Online Appendix for: Efficient Consolidation of Incentives for Education and Retirement Savings

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A Proofs

Proof of Lemma 1: First, to prove necessity, we have to show that Parts (i.) and (ii.) follow from incentive compatibility. To prove Part (i.), let $\Phi(\gamma,\theta) = u(c_1(\gamma,\theta)) + \beta\delta_2 u(c_2(\gamma,\theta))$. For a fixed γ and productivities θ and θ' with $\theta > \theta'$, incentive compatibility requires

$$\Phi(\gamma,\theta) - h\left(\frac{y(\gamma,\theta)}{\theta}\right) \geq \Phi(\gamma,\theta') - h\left(\frac{y(\gamma,\theta')}{\theta}\right)$$

and

$$\Phi(\gamma,\theta') - h\left(\frac{y(\gamma,\theta')}{\theta'}\right) \geq \Phi(\gamma,\theta) - h\left(\frac{y(\gamma,\theta)}{\theta'}\right).$$

Adding these two inequalities yields

$$h\left(\frac{y(\gamma,\theta)}{\theta'}\right) - h\left(\frac{y(\gamma,\theta)}{\theta}\right) \geq h\left(\frac{y(\gamma,\theta')}{\theta'}\right) - h\left(\frac{y(\gamma,\theta')}{\theta}\right). \tag{14}$$

Since h is strictly increasing and strictly convex, for (14) to hold, it must be the case that $y(\gamma,\theta) \geq y(\gamma,\theta')$.

Next, we will prove Part (ii.). For a fixed γ and θ, the incentive constraint is

$$U_1(\gamma,\theta) = \max_{\theta'\in\Theta} u(c_1(\gamma,\theta')) - h\left(\frac{y(\gamma,\theta')}{\theta}\right) + \beta\delta_2 u(c_2(\gamma,\theta')).$$

Theorem 2 of Milgrom and Segal (2002) applies, so $U_1(\gamma,\theta)$ is absolutely continuous in θ. Finally, since $U_1(\gamma,\theta)$ is absolutely continuous in θ, it is differentiable in θ almost everywhere.

Hence, we have $\frac{\partial U_1(\gamma,\theta)}{\partial\theta} = \frac{y(\gamma,\theta)}{\theta^2} h'\left(\frac{y(\gamma,\theta)}{\theta}\right)$.

Finally, to prove sufficiency, we have to show that, for a given γ, none of the agents would want to misreport θ when Parts (i.) and (ii.) hold. Consider any productivities θ and θ' with $\theta > \theta'$, then

$$\begin{aligned}
\int_{\theta'}^{\theta} \frac{y(\gamma,x)}{x^2} h'\left(\frac{y(\gamma,x)}{x}\right) dx \geq \int_{\theta'}^{\theta} \frac{y(\gamma,\theta')}{x^2} h'\left(\frac{y(\gamma,\theta')}{x}\right) dx& \\
\iff U_1(\gamma,\theta) - U_1(\gamma,\theta') \geq -h\left(\frac{y(\gamma,\theta')}{\theta}\right) + h\left(\frac{y(\gamma,\theta')}{\theta'}\right)& \\
\iff U_1(\gamma,\theta) \geq \Phi(\gamma,\theta') - h\left(\frac{y(\gamma,\theta')}{\theta}\right).&
\end{aligned}$$

The first inequality comes from Part (i.), $y(\gamma,\theta)$ is non-decreasing in θ, and h is strictly convex. The left-hand side of the second inequality comes from Part (ii.). Rearranging the terms in the second inequality yields the third inequality, which implies incentive compatibility. ■

Proof of Proposition 1: Conditions (6) and (7) and μ follow from the first order conditions. The inverse Euler equations (4) and (5) are derived using the perturbation argument.

Let $P = \left\{c_0(\gamma), [c_t(\gamma,\theta), y(\gamma,\theta)]_{t>0,\theta\in\Theta}\right\}_\gamma$ be the allocation that solves the constrained efficient planning problem. We first derive (5) by considering a small increase in $c_2(\gamma,\theta)$ across θ for a fixed γ. That is, for all θ, define $u(\tilde{c}_2(\gamma,\theta)) = u(c_2(\gamma,\theta)) + \Delta$ for some small Δ. We simultaneously decrease $c_1(\gamma,\theta)$ for all θ such that $u(\tilde{c}_1(\gamma,\theta)) = u(c_1(\gamma,\theta)) - \delta_2\Delta$. Such perturbations do not affect the objective function, the ex-ante incentive compatibility, and the ex-post incentive compatibility. It only affects the resource constraint. Note that the perturbation must be the same for all θ or else it may violate ex-post incentive compatibility, which is not the case if $\beta = 1$. If P is optimal, then it must be that $\Delta = 0$ minimizes the resource used, i.e.,

$$0 = \arg\min_{\Delta} \int_\Theta \left[-u^{-1}\left[u(c_1(\gamma,\theta)) - \delta_2\Delta\right] - \frac{1}{R_2} u^{-1}\left[u(c_2(\gamma,\theta)) + \Delta\right]\right] f(\theta|\kappa_\gamma)\, d\theta.$$

Evaluating the first order condition of this problem at $\Delta = 0$ yields (5).

Similarly, to derive (4), we consider a small decrease in $c_1(\gamma,\theta)$ for all θ and γ such that $u(\tilde{c}_1(\gamma,\theta)) = u(c_1(\gamma,\theta)) - \frac{1}{\delta_1(e_\gamma)}\Delta$ for some small Δ. We simultaneously increase $c_0(\gamma)$ for all γ such that $u(\tilde{c}_0(\gamma)) = u(c_0(\gamma)) + \frac{1}{\delta_0(e_\gamma)}\Delta$. Since it is perturbed for all θ, the ex-post incentive compatibility constraint is not affected. Also, notice that the ex-ante incentive compatibility constraint and objective function are not affected, but the resource constraint changes. Crucially, the perturbation must be the same for all γ or it may violate ex-ante incentive compatibility, which is not the case if $\beta = 1$. If P is optimal, then $\Delta = 0$ solves,

$$\min_{\Delta} \sum_{\gamma} \pi_\gamma \left\{ \frac{-u^{-1}\left[u\left(c_0\left(\gamma\right)\right) + \frac{\Delta}{\delta_0(e_\gamma)}\right]}{R_0\left(e_\gamma\right)} - \frac{1}{R_1\left(e_\gamma\right)} \int_\Theta u^{-1}\left[u\left(c_1\left(\gamma,\theta\right)\right) - \frac{\Delta}{\delta_1\left(e_\gamma\right)}\right] f\left(\theta|\kappa_\gamma\right) d\theta \right\}.$$

Evaluating the first order condition of this problem at $\Delta = 0$ yields (4). ■

Proof of Proposition 2: From the first order conditions, we have

$$\xi_H\left(\theta\right) = \int_\theta^{\overline{\theta}} \left[\lambda_H\left(x\right) - \left(\pi_H + \beta\mu\right)\delta_1\left(e_H\right) f\left(x|\kappa_H\right)\right] dx,$$

$$\xi_L\left(\theta\right) = \int_\theta^{\overline{\theta}} \left[\lambda_L\left(x\right) - \left[\pi_L f\left(x|\kappa_L\right) - \beta\mu f\left(x|\kappa_{L,H}\right)\right]\delta_1\left(e_L\right)\right] dx.$$

First, we derive (9). Since $\lambda_\gamma\left(\theta\right) = \frac{\phi\pi_\gamma\delta_1(e_\gamma)f(\theta|\kappa_\gamma)}{u'(c_1(\gamma,\theta))}$, we rewrite the first order condition on $y\left(H,\theta\right)$ as

$$\phi\pi_H\delta_1\left(e_H\right) f\left(\theta|\kappa_H\right)\left[1 - \frac{\frac{1}{\theta}h'\left(\frac{y(H,\theta)}{\theta}\right)}{u'\left(c_1\left(H,\theta\right)\right)}\right]$$
$$= \left[\frac{1}{\theta^2}h'\left(\frac{y\left(H,\theta\right)}{\theta}\right) + \frac{y\left(H,\theta\right)}{\theta^3}h''\left(\frac{y\left(H,\theta\right)}{\theta}\right)\right]\int_\theta^{\overline{\theta}} \left[\lambda_H\left(x\right) - \left(\pi_H + \beta\mu\right)\delta_1\left(e_H\right) f\left(x|\kappa_H\right)\right] dx.$$

Let $A_\gamma(\theta) = \frac{1-F(\theta|\kappa_\gamma)}{\theta f(\theta|\kappa_\gamma)}$ and $B_\gamma(\theta) = 1 + \frac{\frac{y(\gamma,\theta)}{\theta}h''\left(\frac{y(\gamma,\theta)}{\theta}\right)}{h'\left(\frac{y(\gamma,\theta)}{\theta}\right)}$, then dividing both sides by $\frac{1}{\theta}h'\left(\frac{y(H,\theta)}{\theta}\right)\phi\pi_H\delta_1\left(e_H\right) f\left(\theta|\kappa_H\right)$ yields

$$\frac{1}{\frac{1}{\theta}h'\left(\frac{y(H,\theta)}{\theta}\right)} - \frac{1}{u'\left(c_1\left(H,\theta\right)\right)}$$
$$= A_H\left(\theta\right) B_H\left(\theta\right)\int_\theta^{\overline{\theta}}\left[\frac{\lambda_H\left(x\right)}{\phi\pi_H\delta_1\left(e_H\right) f\left(x|\kappa_H\right)} - \frac{\pi_H + \beta\mu}{\phi\pi_H}\right]\frac{f\left(x|\kappa_H\right)}{1 - F\left(\theta|\kappa_H\right)} dx.$$

By definition $\frac{1}{\theta}h'\left(\frac{y(\gamma,\theta)}{\theta}\right) = \left(1 - \tau^w(\gamma,\theta)\right)u'\left(c_1\left(\gamma,\theta\right)\right)$ and from the first order condition, $\frac{\lambda_\gamma(x)}{\phi\pi_\gamma\delta_1(e_\gamma)f(x|\kappa_\gamma)} = \frac{1}{u'(c_1(\gamma,x))}$, so we have

$$\frac{1}{u'\left(c_1\left(H,\theta\right)\right)}\left(\frac{\tau^w\left(H,\theta\right)}{1 - \tau^w\left(H,\theta\right)}\right) = A_H\left(\theta\right) B_H\left(\theta\right)\left[\int_\theta^{\overline{\theta}} \frac{1}{u'\left(c_1\left(H,x\right)\right)}\frac{f\left(x|\kappa_H\right)}{1 - F\left(\theta|\kappa_H\right)} dx - \frac{\pi_H + \beta\mu}{\phi\pi_H}\right].$$

Observe that $\frac{\beta\mu}{\phi\pi_H} = \frac{\mu}{\phi\pi_H} - \frac{(1-\beta)\mu}{\phi\pi_H}$, then by the first order conditions, we can substitute in $\frac{\mu}{\phi\pi_H} = \frac{1}{u'(c_0(H))} - \frac{1}{\phi}$. Define $C_\gamma(\theta) = \int_\theta^{\overline{\theta}} \frac{u'(c_1(\gamma,\theta))}{u'(c_1(\gamma,x))}\left[1 - \frac{u'(c_1(\gamma,x))}{\phi}\right]\frac{f(x|\kappa_\gamma)}{1-F(\theta|\kappa_\gamma)}dx,\ D_\gamma(\theta) = u'(c_1(\gamma,\theta))\left[\frac{1}{u'(c_0(\gamma))} - \frac{1}{\phi}\right]$, and $E_\gamma(\theta) = (1-\beta)D_\gamma(\theta)$, then multiply both sides by $u'(c_1(H,\theta))$ to yield (9).

Using a similar process as above, we have the following expression for $\gamma = L$

$$\frac{\tau^w(L,\theta)}{1-\tau^w(L,\theta)} = A_L(\theta)B_L(\theta)\left[C_L(\theta) + \int_\theta^{\overline{\theta}} \frac{\beta\mu f(x|\kappa_{L,H})}{\phi\pi_L}\frac{u'(c_1(L,\theta))}{1-F(\theta|\kappa_L)}dx\right].$$

Since $\frac{\beta\mu}{\phi\pi_L} = \frac{\mu}{\phi\pi_L} - \frac{(1-\beta)\mu}{\phi\pi_L}$ and from the first order conditions, we get (10). Furthermore, from the first order condition for c_0, we have $\phi = \left[\frac{\pi_H}{u'(c_0(H))} + \frac{\pi_L}{u'(c_0(L))}\right]^{-1}$, combining it with (4) yields $\phi = \left\{\mathbb{E}_\gamma\left[\mathbb{E}_\theta\left(\frac{1}{u'(c_1(\gamma,\theta))}\middle|\gamma\right)\right]\right\}^{-1}$. ■

Proof of Lemma 2: For a fixed γ, suppose there exists $\tilde{\theta}$ and $\hat{\theta}$ such that $y\left(\gamma,\tilde{\theta}\right) = y\left(\gamma,\hat{\theta}\right)$. Let $\Phi(\gamma,\theta) = u(c_1(\gamma,\theta)) + \beta\delta_2 u(c_2(\gamma,\theta))$. There are two cases to consider. First, suppose $\Phi\left(\gamma,\tilde{\theta}\right) \neq \Phi\left(\gamma,\hat{\theta}\right)$, then clearly the allocations are not incentive compatible. Next, suppose $\Phi\left(\gamma,\tilde{\theta}\right) = \Phi\left(\gamma,\hat{\theta}\right)$, and without loss of generality $c_1\left(\gamma,\tilde{\theta}\right) > c_1\left(\gamma,\hat{\theta}\right)$ and $c_2\left(\gamma,\tilde{\theta}\right) < c_2\left(\gamma,\hat{\theta}\right)$. Let $\tilde{\pi}$ and $\hat{\pi}$ denote the measure of $\left(\gamma,\tilde{\theta}\right)$ and $\left(\gamma,\hat{\theta}\right)$ agents. Let $\bar{u}_t = \frac{1}{\tilde{\pi}+\hat{\pi}}\left[\tilde{\pi}u\left(c_t\left(\gamma,\tilde{\theta}\right)\right) + \hat{\pi}u\left(c_t\left(\gamma,\hat{\theta}\right)\right)\right]$. By assigning these agents the average utility, the total welfare is unchanged and incentive compatibility is preserved (because $\Phi\left(\gamma,\tilde{\theta}\right) = \Phi\left(\gamma,\hat{\theta}\right)$). However, since u is strictly concave, the consumption level that gives $\bar{u}_1$ and $\bar{u}_2$ relaxes the resource constraint. This means that it is not optimal for $c_1\left(\gamma,\tilde{\theta}\right) > c_1\left(\gamma,\hat{\theta}\right)$ and $c_2\left(\gamma,\tilde{\theta}\right) < c_2\left(\gamma,\hat{\theta}\right)$ with $\Phi\left(\gamma,\tilde{\theta}\right) = \Phi\left(\gamma,\hat{\theta}\right)$. In other words, the consumption paths are equivalent for agents of the same level of income. ■

Proof of Proposition 3: By Lemma 2, we can define the optimal consumption derived from the direct mechanism as $(c_0(e), c_1(e,y), c_2(e,y))$. Next, following Werning (2011), we construct bond savings tax $T^k(b)$ such that agents do not double deviate—misreport and buy too much bonds. To see how, consider the government assigning the optimal allocation from the direct revelation mechanism given past and current reports, while agents are allowed to purchase any desired amount of bonds. Define a fictitious tax $T_1^k(b_2,\tilde{r},\theta)$ paid in $t=1$ for each productivity realization θ, current bond level b_1, past report $\tilde{r}_\gamma$, current report $\tilde{r}_\theta$, and bond savings b_2, where $\tilde{r} = (\tilde{r}_\gamma, \tilde{r}_\theta)$. The tax $T_1^k(b_2, \tilde{r}, \theta)$ is set such that

$$u\left(c_1(\tilde{r}) + \tilde{R}_1(e(\tilde{r}_\gamma))b_1 - b_2 - T_1^k(b_2, \tilde{r}, \theta)\right) - h\left(\frac{y(\tilde{r})}{\theta}\right) + \beta\delta_2 u(c_2(\tilde{r}) + R_2 b_2)$$
$$= u(c_1(\gamma, \theta)) - h\left(\frac{y(\gamma, \theta)}{\theta}\right) + \beta\delta_2 u(c_2(\gamma, \theta)).$$

Next, by taking the supremum over all $\theta \in \Theta$, we obtain a bond savings tax $T_1^k(b_2, \tilde{r}) = \sup_{\theta\in\Theta} T_1^k(b_2, \tilde{r}, \theta)$ that is independent of productivity. Before we derive the bond savings tax in $t = 0$, let

$$V(b_1, \tilde{r}_\gamma, \theta) = u\left(c_1(\tilde{r}_\gamma, \hat{r}_\theta) + \tilde{R}_1(e(\tilde{r}_\gamma))b_1 - \hat{b}_2 - T_1^k\left(\hat{b}_2, \tilde{r}_\gamma, \hat{r}_\theta\right)\right)$$
$$- h\left(\frac{y(\tilde{r}_\gamma, \hat{r}_\theta)}{\theta}\right) + \delta_2 u\left(c_2(\tilde{r}_\gamma, \hat{r}_\theta) + R_2\hat{b}_2\right),$$

where

$$\left(\hat{r}_\theta, \hat{b}_2\right) \in \arg\max_{\tilde{r}_\theta, b_2}\Bigg\{u\left(c_1(\tilde{r}) + \tilde{R}_1(e(\tilde{r}_\gamma))b_1 - b_2 - T_1^k(b_2, \tilde{r})\right)$$
$$- h\left(\frac{y(\tilde{r})}{\theta}\right) + \beta\delta_2 u(c_2(\tilde{r}) + R_2 b_2)\Bigg\}.$$

Next, define $T_0^k(b_1, \tilde{r}_\gamma) = \sup_{\gamma\in\{H,L\}} T_0^k(b_1, \tilde{r}_\gamma, \gamma)$ with $T_0^k(b_1, \tilde{r}_\gamma, \gamma)$ chosen such that

$$\delta_0(e_\gamma)\, u\left(c_0(\tilde{r}_\gamma) - b_1 - T_0^k(b_2, \tilde{r}_\gamma, \gamma)\right) + \beta\delta_1(e(\tilde{r}_\gamma))\,\mathbb{E}\left[V(b_1, \tilde{r}_\gamma, \theta)\,|\gamma\right] = \delta_0(e_\gamma)\, u(c_0(\gamma))$$
$$+ \beta\delta_1(e_\gamma)\int_{\underline{\theta}}^{\overline{\theta}}\left[u(c_1(\gamma, \theta)) - h\left(\frac{y(\gamma, \theta)}{\theta}\right) + \delta_2 u(c_2(\gamma, \theta))\right] dF(\theta|\kappa(e_\gamma, \gamma)).$$

Finally, by taking the supremum over all reports, we obtain a bond savings tax $T^k(b) = \sup_{\tilde{r}^t} T_t^k(b, \tilde{r}^t)$, where $\tilde{r}^1 = \tilde{r}$ and $\tilde{r}^0 = \tilde{r}_\gamma$, that only depends on bond purchases. With $T^k(b)$, agents do not purchase bonds while misreporting in equilibrium.

