Online Appendix to

"Estimating the Optimal Inflation Target from Trends in Relative Prices"

by Klaus Adam and Henning Weber, American Economic Journal: Macroeconomics

A Price Change Frequencies and Product Age

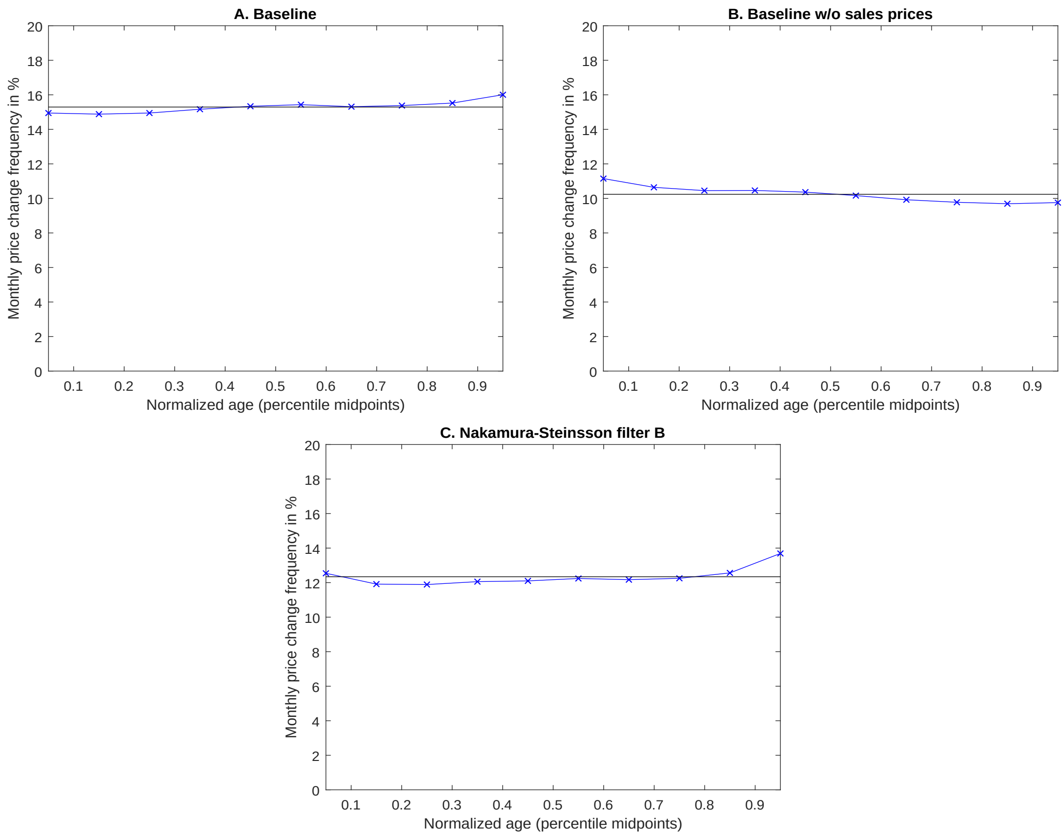
This appendix shows that average price change frequencies do not depend on product age in our sample, in line with the assumed age-independent price adjustment frequency/cost in the sticky price models. To illustrate this fact, we normalize product age of all products from zero to one and record where over the lifetime of the product we observe prices adjustments. Performing this operation for all products in our sample, we average the price adjustment frequencies across products to see whether there are age-dependent patterns.

Figure 10 reports the aggregated monthly price adjustment frequencies over the (normalized) product life for various price change measures. Each graph in the figure depicts the unconditional mean of price changes over the whole product life (solid horizontal line), as well as the prices change frequencies at various points in the normalized product life (averaging over ten equal-sized age bins).

Panel A in figure 10 depicts the price change frequencies for all observed price changes in our baseline sample. Panel B uses only price changes that ONS does not flag as being due to sales. Panel C shows price change frequencies after applying the Nakamura-Steinsson sales filter (NSB) to our baseline sample.

