)\right]^{-1}\lambda_0$ and

$$\lambda_0 = \frac{\underline{W^{ss}}(1+\eta)}{\frac{\psi}{\rho}(1+\beta) + \underline{W^{ss}}(1+\eta)}, \quad \lambda_{-1} = \frac{1}{1+\beta}\frac{\frac{\psi}{\rho}(1+\beta)}{\frac{\psi}{\rho}(1+\beta) + \underline{W^{ss}}(1+\eta)}, \tag{A10}$$

$$\lambda_1 = \frac{\beta}{1+\beta}\frac{\frac{\psi}{\rho}(1+\beta)}{\frac{\psi}{\rho}(1+\beta) + \underline{W^{ss}}(1+\eta)}. \tag{A11}$$

This completes the second step.

Step 3: We start with the conjecture that (A9) collapses to

$$p_t = \nu_{-1} p_{t-1} + \nu_0 \mathbb{E}_{ft}\left[m_t - a_t\right] + \nu_1 \mathbb{E}_{ft}\left[\mu_t\right]. \tag{A12}$$

Inserting this conjecture into (A9) and matching terms shows that the conjecture is true *iff*.

$$\nu_{-1}=\frac{\lambda_{-1}}{1-\lambda_1\nu_{-1}},\quad \nu_0=\frac{\lambda_0}{1-\lambda_1\nu_{-1}}+\frac{\lambda_1}{1-\lambda_1\nu_{-1}}\nu_0,\quad \nu_1=\frac{\lambda_2}{1-\lambda_1\nu_{-1}}+\frac{\lambda_1}{1-\lambda_1\nu_{-1}}\nu_1. \tag{A13}$$

The fixed point equation for ν_{-1} equals from (A13)

$$g_{(-1)}\left(\nu_{-1}\right)=-\lambda_1\nu_{-1}^2+\nu_{-1}-\lambda_{-1}=0.$$

Now, since $g_{(-1)}$ is globally concave, $g_{(-1)}\left(0\right)=-\lambda_{-1}<0$, and $g_{(-1)}\left(1\right)=1-\lambda_1-\lambda_{-1}>0$, we conclude that there exist two positive solutions for ν_{-1}, one of which is stable $\nu_{-1}\in(0,\,1)$.

Consider now the remaining two fixed point equations in (A13):

$$\begin{aligned} g_0\left(\nu_0\right)= &\ \tfrac{\lambda_0}{1-\lambda_1\nu_{-1}}+\tfrac{\lambda_1}{1-\lambda_1\nu_{-1}}\nu_0 &= \nu_0\\ g_1\left(\nu_1\right)= &\ \tfrac{\lambda_2}{1-\lambda_1\nu_{-1}}+\tfrac{\lambda_1}{1-\lambda_1\nu_{-1}}\nu_1 &= \nu_1. \end{aligned}$$

Because $g_0(0)=\frac{\lambda_0}{1-\lambda_1\nu_{-1}}>0$, $0<g_0(1)<\frac{\lambda_0+\lambda_1}{1-\lambda_1\nu_{-1}}=\frac{1-\lambda_{-1}}{1-\lambda_1\nu_{-1}}<1$ and $\frac{dg_0}{d\nu_0}>0$, it follows that there exists a unique $\nu_0\in(0,1)$. A similar argument then shows that there exists a unique, positive solution for ν_1. It also immediately follows from (A10) and (A13) that if $\psi=0$,

$$\nu_{-1}=0,\quad \nu_0=1,\quad \nu_1=\left[\underline{W}^{ss}(1+\eta)\right]^{-1}.$$

Finally, notice that from (A10), (A11), and (A13) we have that

$$\nu_{-1}=\frac{\frac{\psi}{\rho}}{\underline{W}^{ss}(1+\eta)+\frac{\psi}{\rho}+\beta\frac{\psi}{\rho}\left(1-\nu_{-1}\right)},\quad \nu_0=\frac{\underline{W}^{ss}(1+\eta)}{\left(\frac{1}{\nu_{-1}}-\beta\right)\psi}, \tag{A14}$$

so that

$$\frac{\partial\nu_{-1}}{\partial\psi}>0\quad\text{and}\quad\frac{\partial\left(\frac{1}{\nu_{-1}}-\beta\right)\psi}{\partial\psi}=\frac{1}{\nu_{-1}}-\beta-\frac{\psi}{\nu_{-1}^2}\frac{\partial\nu_{-1}}{\partial\psi}>0,$$

where the last inequality follows from ν_{-1} in (A14). Thus, $\partial\nu_{-1}/\partial\psi>0$ and $\partial\nu_0/\partial\psi<0$. $\square$

Proof of Proposition 2: The steps are well-known (see, for instance, Gali 2008, or Nistico 2007 for the case with Rotemberg 1982 quadratic nominal cost). Define

$$\mathcal{U}_t\equiv\log C_t-\frac{1}{1+\eta}L_t^{1+\eta}.$$

A second-order approximation around the non-stochastic full information steady state then shows that

$$\mathcal{U}_t\approx\mathcal{U}_C C^{ss}\left[c_t+\frac{\mathcal{U}_C}{\mathcal{U}_L}\frac{L^{ss}}{Y^{ss}}\left(l_t+\frac{1+\eta}{2}l_t^2\right)\right],$$

where all derivates are evaluated at their steady state values (ss). Now, employing the resource constraint in its log-linear form (see the final subsection in this Appendix) $y_t=c_t$, the economy-wide production function $y_t=a_t+l_t$, and using that at the *first best* allocation $\mathcal{U}_C/\mathcal{U}_L=-A^{ss}$, we arrive at

$$\mathcal{W}_t=\frac{\mathcal{U}_t}{\mathcal{U}_C C^{ss}}\approx y_t-\left[y_t-a_t+\frac{1+\eta}{2}\left(y_t-a_t\right)^2\right]=\frac{1+\eta}{2}\left(y_t-a_t\right)^2+t.i.p,$$

where $t.i.p$ denotes *terms independent of policy*. Since $\mathcal{W} = \frac{1}{1-\beta}\mathbb{E}_{t-1}[\mathcal{W}_t]$, this completes the proof. $\square$

On the Constancy of $\mathbb{E}_{t-1}[y_t - a_t]^2$: An implication of Proposition 1 is that $q_t \equiv \begin{bmatrix} p_t & m_t & a_t & \mu_t \end{bmatrix}$ follows a VAR(1) in equilibrium. This is because $q_t = Mq_{t-1} + Nu_t$ can be written as:[3]

$$q_t = \begin{bmatrix} p_t \\ m_t \\ a_t \\ \mu_t \end{bmatrix} = \begin{bmatrix} \nu_{-1} & \nu_0 & -\nu_0 & \nu_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{t-1} \\ m_{t-1} \\ a_{t-1} \\ \mu_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & -\nu_0 & \nu_1 & 0 & 0 & \nu_0\phi_\theta & \nu_0\phi_\xi \\ 0 & 0 & 0 & 0 & \phi_\theta & \phi_\xi & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} u_t,$$

where

$$u_t \equiv \begin{bmatrix} \theta_t & \xi_t & \mathbb{E}_t^f\theta_t & \mathbb{E}_t^f\xi_t & \mathbb{E}_t^{cb}\theta_t & \mathbb{E}_t^{cb}\xi_t & \mathbb{E}_t^f\mathbb{E}_t^{cb}\theta_t & \mathbb{E}_t^f\mathbb{E}_t^{cb}\xi_t \end{bmatrix}',$$

and the matrices M and N are implicitly defined.

Consequently, the output gap $y_t - a_t = m_t - p_t - a_t$ can be written as

$$y_t - a_t = \begin{bmatrix} -1 & 1 & -1 & 0 \end{bmatrix} Mq_{t-1} + \begin{bmatrix} -1 & 1 & -1 & 0 \end{bmatrix} Nu_t.$$

Notice that the output gap depends only upon previous period terms q_{t-1} and white noise shocks and expectations u_t. Conditional on full $t-1$ information, the expected squared value of the output gap $\mathbb{E}_{t-1}[y_t - a_t]^2$ is therefore constant and equals

$$\mathbb{E}_{t-1}[y_t - a_t]^2 = \begin{bmatrix} -1 & 1 & -1 & 0 \end{bmatrix} N\mathbb{V}[u_t]N' \begin{bmatrix} -1 & 1 & -1 & 0 \end{bmatrix}'.$$

Real Resource Cost of Inflation: The aggregate resource constraint is in levels

$$Y_t = C_t + \frac{\psi}{2}\tilde{\pi}_t^2 Y_t, \tag{A15}$$

where $\tilde{\pi}_t$ denotes the (non-log-linearized) level of inflation.

Log-linearizing (A15) around the full information steady state immediately shows that

$$y_t = c_t + \frac{3}{2}\psi\left[\tilde{\pi}_t^2 Y_t\right]_{|ss}(\pi_t + y_t),$$

where $[\cdot]_{|ss}$ denotes an expression evaluated at its steady state level. But since the steady state rate of inflation is zero ($[\tilde{\pi}_t]_{|ss} = 0$), it follows that, to a first order,

$$y_t = c_t. \tag{A16}$$

[3] I here ignore unimportant constant terms.

Online Appendix B: Inefficient Disturbances

This Appendix details the proofs of the results described in Section 4.[4]

Online Appendix B.1: Equilibrium Prices and Money Supply

I consider the general case in which $\tau_p \in \mathbb{R}_+$. I first solve for the equilibrium price level and money supply. I then later (in the proof of Proposition 3) derive the optimal policy.

To start with, I conjecture that

$$\begin{aligned} m_t &= m_{t-1} + \phi_\xi \mathbb{E}_{cbt}[\xi_t] && \text{(A17)} \\ &= m_{t-1} + q_0 z_t + q_1 \underline{p}_t && \text{(A18)} \end{aligned}$$

$$\begin{aligned} p_t &= \nu_{-1} p_{t-1} + \nu_0 \mathbb{E}_{ft}[m_t] + \nu_1 \mathbb{E}_{ft}[\mu_t] && \text{(A19)} \\ &= \nu_{-1} p_{t-1} + \nu_0 m_{t-1} + \nu_1 \mu_{t-1} + \nu_0 \phi_\xi \mathbb{E}_{ft}[\mathbb{E}_{cbt}\xi_t] + \nu_1 \mathbb{E}_{ft}[\xi_t] \\ &= \nu_{-1} p_{t-1} + \nu_0 m_{t-1} + \nu_1 \mu_{t-1} + k_0 x_t + k_1 \omega_t + k_2 \underline{p}_t, && \text{(A20)} \end{aligned}$$

where the noisy signal of the economy-wide price level $\bar{p}_t$ is equivalent to the observation of

$$\begin{aligned} \underline{p}_t &\equiv \frac{1}{\underline{k}_0} \left[\bar{p}_t - \nu_{-1} p_{t-1} - \nu_0 m_{t-1} - \nu_1 \mu_{t-1} - k_1 \omega_t - k_2 \underline{p}_t \right] \\ &= x_t + \frac{1}{k_0} \epsilon_{pt} = \xi_t + \epsilon_{xt} + \frac{1}{k_0} \epsilon_{pt}. \end{aligned}$$

I note that $\underline{p}_t$ by design is independent of the other signals in Ω_{cbt} and Ω_{ft} conditional on ξ_t.

To verify the conjecture in (A18) and (A20), we need to derive expressions for firm and central bank expectations, in addition to firm expectations of the central bank's private information. Due to the linear-normal information structure:

$$\begin{aligned} \mathbb{E}_{ft}[\xi_t] &= w_x x_t + w_\omega \omega_t, \quad w_x \equiv \frac{\tau_x (\tau_\omega + \tau_z)}{(\tau_\omega + \tau_z)(\tau_\xi + \tau_x) + \tau_\omega \tau_z} && \text{(A21)} \\ \mathbb{E}_{ft}[z_t] &= v_x x_t + v_\omega \omega_t, \quad v_x \equiv \frac{\tau_x \tau_z}{(\tau_\omega + \tau_z)(\tau_\xi + \tau_x) + \tau_\omega \tau_z} \\ \mathbb{E}_{cbt}[\xi_t] &= \beta_z z_t + \beta_p \underline{p}_t, \quad \beta_z \equiv \frac{\tau_z \left(\tau_x + \tau_p k_0^2\right)}{\left(\tau_x + \tau_p k_0^2\right)(\tau_\xi + \tau_z) + \tau_x \tau_p k_0^2}, \end{aligned}$$

where w_ω, v_ω and β_p are implicitly defined and follow the standard expressions.