For the other policy instruments, we focus on an implementation where none of the agents save in the retirement savings account, so $s_2 = 0$. Agents with education e_L rely on social security for retirement consumption while agents with education e_H depend on social security benefits plus student loan repayment contributions in the retirement account. Let $y(\gamma, \theta)$ be the optimal output of type (γ, θ) agents in a direct revelation mechanism and define $Y = \{y | y = y(\gamma, \theta) \text{ with } \gamma \in \{L, H\} \text{ and } \theta \in \Theta\}$ to be the set of admissible income.

First, we construct the matching rate α to be

$$1+\alpha=\inf_{y\in Y,e\in\{e_L,e_H\}}\frac{u'\left(c_1\left(e,y\right)\right)}{\beta u'\left(c_2\left(e,y\right)\right)}.$$

Next, we construct the social security benefit $a\left(y\right)=c_2\left(e_L,y\right).$ We set the income tax to be $T\left(y-s_2\right)=y-s_2-c_1\left(e_L,y\right),$ and the tax deduction from student loan repayment is

$$g\left(r\left(e_H,y\right)\right)=r\left(e_H,y\right)-\left[c_1\left(e_L,y\right)-c_1\left(e_H,y\right)\right]\text{ and }g\left(0\right)=0.$$

Finally, we construct the student loans and its income-contingent repayment schedule along with the tax on retirement savings account. Let the loan amount be defined as

$$L\left(e\right)=\begin{cases}c_0\left(e\right)+e & \text{if } e\in\{e_L,e_H\}\\ 0 & \text{otherwise}\end{cases},$$

and the income-contingent repayment schedule is $r\left(e_L,y\right)=0$ and

$$r\left(e_H,y\right)=\frac{1}{\alpha R_2}\left[c_2\left(e_H,y\right)-c_2\left(e_L,y\right)+T^{ra}\right].$$

We choose T^{ra} such that $r\left(e_H,y\right)$ and $g\left(R_1r\right)$ are weakly positive. Let $T^{ra}\left(y\right)$ be a fictitious tax schedule defined as

$$T^{ra}\left(y\right)=\max\left\{0,c_2\left(e_L,y\right)-c_2\left(e_H,y\right),c_2\left(e_L,y\right)-c_2\left(e_H,y\right)+\alpha R_2\left[c_1\left(e_L,y\right)-c_1\left(e_H,y\right)\right]\right\}.$$

Observe that given $T^{ra}\left(y\right),$ both the repayment schedule and the tax deduction are weakly positive for any income. Lastly, by taking the supremum over all income, we obtain an income-independent lump-sum tax:

$$T^{ra}=\sup_{y\in Y}T^{ra}\left(y\right).$$

For our last step, we check that the policy instruments implement the optimum. First, notice that all agents would choose $e\in\{e_L,e_H\}$, otherwise $c_0=0$. Next, due to the low matching rate, all agents choose $s_2=0$. As a result, given the taxes and social security benefit, agents who invested e_L consume $c_1=c_1\left(e_L,y\right)$ and $c_2=c_2\left(e_L,y\right).$ Next, for agents who invested e_H, given the taxes, $c_1=y-T(y)+g\left(r\left(e_H,y\right)\right)-r\left(e_H,y\right)$ and $c_2=a\left(y\right)+\alpha R_2r\left(e_H,y\right)-T^{ra}$, so they optimally choose $c_1=c_1\left(e_H,y\right)$ and $c_2=c_2\left(e_H,y\right).$ Also, by the taxation principle, agents with productivity θ choose $y=y\left(e,\theta\right).$ Finally, notice that given $L(e)$, agents with innate ability γ optimally choose education level e_γ. ■

Proof of Proposition 4: With heterogeneous β, the government's problem remains the same except (12) is now

$$U_1(\gamma,\theta) = u(c_1(\gamma,\theta)) - h\left(\frac{y(\gamma,\theta)}{\theta}\right) + \beta_\gamma \delta_2 u(c_2(\gamma,\theta))$$

for all γ, and the ex-ante incentive constraint is

$$\begin{aligned}\delta_0(e_H)\,u(c_0(H)) + \beta_H \delta_1(e_H)\int_{\underline{\theta}}^{\overline{\theta}} \left[U_1(H,\theta) + (1-\beta_H)\,\delta_2 u(c_2(H,\theta))\right] f(\theta|\kappa_H)\,d\theta \\ \geq \delta_0(e_L)\,u(c_0(L)) + \beta_H \delta_1(e_L)\int_{\underline{\theta}}^{\overline{\theta}} \left[U_1(L,\theta) + (1-\beta_L)\,\delta_2 u(c_2(L,\theta))\right] f(\theta|\kappa_{L,H})\,d\theta.\end{aligned}$$

The results follow from the procedures outlined in the proofs for Proposition 1 and Proposition 2. ■

B Approximating Current Policies

To approximate current income taxes in the United States, we follow Heathcote et al. (2017) and assume an income tax function $T(y) = y - \lambda y^{1-\tau}$. College students have access to low-interest federal loans. We introduce such loans by assuming that agents may borrow L in period $t = 0$ to finance consumption and tuition. Then, agents start repaying these loans at the beginning of working period $t = 1$, and we assume they repay them in ten years. Loans up to a limit $\bar{L}$ carry the government-subsidized interest rate r_g; any amount above $\bar{L}$ carry a market interest rate $r_m > r_g$. Similarly as in the main calibration in Section 4, we assume that the education, work, and retirement periods last for 5.12, 43, and 20 years, respectively (in the case of high school graduates, the education period lasts for zero years).

Upon retirement, agents receive social security benefits, which are income-dependent. The regulation below has been translated to fit the context of our model. To derive an agent's social security benefits, first calculate the agent's average indexed monthly earnings (AIME) which is defined as $AIME = \frac{y}{12}$ for annual income y. In practice, the social security administration takes 35 of the highest annual incomes from the 45 years of the agent's work life and calculate the average monthly earnings. Next, based on 2015 social security regulations, the agent's monthly benefit $a(AIME)$ is determined by the following replacement rates and bend points:

$$a(AIME) = \begin{cases} 0.9 \times AIME & \text{if } AIME \leq 826 \\ 743.4 + 0.32 \times (AIME - 826) & \text{if } 826 < AIME \leq 4,980 \\ 2,072.68 + 0.15 \times (AIME - 4,980) & \text{if } 4,980 < AIME \leq 9,875 \\ 2,806.93 & \text{if } AIME > 9,875 \end{cases}.$$

This immediately implies that the agent receives $A(y) = 12 \times a\,(AIME)$ every year in social security benefits.

Using the 2015 regulations, agents are subject to a flat social security tax $T_s\,(y)$, which is defined as

$$T_s\,(y) = \begin{cases} 0.124 \times y & \text{if } y \leq 118,500 \\ 14,694 & \text{if } y > 118,500 \end{cases}.$$

The tax is capped at an annual income of 118,500. Furthermore, the social security benefits are distributed from the social security tax.

We assume that agents accumulate retirement savings in a 401(k) account and a regular savings account which pays a gross interest of R_2. Let s_2 denote savings in a 401(k) account and b_2 in the regular savings account. Contributions to the 401(k) account are capped at an annual amount of 18,000. We also assume an employer matching rate of 50%. Contributions to defined contribution plans, such as 401(k), are pre-tax. This means that income tax payments are deferred upon withdrawal when retiring. However, social security tax is not deferred. Since contributions to 401(k) are matched, agents would first save in their 401(k) accounts until the cap binds, before saving in their regular accounts.

B.1 Deriving Allocations for Current Policies

To determine the allocation of present-biased agents under the current policy, we adopt subgame perfect Nash equilibrium as our solution concept.

B.1.1 The Working Period Problem

By backward induction, agents with productivity θ who took out a total loan of L in $t = 0$ and invested e in education solve the following problem:

$$\max u\,(c_1) - h\,(l) + \beta\delta_2 u\,(c_2)$$

subject to

$$c_1 + b_2 + s_2 = \theta l - T\left(\theta l - s_2\right) - T_s\left(\theta l\right) - i,$$

$$c_2 = 1.5 R_2 s_2 + R_2 b_2 + A\left(\theta l\right) - T\left(1.5 R_2 s_2\right),$$

$$s_2 \leq \overline{c},$$

where $\overline{c}$ is the upper-bound on contributions to the 401(k) account and i is an installment of the student loan defined as follows:

$$i = \frac{1-\delta_a^{10}}{1-\delta_a^{43}}\left[\frac{r_g(1+r_g)^{10}}{(1+r_g)^{10}-1}\min\left\{L,\bar{L}\right\} + \frac{r_m(1+r_m)^{10}}{(1+r_m)^{10}-1}\left(\max\left\{L,\bar{L}\right\} - \bar{L}\right)\right]$$

Agents start repaying their students loans at the beginning of work period and take ten years to pay them down. Loans up to the upper bound of $\bar{L}$ carry a government-subsidized interest rate r_g, while loan amounts above it carry a market interest rate of $r_m > r_g$. The effective installment i is spread out over the entire working-age period using the baseline annual discount factor of δ_a.

To analyze the solution of this model, let $\overline{\chi}_t\left(\theta\right)$ denote the multiplier on the period t budget constraint for agents who invested e_H, and $\underline{\chi}_t\left(\theta\right)$ be the multiplier for low-educated agents.

Using Only 401(k): When agents only use 401(k), then it means that agents choose to save $s_2 < \overline{c}$.

We first look at agents who invested e_H. The first order conditions for consumption and savings s_2 are

$$u'\left(c_1\right) = \overline{\chi}_1\left(\theta\right),\ \beta\delta_2 u'\left(c_2\right) = \overline{\chi}_2\left(\theta\right) \text{ and } \overline{\chi}_1\left(\theta\right) = \overline{\chi}_2\left(\theta\right) 1.5 R_2\left(\frac{\theta l - s_2}{1.5 R_2 s_2}\right)^{\tau}.$$

This provides us with the following Euler equation:

$$u'\left(c_1\right) = 1.5\beta\left(\frac{\theta l - s_2}{1.5 R_2 s_2}\right)^{\tau} u'\left(c_2\right).$$

For labor supply, we have four different income regions to consider:

$$h'(l) = \begin{cases} \overline{\chi}_2(\theta)\left\{1.5R_2\left(\frac{\theta l - s_2}{1.5R_2 s_2}\right)^{\tau} B(\theta, y, s_2) + 0.9\theta\right\} & \text{if } y \leq 9,912 \\ \overline{\chi}_2(\theta)\left\{1.5R_2\left(\frac{\theta l - s_2}{1.5R_2 s_2}\right)^{\tau} B(\theta, y, s_2) + 0.32\theta\right\} & \text{if } 9,912 < y \leq 59,760 \\ \overline{\chi}_2(\theta)\left\{1.5R_2\left(\frac{\theta l - s_2}{1.5R_2 s_2}\right)^{\tau} B(\theta, y, s_2) + 0.15\theta\right\} & \text{if } 59,760 < y \leq 118,500 \\ \overline{\chi}_2(\theta)\, 1.5R_2\left(\frac{1}{1.5R_2 s_2}\right)^{\tau} \theta\lambda(1-\tau) & \text{if } y > 118,500 \end{cases},$$

where $B(\theta, y, s_2) = \theta\lambda(1-\tau)(y - s_2)^{-\tau} - 0.124\theta$ As for agents who invested e_L, the first order conditions are the same except for replacing $\overline{\chi}_t(\theta)$ with $\underline{\chi}_t(\theta)$.

Using Both 401(k) and Savings: When agents start saving in the regular savings account—$b_2 > 0$, then it means that $s_2 = \overline{c}$.

We first analyze the case where agents invested e_H in $t = 0$. Suppose the agent has saved $s_2 = \overline{c}$, then the agent can only continue to save with the standard savings account. We can rewrite the sequential budget constraint into its present value terms:

$$c_1 + \frac{c_2 - \lambda(1.5R_2\overline{c})^{1-\tau} - A(\theta l)}{R_2} = \lambda(\theta l - \overline{c})^{1-\tau} - T_s(\theta l) - i.$$

Let $\overline{\chi}(\theta)$ denote the multiplier on the present-valued budget constraint. The first order conditions on consumption are

$$u'(c_1) = \overline{\chi}(\theta) \text{ and } \beta u'(c_2) = \overline{\chi}(\theta).$$

The first order condition for labor is

$$h'(l) = \begin{cases} \overline{\chi}(\theta)\left[\theta\lambda(1-\tau)(\theta l - \overline{c})^{-\tau} - 0.124\theta + \frac{0.9}{R_2}\theta\right] & \text{if } y \leq 9,912 \\ \overline{\chi}(\theta)\left[\theta\lambda(1-\tau)(\theta l - \overline{c})^{-\tau} - 0.124\theta + \frac{0.32}{R_2}\theta\right] & \text{if } 9,912 < y \leq 59,760 \\ \overline{\chi}(\theta)\left[\theta\lambda(1-\tau)(\theta l - \overline{c})^{-\tau} - 0.124\theta + \frac{0.15}{R_2}\theta\right] & \text{if } 59,760 < y \leq 118,500 \\ \overline{\chi}(\theta)\,\theta\lambda(1-\tau)(\theta l - \overline{c})^{-\tau} & \text{if } y > 118,500 \end{cases},$$

We can derive a similar set of first order conditions for agents who obtained education level e_L.

B.1.2 The Schooling Period Problem

Let $(\check{c}_1(e,\theta), \check{y}(e,\theta), \check{c}_2(e,\theta))$ denote the solution to the problem in Section B.1.1, which is the optimal consumption path and output agents choose in $t=1$ given education e and productivity θ. Agents with innate ability γ solve the following problem:

$$\max_{c_0,e,b_1} \delta_0(e)\, u(c_0) + \beta\delta_1(e) \int_{\underline{\theta}}^{\overline{\theta}} \left[u(\check{c}_1(e,\theta)) - h\left(\frac{\check{y}(e,\theta)}{\theta}\right) + \delta_2 u(\check{c}_2(e,\theta)) \right] f(\theta|\kappa(e,\gamma))\, d\theta$$

subject to

$$c_0 + e = b_1 \text{ and } e \in \{e_L, e_H\}.$$

In essence, agents take out a yearly loan of b_1 to pay for their schooling and consumption in $t=0$. The total amount of student loans L that carries into the working-age period $t=1$ is defined as

$$L = \left(\frac{1}{\delta_a}\right)^{5.12} b_1 \frac{1-\delta_a^{5.12}}{1-\delta_a}$$

B.2 Deriving Allocations for Proposed Reform

Following Section B.1, we derive the allocations from the proposed policy reform that treats student loan repayments as contributions to retirement savings in this section.

In the proposed reform, student loan repayments may qualify as a contribution, even without agents making a direct contribution to their own accounts. Let m denote the match received from student loan repayments. Assuming a 50% matching rate, these agents receive a contribution of $m = 0.5\min\{i, \overline{c} - s_2\}$. In essence, student loan repayment i is treated as a contribution, but only to the extent that it does not put the total contribution over the limit $\overline{c}$ of what is qualified for matching. Agents can also elect to forgo this option. This happens if agents save $s_2 = \overline{c}$. By contrast, for any $s_2 < \overline{c}$, agents will benefit, at least partially, from receiving a match on their student loan repayments.

During the working period, agents with productivity θ who took out a total loan of L in $t=0$, resulting in an annual loan installment of i (as defined in subsection B.1.1), and invested e in education solve the following problem:

$$\max u(c_1) - h(l) + \beta\delta_2 u(c_2)$$

subject to

$$c_1 + b_2 + s_2 = \theta l - T(\theta l - s_2) - T_s(\theta l) - i,$$

$$c_2 = 1.5R_2 s_2 + R_2 m + R_2 b_2 + A(\theta l) - T(1.5R_2 s_2 + R_2 m),$$

$$s_2 \leq \overline{c}.$$

There are three cases to consider. In the first case, agents do not benefit from the new policy proposal because they choose to save in the retirement account up to the limit: $s_2 = \overline{c}$ and $b_2 \geq 0$. For the last two cases, $s_2 < \overline{c}$ and student loan repayments act as contributions to their retirement savings. Notice that if agents elect to receive a match on their student loans, they would not save in their regular savings account, so $b_2 = 0$.[1] In the second case, the agents receive a match on the full student loan repayment amount with total contributions below the limit: $i + s_2 \leq \overline{c}$ and $b_2 = 0$. In the last case, agents choose to primarily save on their own and only receive a match on a fraction of the student loan repayment: $i + s_2 = \overline{c}$ and $b_2 \geq 0$.

For the schooling period, by backward induction, agents solve the same problem as the one presented in Section B.1.2.

For our quantitative implementation of this model, we assume that the new policy is introduced in a revenue-neutral way. This means that the additional matching provided to employees based on their repayment of student loans is financed by a simultaneous increase in income taxes on everyone. We find that the reform is fully financed by reducing the λ parameter of the tax function from the baseline value of 0.839 to 0.826.