Common to all panels is the fact that price change frequencies do not vary substantially over the (normalized) product life time. Also, no clear pattern emerges of how price change frequencies move over the (normalized) product life: while panel A shows an upward sloping pattern, panel B shows a downward sloping pattern, and panel C shows a U-shaped pattern. In light of these observations, it seems reasonable to assume that the price adjustment frequency/cost is independent of product age. This said, we acknowledge that the aggregate evidence in figure 10 may mask underlying heterogeneity which might be worth exploring further in future research.

Figure 10: Frequency of price changes as function of normalized product age



B Key Model Derivations

B.1 First-Order Conditions of the Household Problem

The representative household maximizes expected discounted utility in equation (2) subject to the budget constraint (3). The first-order conditions to this maximization problem comprise

$$\frac{W_t}{P_t} = -C_t \frac{\partial V(L_t)/\partial L_t}{V(L_t)} \tag{44}$$

$$\Omega_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{V(L_{t+1})}{V(L_t)}\right)^{1-\sigma} \tag{45}$$

$$1 = E_t \left[\Omega_{t,t+1} \left(\frac{1+i_t}{P_{t+1}/P_t}\right)\right] \tag{46}$$

$$1 = E_t \left[\Omega_{t,t+1}(r_{t+1} + 1 - d)\right], \tag{47}$$

a no-Ponzi scheme condition, the transversality condition and the household's budget constraint.

B.2 Derivation of Firms' Marginal Cost Expression (21)

Let

$$I_{jzt} \equiv Y_{jzt}/(A_{zt} Q_{jzt} G_{jzt})$$

denote the units of factor inputs ($K_{jzt}^{1-\frac{1}{\phi}} L_{jzt}^{\frac{1}{\phi}}$) required to produce Y_{jzt} units of (quality-adjusted) output. We now show that cost minimization yields the expression for nominal marginal costs of I_{jzt} provided in equation (21). Firm j chooses the factor input mix to minimize production costs subject to the constraint imposed by the production function (6),

$$\min_{K_{jzt}, L_{jzt}} K_{jzt} r_t + L_{jzt} W_t / P_t \quad s.t. \quad Y_{jzt} = A_{zt} Q_{jzt} G_{jzt} K_{jzt}^{1-\frac{1}{\phi}} L_{jzt}^{\frac{1}{\phi}} .$$

Denoting the Lagrange multiplier by λ_t, this cost minimization problem yields first-order conditions

$$0 = r_t + \left(1 - \frac{1}{\phi}\right) \lambda_t A_{zt} Q_{jzt} G_{jzt} \left(\frac{L_{jzt}}{K_{jzt}}\right)^{\frac{1}{\phi}}$$

$$0 = W_t / P_t + \frac{1}{\phi} \lambda_t A_{zt} Q_{jzt} G_{jzt} \left(\frac{L_{jzt}}{K_{jzt}}\right)^{\frac{1}{\phi}-1} .$$

These conditions imply that the optimal capital labor ratio is the same for all firms $j \in [0,1]$ and all items $z = 1, \ldots Z_t$, i.e.,

$$\frac{K_{jzt}}{L_{jzt}} = \frac{W_t}{P_t r_t} (\phi - 1). \tag{48}$$

Substituting the optimal factor input mix into the production function (6) and solving for the factor inputs yields the factor demand functions

$$L_{jzt} = \left(\frac{W_t}{P_t r_t} (\phi - 1)\right)^{\frac{1}{\phi}-1} I_{jzt} \tag{49}$$

$$K_{jzt} = \left(\frac{W_t}{P_t r_t} (\phi - 1)\right)^{\frac{1}{\phi}} I_{jzt}, \tag{50}$$

where I_{jzt} is defined in the text. Firm j demands these amounts of labor and capital, respectively, to combine them to Y_{jzt} units of (quality-adjusted) output. Thus, the firm's cost function is

$$MC_t I_{jzt} = W_t \left(\frac{W_t}{P_t r_t} (\phi - 1)\right)^{\frac{1}{\phi}-1} I_{jzt} + P_t r_t \left(\frac{W_t}{P_t r_t} (\phi - 1)\right)^{\frac{1}{\phi}} I_{jzt}, \tag{51}$$

where MC_t denotes nominal marginal (or average) costs. The previous equation can be rearranged to obtain equation (21).