Inserting these results into (A17) and (A19) then demonstrates that

$$m_t = m_{t-1} + \phi_\xi \left(\beta_z z_t + \beta_p \underline{p}_t \right) \quad \text{(A22)}$$

$$\begin{aligned} p_t &= \nu_{-1} p_{t-1} + \nu_0 m_{t-1} + \nu_1 \mu_{t-1} + \nu_0 \left(q_0 \mathbb{E}_{ft}[z_t] + q_1 \underline{p_t} \right) + \nu_1 \mathbb{E}_{ft}[\xi_t] && \text{(A23)} \\ &= \nu_{-1} p_{t-1} + \nu_0 m_{t-1} + \nu_1 \mu_{t-1} + \nu_0 \left(q_0 v_x + \frac{\nu_1}{\nu_0} w_x \right) x_t + \nu_0 \left(q_0 v_\omega + \frac{\nu_1}{\nu_0} w_\omega \right) \omega_t + \nu_0 q_1 \underline{p}_t && \text{(A24)} \end{aligned}$$

[4] I below dispense with all superscripts to ease notation when it does not cause confusion. Therefore, the variable x_t, for example, refers to x_t^ξ in this appendix.

which verifies our conjecture *iff.* there exists a solution to the system of equations

$$q_0 = \phi_\xi \beta_z, \quad q_1 = \phi_\xi \beta_p \tag{A25}$$

$$k_0 = \nu_0 \left(q_0 v_x + \frac{\nu_1}{\nu_0} w_x \right), \quad k_1 = \nu_0 \left(q_0 v_\omega + \frac{\nu_1}{\nu_0} w_\omega \right), \quad k_2 = \nu_0 q_1, \tag{A26}$$

where $q_h \in \mathbb{R}$ and $k_j \in \mathbb{R}$, $h = \{0, 1\}$ and $j = \{0, 1, 2\}$.

Since all fixed point equations in (A25) and (A26) ultimately depend *only* on k_0, all we need to show is that the equation for k_0 has a solution. We can re-write this equation as

$$\begin{aligned} Q(k_0) &= (\tau_\xi + \tau_x + \tau_z)\, \tau_p k_0^3 - \nu_0 \left[\phi_\xi \tau_z v_x + \frac{\nu_1}{\nu_0} (\tau_\xi + \tau_x + \tau_z)\, w_x \right] \tau_p k_0^2 \\ &+ \tau_x (\tau_\xi + \tau_z)\, k_0 - \nu_0 \tau_x \left[\phi_\xi \tau_z v_x + \frac{\nu_1}{\nu_0} (\tau_\xi + \tau_z)\, w_x \right] = 0. \end{aligned} \tag{A27}$$

Because $\phi_\xi > 0$, *Descartes' Rule of Signs* establishes that there exists either one or three *positive* solutions for k_0 to (A27), depending on parameter values. I deal with this multiplicity of equilibria by focusing on the highest welfare equilibrium (see also my remarks on this point in Section 5). □

Online Appendix B.2: The Cost of Disclosure

I show that complete opacity is optimal when $\phi_\xi = 0$. The economy-wide output gap is

$$\begin{aligned} y_t &= m_t - p_t \\ &= \phi_\xi \mathbb{E}_{cbt} [\xi_t] - \nu_0 \mathbb{E}_{ft} \left[\phi_\xi \mathbb{E}_{cbt} \xi_t + \frac{\nu_1}{\nu_0} \xi_t \right] + t.l.p, \end{aligned}$$

where *t.l.p* denotes terms *from last period* irrelevant to current welfare. Thus, when $\phi_\xi = 0$

$$y_t = -\nu_1 \mathbb{E}_{ft} [\xi_t] + t.l.p,$$

and therefore

$$(1 - \beta)\, \mathcal{W} = \nu_1^2 \mathbb{V} [\mathbb{E}_{ft} \xi_t] = \nu_1^2 \left(\frac{1}{\tau_\xi} - \frac{1}{(\tau_\omega + \tau_z)(\tau_\xi + \tau_x) + \tau_\omega \tau_z} \right),$$

where I have used the expression for $\mathbb{E}_{ft} [\xi_t]$ from (A21). It follows that $\frac{d\mathcal{W}}{d\tau_\omega} > 0$. □

Online Appendix B.3: The Benefit of Disclosure

Proof of Corollary 1: I consider the case in which $\tau_p \to 0$. The economy-wide output gap is

$$\begin{aligned} y_t &= m_t - p_t \\ &= \phi_\xi \mathbb{E}_t^{cb} [\xi_t] - \nu_0 \phi_\xi \mathbb{E}_t^f \mathbb{E}_t^{cb} [\xi_t] - \nu_1 \mathbb{E}_t^f [\xi_t] + l.p.t. \\ &= [\phi_\xi \beta_z - \nu_0 \phi_\xi \beta_z (v_x + v_\omega) - \nu_1 (w_x + w_\omega)]\, \xi_t - [\nu_0 \phi_\xi \beta_z v_x + \nu_1 w_x]\, \epsilon_{xt} + l.p.t. \\ &+ [\phi_\xi \beta_z - \nu_0 \phi_\xi \beta_z v_\omega - \nu_1 w_\omega]\, \epsilon_{zt} - [\nu_0 \phi_\xi \beta_z v_w + \nu_1 w_\omega]\, \epsilon_{\omega t} + l.p.t.. \end{aligned}$$

Thus, it follows from Proposition 2 that

$$\begin{aligned}(1-\beta)\,\mathcal{W} &= \left[\phi_\xi\beta_z-\nu_0\phi_\xi\beta_z\frac{\tau_x\tau_z+\tau_\omega\left(\tau_\xi+\tau_x+\tau_z\right)}{\left(\tau_\omega+\tau_z\right)\left(\tau_\xi+\tau_x\right)+\tau_\omega\tau_z}-\nu_1\frac{\tau_x\left(\tau_z+\tau_\omega\right)+\tau_\omega\left(\tau_\xi+\tau_x+\tau_z\right)}{\left(\tau_\omega+\tau_z\right)\left(\tau_\xi+\tau_x\right)+\tau_\omega\tau_z}\right]^2\frac{1}{\tau_\xi}\\ &- \left[\nu_0\phi_\xi\beta_z\frac{\tau_x\tau_z}{\left(\tau_\omega+\tau_z\right)\left(\tau_\xi+\tau_x\right)+\tau_\omega\tau_z}+\nu_1\frac{\tau_x\left(\tau_z+\tau_\omega\right)}{\left(\tau_\omega+\tau_z\right)\left(\tau_\xi+\tau_x\right)+\tau_\omega\tau_z}\right]^2\frac{1}{\tau_x}\\ &+ \left[\phi_\xi\beta_z-\nu_0\phi_\xi\beta_z\frac{\tau_\omega\left(\tau_\xi+\tau_x+\tau_z\right)}{\left(\tau_\omega+\tau_z\right)\left(\tau_\xi+\tau_x\right)+\tau_\omega\tau_z}-\nu_1\frac{\tau_\omega\left(\tau_\xi+\tau_x+\tau_z\right)}{\left(\tau_\omega+\tau_z\right)\left(\tau_\xi+\tau_x\right)+\tau_\omega\tau_z}\right]^2\frac{1}{\tau_z}\\ &- \left[\nu_0\phi_\xi\beta_z\frac{\tau_\omega\left(\tau_\xi+\tau_x+\tau_z\right)}{\left(\tau_\omega+\tau_z\right)\left(\tau_\xi+\tau_x\right)+\tau_\omega\tau_z}+\nu_1\frac{\tau_\omega\left(\tau_\xi+\tau_x+\tau_z\right)}{\left(\tau_\omega+\tau_z\right)\left(\tau_\xi+\tau_x\right)+\tau_\omega\tau_z}\right]^2\frac{1}{\tau_\omega}.\end{aligned}$$

In turn, this shows after a few simple but tedious derivations that $\frac{\partial\mathcal{W}}{\partial\tau_\omega}\gtrless 0$ *iff.*

$$\nu_0\left(\nu_0-2\right)\left[\left(\tau_\xi+\tau_x+\tau_z\right)\phi_\xi\beta_z\right]^2+2\nu_1\tau_z\left(\nu_0-1\right)\left[\left(\tau_\xi+\tau_x+\tau_z\right)\phi_\xi\beta_z\right]+\nu_1^2\tau_z^2\gtrless 0. \tag{A28}$$

Equation (A28) is second-degree polynomial in $\left(\tau_\xi+\tau_x+\tau_z\right)\phi_\xi\beta_z$ with a unique positive solution, which follows from *Descartes' Rule of Signs*. We can now use this solution to show that $\frac{\partial\mathcal{W}}{\partial\tau_\omega}\gtrless 0$ *iff.*

$$\phi_\xi\lessgtr\frac{\nu_1}{2-\nu_0}\frac{\tau_x^\xi+\tau_z^\xi}{\tau_\xi+\tau_x^\xi+\tau_z^\xi}\equiv\bar{\phi}_\xi.$$

□

Proof of Proposition 3: I once more consider the general case in which $\tau_p\in\mathbb{R}_+$. The proof has three steps: The first step uses the equilibrium price level (A20) and Proposition 2 to derive a convenient expression for $\mathcal{W}$ as a function of k_0, q_0, and τ_ω. The second step then uses that expression to find the unique optimal values for these variables. Lastly, I use the expression for q_0 from (A25) to translate the optimal q_0 coefficient back into the level of ϕ_ξ that it entails.

Step 1: Equilibrium welfare.

The economy-wide output gap is

$$\begin{aligned}y_t &= m_t-p_t\\ &= \left[q_0+\left(1-\nu_0\right)q_1-k_0-k_1\right]\xi_t+\left[\left(1-\nu_0\right)q_1-k_0\right]\epsilon_{xt}+\left(1-\nu_0\right)q_1\frac{1}{k_0}\epsilon_{pt}+t.l.p\\ &+ \left(q_0-k_1\right)\epsilon_{zt}-k_1\epsilon_{\omega t}+t.l.p.\end{aligned}$$

Thus, after a few, simple derivations

$$\begin{aligned}(1-\beta)\mathcal{W} &= \left[-\nu_1+\left(1-\nu_0\right)q_0\frac{\tau_z+\alpha}{\tau_z}+\frac{\tau_\xi}{\tau_x}k_0\right]^2\frac{1}{\tau_\xi}+\left[\left(1-\nu_0\right)q_0\left(\frac{\alpha}{\tau_z}\right)-k_0\right]^2\frac{1}{\tau_x}\\ &+ \left(1-\nu_0\right)^2q_0^2\left(\frac{\alpha}{\tau_z}\right)^2\frac{1}{\tau_pk_0^2}+\left[-\nu_1+(1-\nu_0)q_0+\frac{\tau_\xi+\tau_x}{\tau_x}k_0\right]^2\frac{1}{\tau_z}+k_1^2\frac{1}{\tau_\omega},\end{aligned} \tag{A29}$$

where $\alpha\equiv\frac{\tau_x\tau_pk_0^2}{\tau_x+\tau_pk_0^2}$ and $k_1=\nu_0\tau_\omega\frac{q_0(\tau_\xi+\tau_x+\tau_z)+\frac{\nu_1}{\nu_0}\tau_z}{(\tau_\omega+\tau_z)(\tau_\xi+\tau_x)+\tau_\omega\tau_z}$.

Step 2: The unique optimal values of k_0, τ_ω and q_0.

Equations (A25) and (A26) show that there exists a one-to-one relationship between ϕ_ξ and $\bar{q}_0=(1-\nu_0)q_0\left(\frac{\alpha}{\tau_z}\right)=(1-\nu_0)\phi_\xi\beta_z\left(\frac{\alpha}{\tau_z}\right)$ for given τ_ω and k_0. Instead of optimally choosing $\phi_\xi>0$, I therefore choose $\bar{q}_0$ instead, implicitly defining the associated optimal q_0 and ϕ_ξ. This simplifies the derivations.

Consider the Lagrangian associated with the optimal policy problem[5]

$$\mathcal{L} = \mathcal{W} + \lambda \left[k_0 - \nu_0 \tau_x \frac{q_0 \tau_z + \frac{\nu_1}{\nu_0} (\tau_\omega + \tau_z)}{(\tau_\omega + \tau_z)(\tau_\xi + \tau_x) + \tau_\omega \tau_z} \right]. \tag{A30}$$

Thus,

$$\frac{\partial \mathcal{L}}{\partial \tau_\omega} = 0: \quad \frac{\nu_0 \tau_z}{[(\tau_\omega + \tau_z)(\tau_\xi + \tau_x) + \tau_\omega \tau_z]^2} \left(2k_1 \frac{\tau_\xi + \tau_x}{\tau_\omega} + \lambda \tau_x \right) \left[q_0 (\tau_\xi + \tau_x + \tau_z) + \frac{\nu_1}{\nu_0} \tau_z \right] = 0. \tag{A31}$$

This equation is satisfied for $\tau_\omega \to \infty$.[6]

We can now return to (A29) with $\tau_\omega \to \infty$. Minimizing (A29) with respect to $\bar{q}_0$ when $k_0 = \nu_1 \frac{\tau_x}{\tau_\theta + \tau_x + \tau_z} > 0$ and $k_1 = 0$ because $\tau_\omega \to \infty$ yields

$$\begin{aligned} \frac{\partial \mathcal{W}}{\partial \bar{q}_0} = 0: \quad & \left[-\nu_1 + \bar{q}_0 \left(\tfrac{\tau_z + \alpha}{\alpha} \right) + \tfrac{\tau_\xi}{\tau_x} k_0 \right] \tfrac{\tau_z + \alpha}{\alpha \tau_\xi} + (\bar{q}_0 - k_0) \tfrac{1}{\tau_x} + \bar{q}_0 \tfrac{1}{\tau_p k_0^2} \\ & + \left[-\nu_1 + \bar{q}_0 \tfrac{\tau_z}{\alpha} + \tfrac{\tau_\theta + \tau_x}{\tau_x} k_0 \right] \tfrac{1}{\alpha} = 0, \end{aligned} \tag{A32}$$

which is affine in $\bar{q}_0$. Thus, after a few simple steps:

$$\bar{q}_0^\star = \nu_1 \frac{\alpha^\star}{\tau_\xi + \alpha^\star + \tau_z} > 0,$$

where $\alpha^\star = \frac{\tau_x \tau_p k_0^{\star,2}}{\tau_x + \tau_p k_0^{\star,2}}$, $\tau_\omega^\star \to \infty$, and $k_0^\star = \nu_1 \frac{\tau_x}{\tau_\xi + \tau_x + \tau_z}$.