B.3 Calibration

In this section we calibrate the model to resemble the "real world" as closely as possible. The goal is to back out the distribution of productivities across different education groups. To this extent, we first pick a number of parameters externally and summarize them in Table 5. Then, we calibrate the distributions of skills internally to match the evidence on lifetime earning provided by Cunha and Heckman (2007).

The values of risk aversion and Frisch elasticity of labor are standard and set to 2 and 0.5, respectively. Next, we discuss the calibration of the current tax system. The parameters of the income tax function τ and λ are borrowed from Heathcote and Tsujiyama (2021) and apply to income level normalized by average income in the economy.[2] The upper bound for 401(k) contributions $\overline{c}$ is set to \$18,000 based on the limit in 2015. As for the financing of student loans, we assume for simplicity that the annual interest rates an agent may obtain through private market and through a government-subsidized scheme are 10% and 5%,

[1] It is always optimal to move some of the savings from the regular account into the 401(k) and receive a tax deferral, even when this shift causes a decrease in the match from student loans.

[2] We calculate average income directly using the factual distributions of lifetime income from Cunha and Heckman (2007) and the shares of high school and college graduates (and beyond) of 0.68 and 0.32, respectively, from the CPS.

Table 5: Parameter values in the model

Symbol	Meaning	Value	Source
σ	Risk aversion	2	Standard values
η	Frisch elasticity	0.5	
τ	Tax progressivity	0.161	Heathcote and Tsujiyama (2021)
λ	Taxation level	0.839	
$\bar{c}$	401(k) contribution limit	1.80	Approximated from data
e_H	Cost of college	1.57	
r_m	Commercial interest on student loans	0.1	
r_g	Government interest on student loans	0.05	
$\bar{L}$	Cap on government-subsidized student loans	8.75	
β	Short-term discount factor	0.7	Based on Nakajima (2012)
$\delta_0(e_L)$	High school period 0 long-term discount factor	0.00	
$\delta_1(e_L)$	High school period 1 long-term discount factor	1.00	
$\delta_0(e_H)$	College period 0 long-term discount factor	0.16	
$\delta_1(e_H)$	College period 1 long-term discount factor	0.93	
δ_2	Retirement discount factor	0.29	
Time-consistent benchmark ($\beta = 1$)			Based on Nakajima (2012)
$\delta_0(e_L)$	High school period 0 long-term discount factor	0.00	
$\delta_1(e_L)$	High school period 1 long-term discount factor	1.00	
$\delta_0(e_H)$	College period 0 long-term discount factor	0.20	
$\delta_1(e_H)$	College period 1 long-term discount factor	0.85	
δ_2	Retirement discount factor	0.17	

Note: All monetary parameters are denominated in 10,000 of 2015 US dollars.

respectively. The amount of subsidized loan is capped at \$87,500, in line with the regulations for Stafford loans in the US (weighted by the shares of undergraduate and professional degrees from Table 6). We further assume that an agent takes ten years to repay the student loans.

The annual cost of higher education e_H is assumed to be \$15,700, which is calculated for 2015 based on average tuition costs of private and public colleges plus different types of graduate degrees[3] as well as relative enrollment data for both types of college.[4] Table 6 presents a breakdown of different higher education outcomes, along with average costs and durations, which we use to calculate this parameter.

In calibrating the short- and long-term discount factors we primarily follow Nakajima (2012) who uses a general equilibrium model with present-biased agents and targets a capital-

[3]Source: College Board, Annual Survey of Colleges and NCES, Digest of Education Statistics
[4]Source: NCES, Digest of Education Statistics, and Current Population Survey 2015

Table 6: Breakdown of higher education outcomes

Degree type	*% of population*	*Duration*	*Annual cost*
Associate's and less	67.7	0	0
Bachelor's only	20.3	4	15,396
Master's	8.0	6	16,140
Professional	1.9	8	27,210
Doctoral	2.1	10	6,158
Total	100	5.12	15,695

Note: distribution of educational attainment is from CPS 2015. The durations and annual costs are cumulative. The data on costs of various higher degrees are taken from NCES, Digest of Education Statistics and expressed in 2015 dollars. We ignore the cost and duration of Associate's degrees as those are often combined with jobs.

output ratio of 3. We adopt his assumed value of the short-term discount factor of 0.7 which places in the midrange of estimates found by Laibson et al. (2017). The annual long-run discount factor is $\delta_a = 0.9852$ following Nakajima (2012) which we in turn use to calculate effective discount factors across the three periods in our model. These effective discount factors also reflect the relative lengths of the periods, which may differ across agents of different education groups. Because high school graduates start working right away, they never actually experience the education period 0; hence their parameter $\delta_0(e_L)$ is zero and $\delta_1(e_L)$ is one. On the other hand, college graduates spend 5.12 years in period 0, which reflects the average duration of undergraduate and graduate studies in the US (Table 6 presents a detailed breakdown), and then another 43 years in period 1. This yields $\delta_0(e_H) = \frac{1-\delta_a^{5.12}}{1-\delta_a^{43}} = 0.16$ and $\delta_1(e_H) = \frac{\delta_a^{5.12}-\delta_a^{48.12}}{1-\delta_a^{43}} = 0.93$. We assume that both education types spend 43 years working and 20 years in retirement. This yields a common retirement period discount factor of $\delta_2 = \frac{\delta_a^{43}-\delta_a^{63}}{1-\delta_a^{43}} = 0.29$.[5]

For our analysis in the main body of the paper we also use the benchmark of time-consistent agents, i.e. the world where $\beta = 1$. For reference, we present here the analogous derivations of the effective long-run discount factors for that case. Once again following Nakajima (2012) we assume an annual discount factor $\delta_a = 0.9698$. Then, with the same reasoning we assume $\delta_0(e_L) = 0$ and $\delta_1(e_L) = 1$ for high school graduates, compared to $\delta_0(e_H) = 0.20$ and $\delta_1(e_H) = 0.85$ for college graduates. The discount factor for retirement amounts to $\delta_2 = 0.17$.

Having established the external parameters, we turn to the parameters governing the

[5] Because the college type first spends around five years on education before they start to work, we assume that they also retire later and live longer for the same number of years. This is consistent with a significant body of research which shows college graduates live longer than non-college graduates (Meara et al., 2008).

distribution of skills which are set through solving and simulating the model. For each of the four groups of agents: (i.) factual high school graduates, (ii.) high school graduates had they gone to college, (iii.) factual college graduates, and (iv.) college graduates had they not gone to college, we observe the empirical distributions of lifetime earnings reported by Cunha and Heckman (2007). Roughly speaking, these distributions are obtained by estimating a Roy-type model on combined NLSY and PSID data and generating counterfactuals for both education groups. As it is commonly known, panel surveys such as these tend to under-represent the upper tail of the earnings distribution. For this reason, similar to Findeisen and Sachs (2016), we add an upper Pareto-tail with the shape parameter of 1.5 (Saez, 2001). For each distribution, we select an income threshold at which we attach the Pareto tail such that the upper 10% of the mass is distributed according to it. We pick the scale parameter such that the (smoothed out) PDF of the empirical distribution of earnings from Cunha and Heckman (2007) intersects at the threshold with the Pareto PDF. Table 7 summarizes the parameters of the Pareto tail added to each of the empirical distribution of lifetime earnings.

Table 7: Adding a Pareto tail to lifetime income distributions

	HS fact.	*HS counter.*	*COL fact.*	*COL counter.*
Threshold	205.5	269.2	315.4	223.2
Scale parameter	63.4	86.0	104.5	69.5

Note: The thresholds refer to present value of lifetime earnings and are expressed in \$10,000s of 2015 dollars. Thresholds are selected in each case such that 10% of total mass is distributed according to Pareto distribution with the shape parameter of 1.5.

To capture the earnings distribution with a fat upper tail in our model, we assume that agents' skills θ follow a mixture of two distributions, a normal distribution and a two-piece distribution (lognormal-Pareto) as described in Nigai (2017). The probability density function of our mixture is then given by

$$\begin{aligned} f(\theta) =& p \times \left[\frac{1}{2\pi\psi_1}\exp\left\{\frac{(\theta-\mu_1)^2}{2\psi_1^2}\right\}\right] \\ &+(1-p)\times\begin{cases}\frac{\rho}{\Phi\left(\alpha s(\alpha,\rho)\right)}\frac{1}{\sqrt{2\pi}s(\alpha,\rho)\theta}\exp\left\{-\frac{1}{2}\left(\alpha s(\alpha,\rho)-\frac{\log(\theta^T)-\log(\theta)}{s(\alpha,\rho)}\right)\right\}, & \text{if } \theta\in(0,\theta^T) \\ (1-\rho)\frac{\alpha(\theta^T)^\alpha}{\theta^{\alpha+1}}, & \text{if } \theta\in[\theta^T,\infty)\end{cases} \end{aligned} \tag{15}$$

In equation (15), μ and ψ are the mean and standard deviation of the normal distribution, and p is the probability of drawing it. The two-piece lognormal-Pareto distribution comes with a shape parameter α, which we fix at 1.5, and two scale parameters, ρ and θ^T.

Intuitively, θ^T is the threshold value at which the standard lognormal distribution turns into Pareto, while $\rho \in (0, 1)$ represents the fraction of total mass that is distributed according to lognormal. We have hence 5 parameters to pin down for each of the four groups of agents, $(\mu, \psi, \rho, \theta^T, p)$, in order to replicate the empirical distributions of earning provided by Cunha and Heckman (2007) and augmented with the Pareto tail. To do so, we solve for the optimal policy functions in each of the four cases and simulate random draws for 100,000 agents. We use a global optimization algorithm to minimize the distance between the simulated CDF of lifetime earnings and the targeted one. Table 8 shows all components of our mixture density defined in (15) matter quantitatively and altogether result in a good fit for model-derived distributions of earnings in each group. Figures 10-11 depict the PDFs of lifetime earnings in the model and their empirical targets across the four groups of agents. Notice that all estimations result in an excellent fit to the data, with perhaps a slight exception for High School counterfactual. However, this distribution does not affect the model solution in any way and it is only necessary to verify that the low type indeed prefers to report truthfully.

Table 8: Parameters of productivity distributions

Symbol	**Meaning**	**Value**			
		HS fact.	*HS counter.*	*COL fact.*	*COL counter.*
μ	Mean of normal	8.05	9.74	11.33	8.81
ψ	St. dev. of normal	2.19	2.74	3.66	2.29
θ^T	Threshold for Pareto	7.79	9.97	14.70	8.60
ρ	Fraction of lognormal	0.35	0.61	0.63	0.43
p	Probability of normal	0.62	0.39	0.53	0.56

Note: Productivities drawn from these distributions are in annual terms.

C Sensitivity Analyses for Efficiency Wedges

In this section, we conduct various sensitivity checks. We show how the efficiency wedge, varies with respect to the two main preference parameters, namely the short-term discount factor and the risk aversion. Figures 12(a) and 12(b) present the efficiency wedges (analogous to Figure 2(a)) when $\beta = 0.5$ and $\beta = 0.9$, respectively. To make the comparison easier, we keep the scale of the y-axes unchanged. The results are quite intuitive in that the two wedges become much steeper (and separated from each other) as β decreases. On the other hand, for a small degree of present bias, the two wedges flatten out and converge to zero.

Figures 13(a) and 13(b) show a similar sensitivity analysis for the efficiency wedges with respect to risk aversion. Intuitively, as σ moves towards risk neutrality the wedges become flatter, while higher risk aversion makes them steeper.

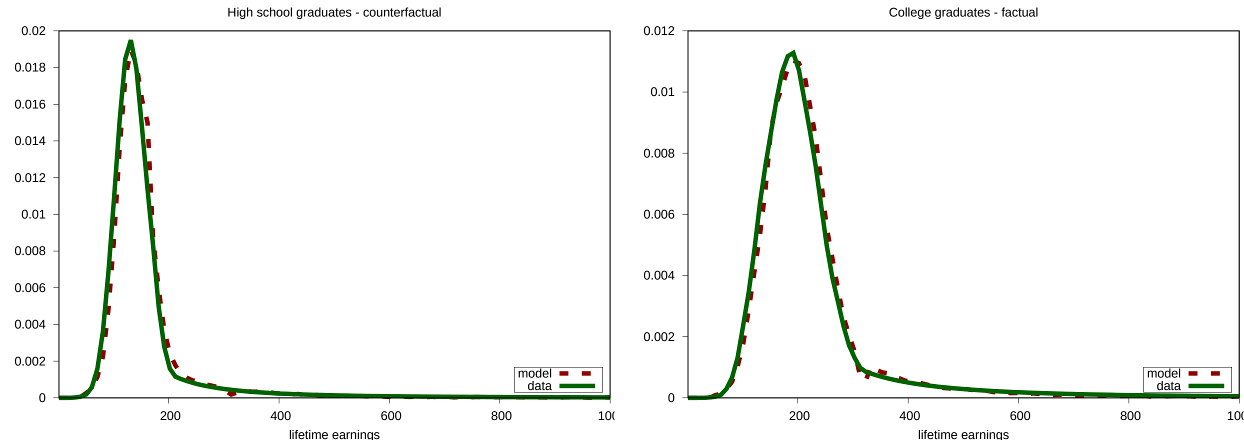


Figure 10: Distributions of lifetime income - factual

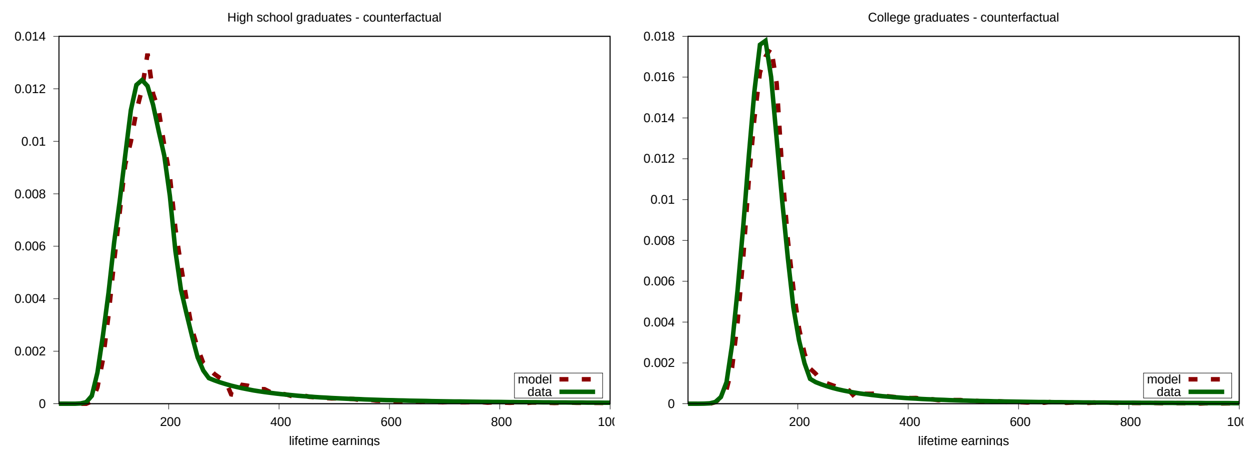
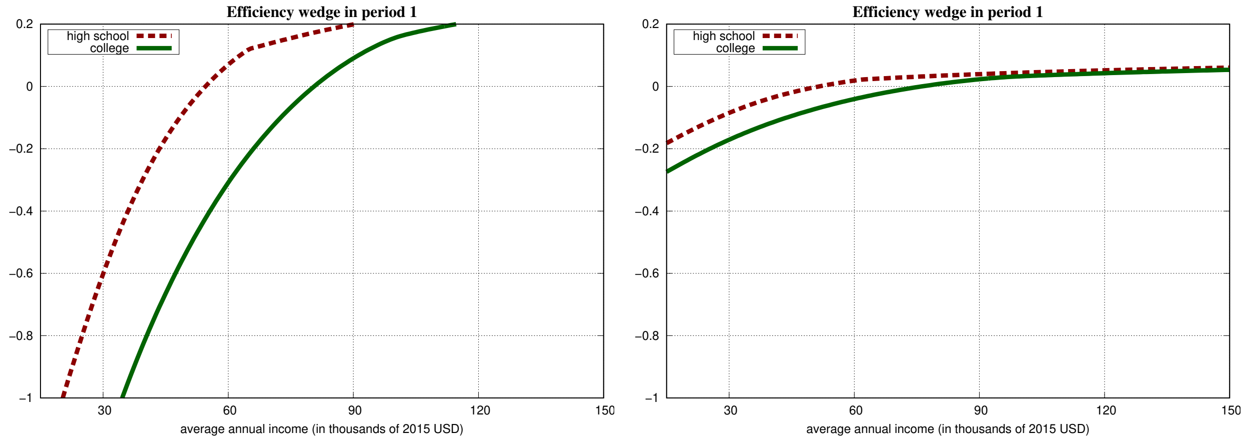


Figure 11: Distributions of lifetime income - counterfactual

Finally, it should be emphasized that the labor wedge remains virtually unaffected by varying the degree of present bias, which is reminiscent of the result in Figure 3. By contrast, the labor wedge is impacted by the degree of risk aversion, but this issue has been studied extensively by the previous literature and is not the main topic of interest in this paper.