B.3 Derivation of the Optimal Price Setting Equation (25)

The first order condition to the firm's price setting problem (22) yields

$$0 = E_t \sum_{i=0}^{\infty} (\alpha_z (1 - \delta_z))^i \frac{\Omega_{t,t+i}}{P_{t+i}} Y_{jzt+i} \left[P_{jzt}^{\star} - \frac{\theta}{(1+\tau)(\theta - 1)} \left(\frac{MC_{t+i}}{A_{zt+i} Q_{zt+i} \mathcal{Q}_{jzt+i}} \right) \right],$$

where we use the short-hand notation $\mathcal{Q}_{jzt} = Q_{jzt} G_{jzt} / Q_{zt}$. Solving this equation for $P_{jzt}^{\star}$ yields

$$\frac{P_{jzt}^{\star}}{P_t} \mathcal{Q}_{jzt} = \left(\frac{1}{1+\tau} \frac{\theta}{\theta - 1} \right) \tag{52}$$

$$\frac{E_t \sum_{i=0}^{\infty} (\alpha_z (1-\delta_z))^i \Omega_{t,t+i} \left(\frac{P_{zt+i}}{P_{zt}}\right)^{\theta - 1} \left(\frac{P_{t+i}}{P_t} \frac{Y_{t+i}}{Y_t}\right) \left(\frac{MC_{t+i}}{P_{t+i} A_{zt+i} Q_{zt+i}}\right) \left(\frac{\mathcal{Q}_{jzt}}{\mathcal{Q}_{jzt+i}}\right)}{E_t \sum_{i=0}^{\infty} (\alpha_z (1-\delta_z))^i \Omega_{t,t+i} \left(\frac{P_{zt+i}}{P_{zt}}\right)^{\theta - 1} \left(\frac{Y_{t+i}}{Y_t}\right)} .$$

We can express the ratio $\mathcal{Q}_{jzt}/\mathcal{Q}_{jzt+i}$ in the previous equation as

$$\frac{\mathcal{Q}_{jzt}}{\mathcal{Q}_{jzt+i}} = \frac{G_{jzt} Q_{zt+i}}{G_{jzt+i} Q_{zt}},$$

because quality remains constant over the lifetime of product j, so that $Q_{jzt} = Q_{jzt+i}$. Using equation (7) to substitute for productivity G_{jzt} and the fact that the idiosyncratic component ϵ^G_{jzt} remains constant of the product lifetime further yields

$$\frac{\mathcal{Q}_{jzt}}{\mathcal{Q}_{jzt+i}} = \frac{\overline{G}_{jzt}}{\overline{G}_{jzt+i}} \frac{Q_{zt+i}}{Q_{zt}}.$$

Given the evolution of $\overline{G}_{jzt}$ implied by equation (9), this equation can be rearranged to obtain

$$\frac{\mathcal{Q}_{jzt}}{\mathcal{Q}_{jzt+i}} = \frac{\prod_{k=1}^{i} q_{zt+k}}{\prod_{k=1}^{i} g_{zt+k}},$$

which is independent of the product index j and reduces to $\mathcal{Q}_{jzt}/\mathcal{Q}_{jzt+i} = 1$ for $i = 0$. Using the previous equation, we can express the numerator on the r.h.s. of equation (52), denoted by N_{zt}, recursively as

$$N_{zt} = \frac{MC_t}{P_t A_{zt} Q_{zt}} + \alpha_z(1-\delta_z) E_t \left[\Omega_{t,t+1} \left(\frac{P_{zt+1}}{P_{zt}} \right)^{\theta-1} \left(\frac{P_{t+1}}{P_t} \right) \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{q_{zt+1}}{g_{zt+1}} \right) N_{zt+1} \right]. \tag{53}$$

We can also express the denominator on the r.h.s. of equation (52), denoted by D_{zt}, recursively as

$$D_{zt} = 1 + \alpha_z(1-\delta_z) E_t \left[\Omega_{t,t+1} \left(\frac{P_{zt+1}}{P_{zt}} \right)^{\theta-1} \left(\frac{Y_{t+1}}{Y_t} \right) D_{zt+1} \right], \tag{54}$$

which then leads to equation (25) for the optimal price.

