Online Appendix – On the Macroeconomic Consequences of Over-Optimism

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When can one identify the effect of over-optimism on the economy? To tackle this problem, it is useful to consider a framework similar to that used in the news-noise literature, where we interpret over-optimism as a positive, but false, signal about the economy's future fundamentals. To clarify ideas, consider the two following simple environments. They are chosen so that one setup has over-optimism as an unambiguous positive (only creating a boom), while over-optimism also carries costs in the other setup (where it generates a recession after a temporary boom). The relevant question is whether our econometric approach would be able to distinguish between these two cases.

Environment (A) is a situation in which the *level* of GDP (possibly around a trend) is stationary and determined by:

$$y_t = \alpha E_t y_{t+1} + z_t, \ 0 < \alpha < 1.$$

Environment (B) is a situation in which the *growth rate* of GDP is stationary:

$$g_t = \alpha E_t g_{t+1} + z_t, \ 0 < \alpha < 1.$$

Here, y_t represents a country's (log) GDP and g_t represents its growth rate ($g_t \equiv y_t - y_{t-1}$), E_t is the expectations operator based on information available at time t, and z_t is an exogenous driving force.

To create an environment where agents are always unsure about fundamentals (so that over-optimism can be well-defined), let us assume that z_t is the sum of a persistent process $\theta_t = \rho \theta_{t+1} + \nu_t$ ($0 < \rho < 1$), plus a white noise process ϵ_t (where all stochastic elements are assumed to be Gaussian). Agents are assumed to observe z_t but not its components.

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Moreover, let us assume that at time t agents get a noisy signal s_t regarding ν_{t+1} , such that $s_t = \nu_{t+1} + \omega_t$. Given this setup, the expectation of θ_{t+1} conditional on information at time t can be expressed as:

$$E_t \theta_{t+1} = \rho \lambda E_{t-1} \theta_t + (1-\lambda)(\theta_t + \epsilon_t) + \psi(\nu_{t+1} + \omega_t)$$
$$= \rho(1-\lambda) \sum_{i=0}^{\infty} (\rho \lambda)^i (\theta_{t-i} + \epsilon_{t-i}) + \psi \sum_{i=0}^{\infty} (\rho \lambda)^i (\nu_{t+1} + \omega_t),$$

where λ and ψ reflect the signal extraction problems faced by the agents. These two parameters depend on the variances of the driving forces and can be shown to lie between 0 and 1. For each of these two environments we can derive the impulse-response of the growth in GDP induced by a shock ω_t (making agents more optimistic about ν_{t+1}), which will be referred to as a noise shock.

In environment (A), the impulse-response shows an initial positive effect on growth of size $\frac{\alpha\psi}{1-\alpha\rho}$, followed by a contractionary phase where the effects on growth are negative and given by $-\frac{\alpha\psi}{1-\alpha\rho}(\rho\lambda)^i$ for *i* periods after the receipt of the signal $(i \ge 1)$. Hence, in environment (1), noise (over-optimism) causes a boom followed by a recessionary period.

Under (B), the impulse-response of GDP growth is given by a declining series of positive effects of size $\frac{\alpha\psi}{1-\alpha\rho}(\rho\lambda)^i$ for periods $i \ge 0$. So in this case, there is no induced recession.

While in both environments a noise shock leads to an initial period of positive growth, the first environment leads to a subsequent recession while the second environment simply exhibits a petering out boom. How could we tell these two environments apart? Obviously, this would be rather easy if we could observe ω_t . However, this is generally not the case. Instead, let us suppose that we can observe agents' forecasts of next period's growth rate, as denoted by $E_t g_{t+1}$ and we have an instrument I_t that is positively correlated with ω_t and uncorrelated with ν_{t-i} and ϵ_{t-i} (for all i). Our Mission Chief fixed effects are a good candidate for such an instrument as Mission Chiefs get re-allocated across countries in a pseudo-random fashion, while econometric estimates indicate that a higher fixed effect is associated with over-optimism (i.e.: a positive noise shock ω_t). However, because Mission Chiefs typically serve on each country for several years, this positive correlation will persist over time. Such positive correlation is also likely to be present for our second instrument (one-year ahead forecast errors determined by a "bullish" or "bearish" atmosphere in the IMF country team). The remaining question therefore becomes: can we still use such instruments to differentiate between these two environments when I_t is correlated with future noise shocks ω_{t+1} (with a particular interest in the most plausible case of positive correlation)?