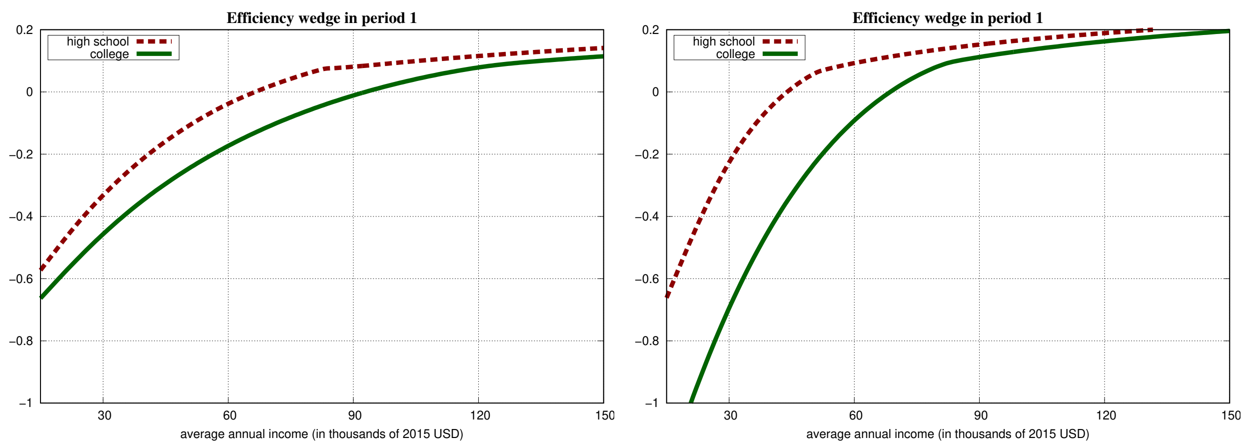
D Decomposition of the Labor Wedge

In this section, we quantify the decomposition of the labor wedge introduced in Section 3.2. Figure 14 presents the numerical approximation of the labor wedge components A,C,D



(a) Case $\beta = 0.5$ (b) Case $\beta = 0.9$

Figure 12: Efficiency wedges for alternative values of short-term discount factor



(a) Case $\sigma = 1.5$ (b) Case $\sigma = 2.5$

Figure 13: Efficiency wedges for alternative values of relative risk aversion

and E as function of income for the high innate ability type.[6] The A component depends on the inverse hazard rate of the distribution of θ and declines at first, before increasing and converging to a constant due to the presence of a Pareto tail. By contrast, the intratemporal component C increases and then converges, resulting in the overall convergence of the labor wedge at the top of the distribution. The offsetting role that comes from the intertemporal component D is much smaller in size and decreases monotonically.

The novel aspect of our paper is the introduction of E, the present bias component. Since

[6] We ignore the B components because, given the functional forms we impose, it reduces to a constant. We also omit the decomposition for the low innate ability type because in our calibrated model the period-zero consumption of L-agents is not pinned down. As a result, the intertemporal component is not well-defined.

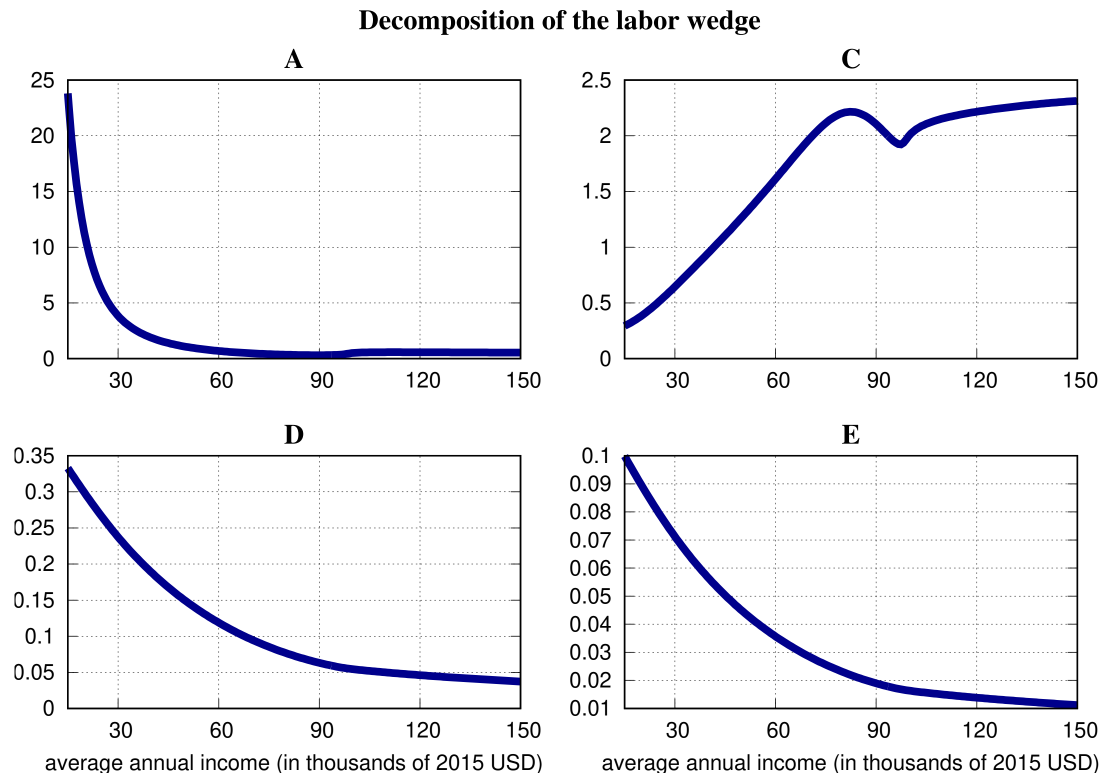


Figure 14: Labor wedge components for the high innate ability type

$E = (1-\beta) D$, this component declines monotonically but its magnitude is also very small compared to components D, or especially C. Consequently, the labor wedge is generally not much affected by the present bias, and any difference shows up most prominently at the lowest levels of income, as evident in Figure 3.

E Time-Consistent Benchmarks

In this section, we present details of the implementation of the time-consistent optimal policies, which we use as benchmark for welfare calculations in Section 4.3. We consider two ways to implement the optimal allocation for time-consistent agents: mandatory retirement savings and laissez faire retirement savings. The two different implementations lead to different measures of welfare improvement.

First, we characterize the optimal allocations for time-consistent agents in a direct mechanism. Let $\left\{\tilde{c}_0(\gamma), [\tilde{c}_t(\gamma,\theta), \tilde{y}(\gamma,\theta)]_{t>0,\theta\in\Theta}\right\}$ be the optimal allocation for time-consistent agents. The optimal allocation for time-consistent agents satisfies the following:

$$u'(\tilde{c}_1(\gamma,\theta)) = u'(\tilde{c}_2(\gamma,\theta)).$$

This implies that $\tilde{c}_1(\gamma,\theta) = \tilde{c}_2(\gamma,\theta) = \tilde{c}(\gamma,\theta)$. For $t=0$, the government implements $\tilde{c}_0(\gamma)$ by providing agents a student loan of

$$L(e_H) = \tilde{c}_0(H) + e_H \text{ and } L(e_L) = \tilde{c}_0(L) + e_L.$$

Next, we proceed to consider two different methods to decentralize the optimal allocations in $t=1$ and $t=2$.

E.1 Mandatory Savings

Consider a mandatory minimum savings rule that forces agents to smooth consumption: $\tilde{c}_1(\gamma,\theta) = \tilde{c}_2(\gamma,\theta)$. For time-consistent agents, the policy implements the optimum. However, for present-biased agents, the minimum savings rule is not incentive compatible.

To see how the minimum savings rule changes the behavior of present-biased agents, we first analyze how agents would change their reports of θ. Since for our quantitative exercise, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and $h\left(\frac{y}{\theta}\right) = \frac{1}{1+\frac{1}{\eta}}\left(\frac{y}{\theta}\right)^{1+\frac{1}{\eta}}$. Then, for a given report of innate ability $\hat{\gamma}$ and the time-consistent allocations, present-biased agents choose a report $\hat{\theta}$ to maximize the utility at $t=1$. In essence, a θ-agent solves

$$\max_{\hat{\theta}} u\left(\tilde{c}_1\left(\hat{\gamma},\hat{\theta}\right)\right) - h\left(\frac{\tilde{y}\left(\hat{\gamma},\hat{\theta}\right)}{\theta}\right) + \beta\delta_2 u\left(\tilde{c}_2\left(\hat{\gamma},\hat{\theta}\right)\right).$$

From the argument above and the assumptions on the utility function, the problem can be rewritten as

$$\max_{\hat{\theta}} u\left(\tilde{c}\left(\hat{\gamma},\hat{\theta}\right)\right) - \frac{1}{1+\frac{1}{\eta}}\left(\frac{\tilde{y}\left(\hat{\gamma},\hat{\theta}\right)}{(1+\beta\delta_2)^{\frac{1}{1+\frac{1}{\eta}}}\theta}\right)^{1+\frac{1}{\eta}}.$$

We know that when $\beta = 1$, the solution to the problem above is $\hat{\theta} = \theta$, because the mechanism satisfies incentive compatibility for time-consistent agents by assumption. Thus, we can transform the problem into the following alternative problem:

$$\max_{\hat{\theta}} u\left(\tilde{c}\left(\hat{\gamma},\hat{\theta}\right)\right) - \frac{1}{1+\frac{1}{\eta}}\left(\frac{\tilde{y}\left(\hat{\gamma},\hat{\theta}\right)}{\alpha(1+\delta_2)^{\frac{1}{1+\frac{1}{\eta}}}\theta}\right)^{1+\frac{1}{\eta}},$$

where $\alpha = \left(\frac{1+\beta\delta_2}{1+\delta_2}\right)^{\frac{1}{1+\frac{1}{\eta}}}$. Immediately, we can see that agents optimally report $\hat{\theta} = \alpha\theta$, because the problem is similar to a time-consistent agent with productivity $\alpha\theta$. As a result, the present-biased agents with productivity θ do not report truthfully and instead report

$$\left(\frac{1+\beta\delta_2}{1+\delta_2}\right)^{\frac{1}{1+\frac{1}{\eta}}}\theta.$$

This result is intuitive, because the reward for working is spread evenly between the two periods with mandatory savings. Since present-biased agents put less weight on retirement consumption, the mandatory savings policy provides less incentives for them to work. Their optimal strategy is to under-report their productivity to work less.

Finally, in $t=0$, agents know that they will report $\left(\frac{1+\beta\delta_2}{1+\delta_2}\right)^{\frac{1}{1+\frac{1}{\eta}}}\theta$ in $t=1$. As a result, given the optimal time-consistent allocation, H-agents solve the following:

$$\max\left\{u\left(\tilde{c}_0\left(H\right)\right)+\beta\frac{\delta_1\left(e_H\right)}{\delta_0\left(e_H\right)}\int_{\underline{\theta}}^{\overline{\theta}}\left[u\left(\tilde{c}_1\left(H,\hat{\theta}\right)\right)-h\left(\frac{\tilde{y}\left(H,\hat{\theta}\right)}{\theta}\right)+\delta_2 u\left(\tilde{c}_2\left(H,\hat{\theta}\right)\right)\right]dF\left(\theta|\kappa_H\right),\right.$$
$$\left.\frac{\delta_0\left(e_L\right)}{\delta_0\left(e_H\right)}u\left(\tilde{c}_0\left(L\right)\right)+\beta\frac{\delta_1\left(e_L\right)}{\delta_0\left(e_H\right)}\int_{\underline{\theta}}^{\overline{\theta}}\left[u\left(\tilde{c}_1\left(L,\hat{\theta}\right)\right)-h\left(\frac{\tilde{y}\left(L,\hat{\theta}\right)}{\theta}\right)+\delta_2 u\left(\tilde{c}_2\left(L,\hat{\theta}\right)\right)\right]dF\left(\theta|\kappa_{L,H}\right)\right\},$$

where $\hat{\theta}=\left(\frac{1+\beta\delta_2}{1+\delta_2}\right)^{\frac{1}{1+\frac{1}{\eta}}}\theta$.

E.2 Laissez Faire Savings

Another way to implement the optimum is for the government to allow agents to save freely for retirement. This is because with time-consistent agents, it is not necessary for the government to introduce any additional incentives for retirement savings. Hence, to implement the optimal allocation for time-consistent agents, the government only needs to introduce appropriate income taxes at $t=1$ and student loans in $t=0$. However, under laissez faire savings, present-biased agents do not smooth consumption and it is also not incentive compatible.

To find out how present-biased agents behave, we first derive the income tax $\tilde{T}\left(y\right)$ that implements the optimum for time-consistent agents. At $t=1$, time-consistent agents solves the following:

$$\max_{c_1,c_2,y} u\left(c_1\right)-h\left(\frac{y}{\theta}\right)+\delta_2 u\left(c_2\right)$$

subject to

$$c_1+s_2=y-\tilde{T}\left(y\right)\text{ and }c_2=R_2 s_2.$$

Let $\tilde{Y}$ be the set of optimal income for time-consistent agents:

$$\tilde{Y} = \{y | y = \tilde{y}(\gamma, \theta), \forall \gamma \in \{L, H\}, \theta \in \Theta\}.$$

By Lemma 2, we can rewrite the allocations in terms of income: $\tilde{c}_t(\tilde{y}(\gamma, \theta)) = \tilde{c}(\gamma, \theta)$. As a result, we can define the following income tax, which implements the optimum for time-consistent agents:

$$\tilde{T}(y) = \begin{cases} y & \text{if } y \notin \tilde{Y} \\ y - \tilde{c}_1(y) - \frac{1}{R_2}\tilde{c}_2(y) & \text{if } y \in \tilde{Y}. \end{cases}.$$

For simplicity, we assume that if the government observes an off-path income level that it did not expect, it usurps all of the output and leaves the agent without any consumption.

Next, we outline how present-biased agents behave under laissez faire savings. Given laissez faire savings and the income tax above, present-biased agents solve the following at $t = 1$,

$$\max_{c_1, c_2, y} u(c_1) - h\left(\frac{y}{\theta}\right) + \beta\delta_2 u(c_2)$$

subject to

$$c_1 + s_2 = y - \tilde{T}(y) \text{ and } c_2 = R_2 s_2.$$

We can rewrite the problem as

$$\max_{c_1, c_2, y} u(c_1) - h\left(\frac{y}{\theta}\right) + \beta\delta_2 u(c_2)$$

subject to

$$c_1 + \frac{1}{R_2}c_2 = \left(1 + \frac{1}{R_2}\right)\tilde{c}(y) \text{ and } y \in \tilde{Y}.$$

It is clear that agents never choose $y \notin \tilde{Y}$, because all of the output would be confiscated. As a result, for any given $y \in \tilde{Y}$, present-biased agents choose consumption $(\hat{c}_1(y), \hat{c}_2(y))$ to satisfy

$$u'(\hat{c}_1(y)) = \beta u'(\hat{c}_2(y))$$

and

$$\hat{c}_1(y) + \frac{1}{R_2}\hat{c}_2(y) = \tilde{c}(y) + \frac{1}{R_2}\tilde{c}(y).$$

It is obvious that there will be intertemporal inefficiencies. Specifically, given $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, we have

$$\hat{c}_1(y) = \beta^{-\frac{1}{\sigma}} \left(\frac{1+\delta_2}{\beta^{-\frac{1}{\sigma}} + \delta_2} \right) \tilde{c}(y) \text{ and } \hat{c}_2(y) = \left(\frac{1+\delta_2}{\beta^{-\frac{1}{\sigma}} + \delta_2} \right) \tilde{c}(y).$$

In addition to the intertemporal inefficiencies, the agents might also choose suboptimal output. The choice in output y is equivalent to a choice in the report $\hat{\theta}$ in a direct mechanism. Given the savings decision derived above, the agent would solve the following problem

$$\max_{\hat{\theta}} \frac{\left[\beta^{-\frac{1}{\sigma}} \left(\frac{1+\delta_2}{\beta^{-\frac{1}{\sigma}}+\delta_2}\right) \tilde{c}\left(\hat{\gamma}, \hat{\theta}\right)\right]^{1-\sigma}}{1-\sigma} - \frac{1}{1+\frac{1}{\eta}} \left(\frac{\tilde{y}\left(\hat{\gamma}, \hat{\theta}\right)}{\theta} \right)^{1+\frac{1}{\eta}} + \beta\delta_2 \frac{\left[\left(\frac{1+\delta_2}{\beta^{-\frac{1}{\sigma}}+\delta_2}\right) \tilde{c}\left(\hat{\gamma}, \hat{\theta}\right)\right]^{1-\sigma}}{1-\sigma},$$

which can be rewritten as

$$\max_{\hat{\theta}} u\left(\tilde{c}\left(\hat{\gamma}, \hat{\theta}\right)\right) - \frac{1}{1+\frac{1}{\eta}} \left(\frac{\tilde{y}\left(\hat{\gamma}, \hat{\theta}\right)}{\left[\beta (1+\delta_2)^{1-\sigma} \left(\beta^{-\frac{1}{\sigma}} + \delta_2\right)^{\sigma}\right]^{\frac{1}{1+\frac{1}{\eta}}} \theta} \right)^{1+\frac{1}{\eta}}.$$

We can compare this problem to the time-consistent agent's problem:

$$\max_{\hat{\theta}} u\left(\tilde{c}\left(\hat{\gamma}, \hat{\theta}\right)\right) - \frac{1}{1+\frac{1}{\eta}} \left(\frac{\tilde{y}\left(\hat{\gamma}, \hat{\theta}\right)}{(1+\delta_2)^{\frac{1}{1+\frac{1}{\eta}}} \theta} \right)^{1+\frac{1}{\eta}}.$$

Since the allocations are incentive compatible for time-consistent agents, the time-consistent agents choose $\hat{\theta} = \theta$. This implies that we can rewrite the problem for the present-biased agent as follows:

$$\max_{\hat{\theta}} u\left(\tilde{c}\left(\hat{\gamma}, \hat{\theta}\right)\right) - \frac{1}{1+\frac{1}{\eta}} \left(\frac{\tilde{y}\left(\hat{\gamma}, \hat{\theta}\right)}{(1+\delta_2)^{\frac{1}{1+\frac{1}{\eta}}} T\theta} \right)^{1+\frac{1}{\eta}},$$

where

$$T = \left[\beta \left(\frac{\beta^{-\frac{1}{\sigma}} + \delta_2}{1+\delta_2}\right)^{\sigma}\right]^{\frac{1}{1+\frac{1}{\eta}}}.$$

This implies that present-biased agents with productivity θ would misreport as

$$T\theta = \left[\beta \left(\frac{\beta^{-\frac{1}{\sigma}} + \delta_2}{1+\delta_2}\right)^{\sigma}\right]^{\frac{1}{1+\frac{1}{\eta}}} \theta.$$

After solving for the optimal allocations for $t = 1, 2$, we can solve for the agent's education choices in $t = 0$. The process is the same as the one for mandatory savings.

E.3 Welfare Comparisons

To evaluate the welfare improvement of the paper's proposed policies, we measure the change of moving from mandatory savings or laissez faire savings to the policies introduced in Section 5.