B.4 Item Price Level and Its Recursive Evolution Equation

We derive a recursive representation of the item price level P_{zt} in two steps. First, we decompose the price level into the prices of newly entering products, the prices of existing products that are optimally reset in period t, and all remaining prices. Second, we show that optimal reset prices for existing products with age $s \geq 1$ can be expressed as a function of the optimal prices of newly entering products. This relationship allows us to derive the recursive price-level representation. The derivation in the present section follows similar steps as in Adam and Weber (2019) but generalizes it by allowing for idiosyncratic components in productivity and product quality.

From equation (15), we have

$$P_{zt}^{1-\theta} = \int_0^1 P_{jzt}^{1-\theta} \mathrm{dj},$$

where $P_{jzt} = \widetilde{P}_{jzt}/Q_{jzt}$ denotes the quality-adjusted price of product j in item z. We decompose this price level into (i) all prices that are adjusted in period t, including prices for newly entering products; (ii) the sticky prices of continuing products. The share of the latter is equal to $\alpha_z(1-\delta_z)$

and their average price is equal to the lagged item price level. Thus, applying this decomposition to the previous equation yields

$$P_{zt}^{1-\theta} = \sum_{s=0}^{\infty} \int_{J^{\star}_{t-s,t}} (P^{\star}_{jzt})^{1-\theta} \mathrm{dj} + \alpha_z (1-\delta_z)(P_{zt-1})^{1-\theta}, \tag{55}$$

where $J^{\star}_{t-s,t}$ denotes the set of products with age s in period t that can adjust prices in t. The share of products that can adjust prices in t is equal to $\delta_z + (1-\delta_z)(1-\alpha_z)$, where δ_z is the share of newly entering products (all with optimal prices) and $(1-\delta_z)(1-\alpha_z)$ is the share of continuing products that can adjust prices. We can define the average optimal price of products newly entering in t as

$$P^{\star}_{z,t,t} \equiv \left(\frac{1}{\delta_z} \int_{J^{\star}_{t,t}} (P^{\star}_{jzt})^{1-\theta} \mathrm{dj} \right)^{\frac{1}{1-\theta}}, \tag{56}$$

and the average optimal price of products that entered in $t-s$ (for $s \geq 1$) and reset prices in t as

$$P^{\star}_{z,t-s,t} \equiv \left(\frac{1}{(1-\alpha_z)\delta_z(1-\delta_z)^s} \int_{J^{\star}_{t-s,t}} (P^{\star}_{jzt})^{1-\theta} \mathrm{dj} \right)^{\frac{1}{1-\theta}}. \tag{57}$$

Substituting the previous two definitions into equation (55) yields

$$P_{zt}^{1-\theta} = \delta_z (P^{\star}_{z,t,t})^{1-\theta} + (1-\alpha_z)\delta_z \sum_{s=1}^{\infty} (1-\delta_z)^s (P^{\star}_{z,t-s,t})^{1-\theta} + \alpha_z(1-\delta_z)(P_{zt-1})^{1-\theta}, \tag{58}$$

where $(1-\alpha_z)\delta_z \sum_{s=1}^{\infty}(1-\delta_z)^s + \alpha_z(1-\delta_z) = 1-\delta_z$ is equal to the share of continuing products.