Step 3: The optimal level of instrument policy, $\phi_\xi^\star$.

In turn, this optimal value of $\bar{q}_0$ can uniquely be implemented with

$$\phi_\xi^\star = \frac{\nu_1}{1 - \nu_0} > 0, \tag{A33}$$

where I have used the definition of $\bar{q}_0$ in addition to (A25). □

Online Appendix B.4: Extensions

Dispersed Information: Consider the mass $1-\alpha$ of *flexible price* firms. Log-linearizing equation (A6) when $\psi = 0$ shows that these firms' optimal prices are characterized by

$$\begin{aligned} p_{it}^{\text{flex}} &= \mathbb{E}_{it}^f [p_t] + (1+\eta) \mathbb{E}_{it}^f [y_t - a_t] + \mathbb{E}_{it}^f [\mu_t] \\ &= (1+\eta) \mathbb{E}_{it}^f [m_t] - \eta \mathbb{E}_{it}^f [p_t] + \mathbb{E}_{it}^f [\mu_t], \end{aligned} \tag{A34}$$

where the second equality uses that $y_t - a_t = m_t - p_t$.

Turning to the mass α of *fixed price* firms, their prices are instead simply

$$p_{it}^{\text{fixed}} = p_{it-1}^{\text{fixed}}, \tag{A35}$$

[5]The social welfare loss function in (A29) is *strictly pseudo-convex*. When combined with the expression for k_0 in (A26), we thus have that an interior solution $\frac{d\mathcal{L}}{dx} = 0$, where $x = \{\bar{q}_0, \tau_\omega, k_0\}$, is the unique global minimum.

[6]Notice that $q_0 = -\frac{\nu_1}{\nu_0} \frac{\tau_z}{\tau_\xi + \tau_x + \tau_z}$ is not an option in (A31) since that would imply that $\phi_\xi < 0$, and neither is $k_1 = \frac{-\lambda \tau_x \tau_\omega}{2(\tau_\xi + \tau_x)}$ since that would imply that $k_1 < 0$.

We can now combine (A34) and (A35) to show that

$$\begin{aligned} p_t &= \int_\alpha^1 p_{it}^{\text{flex}} di + \int_0^\alpha p_{it}^{\text{fixed}} di \\ &= \nu_0 \bar{\mathbb{E}}_t^f [m_t] + \nu_1 \bar{\mathbb{E}}_t^f [\mu_t] + \nu_p \bar{\mathbb{E}}_t^f [p_t] + \alpha p_{t-1}^{\text{fixed}}, \end{aligned} \tag{A36}$$

where $\nu_0 = (1-\alpha)(1+\eta)$, $\nu_1 = 1-\alpha$, $\nu_p = (1-\alpha)\eta$, and $\bar{\mathbb{E}}_t^f [\cdot] = \frac{1}{1-\alpha} \int_\alpha^1 \mathbb{E}_{it}^f [\cdot] \, di$.

Iterating on (A36), we can now show that

$$p_t = \sum_{j=0}^{\infty} \nu_p^j \left[\nu_0 \phi \bar{\mathbb{E}}_t^{f,j+1} \left[\mathbb{E}_t^{cb} \xi_t \right] + \nu_1 \bar{\mathbb{E}}_t^{f,j+1} [\xi_t] \right] + l.p.t.,$$

where $\bar{\mathbb{E}}_t^{f,j+1} [\cdot] = \bar{\mathbb{E}}_t^f \left[\bar{\mathbb{E}}_t^{f,j} [\cdot] \right]$, so that the output gap that results from (A36) is that in (4.8).

An informative special case is once more that in which the central bank's private signal is perfectly accurate $\tau_z \to \infty$. In this case, the output gap in (4.8) under full disclosure collapses to

$$y_t - a_t = \left(\phi - \frac{\nu_0 \phi - \nu_1}{1 - \nu_p} \right) \xi_t + l.p.t.,$$

because $\bar{\mathbb{E}}_t^{f,j+1} \left[\mathbb{E}_t^{cb} \xi_t \right] = \bar{\mathbb{E}}_t^{f,j+1} [\xi_t] = \xi_t$. This shows that $\phi^\star = \frac{\nu_1}{1-\nu_0-\nu_p}$ minimizes $\mathcal{W} = \frac{1}{1-\beta} \mathbb{E}_{t-1} [y_t - a_t]^2$.

An identical result is straightforward to derive for the $\tau_z \in \mathbb{R}_+$ case.

Learning from Prices and Signals of Central Bank Information: First, notice that Proposition 3 has been proved already in the case in which the central bank observes the noisy signal of the price level in (2.12). Proposition 3 thus extends to the case in which the central bank observes the *endogenous* signal $\underline{p}_t$ of firms' private sector information x_t. I therefore do not comment further on this case.

Second, notice that if the central bank were instead to observe a noisy *exogenous* signal of x_t of the form $x_t + \epsilon_{pt}$, where $\epsilon_{pt} \sim \mathcal{N}\left(0, \tau_p^{-1}\right)$, the only difference that this would make to the welfare expression (A29) is that $\tau_p k_0^2$ would be replaced with τ_p. Importantly, (A30) and (A31) would still hold, and hence so too would the above argument showing the optimality of full disclosure. The presence of a direct, exogenous signal of firms' private sector information does not alter the optimality of full disclosure.

Finally, notice that full disclosure is optimal for all $\tau_p > 0$. Thus, even if the central bank could choose the noise τ_p^{-1} in its direct signal of x_t (behind a veil of ignorance), so long as attention costs ensure that an interior optimum exists, full disclosure would still be optimal. This shows that the optimality of full disclosure also extends to the case in which the central bank chooses the precision of its information.

Alternative Monetary Policy Rules: Consider the economy-wide output gap for any m_t,

$$\begin{aligned} y_t - a_t = m_t - p_t &= m_t - \nu_0 \mathbb{E}_t^f [m_t] - \nu_1 \mathbb{E}_t^f [\xi_t] + l.p.t. && \text{(A37)} \\ &= (1-\nu_0) m_t - \nu_0 \eta_t^{mp} - \nu_1 \mathbb{E}_t^f [\xi_t] + l.p.t., && \text{(A38)} \end{aligned}$$

where $\eta_t^{mp} = m_t - \mathbb{E}_t^f [m_t]$ denotes firms' prediction error about monetary policy.

Further, notice that we can write

$$\begin{aligned} \mathbb{E}_t^f [\xi_t] &= w_x x_t + w_\omega \omega_t \\ &= (w_x + w_\omega) \omega_t + w_x (\epsilon_{xt} - \epsilon_{\omega t} - \epsilon_{zt}) \equiv w \omega_t + e_t, \end{aligned} \tag{A39}$$

where $w = \frac{\tau_x(\tau_\omega + \tau_z) + \tau_\omega \tau_z}{(\tau_\omega + \tau_z)(\tau_\xi + \tau_x) + \tau_\omega \tau_z}$ and $e_t = w_x (\epsilon_{xt} - \epsilon_{\omega t} - \epsilon_{zt})$.

I consider two alternative monetary policy rules in the case in which $\tau_p \to 0$.

(i) Suppose monetary policy also responds to the central bank's (potentially noisy) disclosure,

$$m_t = \phi_z \mathbb{E}_t^{cb}[\xi_t] + \phi_\omega \omega_t + l.p.t. = \phi_z z_t + \phi_\omega \omega_t + l.p.t. \tag{A40}$$

Because $\omega_t = z_t + \epsilon_{\omega t}$, this case is naturally equivalent to the central bank independently responding to the noise in its disclosure $\epsilon_{\omega t}$. Inserting (A40) and (A39) into (A38) now shows that

$$y_t - a_t = (1-\nu_0)\phi_z z_t + [(1-\nu_0)\phi_\omega - \nu_1 w]\omega_t - \nu_0 \eta_t^{mp} - \nu_1 e_t. \tag{A41}$$

But now notice that $\phi_z = 0$, $\phi_\omega = \frac{\nu_1}{1-\nu_0}$, and $\tau_\omega \to \infty$ completely eliminate the first three components, while also setting the variance of e_t to its minimal value. This illustrates how Proposition 3 extends to the case in which monetary policy also responds to the central bank's (potentially noisy) disclosure. (Notice that $\omega_t \to z_t$ when $\tau_\omega \to \infty$. Thus, responding to ω_t or z_t becomes the same.)

(ii) Suppose now instead that monetary policy directly targets the variable that causes fluctuations in the output gap, the price level,

$$m_t = \phi_p \mathbb{E}_t^{cb}[p_t] + l.p.t., \tag{A42}$$

and suppose moreover that the cash-in-advance constraint always binds.

Notice that the central bank can with (A42) still replicate the flex-price, first-best outcome when it itself has full information about the mark-up shock (with $\phi_p^{\star,full} = 1$ and $\tau_\omega^\star \to \infty$).

Equilibrium prices from Proposition 1 can be combined with (A42) to show that

$$p_t = \nu_{-1} p_{t-1} + \nu_0 \mathbb{E}_t^f[m_t] + \nu_1 \mathbb{E}_t^f[\mu_t] = \nu_1 \mathbb{E}_t^f \sum_{j=0}^{\infty} (\nu_0 \phi_p)^j \left(\mathbb{E}_t^{cb}\mathbb{E}_t^f\right)^j [\xi_t] + l.p.t., \tag{A43}$$

where $\left(\mathbb{E}_t^{cb}\mathbb{E}_t^f\right)^j [\xi_t]$ is defined by the recursion $\left(\mathbb{E}_t^{cb}\mathbb{E}_t^f\right)^j [\xi_t] = \mathbb{E}_t^{cb}\mathbb{E}_t^f \left\{\left(\mathbb{E}_t^{cb}\mathbb{E}_t^f\right)^{j-1}[\xi_t]\right\}$ with $\left(\mathbb{E}_t^{cb}\mathbb{E}_t^f\right)^0 [\xi_t] = \xi_t$, and I abstract from irrelevant constant terms throughout.

The corresponding output gap in (A37), in this case, becomes

$$\begin{aligned} y_t - a_t &= \phi_p \nu_1 \mathbb{E}_t^{cb}\mathbb{E}_t^f \sum_{j=0}^{\infty} (\nu_0\phi_p)^j \left(\mathbb{E}_t^{cb}\mathbb{E}_t^f\right)^j [\xi_t] - \nu_1 \mathbb{E}_t^f \sum_{j=0}^{\infty} (\nu_0\phi_p)^j \left(\mathbb{E}_t^{cb}\mathbb{E}_t^f\right)^j [\xi_t] + l.p.t. \\ &= \nu_1 \left\{ \phi_p \mathbb{E}_t^{cb}\mathbb{E}_t^f \sum_{j=0}^{\infty} (\nu_0\phi_p)^j \left(\mathbb{E}_t^{cb}\mathbb{E}_t^f\right)^j [\xi_t] - \mathbb{E}_t^f \sum_{j=0}^{\infty} (\nu_0\phi_p)^j \left(\mathbb{E}_t^{cb}\mathbb{E}_t^f\right)^j [\xi_t] \right\} + l.p.t. \end{aligned} \tag{A44}$$

Equation (A44) shows that when the central bank sets monetary policy to its optimal full-information value ($\phi_p = 1$) welfare losses once more only arise from a lack of common knowledge. When the central bank now also sets $\tau_\omega \to \infty$, the expression for the output gap collapses to

$$y_t - a_t = m_t - p_t = \frac{\nu_1}{1 - v_0}\left(\mathbb{E}_t^{cb}\mathbb{E}_t^f[\xi_t] - \mathbb{E}_t^f[\xi_t]\right) + l.p.t. \tag{A45}$$

Indeed, following a similar approach as in the proof of Proposition 3 (see also the previous working paper version of this paper) shows that $\phi_p^\star = 1$ and $\tau_\omega^\star \to \infty$ describe the combined optimal policy.

Online Appendix C: Efficient Disturbances

This Appendix details the proofs of the Propositions and results in Sections 5.