However, this welfare evaluation is not straightforward. We need to guarantee the allocations chosen by present-biased agents under mandatory savings or laissez faire savings are feasible. This is because, from the analysis above, output of present-biased agents is further distorted under policies designed for TC agents. Therefore, the government budget constraint does not hold with present-biased agents under mandatory savings or laissez faire savings.

To facilitate the welfare comparison, we introduce an external government expenditure $G > 0$ in the time-consistent setup, so that the resource constraint becomes

$$\sum_{\gamma} \pi_{\gamma} \left\{ -\frac{\tilde{c}_0(\gamma)}{R_0(e_{\gamma})} - e_{\gamma} + \frac{1}{R_1(e_{\gamma})} \int_{\Theta} \left[\tilde{y}(\gamma,\theta) - \tilde{c}_1(\gamma,\theta) - \frac{1}{R_2} \tilde{c}_2(\gamma,\theta) \right] f(\theta|\kappa_{\gamma})\, d\theta \right\} \geq G.$$

We interpret G as an emergency fund the government uses to supplement the agents' consumption when total output is lower than expected. Hence, we require G to be sufficiently large so that the allocations chosen by the present-biased agents, $\left\{ \hat{c}_0(\gamma), [\hat{c}_t(\gamma,\theta), \hat{y}(\gamma,\theta)]_{t>0,\theta\in\Theta} \right\}$, are feasible:

$$\sum_{\gamma} \pi_{\gamma} \left\{ -\frac{\hat{c}_0(\gamma)}{R_0(e_{\gamma})} - e_{\gamma} + \frac{1}{R_1(e_{\gamma})} \int_{\Theta} \left[\hat{y}(\gamma,\theta) - \hat{c}_1(\gamma,\theta) - \frac{1}{R_2} \hat{c}_2(\gamma,\theta) \right] f(\theta|\kappa_{\gamma})\, d\theta \right\} \geq 0. \quad (16)$$

E.4 Quantitative Implementation

In our quantitative exercise, we design a fixed-point algorithm to find the value of G such that the resource constraint in (16) binds. The algorithm can be summarized as follows:

1. Start with an initial value for government spending G_0.
2. Solve for the optimal allocations with time-consistent agents.
3. Use the allocations, implemented either through mandatory savings or laissez-faire arrangement, to solve for the best response of present-biased agents. Calculate the resulting gap in the resource constraint which stems from present-biased agents underreporting their productivity type. Denote the gap G_1.
4. Check if $|G_0 - G_1| < \varepsilon$, where ε is a tolerance criterion. If yes, we have found a fixed point. If not, update G_0 and go back to step 1.

Table 9 summarizes the fixed-point amount of government spending G which balances the resource constraint under present-biased agents, under both implementations and for all parameter combinations considered in Table 3.

Table 9: Fixed-point amount of government spending that balances the resource constraint

	Mandatory savings		
	$\sigma = 1.5$	$\sigma = 2$	$\sigma = 2.5$
$\beta = 0.5$	27.05	21.14	17.52
$\beta = 0.7$	13.16	**10.15**	8.31
$\beta = 0.9$	3.60	2.74	2.23

	Laissez-faire		
	$\sigma = 1.5$	$\sigma = 2$	$\sigma = 2.5$
$\beta = 0.5$	27.29	21.46	17.86
$\beta = 0.7$	13.23	**10.23**	8.40
$\beta = 0.9$	3.61	2.75	2.23

To gain a better understanding of where the difference in welfare gains between the two implementations comes from, Table 10 calculates the welfare gains relative to the laissez-faire allocations, where we assume that agents are able to smooth consumption over the life cycle. In other words, the government spending amount G is the same as in the right panel of Table 9 (because agents report their productivity as in the laissez-faire implementation), but consumption is smoothed over time as in the mandatory savings world. Comparing Table 10 to Table 3, we learn that while the laissez-faire allocations lead to a minor efficiency loss, most of the additional welfare loss in this scenario is due to the agents' inability to smooth consumption over time.

Table 10: Welfare gains relative to time-consistent laissez-faire allocations with perfect consumption smoothing

	Laissez-faire with smoothing		
	$\sigma = 1.5$	$\sigma = 2$	$\sigma = 2.5$
$\beta = 0.5$	2.51	3.41	3.26
$\beta = 0.7$	1.04	**1.43**	1.40
$\beta = 0.9$	0.27	0.37	0.38

F Solving for Optimal Education-Independent Policies

In this section, we describe the computational algorithm used to solve for the benchmark case of optimal allocations conditional on the intertemporal wedge being education-independent. The challenge lies in the fact that while the allocations that we solve for are a function of productivity θ, the education-independence constraint on the wedge is imposed for each observable income $y(\gamma, \theta)$ which itself is an allocation.

To overcome this challenge we adopt the following approach:

1. Consider a generic set of allocations $\{c_1(\gamma,\theta), c_2(\gamma,\theta), y(\gamma,\theta)\}_{\gamma\in\{H,L\}}$. For each type $\gamma \in \{H, L\}$, and for each productivity θ, find $\hat{\theta}_\gamma$ such that:

$$y(\gamma,\theta) = y(\hat{\gamma},\hat{\theta}_\gamma)$$

   Here, $\hat{\gamma}$ is the innate ability type other than γ. In essence, for each type and productivity level we find an off-grid productivity level that yields the same output for the other innate ability type. We use linear interpolation to evaluate income at off-grid productivity values.

2. Solve for the optimal allocations under an additional set of $2N$ constraints, one for each pair of innate ability and productivity, such that for each γ and θ

$$\frac{u'\Big(c_1(\gamma,\theta)\Big)}{u'\Big(c_2(\gamma,\theta)\Big)} = \frac{u'\Big(c_1(\hat{\gamma},\hat{\theta}_\gamma)\Big)}{u'\Big(c_2(\hat{\gamma},\hat{\theta}_\gamma)\Big)}$$

   Once again, $\hat{\gamma}$ is the innate ability type other than γ and we use linear interpolation to evaluate consumption in both periods at off-grid values of productivity. In essence, for each productivity level we require that the efficiency wedge be equalized with the efficiency wedge of the other innate ability type, at the productivity level which yields the same output.

G Education-Contingent Retirement Savings Subsidy

In this section, we consider an alternative implementation with income and education contingent retirement savings subsidies. Agents are offered a student loan $L(e)$ in $t = 0$. They are required to make income contingent repayments of $[1 - \tau^e(e,y)]\, L(e)$ in $t = 1$, where the subsidy $\tau^e(e,y)$ is a function of education expenses and income. In $t = 1$, agents also face an income tax $T(y)$ independent of education. Importantly, agents can save s_2 in a retirement account at $t = 1$, where the savings are subsidized at a rate $\tau^s(e,y)$ which is a function of income and education investment. Furthermore, retirement savings s_2 come from after-tax funds, so the income and education dependent retirement savings account is similar to a Roth 401(k). Finally, in each period, agents can save via the risk-free bond b, which are taxed with a history-independent bond savings tax $T^k(b)$.

Given the proposed policies, at $t = 1$, agents with education level e and productivity θ

solve the following:

$$\max_{c_1,y,c_2,s_2,b_2} u(c_1) - h\left(\frac{y}{\theta}\right) + \beta\delta_2 u(c_2)$$

subject to

$$c_1 + s_2 + b_2 + \tilde{R}_1(e)(1-\tau^e(e,y))L(e) = y - T(y) + \tilde{R}_1(e)b_1 - T^k(b_2),$$

$$c_2 = R_2(1+\tau^s(e,y))s_2 + R_2 b_2,$$

where $\tilde{R}_1(e) = \frac{R_1(e)}{R_0(e)}$ is the gross interest rate normalized by the difference between the period lengths of $t=0$ and $t=1$. Let $\{c_1^*(e,\theta), y^*(e,\theta), c_2^*(e,\theta)\}$ denote the solution to the agents' problem at $t=1$ for any $\theta \in \Theta$ and $e \in \{e_L, e_H\}$. Also, let $U_1(e,\theta)$ denote the value function for the agents' problem at $t=1$. The agents' problem with innate ability γ at $t=0$ is

$$\max_{c_0,e,b_1} \delta_0(e)u(c_0) + \beta\delta_1(e)\int_{\underline{\theta}}^{\overline{\theta}} [U_1(e,\theta) + (1-\beta)\delta_2 u(c_2^*(e,\theta))] f(\theta|\kappa(e,\gamma))\,d\theta$$

subject to

$$c_0 + e + b_1 = L(e) - T^k(b_1) \text{ and } e \in \{e_L, e_H\}.$$

Let $P^{ss} = \left\{[L(e), \tau^e(e,y)], \tau^s(e,y), \left[T(y), T^k(b)\right]\right\}$. The following proposition states that the optimum can be decentralized with an income-contingent student loans policy $(L(e), \tau^e(e,y))$ combined with an income and education dependent retirement subsidy $\tau^s(e,y)$ and tax policy $\left(T(y), T^k(b)\right)$.

Proposition 5 *The optimum can be implemented with* P^{ss}.

Proof Similar to Section 5.1, we focus on an implementation where agents do not double deviate (misreport and purchase bonds) due to the bond savings tax $T^k(b)$, which is constructed in the proof of Proposition 3.

Next, we construct the other policy instruments. By Lemma 2, we can define the optimal consumption derived from the direct mechanism as $(c_0(e), c_1(e,y), c_2(e,y))$. First, we construct the student loans and its income-contingent repayment schedule along with the income tax. Let the loan amount be defined as

$$L(e) = \begin{cases} c_0(e) + e & \text{if } e \in \{e_L, e_H\} \\ 0 & \text{otherwise} \end{cases},$$

and the income-contingent repayment subsidy is $\tau^e(e_L, y) = 1$ and

$$\tau^e(e_H, y) = 1 + \frac{1}{\tilde{R}_1(e_H) L(e_H)} \left[c_1(e_H, y) - c_1(e_L, y) + \frac{c_2(e_H, y)}{R_2(1 + \tau^s(e_H, y))} - \frac{c_2(e_L, y)}{R_2(1 + \tau^s(e_L, y))} \right].$$

Let $y(\gamma, \theta)$ be the optimal output of type (γ, θ) agents in a direct revelation mechanism and define $Y = \{y | y = y(\gamma, \theta) \text{ with } \gamma \in \{L, H\} \text{ and } \theta \in \Theta\}$ to be the admissible set of income. The income tax is

$$T(y) = \begin{cases} y - c_1(e_L, y) - \frac{c_2(e_L, y)}{R_2(1+\tau^s(e_L, y))} & \text{if } y \in Y \\ y & \text{if } y \notin Y \end{cases}.$$

Next, we define the income and education contingent retirement savings subsidy as

$$1 + \tau^s(e, y) = \begin{cases} \frac{u'(c_1(e,y))}{\beta u'(c_2(e,y))} & \text{if } e \in \{e_L, e_H\} \\ 0 & \text{otherwise} \end{cases}.$$

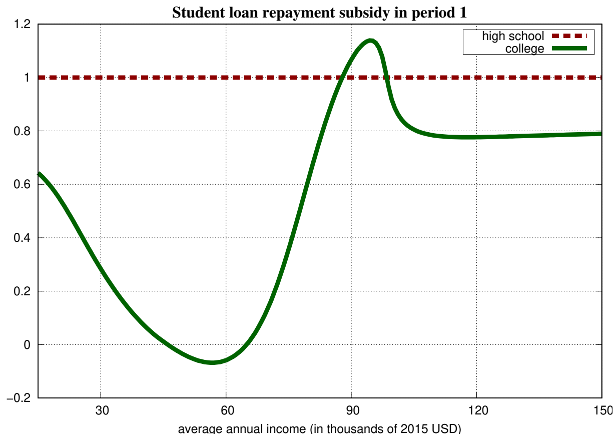
Finally, we check that the policy instruments implement the optimum. First, notice that all agents choose $e \in \{e_L, e_H\}$, otherwise they will not have any retirement consumption. Similarly, due to the income tax, all agents produce output $y \in Y$. Next, for any $e \in \{e_L, e_H\}$ and $y \in Y$, agents at $t = 1$ choose consumption to satisfy

$$\frac{u'(c_1)}{\beta u'(c_2)} = 1 + \tau^s(e, y) \text{ and } c_1 + \frac{c_2}{R_2(1 + \tau^s(e, y))} = c_1(e, y) + \frac{c_2(e, y)}{R_2(1 + \tau^s(e, y))}.$$

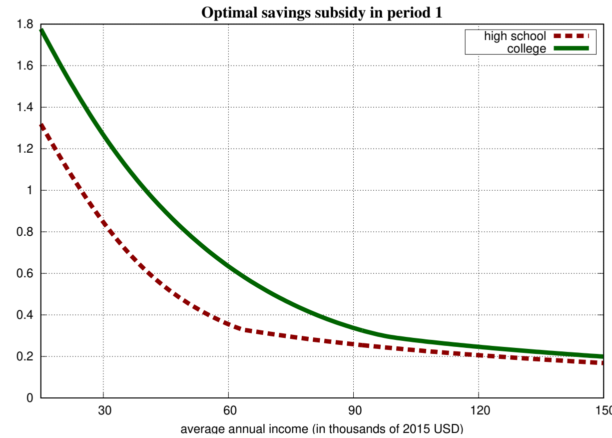
Clearly, agents optimally choose $c_1 = c_1(e, y)$ and $c_2 = c_2(e, y)$. Also, by the taxation principle, agents with productivity θ choose $y = y(e, \theta)$. For the final step, notice that given $L(e)$, agents with innate ability γ optimally choose education level e_γ. ■

Figure 15 presents the optimal student loan repayment and retirement savings subsidies for the two education groups as function of income. Panel 15(a) shows that for the H-agents with income below 60,000 in present value, the repayment subsidy starts at over 60% and decreases with income. Once the Pareto tail kicks in, the trend reverts and the optimal subsidy increases and then drops again before settling at around 80%. Panel 15(b) shows the savings subsidy schedules. The optimal savings subsidies are chosen such that they are the negative of the corresponding decision wedges $\hat{\tau}_1^k$, which is why its shape is similar to the efficiency wedges τ_1^k depicted in Figure 2, where lower income levels receive more subsidies and the subsidy for college graduates is higher for virtually all income levels.

It is worth pointing out that the optimal student loans subsidy is determined by the labor



(a) Optimal student loan subsidy $\tau^e(e, y)$



(b) Optimal savings subsidy $\tau^s(e, y)$

Figure 15: Optimal education-contingent subsidies

wedges. We set the income tax to match the labor wedge for L-agents, while the income contingent student loans subsidies coupled with the marginal income tax rate replicate the optimal labor wedge for H-agents. Since the optimal labor wedge for H-agents is larger with the difference growing until income $60,000$, the student loans subsidy is decreasing up to that amount. Beyond $60,000$, the difference in the labor wedges decreases initially and with the optimal labor wedge for L-agents eventually rising above the labor wedge of H-agents, which causes the student loans subsidy to increase. Also, since we appended the Pareto-tail to the top 10% of the productivity distribution for each education group, the U-shaped dip in the labor wedge for H-agents is much higher than the one for L-agents. As a result, the labor wedge for L-agents is much larger than the labor wedge for H-agents with incomes between $70,000$ and $90,000$. This drives the significant increase in student loans subsidy beyond $60,000$.

H Off-Path Mechanisms

In this section, we show how mechanisms with off-path options can relax the ex-ante incentive constraint. First, we discuss how threats can be constructed off the equilibrium path when agents are sophisticated and the government can perfectly identify some of the agents who misreported in the previous period. Then, we illustrate the off-path mechanism for non-sophisticated agents. Finally, we characterize the optimum when the ex-ante incentive constraint is fully relaxed.

H.1 Off-Path Mechanism for Sophisticated Agents

Our paper focuses on a setting where all productivity distributions span the whole range of Θ regardless of agents' human capital κ. Here, we will show how off-path threats can fully relax the ex-ante incentive constraint when there exists a positive measure of productivities that only H-agents can have. Though off-path threats can also be introduced in a setting where all the productivity distributions span the whole range of Θ, it is not apparent why only the dishonest agents choose the off-path threats while the honest agents choose the on-path allocations. This is because, for the off-path threats to work, both H-agents with productivity θ who misreported as low innate ability and actual L-agents with productivity θ would have to be indifferent between the off-path threat and the on-path allocation (Amador et al., 2003; Halac and Yared, 2014). Finally, for off-path threats to be effective, agents have to be aware of their present bias, which is the case with sophisticated agents.

We will assume that only H-agents can have productivities greater than θ_H where $\underline{\theta} < \theta_H < \overline{\theta}$, so $f(\theta|\kappa_H), f(\theta|\kappa_{L,H}) > 0$ and $f(\theta|\kappa_L) = 0$ for any $\theta \in \left(\theta_H, \overline{\theta}\right]$. In other words, only agents with high innate ability can achieve the productivity levels above θ_H. To illustrate how the off-path threats weaken the ex-ante incentive constraint, we also assume that u is unbounded below and above ($u(\mathbb{R}_+) = \mathbb{R}$). When u is unbounded below and above, the ex-ante incentive constraint can be fully relaxed by the off-path threats. In general, off-path threats can weaken the ex-ante incentive constraint—though perhaps not fully—as long as there is a set of productivities with positive measure that only H-agents can achieve.

Following Yu (2021), the government can introduce a conditional commitment mechanism (CCM). CCM features off-path allocations used to exploit the sophisticated present-biased agents' demand for commitment, which are referred to as threat allocations. For L-agents, the government designs the menu

$$\tilde{P}_L = \left\{c_0(L), [c_1(L,\theta), y(L,\theta), c_2(L,\theta)]_{\theta\in\left(\theta_H,\overline{\theta}\right]}, [c_1(L,\theta), y(L,\theta), c_2(L,\theta)]_{\theta\in[\underline{\theta},\theta_H]}\right\}.$$

Since none of the L-agents would end up with a productivity greater than θ_H, the set of allocations $[c_1(L,\theta), y(L,\theta), c_2(L,\theta)]_{\theta\in\left(\theta_H,\overline{\theta}\right]}$ only punishes H-agents who misreported in $t = 0$, so it is the set of threat allocations. The threat allocations are located in the menu for L-agents to deter H-agents from misreporting. Without loss of generality, it is sufficient to assume the government designs a single threat allocation: $c_t(L,\theta) = c_t^T$ and $y(L,\theta) = y^T$ for any $\theta \in \left(\theta_H, \overline{\theta}\right]$. In contrast, we assumed that L-agents do not have an incentive to misreport upwards, so the menu for H-agents stays the same: $\tilde{P}_H = \left\{c_0(H), [c_1(H,\theta), y(H,\theta), c_2(H,\theta)]_{\theta\in\Theta}\right\}$.