In the second step, we use the optimal price setting equation (25) to express the item price level in the previous equation recursively. Consider the pricing equation for product j with age $s_{jzt} = s \geq 1$ and rewrite (25) by substituting G_{jzt} using equation (7) and substituting Q_{jzt} using equation (11). This yields

$$\frac{P^{\star}_{jzt}}{P_t} \left(\frac{Q_{zt-s}\overline{G}_{jzt}}{Q_{zt}} \right) [\epsilon^{Q}_{jz,t-s} \epsilon^{G}_{jz,t-s}] = \left(\frac{\theta}{\theta-1} \frac{1}{1+\tau} \right) \frac{N_{zt}}{D_{zt}}, \tag{59}$$

where the term in brackets captures the idiosyncratic component of the optimal price, which is constant over the product's lifetime. Since the previous equation refers to products with the same age, we can use equation (9) to rewrite $\overline{G}_{jzt}$ and equation (12) to rewrite Q_{zt-s}/Q_{zt}. This yields

$$\frac{P^{\star}_{jzt}}{P_t} \left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}} \right) [\epsilon^{Q}_{jz,t-s} \epsilon^{G}_{jz,t-s}] = \left(\frac{\theta}{\theta-1} \frac{1}{1+\tau} \right) \frac{N_{zt}}{D_{zt}}.$$

Rearranging the previous equation to obtain the average of the optimal prices of products with the same age s, as defined in equation (57), yields

$$P^{\star}_{z,t-s,t} = \left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}} \right)^{-1} \left(\frac{\theta}{\theta-1} \frac{1}{1+\tau} \right) \frac{N_{zt}P_t}{D_{zt}}, \tag{60}$$

where we used $E[(\epsilon^{G}_{jzt})^{\theta-1}] = 1$ and $E[(\epsilon^{Q}_{jzt})^{\theta-1}] = 1$ and the fact that ϵ^{G}_{jzt} and ϵ^{Q}_{jzt} are independent.

Analogous steps for the case of products that newly entering in period t deliver the following expression for the optimal average price $P^\star_{z,t,t}$ of these products, as defined in equation (56):

$$P^\star_{z,t,t} = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{N_{zt}P_t}{D_{zt}}. \tag{61}$$

Equations (60) and (61) jointly deliver

$$P^\star_{z,t-s,t} = P^\star_{z,t,t}\left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}}\right)^{-1}, \tag{62}$$

for $s \geq 1$. This equation shows how the optimal average price of older products is related to the optimal average price of newly entering products. Using the previous equation to substitute for $P^\star_{z,t-s,t}$ in equation (58) and rearranging the result yields

$$\begin{aligned} P_{zt}^{1-\theta} &= (P^\star_{z,t,t})^{1-\theta}\left\{\alpha_z\delta_z + (1-\alpha_z)\left[\delta_z + \sum_{s=1}^{\infty}\delta_z(1-\delta_z)^s\left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}}\right)^{\theta-1}\right]\right\} \\ &\quad + \alpha_z(1-\delta_z)(P_{zt-1})^{1-\theta}. \end{aligned} \tag{63}$$

Now define

$$(\Delta^e_{zt})^{1-\theta} \equiv \delta_z + \sum_{s=1}^{\infty}\delta_z(1-\delta_z)^s\left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}}\right)^{\theta-1}, \tag{64}$$

and substitute this definition into equation (63). This delivers the recursive representation of the item price level:

$$P_{zt}^{1-\theta} = \left\{\alpha_z\delta_z + (1-\alpha_z)(\Delta^e_{zt})^{1-\theta}\right\}(P^\star_{z,t,t})^{1-\theta} + \alpha_z(1-\delta_z)(P_{zt-1})^{1-\theta}, \tag{65}$$

where $P^\star_{z,t,t}$ is defined in equation (56). Finally, we rewrite the definition of Δ^e_{zt} according to

$$\begin{aligned} (\Delta^e_{zt})^{1-\theta} &= \delta_z + (1-\delta_z)\left(\frac{g_{zt}}{q_{zt}}\right)^{\theta-1}\left(\delta_z + \sum_{s=1}^{\infty}\delta_z(1-\delta_z)^s\left(\frac{\prod_{k=0}^{s-1} g_{zt-1-k}}{\prod_{k=0}^{s-1} q_{zt-1-k}}\right)^{\theta-1}\right) \\ &= \delta_z + (1-\delta_z)\left(\Delta^e_{zt-1}q_{zt}/g_{zt}\right)^{1-\theta}, \end{aligned} \tag{66}$$