Online Appendix C.1: Equilibrium Prices and Money Supply

Proof of Proposition 4: I use the same steps as those used for the mark-up shock case. To solve for the symmetric linear rational expectations equilibria, I conjecture that m_t and p_t equal

$$\begin{aligned} m_t &= m_{t-1} + \phi_\theta \mathbb{E}_{cbt}\left[\theta_t\right] && \text{(A46)} \\ &= m_{t-1} + q_0 z_t + q_1 \underline{p}_t && \text{(A47)} \end{aligned}$$

$$\begin{aligned} p_t &= \nu_{-1} p_{t-1} + \nu_0 \mathbb{E}_{ft}\left[m_t - a_t\right] \\ &= \nu_{-1} p_{t-1} + \nu_0 \left(m_{t-1} - a_{t-1}\right) + \nu_0 \mathbb{E}_{ft}\left[\phi_\theta \mathbb{E}_{cbt}\left(\theta_t\right) - \theta_t\right] && \text{(A48)} \\ &= \nu_{-1} p_{t-1} + \nu_0 \left(m_{t-1} - a_{t-1}\right) + k_0 x_t + k_1 \omega_t + k_2 \underline{p}_t, && \text{(A49)} \end{aligned}$$

where the noisy signal of the economy-wide price level is equivalent to the observation of

$$\begin{aligned} \underline{p}_t &= \frac{1}{k_0}\left[\bar{p}_t - \nu_{-1} p_{t-1} - \nu_0 \left(m_{t-1} - a_{t-1}\right) - k_1 \omega_t - k_2 \underline{p}_t\right] \\ &= x_t + \frac{1}{k_0}\epsilon_{pt} = \theta_t + \epsilon_{xt} + \frac{1}{k_0}\epsilon_{pt}. \end{aligned}$$

We need to check the conjectures in (A47) and (A49). To do so, we first compute expressions for firm and central bank expectations of the productivity shock, as well as firm expectations of the central bank's private information. Because of the linear-normal information structure:

$$\begin{aligned} \mathbb{E}_{ft}\left[\theta_t\right] &= w_x x_t + w_\omega \omega_t, \quad w_x = \frac{\tau_x\left(\tau_\omega + \tau_z\right)}{\left(\tau_\omega + \tau_z\right)\left(\tau_\theta + \tau_x\right) + \tau_\omega \tau_z} \\ \mathbb{E}_{ft}\left[z_t\right] &= v_x x_t + v_\omega \omega_t, \quad v_x = \frac{\tau_x \tau_z}{\left(\tau_\omega + \tau_z\right)\left(\tau_\theta + \tau_x\right) + \tau_\omega \tau_z} \\ \mathbb{E}_{cbt}\left[\theta_t\right] &= \beta_z z_t + \beta_p \underline{p}_t, \quad \beta_z = \frac{\tau_z\left(\tau_x + \tau_p k_0^2\right)}{\left(\tau_x + \tau_p k_0^2\right)\left(\tau_\theta + \tau_z\right) + \tau_x \tau_p k_0^2}, \end{aligned}$$

where w_ω, v_ω and β_p are implicitly (*re*-)defined in accordance with the standard expressions.

Inserting these expressions into (A46) and (A48) shows that

$$m_t = m_{t-1} + \phi_\theta\left(\beta_z z_t + \beta_p \underline{p}_t\right) \tag{A50}$$

$$\begin{aligned} p_t &= \nu_{-1} p_{t-1} + \nu_0 \left(m_{t-1} - a_{t-1}\right) + \nu_0 \left(q_0 \mathbb{E}_{ft}\left[z_t\right] + q_1 \underline{p_t} - \mathbb{E}_{ft}\left[z_t\right]\right) && \text{(A51)} \\ &= \nu_{-1} p_{t-1} + \nu_0 \left(m_{t-1} - a_{t-1}\right) + \nu_0 \left(q_0 v_x - w_x\right) x_t + \nu_0 \left(q_0 v_\omega - w_\omega\right)\omega_t + \nu_0 q_1 \underline{p}_t, && \text{(A52)} \end{aligned}$$

which verifies the conjecture *iff.* there exists a solution to the system of equations

$$q_0 = \phi_\theta \beta_z, \quad q_1 = \phi_\theta \beta_p \tag{A53}$$

$$k_0 = \nu_0\left(q_0 v_x - w_x\right), \quad k_1 = \nu_0\left(q_0 v_\omega - w_\omega\right), \quad k_2 = \nu_0 q_1, \tag{A54}$$

in which $q_h \in \mathbb{R}$ and $k_j \in \mathbb{R}$, $h = \{0, 1\}$ and $j = \{0, 1, 2\}$.

Because all fixed point equations in (A53) and (A54) depend *only* on k_0, all that however needs to be shown is that the equation for k_0 has a solution. We can re-write the equation for k_0 as

$$\begin{aligned} Q(k_0) &= (\tau_\theta + \tau_x + \tau_z)\tau_p k_0^3 + \nu_0 [(\tau_\theta + \tau_x + \tau_z) w_x - \phi_\theta \tau_z v_x] \tau_p k_0^2 \\ &+ \tau_x (\tau_\theta + \tau_z) k_0 + \nu_0 \tau_x [(\tau_\theta + \tau_z) w_x - \phi_\theta \tau_z v_x] = 0. \end{aligned} \tag{A55}$$

Since $\phi_\theta \in [0, 1]$ by assumption and $v_x \leq w_x$, *Descartes' Rule of Signs* then establishes that there exists either one or three *negative* solutions for k_0 to (A55), depending on parameter values. □

Online Appendix C.2: Disclosure and Prices

Proof of Proposition 5: The equation determining k_0 is

$$k_0 = \nu_0 \tau_x \frac{q_0 \tau_z - (\tau_\omega + \tau_z)}{(\tau_\omega + \tau_z)(\tau_\theta + \tau_x) + \tau_\omega \tau_z} < 0, \quad q_0 = \phi_\theta \frac{\tau_z (\tau_x + \tau_a k_0^2)}{(\tau_x + \tau_a k_0^2)(\tau_\theta + \tau_z) + \tau_x \tau_a k_0^2}. \tag{A56}$$

The derivative of *the right-hand side (RHS)* of k_0 in (A56) with respect to τ_ω equals

$$\frac{\partial RHS}{\partial \tau_\omega} = \nu_0 \frac{\tau_x \tau_z}{[(\tau_\omega + \tau_z)(\tau_\theta + \tau_x) + \tau_\omega \tau_z]^2} [\tau_z - q_0 (\tau_\theta + \tau_x + \tau_z)].$$

Thus, $\frac{\partial RHS}{\partial \tau_\omega} \geq 0$ and hence $\frac{dk_0}{d\tau_\omega} \geq 0$ *iff.* $q_0 \leq \frac{\tau_z}{\tau_\theta + \tau_x + \tau_z}$. From (A56), the latter is equivalent to

$$\phi_\theta \leq \frac{\tau_\theta + \alpha + \tau_z}{\tau_\theta + \tau_x + \tau_z} = \hat{\phi}_\theta,$$

where $\alpha = \frac{\tau_x \tau_p k_0^2}{\tau_x + \tau_p k_0^2}$ denotes the precision of $\underline{p}_t$. Conversely, $\frac{dk_0}{d\tau_\omega} < 0$ when $\phi_\theta > \hat{\phi}_\theta$. □

Online Appendix C.3: Optimal Use of Information

Proof of Proposition 6: I use the same three step procedure as in the proof of Proposition 3.

Step 1: Equilibrium welfare.

The economy-wide output gap is

$$\begin{aligned} y_t - a_t &= m_t - p_t - a_t \\ &= [q_0 + (1-\nu_0) q_1 - k_0 - k_1 - 1]\theta_t + [(1-\nu_0) q_1 - k_0]\epsilon_{xt} + (1-\nu_0) q_1 \frac{1}{k_0} \epsilon_{pt} + t.l.p \\ &+ (q_0 - k_1)\epsilon_{zt} - k_1 \epsilon_{\omega t} + t.l.p. \end{aligned}$$

Thus, after a few, simple derivations

$$\begin{aligned} (1-\beta)\mathcal{W} &= \left[-(1-\nu_0) + (1-\nu_0) q_0 \frac{\tau_z + \alpha}{\tau_z} + \frac{\tau_\theta}{\tau_x} k_0\right]^2 \frac{1}{\tau_\theta} + \left[(1-\nu_0) q_0 \left(\frac{\alpha}{\tau_z}\right) - k_0\right]^2 \frac{1}{\tau_x} \\ &+ (1-\nu_0)^2 q_0^2 \left(\frac{\alpha}{\tau_z}\right)^2 \frac{1}{\tau_p k_0^2} + \left[\nu_0 + (1-\nu_0) q_0 + \frac{\tau_\theta + \tau_x}{\tau_x} k_0\right]^2 \frac{1}{\tau_z} + k_1^2 \frac{1}{\tau_\omega}, \end{aligned} \tag{A57}$$

where $\alpha = \frac{\tau_x \tau_p k_0^2}{\tau_x + \tau_p k_0^2} > 0$ and $k_1 = \nu_0 \tau_\omega \frac{q_0(\tau_\theta + \tau_x + \tau_z) - \tau_z}{(\tau_\omega + \tau_z)(\tau_\theta + \tau_x) + \tau_\omega \tau_z}$.

Step 2: The unique optimal values of k_0, τ_ω and q_0.

Proposition 4 demonstrates that there is a one-to-one relationship between q_0 and ϕ_θ for given τ_ω and k_0. Instead of optimally choosing $\phi_\theta \in \mathbb{R}_+$, one can therefore choose $q_0 \in \mathbb{R}_+$, implicitly defining the associated optimal ϕ_θ-response. I adopt this approach below.

Consider the Lagrangian associated with our optimal policy problem[7]

$$\mathcal{L} = \mathcal{W} + \lambda \left[k_0 - \nu_0 \tau_x \frac{q_0 \tau_z - (\tau_\omega + \tau_z)}{(\tau_\omega + \tau_z)(\tau_\theta + \tau_x) + \tau_\omega \tau_z} \right].$$

Thus,

$$\frac{\partial \mathcal{L}}{\partial \tau_\omega} = 0: \quad \nu_0 \frac{\tau_z}{\left[(\tau_\omega + \tau_z)(\tau_\theta + \tau_x) + \tau_\omega \tau_z\right]^2} \left(2k_1 \frac{\tau_\theta + \tau_x}{\tau_\omega} + \lambda \tau_x \right) \left[q_0 (\tau_\theta + \tau_x + \tau_z) - \tau_z \right] = 0.$$

This equation is satisfied for $\tau_\omega \to \infty$ for given k_0 and q_0.

We can now return to (A57). Minimizing (A57) with respect to q_0 when $k_0 = -\nu_0 \frac{\tau_x}{\tau_\theta + \tau_x + \tau_z} < 0$ and $k_1 = 0$ (because $\tau_\omega \to \infty$) delivers

$$\begin{aligned} \frac{\partial \mathcal{W}}{\partial q_0} = 0: \quad & \left[-(1-\nu_0) + (1-\nu_0) q_0 \left(\tfrac{\tau_z + \alpha}{\tau_z} \right) + \tfrac{\tau_\theta}{\tau_x} k_0 \right] \left(\tfrac{\tau_z + \alpha}{\tau_z \tau_\theta} \right) + \left[(1-\nu_0) q_0 \left(\tfrac{\alpha}{\tau_z} \right) - k_0 \right] \left(\tfrac{\alpha}{\tau_x \tau_z} \right) \\ & + (1-\nu_0) q_0 \left(\tfrac{\alpha}{\tau_z} \right)^2 \tfrac{1}{\tau_p k_0^2} + \left[\nu_0 + (1-\nu_0) q_0 + \tfrac{\tau_\theta + \tau_x}{\tau_x} k_0 \right] \tfrac{1}{\tau_z} = 0, \end{aligned} \tag{A58}$$

which is linear in q_0. After a few simple derivations, (A58) then shows that

$$q_0^\star = \frac{\tau_z}{\tau_\theta + \tau_x + \alpha} > \frac{\tau_z}{\tau_\theta + \tau_x + \tau_x}. \tag{A59}$$

Hence, $\tau_\omega^\star \to \infty$, $k_0^\star = -\nu_0 \frac{\tau_x}{\tau_\theta + \tau_x + \tau_z}$, and $q_0^\star$ is given by (A59).

Step 3: The optimal level of instrument policy, $\phi_\theta^\star$.

In turn, the optimal value of q_0 can uniquely be implemented by,

$$\phi_\theta^\star = 1 > \hat{\phi}_\theta = \frac{\tau_\theta + \tau_z + \alpha}{\tau_\theta + \tau_z + \tau_x}, \tag{A60}$$

where I have once more used Proposition 4. □

Online Appendix C.4: Extensions

Dispersed Information: Analogous steps to those that lead to (A34), (A35), and (A36) show that the price level with productivity shocks in the dispersed information case equals

$$p_t = \nu_0 \bar{\mathbb{E}}_t^f \left[m_t - a_t \right] + \nu_p \bar{\mathbb{E}}_t^f \left[p_t \right] + l.p.t.,$$

where once again $\bar{\mathbb{E}}_t^f [\cdot] = \frac{1}{1-\alpha} \int_\alpha^1 \mathbb{E}_{it}^f [\cdot] \, di$.