The consumption path of the threat allocations are frontloaded to exacerbate the agents' present bias: $c_1^T > c_2^T$. Since present-biased agents at $t = 0$ hope to prevent their future selves from saving too little for retirement, the frontloaded threat consumption path can help deter H-agents from misreporting downwards. Furthermore, to prevent the actual L-agents from selecting the threat allocations, the threat output y^T is increased. Formally, the threat allocations are disciplined by the threat constraints: for any $\theta \in \left(\theta_H, \overline{\theta}\right]$ and $\theta' \in [\underline{\theta}, \theta_H]$,

$$u\left(c_1^T\right) - h\left(\frac{y^T}{\theta}\right) + \beta\delta_2 u\left(c_2^T\right) \geq u\left(c_1\left(L, \theta'\right)\right) - h\left(\frac{y\left(L, \theta'\right)}{\theta}\right) + \beta\delta_2 u\left(c_2\left(L, \theta'\right)\right),$$

and the executability constraints: for any $\theta \in [\underline{\theta}, \theta_H]$,

$$U_1\left(L, \theta\right) \geq u\left(c_1^T\right) - h\left(\frac{y^T}{\theta}\right) + \beta\delta_2 u\left(c_2^T\right).$$

The threat constraints guarantee that H-agents who misreported downwards in $t = 0$ would end up consuming the threat allocation if their productivity is greater than θ_H. The executability constraints ensure that none of the L-agents would consume the threat allocation. Therefore, the ex-ante incentive constraint is

$$\begin{aligned} U_0\left(H\right) \geq \delta_0\left(e_L\right) u\left(c_0\left(L\right)\right) + \beta\delta_1\left(e_L\right) \Bigg[&\int_{\underline{\theta}}^{\theta_H} \left[U_1\left(L, \theta\right) + \left(1 - \beta\right)\delta_2 u\left(c_2\left(L, \theta\right)\right)\right] f\left(\theta | \kappa_{L,H}\right) d\theta \\ &+ \int_{\theta_H}^{\overline{\theta}} \left[u\left(c_1^T\right) - h\left(\frac{y^T}{\theta}\right) + \delta_2 u\left(c_2^T\right)\right] f\left(\theta | \kappa_{L,H}\right) d\theta \Bigg]. \end{aligned}$$

Finally, the on-path allocations need to be incentive compatible.

Lemma 3 *When $f\left(\theta|\kappa_H\right), f\left(\theta|\kappa_{L,H}\right) > 0$ and $f\left(\theta|\kappa_L\right) = 0$ for any $\theta \in \left(\theta_H, \overline{\theta}\right]$ and u is unbounded above and below, the ex-ante incentive compatibility constraint is non-binding at the optimum.*

Proof First, for any on-path allocations, let

$$\hat{\theta} = \arg\max_{\theta' \in [\underline{\theta}, \theta_H]} \left\{u\left(c_1\left(L, \theta'\right)\right) - h\left(\frac{y\left(L, \theta'\right)}{\overline{\theta}}\right) + \beta\delta_2 u\left(c_2\left(L, \theta'\right)\right)\right\}.$$

In other words, an agent with the highest productivity $\overline{\theta}$ would misreport as $\hat{\theta} \in [\underline{\theta}, \theta_H]$ if it were restricted to choosing the on-path allocations. This implies that $U_1\left(\hat{\theta}; L, \overline{\theta}\right)$ is the highest attainable utility for the right-hand side of the threat constraint for any agent who reported $\gamma = L$ in $t = 0$. As a result, the threat constraint can be rewritten as

$$u\left(c_1^T\right) + \beta\delta_2 u\left(c_2^T\right) \geq U_1\left(\hat{\theta}; L, \overline{\theta}\right) + \sup_{\theta \in \left(\theta_H, \overline{\theta}\right]} h\left(\frac{y^T}{\theta}\right).$$

This construction would imply that all of the agents with $\theta \in \left(\theta_H, \overline{\theta}\right]$ prefer the threat allocation. We can set c_1^T such that

$$u\left(c_1^T\right) = U_1\left(\hat{\theta}; L, \overline{\theta}\right) + \sup_{\theta \in \left(\theta_H, \overline{\theta}\right]} h\left(\frac{y^T}{\theta}\right) - \beta\delta_2 u\left(c_2^T\right). \tag{17}$$

Next, let

$$\begin{aligned} &U_0\left(L; H, \theta \leq \theta_H\right) \\ &\quad = \delta_0\left(e_L\right) u\left(c_0\left(L\right)\right) + \beta\delta_1\left(e_L\right) \int_{\underline{\theta}}^{\theta_H} \left[U_1\left(L, \theta\right) + \left(1 - \beta\right)\delta_2 u\left(c_2\left(L, \theta\right)\right)\right] f\left(\theta | \kappa_{L,H}\right) d\theta, \end{aligned}$$

then the ex-ante incentive constraint can be rewritten as

$$u\left(c_1^T\right) + \delta_2 u\left(c_2^T\right) \leq \frac{\frac{1}{\beta\delta_1(e_L)}\left[U_0\left(H\right) - U_0\left(L; H, \theta \leq \theta_H\right)\right] + \int_{\theta_H}^{\overline{\theta}} h\left(\frac{y^T}{\theta}\right) f\left(\theta | \kappa_{L,H}\right) d\theta}{1 - F\left(\theta | \kappa_{L,H}\right)}.$$

By (17), the ex-ante incentive constraint is

$$\begin{aligned} u\left(c_2^T\right) \leq &\frac{\frac{1}{\beta\delta_1(e_L)}\left[U_0\left(H\right) - U_0\left(L; H, \theta \leq \theta_H\right)\right] + \int_{\theta_H}^{\overline{\theta}} h\left(\frac{y^T}{\theta}\right) f\left(\theta | \kappa_{L,H}\right) d\theta}{\left(1 - \beta\right)\delta_2\left[1 - F\left(\theta | \kappa_{L,H}\right)\right]} \\ &- \frac{U_1\left(\hat{\theta}; L, \overline{\theta}\right) + \sup_\theta H\left(\frac{y^T}{\theta}\right)}{\left(1 - \beta\right)\delta_2}. \end{aligned}$$

Since u is unbounded below and above, we can decrease c_2^T such that the ex-ante incentive constraint always holds for any on-path allocation and increase c_1^T such that (17) is satisfied.

Finally, to satisfy the executability constraints, notice that for any on-path and threat allocations, we can increase y^T such that the executability constraints hold. ■

Lemma 3 shows how the off-path threat can help relax the ex-ante incentive compatibility constraint. With a positive probability, H-agents who misreported downwards in $t = 0$ are caught lying. Those who are caught are punished with less retirement savings. As a result, H-agents voluntarily report truthfully and surrender their information rent in exchange for commitment. A special case of Lemma 3 is when the innate ability and productivity are the same or when private information is not dynamic. In this case, the government learns the agents' productivity when they report their innate ability truthfully in $t = 0$. As a result, the full information efficient optimum is implementable when the off-path threat fully relaxes the ex-ante incentive constraint on innate ability (Yu, 2021).

H.2 Off-Path Mechanism for Non-Sophisticated Agents

The paper has focused on sophisticated present-biased agents. Sophisticated agents fully anticipate the behavior of their future selves, so they have a demand for commitment. On the other hand, non-sophisticated agents underestimate the severity of their bias and tend to demand too little commitment. We explore the implications of non-sophistication on the design of optimal policy in this section.

To model non-sophistication, we follow O'Donoghue and Rabin (2001). Agents at $t = 0$ perceive their present bias in $t = 1$ to be $\hat{\beta} \in [\beta, 1]$. Let $W_1\left(c_1, c_2, y; \theta, \hat{\beta}\right)$ denote the non-sophisticated agents' perceived utility in $t = 1$:

$$W_1\left(c_1, c_2, y; \theta, \hat{\beta}\right) = u\left(c_1\right) - h\left(\frac{y}{\theta}\right) + \hat{\beta}\delta_2 u\left(c_2\right).$$

If $\hat{\beta} = \beta$, agents are *sophisticated* and fully aware of the bias. If $\hat{\beta} = 1$, agent are *fully naïve* and believe their future selves to be time-consistent. *Partially naïve* agents know they are present-biased, $\hat{\beta} < 1$, but they underestimate its severity, $\hat{\beta} > \beta$. For this extension, we assume all agents are non-sophisticated and have heterogeneous and unobservable sophistication distributed within support $\left[\underline{\hat{\beta}}, 1\right]$, where $\underline{\hat{\beta}} \in (\beta, 1]$.

Yu (2021) showed that it is optimal for the government to take advantage of the misspecified beliefs of present-biased agents through the preference arbitrage mechanism (PAM). PAM features off-path allocation used to exploit the incorrect beliefs, which are referred to as the *imaginary allocations* and denoted as (c^I, y^I). The allocation that is implemented on-path is called the *real allocations* denoted as (c, y). To illustrate how PAM works, we assume that u is unbounded below and above $(u(\mathbb{R}_+) = \mathbb{R})$. We will show how the ex-ante incentive constraint can be fully relaxed in this setting. In general, PAM weakens the ex-ante incentive constraint, though perhaps not fully, whenever agents are non-sophisticated.

For H-agents, the government designs the menu

$$\hat{P}_H = \left\{c_0\left(H\right), \left[c_1^I, y^I, c_2^I\right], \left[c_1\left(H, \theta\right), y\left(H, \theta\right), c_2\left(H, \theta\right)\right]_{\theta \in \Theta}\right\}.$$

At $t = 1$, H-agents choose between imaginary and real allocations. The consumption path of the imaginary allocation is backloaded ($c_2^I > c_1^I$), while the consumption path of the real allocation is relatively less back-loaded. It is designed this way so that at $t = 0$, the agents mistakenly believe their future selves will choose the imaginary allocation. However, they end-up selecting the real allocation instead. Since we assumed the ex-ante incentive constraints are non-binding for L-agents, the government does not need to design imaginary allocations for them, so $\hat{P}_L = \left\{c_0(L), [c_1(L,\theta), y(L,\theta), c_2(L,\theta)]_{\theta\in\Theta}\right\}$. Similar to Yu (2021), it is not necessary to design imaginary allocations tailored for each level of sophistication. It is possible to find a single set of imaginary allocations such that it implements the same real allocations for agents of any sophistication.

Lemma 4 *For non-sophisticated present-biased agents, when u is unbounded above and below, the ex-ante incentive compatibility constraint is non-binding at the optimum.*

Proof From Yu (2021), we first choose the imaginary allocation for a fixed $\hat{\beta}$ such that it satisfies the preference arbitrage constraint: for any θ,

$$u\left(c_1^I\right) - h\left(\frac{y^I}{\theta}\right) + \hat{\beta}\delta_2 u\left(c_2^I\right) \geq \max_{\hat{\theta}}\left\{u\left(c_1\left(H,\hat{\theta}\right)\right) - h\left(\frac{y\left(H,\hat{\theta}\right)}{\theta}\right) + \hat{\beta}\delta_2 u\left(c_2\left(H,\hat{\theta}\right)\right)\right\}.$$

In essence, in $t = 0$, agents believe their future selves would choose the imaginary allocation over the real allocation. Notice that the real allocations may not be incentive compatible under the erroneous belief. Next, the imaginary allocation has to satisfy the executability constraints to make sure that agents actually choose the real allocation at $t = 1$: for any θ,

$$U_1(H,\theta) = u\left(c_1(H,\theta)\right) - h\left(\frac{y(H,\theta)}{\theta}\right) + \beta\delta_2 u\left(c_2(H,\theta)\right) \geq u\left(c_1^I\right) - h\left(\frac{y^I}{\theta}\right) + \beta\delta_2 u\left(c_2^I\right).$$

Thus, the ex-ante incentive compatibility constraint is

$$\delta_0(e_H)\, u\left(c_0(H)\right) + \beta\delta_1(e_H)\int_{\underline{\theta}}^{\overline{\theta}}\left[u\left(c_1^I\right) - h\left(\frac{y^I}{\theta}\right) + \delta_2 u\left(c_2^I\right).\right] f\left(\theta|\kappa_H\right) d\theta \geq U_0(L;H).$$

Next, we show how the imaginary allocation can be designed such that the ex-ante incentive constraint is non-binding for all sophistication levels. Without loss of generality, set $y^I = 0$. Choose the imaginary allocation such that $u\left(c_1^I\right) + \beta\delta_2 u\left(c_2^I\right) = \min_{\tilde{\theta}} U_1\left(H,\tilde{\theta}\right)$, so the executability constraints are non-binding except for H-agents with the lowest utility

U_1. Hence, the preference arbitrage constraints can be expressed as

$$u\left(c_2^I\right) \geq J\left(\hat{\beta}\right) \equiv \frac{\max_{\hat{\theta}}\left\{u\left(c_1\left(H,\hat{\theta}\right)\right) - h\left(\frac{y\left(H,\hat{\theta}\right)}{\theta}\right) + \hat{\beta}\delta_2 u\left(c_2\left(H,\hat{\theta}\right)\right)\right\} - \min_{\tilde{\theta}} U_1\left(H,\tilde{\theta}\right)}{\left(\hat{\beta}-\beta\right)\delta_2}.$$

Since the preference arbitrage constraints need to hold for all productivity realizations and sophistication, it is clear that c_2^I is chosen to satisfy

$$u\left(c_2^I\right) \geq \max_{\hat{\beta}} J\left(\hat{\beta}\right).$$

Similarly, the ex-ante incentive constraint can be rewritten as

$$u\left(c_2^I\right) \geq K \equiv \frac{1}{(1-\beta)\,\delta_2}\left\{\frac{U_0\left(L;H\right) - \delta_0\left(e_H\right)u\left(c_0\left(H\right)\right)}{\beta\delta_1\left(e_H\right)} - \min_{\tilde{\theta}} U_1\left(H,\tilde{\theta}\right)\right\}.$$

Since u is unbounded above, for any real allocation, the imaginary retirement consumption c_2^I can be chosen to satisfy

$$u\left(c_2^I\right) \geq \max\left\{\max_{\hat{\beta}} J\left(\hat{\beta}\right), K\right\}.$$

Also, since u is unbounded below, it is possible to adjust c_1^I so that $u\left(c_1^I\right) = \min_{\tilde{\theta}} U_1\left(H,\tilde{\theta}\right) - \beta\delta_2 u\left(c_2^I\right)$. As a result, it is always possible to find a single set of imaginary allocation for all levels of sophistication such that the ex-ante incentive constraints are non-binding for any allocation implemented on the equilibrium path. ■

To understand Lemma 4, note that non-sophisticated agents at $t=0$ overestimate the value of retirement consumption to their future selves. PAM takes advantage of incorrect beliefs by encouraging education investment through an increased imaginary retirement consumption c_2^I, which H-agents believe they will choose in $t=1$. However, their future selves forsake it for more immediate gratification—the relatively less back-loaded real allocations.

H.3 Optimum when Ex-Ante Incentive Constraint is Non-Binding

Lemmas 3 and 4 showed how the government can relax the ex-ante incentive constraint with off-path mechanisms when agents are non-sophisticated or when there are productivities that only H-agents can have. The following proposition describes the optimal wedges when the ex-ante incentive constraint is non-binding.

Proposition 6 *When the ex-ante incentive constraint does not bind, the constrained efficient allocation satisfies*

i. full insurance in $t = 0 : c_0(H) = c_0(L)$,

ii. the inverse Euler equations: for any γ, $\frac{1}{u'(c_0(\gamma))} = \mathbb{E}_\theta\left(\frac{1}{u'(c_1(\gamma,\theta))}\right) = \mathbb{E}_\theta\left(\frac{1}{u'(c_2(\gamma,\theta))}\right)$, *and for any* θ, $\frac{1}{\beta u'(c_2(\gamma,\theta))} = \frac{1}{u'(c_1(\gamma,\theta))} + \left(\frac{1-\beta}{\beta}\right)\frac{1}{u'(c_0(\gamma))}$,

iii. the labor wedge for any γ *and* θ *satisfies* $\frac{\tau^w(\gamma,\theta)}{1-\tau^w(\gamma,\theta)} = A_\gamma(\theta) B_\gamma(\theta) C_\gamma(\theta)$.

Proof When the ex-ante incentive constraint is non-binding, the optimization problem is the same as the original problem except that $\mu = 0$. As a result, the first order conditions are

$$u'(c_0(H)) = u'(c_0(L)) = \phi,$$

and for all γ,

$$\pi_\gamma \delta_1(e_\gamma) f(\theta|\kappa_\gamma) - \xi'_\gamma(\theta) = \lambda_\gamma(\theta),$$

$$(1-\beta)\pi_\gamma \delta_1(e_\gamma) f(\theta|\kappa_\gamma) + \beta\lambda_\gamma(\theta) = \frac{\phi\pi_\gamma\delta_1(e_\gamma) f(\theta|\kappa_\gamma)}{u'(c_2(\gamma,\theta))},$$

$$\lambda_\gamma(\theta) u'(c_1(\gamma,\theta)) = \phi\pi_\gamma\delta_1(e_\gamma) f(\theta|\kappa_\gamma),$$

$$\xi_\gamma(\underline{\theta}) = \xi_\gamma(\overline{\theta}) = 0,$$

$$\lambda_\gamma(\theta)\frac{1}{\theta}h'\left(\frac{y(\gamma,\theta)}{\theta}\right) + \xi_\gamma(\theta)\left[\frac{1}{\theta^2}h'\left(\frac{y(\gamma,\theta)}{\theta}\right) + \frac{y(\gamma,\theta)}{\theta^3}h''\left(\frac{y(\gamma,\theta)}{\theta}\right)\right] = \phi\pi_\gamma\delta_1(e_\gamma) f(\theta|\kappa_\gamma).$$

By rearranging the first order conditions, the results follow. ■

Proposition 6 demonstrates the government's ability to fully insure agents against differences in innate ability γ. This is not surprising, since the ex-ante incentive constraint is non-binding. As a result, the only distortions in the economy stem from the unobserved productivity θ realized in $t = 1$.