which shows that $(\Delta^e_{zt})^{1-\theta}$ is a stationary variable that evolves recursively. We define the item-level (gross) inflation rate as

$$\Pi_{zt} \equiv P_{zt}/P_{zt-1}$$

and the relative price $p^\star_{zt}$ as

$$p^\star_{zt} \equiv P^\star_{z,t,t}/P_{zt}. \tag{67}$$

Using these definitions, we rearrange equation (65) to obtain

$$1 = \left\{\alpha_z\delta_z + (1-\alpha_z)(\Delta^e_{zt})^{1-\theta}\right\}(p^\star_{zt})^{1-\theta} + \alpha_z(1-\delta_z)(\Pi_{zt})^{\theta-1}. \tag{68}$$

The previous equation shows that in a balanced growth path with a constant item-level inflation Π_z, the relative price $p^\star_z$ is also constant.

B.5 Item-Level and Economy-Wide Aggregate Production Functions

We aggregate the model in two steps. In a first step, we aggregate firm-specific production functions to item-level production functions. In a second step, we aggregate the item-level production functions to a economy-wide production function.

To obtain the item-level production function, we substitute (quality-adjusted) output of product j in item z in the production function (6) using the demand function (18). This yields

$$\frac{Y_{zt}}{A_{zt}Q_{jzt}G_{jzt}}\left(\frac{P_{jzt}}{P_{zt}}\right)^{-\theta} = \left(\frac{K_{jzt}}{L_{jzt}}\right)^{1-\frac{1}{\phi}} L_{jzt} \ .$$

Integrating the previous equation over all firms $j \in [0, 1]$ in item z, using the definition

$$L_{zt} \equiv \int L_{jzt} \text{ dj},$$

and equation (48), which shows that capital-to-labor ratio is identical for all products, we obtain the item-level production function for quality-adjusted output in item z

$$Y_{zt} = \frac{A_{zt}Q_{zt}}{\Delta_{zt}}\left(K_{zt}^{1-\frac{1}{\phi}} L_{zt}^{\frac{1}{\phi}}\right), \tag{69}$$

where

$$K_{zt} \equiv \int K_{jzt} \text{ dj}$$

and where we have defined the productivity parameter $1/\Delta_{zt}$ as

$$\Delta_{zt} \equiv \int_0^1 \left(\frac{Q_{zt}}{Q_{jzt}G_{jzt}}\right)\left(\frac{P_{jzt}}{P_{zt}}\right)^{-\theta} \text{ dj} \ , \tag{70}$$

which captures the (detrended) distribution of productivities and qualities across products in item z. The recursive evolution equation for Δ_{zt} is derived in appendix B.6.

To obtain the economy-wide aggregate production function, we rewrite equation (69) to obtain

$$Y_{zt}\frac{\Delta_{zt}}{A_{zt}Q_{zt}} = \left(\frac{K_t}{L_t}\right)^{1-\frac{1}{\phi}} L_{zt}.$$

where we used the fact that the capital-to-labor ratio is the same across items, see equation (48). Summing the previous equation over all items $z = 1,..Z$, and using labor market clearing across items, $L_t = \sum_z L_{zt}$, and the demand function (19) to substitute for item-level output Y_{zt}, we obtain

$$Y_t \sum_{z=1}^{Z_t} \psi_{zt}\left(\frac{P_{zt}}{P_t}\right)^{-1}\left(\frac{\Delta_{zt}}{A_{zt}Q_{zt}}\right) = K_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}}.$$

The aggregate economy-wide production function for quality-adjusted output is thus given by

$$Y_t = \frac{(\Gamma_t^e)^{1/\phi}}{\Delta_t}\left(K_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}}\right), \tag{71}$$

where the aggregate economy-wide productivity parameter $1/\Delta_t$ is defined according to

$$\Delta_t \equiv (\Gamma_t^e)^{1/\phi} \sum_{z=1}^{Z_t} \psi_{zt}\left(\frac{P_{zt}}{P_t}\right)^{-1}\left(\frac{\Delta_{zt}}{A_{zt}Q_{zt}}\right), \tag{72}$$

and where Γ_t^e denotes the trend-growth factor defined in Appendix C.4 and ensures that Δ_t a stationary variable.