We claim that our instrument can still do so, provided that its correlation with future noise shocks is not negative. To see this, consider an IV regression for the growth rate at time t + 1 (or a recession dummy) on the forecast error for growth at t + 1 based on information given at time t (i.e.: $E_tg_{t+1} - g_{t+1}$), where we instrument the forecast error using the instrument I_t . If the resulting estimate yields a negative coefficient, we claim that this constitutes evidence that we are in the first environment (where the noise-driven forecast error is creating a subsequent recession). If, on the other hand, we get a positive coefficient on the instrumented forecast error, we are likely in the second environment (but can't rule out the first environment).

To verify the above claim, one can write the growth rate at t + 1 as follows. In environment (A) we obtain:

$$g_{t+1} = -\frac{\alpha(1-\lambda\rho)}{1-\alpha\rho\lambda} (E_t g_{t+1} - g_{t+1}) + \frac{\alpha\psi(1-\alpha)}{(1-\alpha\rho)(1-\alpha\rho\lambda)} \omega_{t+1} + \xi_{t+1},$$
(1)

where ξ_{t+1} is a complicated sum of ν s and ϵ s that does not include any terms ω_{t-i} for any i. So if we estimate (5) by instrumental variables using I_t to instrument the forecast error $(E_t g_{t+1} - g_{t+1})$, then we should get a consistent estimate of $-\frac{\alpha(1-\lambda\rho)}{1-\alpha\rho\lambda}$ if I_t is uncorrelated with ω_{t+1} and ξ_{t+1} . If I_t is positively correlated with ω_{t+1} , then the IV estimator will be biasing this coefficient towards a positive number. If we nonetheless obtain a negative estimate for the effect of the forecast error on growth, then we can conclude that we are likely in case (A) where optimism causes a delayed recession. A positive estimate, on the other hand, would be less conclusive as that could be driven by the bias resulting from the positive correlation between the instrument and ω_{t+1} .

Things are very similar in environment (B). There we obtain:

$$g_{t+1} = \frac{\alpha \lambda \rho}{1 - \alpha \rho \lambda} (E_t g_{t+1} - g_{t+1}) + \frac{\alpha \psi}{(1 - \alpha \rho)(1 - \alpha \rho \lambda)} \omega_{t+1} + \xi_{t+1}, \qquad (2)$$

where ξ_{t+1} again does not include ω_{t-i} for any *i*. So if we estimate (6) by instrumental variables using I_t to instrument the forecast error $(E_t g_{t+1} - g_{t+1})$, then we get a consistent estimate of $\frac{\alpha \rho}{1-\alpha \rho \lambda}$ if I_t is also uncorrelated with ω_{t+1} . If I_t is positively correlated with ω_{t+1} , then the IV estimate will be biasing the coefficient upwards (more positive). Again, obtaining a negative estimate for the effect of the forecast error on growth is a strong finding given the likely presence of positive correlation between I_t and ω_{t+1} .

If, on the other hand, our instrument were negatively correlated with ω_{t+1} , a symmetric argument to the above implies that negative estimates could arise purely due to

the associated negative bias. However, this case seems implausible as a negative correlation between I_t and ω_{t+1} requires a "flipping" allocation of IMF Mission Chiefs, with overly-optimistic types consistently being replaced by overly-cautious ones at the annual frequency. Apart from the fact that this requires IMF decision makers to know the exact type of each Mission Chief, such an alternating allocation policy is not in line with IMF practices. Similarly, with respect to our second instrument (one-year ahead forecast errors), this would require the team's perspective on a country's economic future to systematically alternate between overly-positive and overly-negative from one year to the next, which seems rather unlikely.