Since innate ability is screened for free but productivity is not, Proposition 6 shows that the efficiency wedge $\tau_0^k(\gamma)$ is characterized by the standard inverse Euler equation for all innate ability types. This is because the government no longer needs the additional intertemporal distortions illustrated in Proposition 1 on $\tau_0^k(\gamma)$ to incentivize investment in human capital when the ex-ante incentive constraint is slack. However, productivity remains unobservable by the government, so savings in $t = 0$ is still restricted and shaped by the inverse Euler equation to relax the ex-post incentive constraints.

More interestingly, Proposition 6 shows that all agents are provided with a commitment device: for any γ and θ, $\frac{u'(c_1(\gamma,\theta))}{u'(c_2(\gamma,\theta))} > \beta$. When the ex-ante incentive constraint is non-binding, the government can focus on its paternalistic goals since it no longer needs to manipulate retirement consumption to screen innate ability.

Finally, Proposition 6 shows that the optimal labor distortion is determined solely by the intratemporal component. This means the economic forces that shape the labor wedge are essentially static. Recall from Proposition 2 that both the intertemporal and present-bias components are integral to the optimal provision of dynamic incentives through labor distortion. Since the ex-ante incentive constraint is non-binding, the forces that determine the provision of dynamic incentives are absent from the labor wedge. As a result, the intertemporal and present-bias components no longer influence labor distortion.

I Model with Multiple Working Periods

In this section, we divide the working period in half and allow for stochastic changes in productivity. In this four-period model, the agent is a student at $t = 0$ and then works for two periods at $t = 1$ and $t = 2$. The agent retires at $t = 3$.

Similar to the three-period model, agents learn their innate ability $\gamma \in \{H, L\}$ and choose their education investment $e \in \{e_L, e_H\}$ at $t = 0$. Human capital κ depends on both γ and e, as before. At $t = 1$, agents privately learn their productivity $\theta_1 \in \left[\underline{\theta}, \overline{\theta}\right]$. Productivity θ_1 is drawn from c.d.f. $F_1(\theta_1|\kappa)$ with p.d.f. $f_1(\theta_1|\kappa)$. Also, as before, F_1 is ranked according to first order stochastic dominance and f_1 is strictly positive for any θ_1 and κ. The innovation here is that agents privately draw a new productivity $\theta_2 \in \left[\underline{\theta}, \overline{\theta}\right]$ at $t = 2$, from c.d.f. $F_2(\theta_2|\theta_1, \kappa)$ with p.d.f. $f_2(\theta_2|\theta_1, \kappa)$. In essence, the distribution of θ_2 depends on past productivity θ_1 and human capital κ. We will assume that f_2 is strictly positive for any θ_2 and past history (θ_1, κ). At $t = 3$, agents retire and consume their savings. We will continue to assume that agents only differ in the number of years spent as a student at $t = 0$, and the length of other periods are the same for all agents.

Our findings show that, even with a finer pattern of timing, retirement savings subsidies are education-dependent to provide present-biased agents a commitment device that encourages education investment. However, the role of education investments in determining the optimal retirement policy is weaker in a model with multiple working periods. This suggests that the quantitative estimates in our paper are likely loose upper bounds for the impact of education on retirement policy.

I.1 The Mechanism and Incentive Compatibility

The government designs the following direct mechanism:

$$P = \{c_0(\gamma), [c_1(\gamma,\theta_1), y_1(\gamma,\theta_1)], [c_2(\gamma,\theta_1,\theta_2), c_3(\gamma,\theta_1,\theta_2), y_2(\gamma,\theta_1,\theta_2)]\}.$$

Following the analysis for the three-period model, we require the mechanism P to be incentive compatible for every period. Let the utility of a type $(\gamma, \theta_1, \theta_2)$ agent who reports $\theta_2' \in \Theta$ in $t = 2$ be denoted as

$$U_2(\theta_2'; \gamma, \theta_1, \theta_2) = u(c_2(\gamma, \theta_1, \theta_2')) - h\left(\frac{y_2(\gamma, \theta_1, \theta_2')}{\theta_2}\right) + \beta\delta_3 u(c_3(\gamma, \theta_1, \theta_2')).$$

The incentive compatibility constraints in $t = 2$ ensure the agents report θ_2 truthfully: for any $\theta_2, \theta_2' \in \Theta$,

$$U_2(\gamma, \theta_1, \theta_2) \equiv U_2(\theta_2; \gamma, \theta_1, \theta_2) \geq U_2(\theta_2'; \gamma, \theta_1, \theta_2).$$

Let the utility of a type (γ, θ_1) agent who reports $\theta_1' \in \Theta$ in $t = 1$ be denote as

$$\begin{aligned} U_1(\theta_1'; \gamma, \theta_1) = u(c_1(\gamma, \theta_1')) - h\left(\frac{y_1(\gamma, \theta_1')}{\theta_1}\right) \\ + \beta\delta_2 \int_\Theta [U_2(\gamma, \theta_1', \theta_2) + (1-\beta)\delta_3 u(c_3(\gamma, \theta_1', \theta_2))]\, dF_2(\theta_2|\theta_1, \kappa_\gamma). \end{aligned}$$

The incentive compatibility constraints in $t = 1$ ensure the agents report θ_1 truthfully: for any $\theta_1, \theta_1' \in \Theta$,

$$U_1(\gamma, \theta_1) \equiv U_1(\theta_1; \gamma, \theta_1) \geq U_1(\theta_1'; \gamma, \theta_1).$$

Finally, let the utility in $t = 0$ of γ-agents who reported innate ability γ' be denoted as

$$\begin{aligned} U_0(\gamma'; \gamma) = \delta_0(e_{\gamma'})\, u(c_0(\gamma')) + \beta\delta_1(e_{\gamma'}) \int_\Theta \Bigg[u(c_1(\gamma', \theta_1)) - h\left(\frac{y_1(\gamma', \theta_1)}{\theta_1}\right) \\ + \delta_2 \int_\Theta [U_2(\gamma', \theta_1, \theta_2) + (1-\beta)\,\delta_3 u(c_3(\gamma', \theta, \theta_2))]\, dF_2(\theta_2|\theta_1, \kappa_{\gamma',\gamma})\Bigg] dF_1(\theta_1|\kappa_{\gamma',\gamma}). \end{aligned}$$

The incentive compatibility constraints at $t = 0$ ensure that the agents report γ truthfully: for any innate ability γ, γ',

$$U_0(\gamma) \equiv U_0(\gamma; \gamma) \geq U_0(\gamma'; \gamma).$$

The following lemma characterizes the set of allocations that are incentive compatible at $t = 2$. Its proof is similar to the proof of Lemma 1 so it is omitted.

Lemma 5 *For any γ and θ_1, P is incentive compatible at $t = 2$ if and only if (i.) $y_2(\gamma, \theta_1, \theta_2)$ is non-decreasing in θ_2, and (ii.) $U_2(\gamma, \theta_1, \theta_2)$ is absolutely continuous in θ_2, so it is differentiable almost everywhere with*

$$\frac{\partial U_2(\gamma, \theta_1, \theta_2)}{\partial \theta_2} = \frac{y_2(\gamma, \theta_1, \theta_2)}{\theta_2^2} h'\left(\frac{y_2(\gamma, \theta_1, \theta_2)}{\theta_2}\right). \tag{18}$$

Unfortunately, it is difficult to simplify the incentive compatibility constraints at $t = 1$. With a continuum of productivities in $t = 1$, local incentive compatibility may not imply global incentive compatibility when agents are time inconsistent (Halac and Yared, 2014; Galperti, 2015; Yu, 2020). This issue and others were discussed in Section 2.2. We will not focus on the theoretical properties that guarantee the sufficiency of local incentive compatibility.[7] Instead, we will follow the standard procedure and replace the incentive constraints in $t = 1$ with the following envelope condition:

$$\begin{aligned}\frac{\partial U_1(\gamma, \theta_1)}{\partial \theta_1} = {} & \frac{y_1(\gamma, \theta_1)}{\theta_1^2} h'\left(\frac{y_1(\gamma, \theta_1)}{\theta_1}\right) \\ & - \beta\delta_2 \int_\Theta \frac{\partial F_2(\theta_2|\theta_1, \kappa_\gamma)}{\partial \theta_1} \frac{y_2(\gamma, \theta_1, \theta_2)}{\theta_2^2} h'\left(\frac{y_2(\gamma, \theta_1, \theta_2)}{\theta_2}\right) d\theta_2 \\ & + \beta(1-\beta)\delta_2\delta_3 \int_\Theta u(c_3(\gamma, \theta_1, \theta_2)) \frac{\partial f_2(\theta_2|\theta_1, \kappa_\gamma)}{\partial \theta_1} d\theta_2, \end{aligned} \tag{19}$$

which is derived with the help of Lemma 5 by assuming P is incentive compatible at $t = 2$. It is important to note that (19) is a necessary condition for incentive compatibility, not sufficient.

Finally, similar to the three-period model, it is difficult to assess which of the incentive constraints on innate ability are relevant at $t = 0$. Following the analysis of the three-period model, we will focus on the case where only the H-agents go to college, so the relevant incentive constraint at $t = 0$ is

$$U_0(H) \geq U_0(L; H). \tag{20}$$

[7] See Appendix B in Yu (2020) for more information on the conditions that guarantee the sufficiency of local incentive compatibility constraints in an environment with quasi-linear utility.

I.2 The Planning Problem

The government maximizes the sum of long-run utilities:

$$\sum_{\gamma}\pi_{\gamma}\Bigg\{\delta_0\left(e_{\gamma}\right)u\left(c_0\left(\gamma\right)\right)+\delta_1\left(e_{\gamma}\right)\int_{\Theta}\Bigg[U_1\left(\gamma,\theta\right)$$
$$+\left(1-\beta\right)\delta_2\int_{\Theta}\left[U_2\left(\gamma,\theta_1,\theta_2\right)+\left(1-\beta\right)\delta_3 u\left(c_3\left(\gamma,\theta,\theta_2\right)\right)\right]dF_2\left(\theta_2|\theta_1,\kappa_{\gamma}\right)\Bigg]dF_1\left(\theta_1|\kappa_{\gamma}\right)\Bigg\}$$

subject to

$$U_2\left(\gamma,\theta_1,\theta_2\right)=u\left(c_2\left(\gamma,\theta_1,\theta_2\right)\right)-h\left(\frac{y_2\left(\gamma,\theta_1,\theta_2\right)}{\theta_2}\right)+\beta\delta_3 u\left(c_3\left(\gamma,\theta_1,\theta_2\right)\right), \tag{21}$$

$$U_1\left(\gamma,\theta_1\right)=u\left(c_1\left(\gamma,\theta_1\right)\right)-h\left(\frac{y_1\left(\gamma,\theta_1\right)}{\theta_1}\right)$$
$$+\beta\delta_2\int_{\Theta}\left[U_2\left(\gamma,\theta_1,\theta_2\right)+(1-\beta)\delta_3 u\left(c_3\left(\gamma,\theta_1,\theta_2\right)\right)\right]dF_2\left(\theta_2|\theta_1,\kappa_{\gamma}\right), \tag{22}$$

the incentive constraints (18), (19), and (20), and the resource constraint

$$\sum_{\gamma}\pi_{\gamma}\Bigg\{\frac{-c_0\left(\gamma\right)-e_{\gamma}}{R_0\left(e_{\gamma}\right)}+\frac{1}{R_1\left(e_{\gamma}\right)}\int_{\Theta}\Bigg[y_1\left(\gamma,\theta_1\right)-c_1\left(\gamma,\theta_1\right)$$
$$+\frac{1}{R_2}\int_{\Theta}\left[y_2\left(\gamma,\theta_1,\theta_2\right)-c_2\left(\gamma,\theta_1,\theta_2\right)-\frac{c_3\left(\gamma,\theta_1,\theta_2\right)}{R_3}\right]dF_2\left(\theta_2|\theta_1,\kappa_{\gamma}\right)\Bigg]dF_1\left(\theta_1|\kappa_{\gamma}\right)\Bigg\}\geq 0.$$

We will assume that $\delta_t R_t = 1$ for all t.

I.2.1 The Optimality Conditions

Let $\left(\lambda_{\gamma}\left(\theta_1,\theta_2\right),\lambda_{\gamma}\left(\theta_1\right),\xi_{\gamma}\left(\theta_1,\theta_2\right),\xi_{\gamma}\left(\theta_1\right),\mu,\phi\right)$ be the multipliers on (21), (22), (18), (19), (20), and the resource constraint respectively. Using standard Hamiltonian techniques, we derive the following necessary conditions for optimality

$$\left(1+\frac{\mu}{\pi_H}\right)u'\left(c_0\left(H\right)\right)=\left(1-\frac{\mu}{\pi_L}\right)u'\left(c_0\left(L\right)\right)=\phi,$$

$$\lambda_H\left(\theta_1\right)+\beta\mu\delta_1\left(e_H\right)f_1\left(\theta_1|\kappa_H\right)=\frac{\phi\pi_H\delta_1\left(e_H\right)f_1\left(\theta_1|\kappa_H\right)}{u'\left(c_1\left(H,\theta_1\right)\right)},$$

$$\lambda_L(\theta_1) - \beta\mu\delta_1(e_L) f_1(\theta_1|\kappa_{L,H}) = \frac{\phi\pi_L\delta_1(e_L) f_1(\theta_1|\kappa_L)}{u'(c_1(L,\theta_1))},$$

$$\begin{aligned}(1-\beta)\left(\pi_H + \frac{\beta\mu}{1-\beta}\right)\delta_1(e_H)\,\delta_2 f_1(\theta_1|\kappa_H)\, f_2(\theta_2|\theta_1,\kappa_H)&\\ + \beta\lambda_H(\theta_1)\,\delta_2 f_2(\theta_2|\theta_1,\kappa_H) - \frac{\partial\xi_H(\theta_1,\theta_2)}{\partial\theta_2} &= \lambda_H(\theta_1,\theta_2),\end{aligned}$$

$$\begin{aligned}(1-\beta)\left[\pi_L - \frac{\beta\mu}{1-\beta}\left(\frac{f_1(\theta_1|\kappa_{L,H})\, f_2(\theta_2|\theta_1,\kappa_{L,H})}{f_1(\theta_1|\kappa_L)\, f_2(\theta_2|\theta_1,\kappa_L)}\right)\right]\delta_1(e_L)\,\delta_2 f_1(\theta_1|\kappa_L)\, f_2(\theta_2|\theta_1,\kappa_L)&\\ + \beta\lambda_L(\theta_1)\,\delta_2 f_2(\theta_2|\theta_1,\kappa_L) - \frac{\partial\xi_L(\theta_1,\theta_2)}{\partial\theta_2} &= \lambda_L(\theta_1,\theta_2),\end{aligned}$$

$$\begin{aligned}&(1-\beta)^2\left(\pi_H + \frac{\beta\mu}{1-\beta}\right)\delta_1(e_H)\,\delta_2 f_1(\theta_1|\kappa_H)\, f_2(\theta_2|\theta_1,\kappa_H)\\ &+ \beta(1-\beta)\lambda_H(\theta_1)\,\delta_2 f_2(\theta_2|\theta_1,\kappa_H) + \beta\lambda_H(\theta_1,\theta_2) - \beta(1-\beta)\,\xi_H(\theta_1)\,\delta_2\frac{\partial f_2(\theta_2|\theta_1,\kappa_H)}{\partial\theta_1}\\ &= \frac{\phi\pi_H\delta_1(e_H)\,\delta_2 f_1(\theta_1|\kappa_H)\, f_2(\theta_2|\theta_1,\kappa_H)}{u'(c_3(H,\theta_1,\theta_2))},\end{aligned}$$

$$\begin{aligned}&(1-\beta)^2\left[\pi_L - \frac{\beta\mu}{1-\beta}\left(\frac{f_1(\theta_1|\kappa_{L,H})\, f_2(\theta_2|\theta_1,\kappa_{L,H})}{f_1(\theta_1|\kappa_L)\, f_2(\theta_2|\theta_1,\kappa_L)}\right)\right]\delta_1(e_L)\,\delta_2 f_1(\theta_1|\kappa_L)\, f_2(\theta_2|\theta_1,\kappa_L)\\ &+ \beta(1-\beta)\lambda_L(\theta_1)\,\delta_2 f_2(\theta_2|\theta_1,\kappa_L) + \beta\lambda_L(\theta_1,\theta_2) - \beta(1-\beta)\,\xi_L(\theta_1)\,\delta_2\frac{\partial f_2(\theta_2|\theta_1,\kappa_L)}{\partial\theta_1}\\ &= \frac{\phi\pi_L\delta_1(e_L)\,\delta_2 f_1(\theta_1|\kappa_L)\, f_2(\theta_2|\theta_1,\kappa_L)}{u'(c_3(L,\theta_1,\theta_2))},\end{aligned}$$

$$\begin{aligned}&\left[\lambda_H(\theta_1) + \beta\mu\delta_1(e_H) f_1(\theta_1|\kappa_H)\right]\frac{1}{\theta_1}h'\left(\frac{y_1(H,\theta_1)}{\theta_1}\right)\\ &+ \xi_H(\theta_1)\left[\frac{1}{\theta_1^2}h'\left(\frac{y_1(H,\theta_1)}{\theta_1}\right) + \frac{y_1(H,\theta_1)}{\theta_1^3}h''\left(\frac{y_1(H,\theta_1)}{\theta_1}\right)\right] = \phi\pi_H\delta_1(e_H) f_1(\theta_1|\kappa_H),\end{aligned}$$

$$\begin{aligned}&\left[\lambda_L(\theta_1)-\beta\mu\delta_1(e_L)f_1(\theta_1|\kappa_{L,H})\right]\frac{1}{\theta_1}h'\left(\frac{y_1(L,\theta_1)}{\theta_1}\right)\\&\quad+\xi_L(\theta_1)\left[\frac{1}{\theta_1^2}h'\left(\frac{y_1(L,\theta_1)}{\theta_1}\right)+\frac{y_1(L,\theta_1)}{\theta_1^3}h''\left(\frac{y_1(L,\theta_1)}{\theta_1}\right)\right]=\phi\pi_L\delta_1(e_L)f_1(\theta_1|\kappa_L),\end{aligned}$$

and for all γ,

$$\pi_\gamma\delta_1(e_\gamma)f_1(\theta_1|\kappa_\gamma)-\xi_\gamma'(\theta_1)=\lambda_\gamma(\theta_1),$$

$$\lambda_\gamma(\theta_1,\theta_2)u'(c_2(\gamma,\theta_1,\theta_2))=\phi\pi_\gamma\delta_1(e_\gamma)\delta_2f_1(\theta_1|\kappa_\gamma)f_2(\theta_2|\theta_1,\kappa_\gamma),$$

$$\begin{aligned}&\lambda_\gamma(\theta_1,\theta_2)\frac{1}{\theta_2}h'\left(\frac{y_2(\gamma,\theta_1,\theta_2)}{\theta_2}\right)+\left[\xi_\gamma(\theta_1,\theta_2)-\beta\delta_2\xi_\gamma(\theta_1)\frac{\partial F_2(\theta_2|\theta_1,\kappa_\gamma)}{\partial\theta_1}\right]\\&\quad\times\left[\frac{1}{\theta_2^2}h'\left(\frac{y_2(\gamma,\theta_1,\theta_2)}{\theta_2}\right)+\frac{y_2(\gamma,\theta_1,\theta_2)}{\theta_2^3}h''\left(\frac{y_2(\gamma,\theta_1,\theta_2)}{\theta_2}\right)\right]\\&\quad=\phi\pi_\gamma\delta_1(e_\gamma)\delta_2f_1(\theta_1|\kappa_\gamma)f_2(\theta_2|\theta_1,\kappa_\gamma),\end{aligned}$$

and the following boundary conditions hold: for all γ,

$$\xi_\gamma(\underline{\theta})=\xi_\gamma(\overline{\theta})=0,$$

and for any γ and $\theta_1\in\Theta$,

$$\xi_\gamma(\theta_1,\underline{\theta})=\xi_\gamma(\theta_1,\overline{\theta})=0.$$

Next, using the conditions above, we present the optimal labor and intertemporal distortions in the four-period life-cycle model. We will discuss the labor wedges before the intertemporal wedges, because, surprisingly, the intertemporal wedges depend on the distortions in labor when agents are present biased.