B.6 Derivation of the Recursive Evolution Equation for Δ_{zt}

To derive a recursive representation for the productivity shifter Δ_{zt}, defined in equation (70), we decompose it in a way that resembles the decomposition of the item price level in Appendix B.4. This yields

$$\frac{\Delta_{zt}}{P_{zt}^{\theta}} = \sum_{s=0}^{\infty} \int_{J^{\star}_{t-s,t}} \left(\frac{Q_{zt}}{Q_{jzt} G_{jzt}} \right) (P^{\star}_{jzt})^{-\theta} \mathrm{dj} + \frac{q_{zt}}{g_{zt}} \int_{J_t} \left(\frac{Q_{zt-1}}{Q_{jzt-1} G_{jzt-1}} \right) (P_{jzt-1})^{-\theta} \mathrm{dj}, \tag{73}$$

where, as before, $J^{\star}_{t-s,t}$ denotes the set of products with age $s \geq 0$ at time t that can adjust prices in t. Let J_t denote the set of all products that can not adjust prices in t. To derive equation (73), we have used the fact that all products in J_t have age $s \geq 1$. We have also used the fact that the productivity component $\overline{G}_{jzt}$ for the products in $J_{t-1,t}$ continues to evolve over time, which yields

$$\begin{aligned} G_{jzt} &= \overline{G}_{jzt} \cdot \epsilon^{G}_{jzt-1} \\ &= \left(\frac{\overline{G}_{jzt}}{\overline{G}_{jzt-1}} \right) \left(\overline{G}_{jzt-1} \cdot \epsilon^{G}_{jzt-1} \right) \\ &= g_{zt} G_{jzt-1}, \end{aligned} \tag{74}$$

where the last line follows from equations (7) and (9) for the case with $s \geq 1$.

Since products in J_t are a representative subset of all products in the economy at date $t-1$ and since J_t has mass $\alpha_z(1-\delta_z)$, we can rewrite equation (73) by shifting equation (70) one period into the past, which yields

$$\frac{\Delta_{zt}}{P_{zt}^{\theta}} = \sum_{s=0}^{\infty} \int_{J^{\star}_{t-s,t}} \left(\frac{Q_{zt}}{Q_{jzt} G_{jzt}} \right) (P^{\star}_{jzt})^{-\theta} \mathrm{dj} + \alpha_z (1-\delta_z) \frac{q_{zt}}{g_{zt}} \frac{\Delta_{zt-1}}{P^{\theta}_{zt-1}}. \tag{75}$$

We now rearrange the infinite sum in the previous equation. The steps involved in this resemble the steps used in in the derivation of the item price level in Appendix B.4, but with slight modifications. We first show how the integrals appearing in the infinite sum on the r.h.s. of equation (75) are related to the average optimal price of newly entering products $P^{\star}_{z,t,t}$. For $s \geq 1$, we obtain

$$\int_{J^{\star}_{t-s,t}} \left(\frac{Q_{zt}}{Q_{jzt} G_{jzt}} \right) (P^{\star}_{jzt})^{-\theta} \mathrm{dj} = \left(\frac{\prod_{k=0}^{s-1} q_{zt-k}}{\prod_{k=0}^{s-1} g_{zt-k}} \right) \int_{J^{\star}_{t-s,t}} \left[\frac{Q_{zt-s}}{Q_{jzt-s} G_{jzt-s}} \right] (P^{\star}_{jzt})^{-\theta} \mathrm{dj}, \tag{76}$$

using $Q_{zt} = (\prod_{k=0}^{s-1} q_{zt-k}) Q_{zt-s}$ and the fact that products in $J^{\star}_{t-s,t}$ have age greater or equal to s. We can rearrange the r.h.s. of equation (76) further using

$$G_{jzt} = \left(\prod_{k=0}^{s-1} g_{zt-k} \right) G_{jzt-s},$$

which follows from (74). The brackets in equation (76) contain only idiosyncratic components and thus simplify as

$$\frac{Q_{zt-s}}{Q_{jzt-s} G_{jzt-s}} = [\epsilon^{Q}_{jz,t-s} \epsilon^{G}_{jz,t-s}]^{-1}.$$