Identical steps to those used to derive Proposition 4 then show that

$$p_t = \nu_{-1} p_{t-1} + \nu_0 (m_{t-1} - a_{t-1}) + k_0 \theta_t + k_1 \omega_t + k_2 \underline{p}_t \tag{A61}$$

$$m_t = m_{t-1} + q_0 z_t + q_1 \underline{p}_t, \tag{A62}$$

[7] The social welfare loss function in (A57) is *strictly pseudo-convex*. When combined with Proposition 4, we thus have that an interior solution $\frac{d\mathcal{L}}{dx} = 0$, where $x = \{\bar{q}_0, \tau_\omega, k_0\}$, is the unique global minimum.

where the key coefficients k_0 and q_0 now solve

$$k_0 = \nu_0 \frac{q_0 v_x - w_x}{1 - \nu_p w_x}, \quad q_0 = \phi_\theta \beta_z, \tag{A63}$$

and the signal extraction weights v_x, w_x, and β_z are identical to before. Notice the two key differences relative to Proposition 4: (i) that θ_t appears in place of x_t in (A61), because the noise in firms' private signals cancels on averages; and (ii) that k_0 in (A63) is scaled by $(1 - \nu_p w_x)^{-1}$, because of the strategic complementarity ν_p in (A61). That said, these differences have only minor consequences for our results.

First, increases in central bank disclosure τ_ω still decrease both v_x and w_x in (A63). Moreover, since the decrease in v_x is still weighted by ϕ_θ, there once more exists a critical value $\hat{\phi}_\theta \in (0, 1)$, such that for all $\phi_\theta > \hat{\phi}_\theta$ disclosure increases the informativeness of the price level. That is, a result akin to Proposition 5 still extends to the case with dispersed information.

Second, because *Steps 1* to *3* in the Proof of Proposition 6 carry over to the case when $\tau_x \to \infty$ and k_0 is instead given by (A63), full disclosure $\tau_\omega \to \infty$ combined with $\phi_\theta = 1$ still minimizes the variance of the output gap with dispersed information. Importantly, disclosure can once more be shown to increase the informativeness of the price level $\phi_\theta = 1 > \hat{\phi}_\theta$ in this case, precisely as in Proposition 6.

Alternative Monetary Policy Rule: Suppose that the central bank instead directly targets what causes changes to the output gap, the price level and labor productivity,

$$\begin{aligned} m_t &= m_{t-1} + \phi_0 + \phi_\theta \mathbb{E}_{cbt}[a_t] + \phi_p \mathbb{E}_{cbt}[p_t] \\ &= \phi_\theta \mathbb{E}_{cbt}[\theta_t] + \phi_p \mathbb{E}_{cbt}[p_t] + l.p.t. \end{aligned} \tag{A64}$$

Similar steps to those used to derive Proposition 4 now show that

$$p_t = \nu_{-1} p_{t-1} + \nu_0 (m_{t-1} - a_{t-1}) + k_0 x_t + k_1 \omega_t + k_2 \underline{p}_t \tag{A65}$$

$$m_t = m_{t-1} + q_0 z_t + q_1 \underline{p}_t + q_2 \omega_t, \tag{A66}$$

where the key coefficients k_0 and q_0 now solve

$$k_0 = \nu_0 (q_0 v_x - w_x), \quad q_0 = \phi_\theta \beta_z + \phi_p k_0 \beta_z^x, \tag{A67}$$

where β_z^x denotes the signal extraction coefficient on z_t in the central bank's expectation of x_t.

We can now continue to derive the optimal policy. The economy-wide output gap equals

$$\begin{aligned} y_t &= m_t - p_t - a_t = \phi_\theta \mathbb{E}_{cbt}[\theta_t] - \theta_t + \phi_p \mathbb{E}_{cbt}[p_t] - p_t + t.l.p. \\ &= \begin{bmatrix} \phi_\theta & \phi_p \end{bmatrix} \begin{bmatrix} \mathbb{E}_{cbt}\theta_t \\ \mathbb{E}_{cbt}p_t \end{bmatrix} - \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \theta_t \\ p_t \end{bmatrix} = \phi \mathbb{E}_{cbt}[\vartheta_t] - \mathbf{1}'\vartheta_t + t.l.p, \end{aligned}$$

where $\vartheta_t' = \begin{bmatrix} \theta_t & p_t \end{bmatrix}$ and $\phi = \begin{bmatrix} \phi_\theta & \phi_p \end{bmatrix}$. Thus,

$$y_t = \mathbf{1}' (\mathbb{E}_{cbt}[\vartheta_t] - \vartheta_t) + (\phi - \mathbf{1}') \mathbb{E}_{cbt}[\vartheta_t] + t.l.p.$$

It follows that

$$\mathcal{W} = \frac{1}{1-\beta} \left\{ \mathbf{1}' MSE_{cb}[\vartheta_t] \mathbf{1} + (\phi - \mathbf{1}') \mathbb{V}[\mathbb{E}_{cbt}[\vartheta_t]] (\phi - \mathbf{1}')' \right\}. \tag{A68}$$

Hence, the sufficient first order conditions are:[8]

$$\phi_\theta: \qquad \frac{\partial \mathbf{1}' MSE_{cb}\left[\vartheta_t\right]\mathbf{1}}{\partial \phi_\theta} + \frac{\partial\left(\phi - \mathbf{1}'\right)\mathbb{V}\left[\mathbb{E}_{cbt}\left[\vartheta_t\right]\right]\left(\phi - \mathbf{1}'\right)'}{\partial \phi_\theta} = 0 \tag{A69}$$

$$\phi_p: \qquad \frac{\partial \mathbf{1}' MSE_{cb}\left[\vartheta_t\right]\mathbf{1}}{\partial \phi_p} + \frac{\partial\left(\phi - \mathbf{1}'\right)\mathbb{V}\left[\mathbb{E}_{cbt}\left[\vartheta_t\right]\right]\left(\phi - \mathbf{1}'\right)'}{\partial \phi_p} = 0 \tag{A70}$$

$$\tau_\omega: \qquad \frac{\partial \mathbf{1}' MSE_{cb}\left[\vartheta_t\right]\mathbf{1}}{\partial \tau_\omega} + \frac{\partial\left(\phi - \mathbf{1}'\right)\mathbb{V}\left[\mathbb{E}_{cbt}\left[\vartheta_t\right]\right]\left(\phi - \mathbf{1}'\right)'}{\partial \tau_\omega} = 0. \tag{A71}$$

Now, notice that since k_0 in (A67) is independent of ϕ_p and τ_ω when $\tau_\omega \to \infty$ (and $v_x \to 0$), it follows that $\frac{\partial \mathbf{1}' MSE_{cb}[\vartheta_t]\mathbf{1}}{\partial \phi_\theta} \to 0$, $\frac{\partial \mathbf{1}' MSE_{cb}[\vartheta_t]\mathbf{1}}{\partial \phi_p} \to 0$, and $\frac{\partial \mathbf{1}' MSE_{cb}[\vartheta_t]\mathbf{1}}{\partial \tau_\omega} \to 0$ when $\tau_\omega \to \infty$. This is because only the informativeness of $\underline{p}_t$ in the central bank's information set, controlled by k_0, is modified by different policy actions. We therefore have that (A69) to (A71) are satisfied for $\phi_\theta = \phi_p = 1$ when $\tau_\omega \to \infty$.

[8] Sufficiency once more follows from the the strict pseudo-convexity of the social welfare loss function in (A68).

Online Appendix D: A Quantitative Extension

This Appendix describes the derivation and solution of the equilibrium conditions in Section 6. I also provide detail on the calibration as well as the optimal policy in the central bank full information limit.

Linearized Equilibrium Conditions: The are three main log-linear equilibrium conditions: the *Euler equation* in (6.5), the *Taylor Rule* in (6.7), and the *New Keynesian Phillip's Curve* in (6.6). First, the maximization of (6.1) subject to (6.2) leads to

$$C_t = \beta(1+i_t)\mathbb{E}_{ht}\left[C_{t+1}\frac{P_{t+1}}{p_t}\right].$$

A simple log-linearization around the full information steady-state then immediately yields the *Euler equation* in (6.5), after imposing market clearing,

$$y_t = \mathbb{E}_{ht}\left[y_{t+1}\right] - \left(i_t - \mathbb{E}_{ht}\left[\pi_{t+1}\right]\right). \tag{A72}$$

Second, direct log-linearization of (6.3) provides us with the *Taylor Rule* in (6.7)

$$i_t = \phi\mathbb{E}_{cbt}\left[y_t - a_t\right] + \epsilon_{mt}. \tag{A73}$$

Lastly, to arrive at the *New Keynesian Phillip's Curve* in (6.6), start with (A7)

$$\mathbb{E}_{ft}\left[-\frac{1}{\mathcal{M}_t} + \left(\frac{W_t}{A_t P_t}\right) - \frac{\psi}{\rho}\left(\frac{P_t}{P_{t-1}} - 1\right)\left(\frac{P_t}{P_{t-1}}\right) + \beta\frac{\psi}{\rho}\left(\frac{P_{t+1}}{P_t} - 1\right)\left(\frac{P_{t+1}}{P_t}\right)\right] \quad = \quad 0. \tag{A74}$$

Log-linearizing this equation yields

$$\pi_t = \beta\mathbb{E}_{ft}\left[\pi_{t+1}\right] + \frac{\rho}{\psi}\underline{W}^{ss}\mathbb{E}_{ft}\left[w_t - p_t - a_t\right] - \frac{1}{\underline{\mathcal{M}}^{ss}}\mathbb{E}_{ft}\left[\mu_t\right],$$

where $\underline{W}^{ss}$ and $\underline{\mathcal{M}}^{ss}$ denote the steady state values of $\frac{W_t}{P_t A_t}$ and $\mathcal{M}_t$, respectively. Using that $w_t - p_t - a_t = (1+\eta)(y_t - a_t)$ from Online Appendix A, redefining the mark-up shock, and normalizing the steady state price level to equal the steady state wage rate then results in (6.6):

$$\pi_t = \beta\mathbb{E}_{ft}\left[\pi_{t+1}\right] + \lambda\mathbb{E}_{ft}\left[y_t - a_t\right] + \mathbb{E}_{ft}\left[\mu_t\right]. \tag{A75}$$

Solution Method: I extend the solution method proposed in Nimark (2017) to the two-sided learning case, to solve the three-equation model described by (6.5), (6.6), (6.7), and the information sets in (6.8) and (6.10) under the baseline calibration in which $\bar{\tau}_x^{\mu}, \bar{\tau}_x^{a} \to \infty$. There are two steps to the solution procedure: First, I start by conjecturing a solution to the model, and then use this conjecture to derive an expression for the endogenous triplet $q_t = [\pi_t\, y_t\, i_t]'$ as a function of the state, taking as given the equation of motion for the state. I then solve agents' signal extraction problem to find the equation of motion for the state, taking as given the function mapping the endogenous triplet into the state. I iterate on these two steps until convergence.

(1) Conjecture and Endogenous Variables as a Function of the State: I conjecture (and later verify) that

q_t can be written as a linear function of the expectational state vector X_t and the vector of shocks u_t,[9]

$$q_t = \alpha_0 X_t + \alpha_1 u_t, \tag{A76}$$

and that the expectational state vector itself follows a *VAR(1)*

$$X_t = M X_{t-1} + N u_t. \tag{A77}$$

Solving the model implies finding expressions for the matrices α_0, α_1, M, and N.

Using (A76) and that $\Omega_{ft} = \Omega_{ht}$, we can stack (6.5), (6.6), (6.7) to arrive at

$$A_0 q_t = A_1 \mathbb{E}_{ft}\left[q_{t+1}\right] + A_x^j X_t + A_u u_t, \quad j = \{a,\, \mu\} \tag{A78}$$

where

$$A_0 = \begin{bmatrix} 1 & 0 & 1 \\ -\lambda & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_u = \begin{bmatrix} \underline{0} \\ \underline{0} \\ \sqrt{\tau_m^{-1}} s_6' \end{bmatrix},$$

in which s_l denotes the 1×6 selector vector with an entry equal to one in the lth colum and A_x^j is either

$$A_x^a = \begin{bmatrix} \underline{0} \\ -\lambda e_2' \\ \phi\left(\alpha_0^{1:} - e_1'\right) H_{cb} \end{bmatrix} \quad \text{or} \quad A_x^\mu = \begin{bmatrix} \underline{0} \\ e_2' \\ \phi \alpha_0^{1:} H_{cb} \end{bmatrix},$$

depending on which of the structural shocks $j = \{a,\, \mu\}$ drive the economy. The vector e_l' is the $1 \times 2\bar{k}+1$ selector vector with an entry equal to one in the lth column and the matrix H_{cb} is defined so that

$$\mathbb{E}_{cbt}\left[X_t\right] = H_{cb} X_t.$$

That is, H_{cb} selects the central bank's expectations in X_t and moves the hierarchy of expectations "one step up". Equation (A78) implies that

$$q_t = F_1 \mathbb{E}_{ft}\left[q_{t+1}\right] + F_x^j X_t + F_u u_t, \tag{A79}$$

where

$$F_1 = A_0^{-1} A_1, \quad F_x^j = A_0^{-1} A_x^j, \quad F_u = A_0^{-1} A_u.$$

Inserting the conjecture in (A76) and (A77) into (A79) then shows that

$$\begin{aligned} q_t &= F_1 \alpha_0 M \mathbb{E}_{ft}\left[X_t\right] + F_x^j X_t + F_u u_t \\ &= \left(F_1 \alpha_0 M H_f + F_x^j\right) X_t + F_u u_t, \end{aligned} \tag{A80}$$

where H_f is defined analogously to H_{cb} so that

$$\mathbb{E}_{ft}\left[X_t\right] = H_f X_t.$$

[9]To ease notation, I refer to $X_t^{(0:\bar{k})}$ simply as X_t when it does not cause confusion.