I.3 Labor Wedges

To separate the economic forces that determine the optimal labor distortions, we first define the elements that influence the wedges in $t = 1$:

$$A_\gamma(\theta_1) = \frac{1 - F_1\left(\theta_1|\kappa_\gamma\right)}{\theta f_1\left(\theta_1|\kappa_\gamma\right)},$$

$$B_\gamma(\theta_1) = 1 + \frac{\frac{y_1(\gamma,\theta_1)}{\theta_1} h''\left(\frac{y(\gamma,\theta_1)}{\theta_1}\right)}{h'\left(\frac{y_1(\gamma,\theta_1)}{\theta_1}\right)},$$

$$C_\gamma(\theta_1) = \int_\theta^{\overline{\theta}} \frac{u'\left(c_1\left(\gamma,\theta_1\right)\right)}{u'\left(c_1\left(\gamma,x\right)\right)}\left[1 - \frac{u'\left(c_1\left(\gamma,x\right)\right)}{\phi}\right]\frac{f_1\left(x|\kappa_\gamma\right)}{1 - F_1\left(\theta_1|\kappa_\gamma\right)}dx,$$

$$D_\gamma(\theta_1) = u'\left(c_1\left(\gamma,\theta_1\right)\right)\left[\frac{1}{u'\left(c_0\left(\gamma\right)\right)} - \frac{1}{\phi}\right],$$

$$E_\gamma(\theta_1) = (1-\beta)\, D_\gamma\left(\theta_1\right).$$

Also, for the wedges in $t = 2$, we define $A_\gamma\left(\theta_1,\theta_2\right)$, $B_\gamma\left(\theta_1,\theta_2\right)$, and $C_\gamma\left(\theta_1,\theta_2\right)$ analogously, and let

$$D_\gamma(\theta_1,\theta_2) = u'\left(c_2\left(\gamma,\theta_1,\theta_2\right)\right)\left[\frac{1}{u'\left(c_1\left(\gamma,\theta_1\right)\right)} - \frac{1}{\phi}\right],$$

$$E_\gamma(\theta_1,\theta_2) = (1-\beta)\, D_\gamma\left(\theta_1,\theta_2\right),$$

$$\tilde{E}_\gamma(\theta_1,\theta_2) = \beta\left(1-\beta\right)\frac{u'\left(c_2\left(\gamma,\theta_1,\theta_2\right)\right)}{u'\left(c_1\left(\gamma,\theta_1\right)\right)} D_\gamma\left(\theta_1\right).$$

Most of the components are the same as the three-period model and represent the same forces, except for $\tilde{E}_\gamma\left(\theta_1,\theta_2\right)$. Similar to the present-bias component $E_\gamma\left(\theta_1,\theta_2\right)$, notice that $\tilde{E}_\gamma\left(\theta_1,\theta_2\right)$ is also zero when agents are time consistent, so it is unique to our environment with present-biased agents.

The following proposition characterizes the optimal labor distortion in the four-period model.

Proposition 7 *The labor wedge at $t = 1$ for any $\theta_1 \in \Theta$ satisfies*

$$\frac{\tau^w\left(H,\theta_1\right)}{1-\tau^w\left(H,\theta_1\right)} = A_H\left(\theta_1\right) B_H\left(\theta_1\right)\left[C_H\left(\theta_1\right) - D_H\left(\theta_1\right) + E_H\left(\theta_1\right)\right],$$

$$\frac{\tau^w(L,\theta_1)}{1-\tau^w(L,\theta_1)} = A_L(\theta_1) B_L(\theta_1) \left[C_L(\theta_1) - \frac{1-F_1(\theta_1|\kappa_{L,H})}{1-F_1(\theta_1|\kappa_L)} \left[D_L(\theta_1) - E_L(\theta_1) \right] \right],$$

and the labor wedge at $t=2$ for any $\theta_1, \theta_2 \in \Theta$ satisfies

$$\begin{aligned} \frac{\tau^w(H,\theta_1,\theta_2)}{1-\tau^w(H,\theta_1,\theta_2)} &= A_H(\theta_1,\theta_2) B_H(\theta_1,\theta_2) \\ &\times \Bigg\{ C_H(\theta_1,\theta_2) - D_H(\theta_1,\theta_2) + E_H(\theta_1,\theta_2) - \tilde{E}_H(\theta_1,\theta_2) \\ &- \frac{\beta u'(c_2(H,\theta_1,\theta_2))}{u'(c_1(H,\theta_1))} \frac{\frac{\partial F_2(\theta_2|\theta_1,\kappa_H)}{\partial \theta_1}}{f_2(\theta_2|\theta_1,\kappa_H)} \frac{f_2(\theta_2|\theta_1,\kappa_H)}{1-F_2(\theta_2|\theta_1,\kappa_H)} \frac{1-F_1(\theta_1|\kappa_H)}{f_1(\theta_1|\kappa_H)} \frac{\frac{\tau^w(H,\theta_1)}{1-\tau^w(H,\theta_1)}}{A_H(\theta_1) B_H(\theta_1)} \Bigg\}, \end{aligned}$$

$$\begin{aligned} \frac{\tau^w(L,\theta_1,\theta_2)}{1-\tau^w(L,\theta_1,\theta_2)} &= A_L(\theta_1,\theta_2) B_L(\theta_1,\theta_2) \\ &\times \Bigg\{ C_L(\theta_1,\theta_2) - D_L(\theta_1,\theta_2) + E_L(\theta_1,\theta_2) - \frac{f_1(\theta_1|\kappa_{L,H})}{f_1(\theta_1|\kappa_L)} \left[\frac{\frac{1-F_1(\theta_1|\kappa_{L,H})}{1-F_1(\theta_1|\kappa_L)} - \beta}{1-\beta} \right] \tilde{E}_L(\theta_1,\theta_2) \\ &- \frac{\beta u'(c_2(L,\theta_1,\theta_2))}{u'(c_1(L,\theta_1))} \frac{\frac{\partial F_2(\theta_2|\theta_1,\kappa_L)}{\partial \theta_1}}{f_2(\theta_2|\theta_1,\kappa_L)} \frac{f_2(\theta_2|\theta_1,\kappa_L)}{1-F_2(\theta_2|\theta_1,\kappa_L)} \frac{1-F_1(\theta_1|\kappa_L)}{f_1(\theta_1|\kappa_L)} \frac{\frac{\tau^w(L,\theta_1)}{1-\tau^w(L,\theta_1)}}{A_L(\theta_1) B_L(\theta_1)} \Bigg\}, \end{aligned}$$

where $\frac{1}{\phi} = \mathbb{E}_\gamma \left[\mathbb{E}_{\theta_1} \left(\frac{1}{u'(c_1(\gamma,\theta_1))} \middle| \gamma \right) \right]$.

Proof The results follow from the first order conditions by using the techniques shown in the proof of Proposition 2. ■

Proposition 7 shows that the labor wedges at $t=1$ are the same as the ones in the three-period model. However, the labor wedges at $t=2$—the period just before retirement—contains new economic forces. First, due to dynamic productivity, the labor wedge at $t=2$ depends on the previous period's labor distortion. This dependence on past labor distortions is closely related to the intertemporal component characterized in Golosov et al. (2016), which serves to relax the incentive constraints in $t=1$. Furthermore, this dependence is weaker than when agents are time consistent, since present-biased agents are less sensitive to future incentives. Finally, the labor wedge at $t=2$ also directly depends on education investment through the new present-bias component $\tilde{E}_\gamma(\theta_1,\theta_2)$, which depends on $D_\gamma(\theta_1)$.

I.4 Intertemporal Wedges

Before we present the optimal intertemporal distortions, let us define the elasticity of the density f_2 with respect to θ_1 as

$$\epsilon_\gamma(\theta_1,\theta_2) = \frac{\partial f_2\left(\theta_2|\theta_1,\kappa_\gamma\right)}{\partial\theta_1}\frac{\theta_1}{f_2\left(\theta_2|\theta_1,\kappa_\gamma\right)}.$$

The following proposition provides the inverse Euler equations for present-biased agents in the four-period model.

Proposition 8 *The constrained efficient allocation satisfies (i.) the inverse Euler equation in aggregate:*

$$\sum_\gamma \frac{\pi_\gamma}{u'\left(c_0\left(\gamma\right)\right)} = \sum_\gamma \pi_\gamma \mathbb{E}_{\theta_1}\left(\frac{1}{u'\left(c_1\left(\gamma,\theta_1\right)\right)}\middle|\gamma\right),$$

$$\mathbb{E}_{\theta_1}\left(\frac{1}{u'\left(c_1\left(\gamma,\theta_1\right)\right)}\middle|\gamma\right) = \mathbb{E}_{\theta_1,\theta_2}\left(\frac{1}{u'\left(c_2\left(\gamma,\theta_1,\theta_2\right)\right)}\middle|\gamma\right) = \mathbb{E}_{\theta_1,\theta_2}\left(\frac{1}{u'\left(c_3\left(\gamma,\theta_1,\theta_2\right)\right)}\middle|\gamma\right),$$

and (ii.) for any $\theta\in\Theta$,

$$\frac{1}{\beta u'\left(c_3\left(H,\theta_1,\theta_2\right)\right)} = \frac{1}{u'\left(c_2\left(H,\theta_1,\theta_2\right)\right)} + \left[1 - \frac{\epsilon_H\left(\theta_1,\theta_2\right)\frac{\tau^w(H,\theta_1)}{1-\tau^w(H,\theta_1)}}{B_H\left(\theta_1\right)}\right]\frac{1-\beta}{u'\left(c_1\left(H,\theta_1\right)\right)} + \frac{(1-\beta)^2}{\beta}\left(\frac{\pi_H+\beta\mu}{\pi_H+\mu}\right)\frac{1}{u'\left(c_0\left(H\right)\right)},$$

$$\frac{1}{\beta u'\left(c_3\left(L,\theta_1,\theta_2\right)\right)} = \frac{1}{u'\left(c_2\left(L,\theta_1,\theta_2\right)\right)} + \left[1 - \frac{\epsilon_L\left(\theta_1,\theta_2\right)\frac{\tau^w(L,\theta_1)}{1-\tau^w(L,\theta_1)}}{B_L\left(\theta_1\right)}\right]\frac{1-\beta}{u'\left(c_1\left(L,\theta_1\right)\right)}$$
$$+\frac{(1-\beta)^2}{\beta}\left\{\frac{\left[\pi_L - \beta\mu\frac{f_1\left(\theta_1|\kappa_{L,H}\right)f_2\left(\theta_2|\theta_1,\kappa_{L,H}\right)}{f_1\left(\theta_1|\kappa_L\right)f_2\left(\theta_2|\theta_1,\kappa_L\right)}\right] - \frac{\beta^2\mu}{1-\beta}\frac{f_1\left(\theta_1|\kappa_{L,H}\right)}{f_1\left(\theta_1|\kappa_L\right)}\left[\frac{f_2\left(\theta_2|\theta_1,\kappa_{L,H}\right)}{f_2\left(\theta_2|\theta_1,\kappa_L\right)} - 1\right]}{\pi_L-\mu}\right\}\frac{1}{u'\left(c_0\left(L\right)\right)},$$

where $\mu = \left[u'\left(c_0\left(L\right)\right) - u'\left(c_0\left(H\right)\right)\right]\left[\frac{u'(c_0(L))}{\pi_L} + \frac{u'(c_0(H))}{\pi_H}\right]^{-1}$.

Proof The results follow from the first order conditions by using the techniques shown in the proof of Proposition 1. ■

Proposition 8 shows that, similar to the three-period model, the best the government can do is to choose consumption such that the inverse marginal utility is equalized in aggregate when agents are present biased. Similarly, the intertemporal distortions at $t = 0$ are also characterized by the inverse Euler inequalities. However, Proposition 8 also shows that there are new economic forces that determine the intertemporal wedges at $t = 1$ and $t = 2$.

From Proposition 8, the optimal intertemporal wedges at $t = 1$ are characterized by

$$\frac{1}{u'\left(c_1\left(H,\theta_1\right)\right)}+\frac{1-\beta}{\beta}\left(\frac{\pi_H+\beta\mu}{\pi_H+\mu}\right)\frac{1}{u'\left(c_0\left(H\right)\right)}=\mathbb{E}_{\theta_2}\left(\frac{1}{\beta u'\left(c_2\left(H,\theta_1,\theta_2\right)\right)}\middle|\theta_1\right),$$

$$\frac{1}{u'\left(c_1\left(L,\theta_1\right)\right)}+\frac{1-\beta}{\beta}\left[\frac{\pi_L-\beta\mu\frac{f_1\left(\theta_1|\kappa_{L,H}\right)}{f_1\left(\theta_1|\kappa_L\right)}}{\pi_L-\mu}\right]\frac{1}{u'\left(c_0\left(L\right)\right)}=\mathbb{E}_{\theta_2}\left(\frac{1}{\beta u'\left(c_2\left(L,\theta_1,\theta_2\right)\right)}\middle|\theta_1\right).$$

Notice that these wedges look similar to the three-period model's inverse Euler equations (6) and (7), with a key difference—future productivity θ_2 is unknown. Due to this uncertainty, it is optimal to restrict savings at $t = 1$ to relax the incentive constraints at $t = 2$. On the other hand, the main mechanism of this paper also exists: A commitment device that helps agents save at $t = 1$. With these two opposing forces, it is unclear whether the optimal efficiency wedge at $t = 1$ is smaller than $1 - \beta$.

Finally, Proposition 8 demonstrates that the retirement savings of present-biased agents at $t = 2$ depends on the labor distortion at $t = 1$ and education investment. The government uses reports on θ_2 to detect possible prior misreports and distorts the allocations according to that likelihood to relax past incentive constraints. Crucially, this dependence is not present for time-consistent agents and when θ_2 is independent of past productivity θ_1 or human capital κ.

To see how this mechanism works, first notice that the degree of dependence on previous period's labor distortion is affected by the elasticity $\epsilon_\gamma\left(\theta_1,\theta_2\right)$, which measures the percentage change of density f_2 in response to a change in θ_1. The consumption path is more frontloaded when the elasticity is positive compared to when it is negative. This is because a positive elasticity implies that the current productivity θ_2 was more likely to have come from a slightly higher past productivity θ_1. Since the relevant deviation is for agents to misreport downwards, a frontloaded consumption path for agents with $\epsilon_\gamma\left(\theta_1,\theta_2\right) > 0$ exacerbates their present bias, which helps deter them from misreporting θ_1. Furthermore, the degree of frontloading or backloading is increasing in the absolute value of $\epsilon_\gamma\left(\theta_1,\theta_2\right)$, the labor wedge at $t = 1$, and consumption $c_1\left(\gamma,\theta_1\right)$.

Most importantly, Proposition 8 shows that the retirement savings of present-biased agents also depend directly on past investments in education through the consumption at $t = 0$ and indirectly through the labor wedge and consumption at $t = 1$. For the four-period model, this direct link between education investment and retirement savings works in a similar fashion as in the three-period model, albeit with two differences. First, the mechanism is the same for H-agents, but for L-agents the savings commitment depends on the whole productivity profile (θ_1, θ_2). Similar to the three-period model, the commitment is stronger for agents' whose productivity profile is more likely to have come from genuine L-agents. Second, this direct effect of education investment on retirement savings is weaker in the four-period model than in the three-period model by a factor of $1 - \beta$. However, together with the indirect effect—through the labor wedge and consumption at $t = 1$, the role of education-dependent policies on retirement savings remains important.

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