Equating coefficients in (A80) with those from the conjecture now shows that

$$\alpha_0 = F_1 \alpha_0 M H_f + F_x^j, \quad \alpha_1 = F_u. \tag{A81}$$

The solution to (A81) provides, for given M, the coefficients that determine the triplet $q_t = [\pi_t \, y_t \, i_t]'$ as a function of the state. This completes the first step of the solution procedure.

(2) State Evolution as a Function of the Endogenous Variables: We still need to determine the equation of motion for the state X_t. To do so, I proceed in two steps. First, I solve the private sector's and the central bank's respective signal extraction problems under the baseline calibration, taking as given (A76). I then stack these expressions and match to the conjecture in (A77).

I start with the private sector. Its measurement equation equals

$$Z_{ft} = \begin{bmatrix} x_t \\ \omega_t \\ \bar{\pi}_t \\ i_t \end{bmatrix} = \begin{bmatrix} e_1' \\ e_1' \\ \alpha_0^{2:} \\ \alpha_0^{3:} \end{bmatrix} X_t + \begin{bmatrix} \sqrt{\tau_x^{-1}} s_2' \\ \sqrt{\tau_z^{-1}} s_3' + \sqrt{\tau_\omega^{-1}} s_4' \\ \alpha_1^{2:} + \sqrt{\tau_p^{-1}} s_5' \\ \alpha_1^{3:} \end{bmatrix} u_t = L X_t + Q u_t. \tag{A82}$$

By the properties of linear projection, the private sector's expectation of X_t can be written as

$$\mathbb{E}_{ft}[X_t] = M \mathbb{E}_{ft-1}[X_{t-1}] + K \left(Z_{ft} - \mathbb{E}_{ft-1}[Z_{ft}] \right), \tag{A83}$$

where the matrix of Kalman Gains is determined by

$$K = (PL' + NQ')\,(LPL' + QQ' + LNQ' + QN'L')^{-1},$$

and the one-step ahead mean squared error P solves the Ricatti equation

$$P = M \left[P - K (PL' + NQ')' \right] M' + NN'.$$

We can then rewrite (A83) as, using (A82),

$$\mathbb{E}_{ft}[X_t] \;=\; (I - KL)\, M \mathbb{E}_{ft-1}[X_{t-1}] + KLMX_{t-1} + K (LN + Q)\, u_t. \tag{A84}$$

Moving on analogously to the central bank, its measurement equation is

$$Z_{cbt} = \begin{bmatrix} z_t \\ \omega_t \\ \bar{\pi}_t \\ i_t \end{bmatrix} = \begin{bmatrix} e_1' \\ e_1' \\ \alpha_0^{2:} \\ \alpha_0^{3:} \end{bmatrix} X_t + \begin{bmatrix} \sqrt{\tau_z^{-1}} s_3' \\ \sqrt{\tau_z^{-1}} s_3' + \sqrt{\tau_\omega^{-1}} s_4' \\ \alpha_1^{2:} + \sqrt{\tau_p^{-1}} s_5' \\ \alpha_1^{3:} \end{bmatrix} u_t = L_{cb} X_t + Q_{cb} u_t. \tag{A85}$$

such that the central bank's expectation equals

$$\mathbb{E}_{cbt}[X_t] = M \mathbb{E}_{cbt-1}[X_{t-1}] + K_{cb} \left(Z_{cbt} - \mathbb{E}_{cbt-1}[Z_{cbt}] \right), \tag{A86}$$

where

$$K_{cb} = (P_{cb} L_{cb}' + N Q_{cb}')\,(L_{cb} P_{cb} L_{cb}' + Q_{cb} Q_{cb}' + L_{cb} N Q_{cb}' + Q N' L_{cb}')^{-1},$$

and P_{cb} solves

$$P_{cb} = M\left[P_{cb} - K_{cb}\left(P_{cb}L_{cb}' + NQ_{cb}'\right)'\right]M' + NN'.$$

Combining (A86) with (A85) then shows that

$$\mathbb{E}_{cbt}\left[X_t\right] \quad = \quad \left(I - K_{cb}L_{cb}\right)M\mathbb{E}_{cbt-1}\left[X_{t-1}\right] + K_{cb}L_{cb}MX_{t-1} + K_{cb}\left(L_{cb}N + Q_{cb}\right)u_t. \tag{A87}$$

Equations (A84) and (A87) describe the evolution of private sector and central bank expectations, taking as given the mapping between the endogenous variables and the state in (A76).

We are now ready to stack these expressions and match them to the conjecture in (A77). To do so, first notice that

$$X_t^{(0:\bar{k}+1)} = \left[\begin{array}{c} X_t^{(0)} \\ \bar{H}\left[\begin{array}{c} \mathbb{E}_{ft}X_t^{(0:\bar{k})} \\ \mathbb{E}_{cbt}X_t^{(0:\bar{k})} \end{array}\right] \end{array}\right],$$

where $\bar{H}$ reorders the elements in $\left[\begin{array}{cc} \mathbb{E}_{ft}X_t^{(0:\bar{k})\prime} & \mathbb{E}_{cbt}X_t^{(0:\bar{k})\prime} \end{array}\right]'$. The final steps to arrive at the conjectured form (A77) are to stack (A84) and (A87) and append the underlying fundamental.

Stacking provides us with

$$\begin{aligned} \bar{H}\left[\begin{array}{c} \mathbb{E}_{ft}X_t^{(0:\bar{k})} \\ \mathbb{E}_{cbt}X_t^{(0:\bar{k})} \end{array}\right] \quad &= \quad \bar{H}\left[\begin{array}{c} (I - KL)M \\ (I - K_{cb}L_{cb})M \end{array}\right]\left[\begin{array}{c} \mathbb{E}_{ft-1}X_{t-1}^{(0:\bar{k})} \\ \mathbb{E}_{cbt-1}X_{t-1}^{(0:\bar{k})} \end{array}\right] + \bar{H}\left[\begin{array}{c} KLM \\ K_{cb}L_{cb}M \end{array}\right]X_{t-1}^{(0:\bar{k})} \\ &+ \quad \bar{H}\left[\begin{array}{c} K(LN + Q) \\ K_{cb}(L_{cb}N + Q_{cb}) \end{array}\right]u_t = \bar{M}_0\left[\begin{array}{c} \mathbb{E}_{ft-1}X_{t-1}^{(0:\bar{k})} \\ \mathbb{E}_{cbt-1}X_{t-1}^{(0:\bar{k})} \end{array}\right] + \bar{M}_1X_{t-1}^{(0:\bar{k})} + \bar{N}u_t, \end{aligned}$$

where $\bar{M}_0$, $\bar{M}_1$, and $\bar{N}$ are implicitly defined. Appending the *AR(1)* process for the fundamental $X_t^{(0)} = \{a_t,\, \mu_t\}$ verifies the conjectured *VAR(1)* form

$$\begin{aligned} X_t^{(0:\bar{k}+1)} \quad &= \quad \left[\begin{array}{c} X_t^{(0)} \\ \bar{H}\left[\begin{array}{c} \mathbb{E}_{ft}X_t^{(0:\bar{k})} \\ \mathbb{E}_{cbt}X_t^{(0:\bar{k})} \end{array}\right] \end{array}\right] = \left[\begin{array}{cc} \rho_j & \underline{0} \\ \underline{0} & \underline{0} \end{array}\right]\left[\begin{array}{c} X_{t-1}^{(0)} \\ X_{t-1}^{(1:\bar{k}+1)} \end{array}\right] \\ &+ \quad \left[\begin{array}{cc} 0 & \underline{0} \\ \underline{0} & \bar{M}_0 \end{array}\right]\left[\begin{array}{c} X_{t-1}^{(0)} \\ X_{t-1}^{(1:\bar{k}+1)} \end{array}\right] + \left[\begin{array}{c} \underline{0} \\ \bar{M}_1 \end{array}\right]\left[\begin{array}{c} X_{t-1}^{(0)} \\ X_{t-1}^{(1:\bar{k}+1)} \end{array}\right] + \left[\begin{array}{c} \sqrt{\tau_j^{-1}}s_1' \\ \bar{N} \end{array}\right]u_t, \end{aligned}$$

where $j = \{a,\, \mu\}$. Finally, equating coefficients with those in the conjecture in (A77) gives the solution for the coefficient matrices in the law of motion for the state

$$M \quad = \quad \left[\begin{array}{cc} \rho_j & \underline{0} \\ \underline{0} & \underline{0} \end{array}\right] + \left[\begin{array}{cc} 0 & \underline{0} \\ \underline{0} & \bar{M}_0 \end{array}\right] + \left[\begin{array}{c} \underline{0} \\ \bar{M}_1 \end{array}\right] \tag{A88}$$

$$N \quad = \quad \left[\begin{array}{c} \sqrt{\tau_j^{-1}}s_1' \\ \bar{N} \end{array}\right], \tag{A89}$$

where the last two rows/columns have been cropped to make the matrices conformable; i.e. implementing the approximation that for all orders of expectation where $k > \bar{k}$, $X_t^{(k)} = \underline{0}$.

Fixed Point Problem: Equations (A81), (A88), and (A89) present a mapping from $\{M,\, N\} \mapsto \{\alpha_0,\, \alpha_1\} \mapsto \{M,\, N\}$, the fixed point of which provides the approximate rational expectations equilibrium to the extended business cycle model. To find the coefficient matrices, I iterate on the described two steps until convergence. To initialize the algorithm, I use the solution to the model with only exogenous private information; that is where $\Omega_{ft} = \{x_{t-j}\}_{j=-\infty}^{j=0}$ and $\Omega_{cbt} = \{z_{t-j}\}_{j=-\infty}^{j=0}$.

Calibration: The match between moments and model outcomes is listed for the productivity shock case in Table 1. The online replication kit contains the analogous results for the mark-up shock case.

Table 1: Model Calibration

	Data	*Model Outcome*	*Source*
Root MSE Output Private Sector	1.96	1.98	El-Shagi et al (2014)
Root MSE Output Central Bank	1.80	1.89	El-Shagi et al (2014
Runkle Statistic	1.97	2.08	Lorenzoni (2009a)

The first two rows of the table use one-period ahead real output growth forecasts for both the private sector and the central bank. Private sector forecasts utilize data from the SPF Survey, while central bank forecasts use the Greenbook data set. One-period ahead forecast are referred to as "now-casts" in El-Shagi et al (2014, Table I). The final row of the table uses the estimated Runkle statistic in Lorenzoni (2009a).

Optimal Policy Limits: Suppose the central bank has full information about y_t and a_t. In this case, we can re-write the Euler equation in (A72) as

$$\begin{aligned} y_t &= \mathbb{E}_{ht}[y_{t+1}] + \mathbb{E}_{ht}[\pi_{t+1}] - i_t \\ &= \mathbb{E}_{ht}[y_{t+1}] + \mathbb{E}_{ht}[\pi_{t+1}] - \phi(y_t - a_t), \end{aligned}$$

such that

$$y_t = \frac{1}{1+\phi}\left(\mathbb{E}_{ht}[y_{t+1}] + \mathbb{E}_{ht}[\pi_{t+1}]\right) + \frac{\phi}{1+\phi}a_t. \tag{A90}$$

It follows that

$$\lim_{\phi\to\infty} y_t = a_t,$$

and hence that $\phi^\star \to \infty$ obtains the first best outcome under full information.

Now, suppose instead that the central bank has imperfect information about y_t and a_t, and define its forecast error of the output gap as

$$\xi_{y_t-a_t}^{cb} = (y_t - a_t) - \mathbb{E}_{cbt}[y_t - a_t].$$

Then,

$$\begin{aligned} y_t &= \mathbb{E}_{ht}[y_{t+1}] + \mathbb{E}_{ht}[\pi_{t+1}] - i_t \\ &= \mathbb{E}_{ht}[y_{t+1}] + \mathbb{E}_{ht}[\pi_{t+1}] - \phi\mathbb{E}_{cbt}[y_t - a_t] \\ &= \mathbb{E}_{ht}[y_{t+1}] + \mathbb{E}_{ht}[\pi_{t+1}] - \phi(y_t - a_t) - \phi\xi_{y_t-a_t}^{cb}, \end{aligned}$$

and hence

$$\lim_{\phi\to\infty} y_t = a_t + \lim_{\phi\to\infty}\xi_{y_t-a_t}^{cb}. \tag{A91}$$

This shows that when $\phi \to \infty$ social welfare losses only arise from the presence of central bank imperfect information. This is precisely as in the baseline model from Section 2.

Online Appendix E: Robustness Checks

Online Appendix E.1: Dispersed Information

A substantial debate has arisen about the social value of public information in models with incomplete common knowledge among private sector agents. Because of strategic complementarities, public signals may namely in such models receive either too little or too much weight (e.g. Angeletos and Pavan, 2007). This depends in part on how monetary policy is set (Angeletos et al, 2016).

To explore how the welfare effects of central disclosure differ with dispersed private sector information, I in this appendix solve the extended model with dispersed private sector information. In particular, consistent with the benchmark calibration, I set the private information parameters in (6.9) to jointly match the observed pre-February 1994 dispersion in one-quarter ahead GNP/GDP forecasts in the Survey of Professional Forecasters (equal to 0.33 percentage points) and the one-quarter ahead accuracy of the average forecast from the same survey, in addition to the one-quarter ahead accuracy from the Greenbook. I find that to match these targets $\sigma_x^\mu = 0.50$, $\sigma_{x,f}^\mu = 0.11$, $\sigma_x^a = 0.60$, $\sigma_{x,f}^a = 0.20$.[10] I then re-compute the optimal policy for both the mark-up and productivity shock case. Table 2 and 3 show that the main insights from my analysis extend to the case with dispersed private sector information.

The introduction of dispersed private sector information further complicates the solution of the model. Because of dispersed information, the Law of Iterated Expectations does not hold for private sector expectations: Average private sector expectations of average private sector expectations, and so on, do not simply equal average private sector expectations. One consequence of this failure of the Law of Iterated Expectations is that the number of higher-order expectations in each order of expectations k in (6.12) follows the Fibonacci sequence rather than simply increases by two, as under the baseline calibration where $\sigma_{x,f}^\mu = \sigma_{x,f}^a = 0$. Already, with $k = 15$ we therefore have to keep a track of 4,179 different expectations. I solve the model for $k = 15$ and re-compute the optimal policies.

Table 2 shows the breakdown of the quantitative results when only mark-up shocks drive the economy. As in Section 7, I find that the combined optimal policy is to set $\tau_\omega^\mu \to \infty$ and $\phi \to \infty$. Consistent with my previous results, Table 2 shows that disclosure improves welfare – both at the calibrated pre-February 1994 benchmark and at the optimal monetary policy. The benefit from the central bank being able to better predict (and counter) private sector actions once more dominates the increase in private sector responses to the mark-up shock. Moreover, compared to the results in Section 7, the benefit from central bank disclosure is somewhat larger at both the optimal monetary policy (c. -50 percent now vs -27 percent previously) and at the calibrated benchmark (c. -130 percent now vs -115 percent before). This shows how the dispersion of private sector information modifies our previous estimates.

Table 3 shows the corresponding breakdown when productivity instead drive the economy. As in Section 7, I find that the optimal monetary policy is again to set $\tau_\omega^a \to \infty$ and $\phi \to \infty$. The results in Table 3 are remarkably close to those in Table III. Going from complete opacity to full disclosure decreases welfare losses by around 31 percent at the optimal monetary policy. Around 28 percentage points of this decrease is alone due to the increase in central bank information about productivity (compared to 29 percentage points before). The benefit from the central bank being able to back out more information once more dominates the learning externality. In turn, this makes disclosure more beneficial and the overall effect similar to those reported in Table III.

In sum, the main insights from Section 7 are robust to the introduction of realistic amounts of dispersed private sector information and the associated absence of common knowledge.

[10] Recall that the relationship between the precision τ and the standard deviation σ is $\tau = \sigma^{-2}$.

Table 2: Dispersed Information (with Mark-up Shocks)

	Parameters		$\%\Delta W_C$
Calibrated benchmark	$\phi = 1.81$	$\tau_\omega^\mu \to 0$	...
	Breakdown of Benefits from Disclosure		
A. Benchmark with full disclosure	$\phi = 1.81$	$\tau_\omega^\mu \to \infty$	-68.90
B. Benchmark with constant h.o. unc.[†]	$\phi = 1.81$	$\tau_\omega^\mu \to \infty$	51.41
A-B. Cost from decrease in h.o. unc.			-120.32
	Breakdown of Benefits from Optimal Policy		
A. Optimal policy	$\phi \to \infty$	$\tau_\omega^\mu \to \infty$	-99.47
B. Benefit from optimal mon. policy	$\phi \to \infty$	$\tau_\omega^\mu \to 0$	-56.12
A-B. Benefit from central bank disclosure			-43.35

(i) W_C denotes the life-time consumption equivalent of W.

(ii) $\%\Delta W_C$ denotes the %change in W_C relative to the calibrated benchmark.

(†) Private sector and central bank higher-order uncertainty fixed at benchmark values.

Table 3: Dispersed Information (with Productivity Shocks)

	Parameters		$\%\Delta W_C$
Calibrated benchmark	$\phi = 1.81$	$\tau_\omega^a \to 0$	...
	Breakdown of Benefits from Disclosure		
A. Benchmark with disclosure	$\phi = 1.81$	$\tau_\omega^a \to \infty$	$+1.02$
B. Private sector benefit of disclosure[†]	$\phi = 1.81$	$\tau_\omega^a \to \infty$	-9.06
A-B. Central bank cost of disclosure			$+10.08$
	Breakdown of Benefits from Optimal Policy		
A. Optimal policy	$\phi \to \infty$	$\tau_\omega^a \to \infty$	-31.02
B. Benefit from optimal mon. policy	$\phi \to \infty$	$\tau_\omega^a \to 0$	-0.76
C. Private sector benefit of disclosure[†]	$\phi \to \infty$	$\tau_\omega^a \to \infty$	-2.44
A-B-C. Central bank benefit of disclosure			-27.83

(i) W_C denotes the life-time consumption equivalent of W.

(ii) $\%\Delta W_C$ denotes the %change in W_C relative to the calibrated benchmark.

(†) Central bank higher-order uncertainty fixed at calibrated benchmark value.

Online Appendix E.2: The Signaling Channel of Monetary Policy

In the model described in Section 6, under complete opacity or partial disclosure, changes to the interest rate provide firms with a noisy signal of the central bank's private information. By contrast, full disclosure separates the interest rate from the signaling channel of monetary policy. A concern could therefore be that the bulk of the quantitative benefits of disclosure reported in Table II and III arise from the resulting freedom of monetary policy rather than from the mechanisms described in the paper.

Table 4 and 5 show that this is not the case.

Table 4 and 5 show the equivalent results to those reported in Table II and III of the main text when one excludes the central bank interest rate from firms' information set; that is, when $i_t \notin \Omega_{it}^f$, which eliminates any signaling effects of monetary policy. For both the mark-up and productivity shock case, the differences in results relative to Table II and III are minor. The welfare effects of disclosure increase by between a couple of tenths of a percentage point to somewhat close to one percentage point. On balance, both at the calibrated benchmark and at the optimal value of monetary policy, the signaling channel of monetary policy has a relatively minor influence on the benefits of central bank disclosure. This is because in both cases the interest rate provides a rather dim indicator of the central bank's private information. This is consistent with the substantial impact of central bank disclosure on financial markets and on private sector uncertainty about future interest rates documented in, for example, Blinder et al (2008). In sum, the signaling channel of monetary policy comprises a relatively minor component of the welfare benefits of central bank disclosure documented in Table II and III of the main text.

Table 4: Signaling Channel of Monetary Policy (with Mark-up Shocks)

	Parameters		$\%\Delta W_C$
Calibrated benchmark	$\phi = 1.81$	$\tau_\omega \to 0$	...
	Breakdown of Benefits from Disclosure		
A. Benchmark with full disclosure	$\phi = 1.81$	$\tau_\omega \to \infty$	-58.95
B. Benchmark with constant h.o. unc.†	$\phi = 1.81$	$\tau_\omega \to \infty$	$+55.65$
A-B. Benefit from decrease in h.o. unc.			-114.60
	Breakdown of Benefits from Optimal Policy		
A. Optimal policy	$\phi \to \infty$	$\tau_\omega \to \infty$	-96.53
B. Benefit from optimal mon. policy	$\phi \to \infty$	$\tau_\omega \to 0$	-69.80
A-B. Benefit from central bank disclosure			-26.73

(i) W_C denotes the life-time consumption certainty equivalent of W.

(ii) $\%\Delta W_C$ denotes the %change in W_c relative to the calibrated benchmark.

(†) Private sector and central bank higher-order uncertainty fixed at benchmark values.

Table 5: Signaling Channel of Monetary Policy (with Productivity Shocks)

	Parameters		$\%\Delta W_C$
Calibrated benchmark	$\phi = 1.81$	$\tau_\omega \to 0$	...
	Breakdown of Benefits from Disclosure		
A. Benchmark with disclosure	$\phi = 1.81$	$\tau_\omega \to \infty$	-2.93
B. Private sector benefit of disclosure†	$\phi = 1.81$	$\tau_\omega \to \infty$	-14.40
A-B. Central bank cost of disclosure			$+11.47$
	Breakdown of Benefits from Optimal Policy		
A. Optimal policy	$\phi \to \infty$	$\tau_\omega \to \infty$	-32.72
B. Benefit from optimal mon. policy	$\phi \to \infty$	$\tau_\omega \to 0$	$+8.94$
C. Private sector benefit of disclosure†	$\phi \to \infty$	$\tau_\omega \to \infty$	-12.38
A-B-C. Central bank cost of disclosure			-29.28

(i) W_C denotes the life-time consumption certainty equivalent of W.

(ii) $\%\Delta W_C$ denotes the %change in W_c relative to the calibrated benchmark.

(†) Central bank higher-order uncertainty fixed at calibrated benchmark value.

Online Appendix E.3: Household Full Information

Unlike the baseline model, the extended model in Section 6 features both household and firm imperfect information. A concern could thus be that the quantitative results in Table II and III are driven primarily by the assumption of household imperfect information. However, as Table 6 and 7 show this is not the case. Although the overall effects are somewhat smaller when the representative household has full information, the overall direction and magnitude are similar to those reported in Table II and III.

To compute the results in Table 6 and 7, I solve the extended model under the additional assumption that the representative household's information set includes (i) the driving forces of the economy a_t and μ_t; (ii) the central bank signal vector z_t; and lastly (iii) firms' signal vector x_t. Combined, the representative household thus has *full information* — both about the driving forces of the economy as well as firm and central bank (higher-order) expectations about them.

Table 6 reports the breakdown of the result in the mark-up shock case. Because of the decreased importance of higher-order expectations (see the main text and Online Appendix E.4), the benefits of disclosure that I have stressed are somewhat smaller. Under the optimal monetary policy, which I once more find to be $\phi \to \infty$, disclosure decreases welfare losses by 12 percentage points (relative to 27 percentage points in Table II). Consistent with the decreased benefit of disclosure, the benefits from the optimal use of monetary policy are also relatively larger.

Turning to the productivity shock case, Table 7 reports the breakdown of the results with household full information. I once more find the optimal monetary policy to be $\phi \to \infty$. As with the mark-up shock case, the benefit of disclosure is somewhat smaller here than in Table III (10 percentage points versus 29 percentage points before). This is once more due to the decreased importance of higher-order expectations. As I also remarked on in the main text, interestingly full disclosure is now also detrimental for welfare at the calibrated benchmark, even when keeping central bank uncertainty constant. This demonstrates the importance of the interaction between monetary and disclosure policy (see also Angeletos et al, 2016). Lastly, the optimal use of monetary policy now also becomes more important than disclosure policy. This once more stresses how the relative benefits of monetary and disclosure policy depend crucially on the nature and extent of information frictions.

Finally, Table 8 and 9 report the results of an additional exercise in the spirit of Chahrour and Ulbricht (2019). Not only do I assume that the representative household has full information but also that firms know aggregate output y_t. The main impact on my results is again, on balance, a slight decrease in the benefit of central bank disclosure.

In sum, assuming household full information (instead of imperfect information) decreases the benefits of disclosure somewhat. That said, both overall direction and magnitude of the effects are similar to those reported in Table II and III.

Table 6: Household Full Information (with Mark-up Shocks)

	Parameters		$\%\Delta W_C$
Calibrated benchmark	$\phi = 1.81$	$\tau_\omega^\mu \to 0$	...
	Breakdown of Benefits from Disclosure		
A. Benchmark with full disclosure	$\phi = 1.81$	$\tau_\omega^\mu \to \infty$	-2.94
B. Benchmark with constant h.o. unc.†	$\phi = 1.81$	$\tau_\omega^\mu \to \infty$	$+10.33$
A-B. Benefit from decrease in h.o. unc.			-13.27
	Breakdown of Benefits from Optimal Policy		
A. Optimal policy	$\phi \to \infty$	$\tau_\omega^\mu \to \infty$	-81.93
B. Benefit from optimal mon. policy	$\phi \to \infty$	$\tau_\omega^\mu \to 0$	-70.22
A-B. Benefit from central bank disclosure			-11.71

(i) W_C denotes the life-time consumption equivalent of W.

(ii) $\%\Delta W_C$ denotes the %change in W_C relative to the calibrated benchmark.

(†) Private sector and central bank higher-order uncertainty fixed at benchmark values.

Table 7: Household Full Information (with Productivity Shocks)

	Parameters		$\%\Delta W_C$
Calibrated benchmark	$\phi = 1.81$	$\tau_\omega^a \to 0$	...
	Breakdown of Benefits from Disclosure		
A. Benchmark with disclosure	$\phi = 1.81$	$\tau_\omega^a \to \infty$	$+3.75$
B. Private sector benefit of disclosure†	$\phi = 1.81$	$\tau_\omega^a \to \infty$	$+3.53$
A-B. Central bank cost of disclosure			$+0.22$
	Breakdown of Benefits from Optimal Policy		
A. Optimal policy	$\phi \to \infty$	$\tau_\omega^a \to \infty$	-88.64
B. Benefit from optimal mon. policy	$\phi \to \infty$	$\tau_\omega^a \to 0$	-77.94
C. Private sector benefit of disclosure†	$\phi \to \infty$	$\tau_\omega^a \to \infty$	-0.31
A-B-C. Central bank benefit of disclosure			-10.40

(i) W_C denotes the life-time consumption equivalent of W.

(ii) $\%\Delta W_C$ denotes the %change in W_C relative to the calibrated benchmark.

(†) Central bank higher-order uncertainty fixed at calibrated benchmark value.

Table 8: Household Full Information (with Mark-up Shocks and Output)

	Parameters		$\%\Delta W_C$
Calibrated benchmark	$\phi = 1.81$	$\tau_\omega^\mu \to 0$	...
	Breakdown of Benefits from Disclosure		
A. Benchmark with full disclosure	$\phi = 1.81$	$\tau_\omega^\mu \to \infty$	-2.29
B. Benchmark with constant h.o. unc.[†]	$\phi = 1.81$	$\tau_\omega^\mu \to \infty$	$+7.29$
A-B. Benefit from decrease in h.o. unc.			-9.58
	Breakdown of Benefits from Optimal Policy		
A. Optimal policy	$\phi \to \infty$	$\tau_\omega^\mu \to \infty$	-81.28
B. Benefit from optimal mon. policy	$\phi \to \infty$	$\tau_\omega^\mu \to 0$	-65.73
A-B. Benefit from central bank disclosure			-15.55

(i) W_C denotes the life-time consumption equivalent of W.

(ii) $\%\Delta W_C$ denotes the %change in W_C relative to the calibrated benchmark.

(†) Private sector and central bank higher-order uncertainty fixed at benchmark values.

Table 9: Household Full Information (with Productivity Shocks and Output)

	Parameters		$\%\Delta W_C$
Calibrated benchmark	$\phi = 1.81$	$\tau_\omega^a \to 0$	...
	Breakdown of Benefits from Disclosure		
A. Benchmark with disclosure	$\phi = 1.81$	$\tau_\omega^a \to \infty$	$+7.71$
B. Private sector benefit of disclosure[†]	$\phi = 1.81$	$\tau_\omega^a \to \infty$	$+9.38$
A-B. Central bank cost of disclosure			-1.67
	Breakdown of Benefits from Optimal Policy		
A. Optimal policy	$\phi \to \infty$	$\tau_\omega^a \to \infty$	-90.81
B. Benefit from optimal mon. policy	$\phi \to \infty$	$\tau_\omega^a \to 0$	-74.75
C. Private sector benefit of disclosure[†]	$\phi \to \infty$	$\tau_\omega^a \to \infty$	-6.43
A-B-C. Central bank benefit of disclosure			-9.64

(i) W_C denotes the life-time consumption equivalent of W.

(ii) $\%\Delta W_C$ denotes the %change in W_C relative to the calibrated benchmark.

(†) Central bank higher-order uncertainty fixed at calibrated benchmark value.

Online Appendix E.4: Limited Number of Higher-Order Expectations

A substantial literature in experimental economics has demonstrated people's limited capacity to form higher-order expectations (see, for instance, Nagel, 1995). One advantage of the computational approach taken in Section 6 and 7 is that it directly allows one to study the consequences of such behavioral limits for the benefits of central bank disclosure. I demonstrate below how the main quantitative insights extend to cases in which firms and the central bank compute only a few higher-order expectations. I for brevity consider the case where $k = 3$, consistent with the upper-bound in Nagel (1995).

Table 10 shows the breakdown of the welfare benefits of disclosure when only mark-up shocks drive the economy. The table corresponds to Table II in the main text. Compared to the case reported in the main text in which firms compute $k = 50$ higher-order expectations, we see that the benefit of disclosure at the calibrated benchmark is somewhat reduced. This is consistent with fewer higher-order expectations contributing to the equilibrium dynamics of the model. That said, the first $k = 3$ higher-order expectations still account for the lion-share of the welfare benefit that follows from disclosure in Table II. Indeed, the welfare benefit with $k = 3$ are quite similar to those with $k = 50$.

Table 11 shows the corresponding breakdown of the welfare benefits of disclosure when productivity shocks instead drive the economy. This table corresponds to Table III in the main text. Compared to Table III, the welfare benefit of disclosure under the optimal monetary policy is smaller, although the decrease in central bank uncertainty is still clearly present: It contributes four percentage points to the overall decrease in welfare losses. Furthermore, with $k = 3$ the decrease in higher-order uncertainty already dominates the standard learning externality at the calibrated value of monetary policy. As a result, increases in central bank disclosure from this calibrated value would decrease central bank uncertainty, leading to better monetary policy.

In sum, the main quantitative insights from Section 7 are robust to decreases in the number of higher-order expectations that firms and the central bank compute. Although plausible limits to the amount of higher-order expectations somewhat dampen the magnitude of the quantitative results, in all cases central bank disclosure is unequivocally beneficial.

Table 10: Limited Higher-Order Expectations (with Mark-up Shocks)

	Parameters		$\%\Delta W_C$
Calibrated benchmark	$\phi = 1.81$	$\tau_\omega \to 0$	...
	Breakdown of Benefits from Disclosure		
A. Benchmark with full disclosure	$\phi = 1.81$	$\tau_\omega \to \infty$	-41.06
B. Benchmark with constant h.o. unc.†	$\phi = 1.81$	$\tau_\omega \to \infty$	$+54.59$
A-B. Benefit from decrease in h.o. unc.			-95.65
	Breakdown of Benefits from Optimal Policy		
A. Optimal policy	$\phi \to \infty$	$\tau_\omega \to \infty$	-74.39
B. Benefit from optimal mon. policy	$\phi \to \infty$	$\tau_\omega \to 0$	-36.26
A-B. Benefit from central bank disclosure			-38.12

(i) W_C denotes the life-time consumption certainty equivalent of W.
(ii) $\%\Delta W_C$ denotes the %change in W_c relative to the calibrated benchmark.
(†) Private sector and central bank higher-order uncertainty fixed at benchmark values.

Table 11: Limited Higher-Order Expectations (with Productivity Shocks)

	Parameters		$\%\Delta W_C$
Calibrated benchmark	$\phi = 1.81$	$\tau_\omega \to 0$	...
	Breakdown of Benefits from Disclosure		
A. Benchmark with disclosure	$\phi = 1.81$	$\tau_\omega \to \infty$	-32.01
B. Private sector benefit of disclosure†	$\phi = 1.81$	$\tau_\omega \to \infty$	-13.22
A-B. Central bank cost of disclosure			-18.88
	Breakdown of Benefits from Optimal Policy		
A. Optimal policy	$\phi \to \infty$	$\tau_\omega \to \infty$	-48.09
B. Benefit from optimal mon. policy	$\phi \to \infty$	$\tau_\omega \to 0$	-17.29
C. Private sector benefit of disclosure†	$\phi \to \infty$	$\tau_\omega \to \infty$	-27.07
A-B-C. Central bank cost of disclosure			-3.73

(i) W_C denotes the life-time consumption certainty equivalent of W.
(ii) $\%\Delta W_C$ denotes the %change in W_c relative to the calibrated benchmark.
(†) Central bank higher-order uncertainty fixed at calibrated benchmark value.

Online Appendix E.5: Strategic Interactions and Higher-Order Expectations

An important driver of the costs and benefits of central bank disclosure is the extent to which households and firms are forward-looking. The less forward-looking households and firms are, the less their expectations about the future actions of others matter for equilibrium outcomes. Since the degree of such dynamic strategic complementarity is tied intimately to the importance of higher-order expectations, changes to the extent to which households and firms are forward-looking could matter for the costs and benefits of central bank disclosure.

To understand the consequences of a decrease in the discount factor, for example, consider to start the *New Keynesian Phillips Curve* in (6.6) from the main text,

$$\pi_t = \beta \mathbb{E}_t^f \left[\pi_{t+1}\right] + \lambda \mathbb{E}_t^f \left[y_t - a_t\right] + \mathbb{E}_t^f \left[\mu_t\right] = \mathbb{E}_t^f \sum_l \beta^l \left\{\lambda \left[y_{t+l} - a_{t+l}\right] + \mu_{t+l}\right\}. \tag{A92}$$

Inserting the forward-solution for output from the Euler equation into (A92) then shows that

$$\pi_t = \lambda \mathbb{E}_t^f \sum_{l=0}^{\infty} \beta^l \left[\sum_{j=0}^{\infty} \pi_{t+l+1+j} - i_{t+l+j}\right] - \frac{\lambda}{1-\beta\rho_a} \mathbb{E}_t^f \left[a_t\right] + \frac{1}{1-\beta\rho_\mu} \mathbb{E}_t^f \left[\mu_t\right] \tag{A93}$$

Thus, the smaller β is the less inflation at time t depends upon firms' expectations of future inflation and interest rates via the output equation. As a result, the less inflation depends upon firms' higher-order expectations of future firm and central bank actions. This, in turn, decreases both the costs and benefits of central bank disclosure. The costs decrease because of the decrease in amplification that arises from firms discounting future firms' responses to, for instance, the inefficient mark-up shock relatively more. By contrast, the benefits decrease because of the reduction in the importance of firms' expectations of central bank beliefs through the decrease in the importance of future interest rates. Table 12 and 13 show the reduction in the quantitive costs and benefits of disclosure when we decrease β from 0.99 to 0.75. Crucially, in both cases the benefits of disclosure still outweigh the costs at the optimal value of monetary policy, and by a comparable amount to that reported in Table II and III of the main text.

Table 12: Strategic Interactions and Higher-Order Expectations (with Mark-up Shocks)

	Parameters		$\%\Delta W_C$
Calibrated benchmark	$\phi = 1.81$	$\tau_\omega \to 0$	...
	Breakdown of Benefits from Disclosure		
A. Benchmark with full disclosure	$\phi = 1.81$	$\tau_\omega \to \infty$	-62.95
B. Benchmark with constant h.o. unc.†	$\phi = 1.81$	$\tau_\omega \to \infty$	$+47.88$
A-B. Benefit from decrease in h.o. unc.			-110.83
	Breakdown of Benefits from Optimal Policy		
A. Optimal policy	$\phi \to \infty$	$\tau_\omega \to \infty$	-95.99
B. Benefit from optimal mon. policy	$\phi \to \infty$	$\tau_\omega \to 0$	-63.62
A-B. Benefit from central bank disclosure			-32.37

(i) W_C denotes the life-time consumption certainty equivalent of W.

(ii) $\%\Delta W_C$ denotes the %change in W_c relative to the calibrated benchmark.

(†) Private sector and central bank higher-order uncertainty fixed at benchmark values.

Table 13: Strategic Interactions and Higher-Order Expectations (with Productivity Shocks)

	Parameters		$\%\Delta W_C$
Calibrated benchmark	$\phi = 1.81$	$\tau_\omega \to 0$	...
	Breakdown of Benefits from Disclosure		
A. Benchmark with disclosure	$\phi = 1.81$	$\tau_\omega \to \infty$	-18.47
B. Private sector benefit of disclosure†	$\phi = 1.81$	$\tau_\omega \to \infty$	-26.64
A-B. Central bank cost of disclosure			$+8.17$
	Breakdown of Benefits from Optimal Policy		
A. Optimal policy	$\phi \to \infty$	$\tau_\omega \to \infty$	-32.34
B. Benefit from optimal mon. policy	$\phi \to \infty$	$\tau_\omega \to 0$	$+8.44$
C. Private sector benefit of disclosure†	$\phi \to \infty$	$\tau_\omega \to \infty$	-15.30
A-B-C. Central bank cost of disclosure			-25.48

(i) W_C denotes the life-time consumption certainty equivalent of W.

(ii) $\%\Delta W_C$ denotes the %change in W_c relative to the calibrated benchmark.

(†) Central bank higher-order uncertainty fixed at calibrated benchmark value.