Substituting the previous two equations into equation (76) and integrating the result over the products in $J^\star_{t-s,t}$ yields

$$\int_{J^\star_{t-s,t}} \left(\frac{Q_{zt}}{Q_{jzt}G_{jzt}}\right)(P^\star_{jzt})^{-\theta}\mathrm{dj} = \left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}}\right)^{-1} \int_{J^\star_{t-s,t}} [\epsilon^Q_{jz,t-s}\epsilon^G_{jz,t-s}]^{-1}(P^\star_{jzt})^{-\theta}\mathrm{dj}. \tag{77}$$

To link the previous equation to the average optimal price of newly entering products $P^\star_{z,t,t}$, we rearrange equation (60) to obtain

$$[\epsilon^Q_{jz,t-s}\epsilon^G_{jz,t-s}]^{-1}(P^\star_{jzt})^{-\theta} = [\epsilon^Q_{jz,t-s}\epsilon^G_{jz,t-s}]^{\theta-1}\left[\left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}}\right)^{-1}\left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{N_{zt}P_t}{D_{zt}}\right]^{-\theta}.$$

Integrating the previous equation over the set of products in $J^\star_{t-s,t}$ and normalizing the result yields

$$\int_{J^\star_{t-s,t}} \frac{[\epsilon^Q_{jz,t-s}\epsilon^G_{jz,t-s}]^{-1}}{(1-\alpha_z)\delta_z(1-\delta_z)^s}(P^\star_{jzt})^{-\theta}\ \mathrm{dj} = \left[\left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}}\right)^{-1}\left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{N_{zt}P_t}{D_{zt}}\right]^{-\theta},$$

where we used $E[(\epsilon^G_{jzt})^{\theta-1}] = 1$ and $E[(\epsilon^Q_{jzt})^{\theta-1}] = 1$ and the fact that ϵ^G_{jzt} and ϵ^Q_{jzt} are independent. We can now use equation (61) to substitute $P^\star_{z,t,t}$ into the previous equation, which yields

$$\int_{J^\star_{t-s,t}} \frac{[\epsilon^Q_{jz,t-s}\epsilon^G_{jz,t-s}]^{-1}}{(1-\alpha_z)\delta_z(1-\delta_z)^s}(P^\star_{jzt})^{-\theta}\ \mathrm{dj} = \left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}}\right)^{\theta}\left(P^\star_{z,t,t}\right)^{-\theta}.$$

Furthermore, substituting the previous equation for the r.h.s. of equation (77) yields

$$\int_{J^\star_{t-s,t}} \left(\frac{Q_{zt}}{Q_{jzt}G_{jzt}}\right)(P^\star_{jzt})^{-\theta}\mathrm{dj} = (1-\alpha_z)\delta_z(1-\delta_z)^s\left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}}\right)^{\theta-1}\left(P^\star_{z,t,t}\right)^{-\theta},$$

which shows how the integral terms on the r.h.s. of equation (75) are related to the average optimal price of newly entering products $P^\star_{z,t,t}$ for $s \geq 1$. For the case with $s = 0$, analogous steps yield

$$\int_{J^\star_{t,t}} [\epsilon^Q_{jzt}\epsilon^G_{jzt}]^{-1}(P^\star_{jzt})^{-\theta}\ \mathrm{dj} = \delta_z(P^\star_{z,t,t})^{-\theta}.$$

Using the preceding two equations to substitute for the integrals in the infinite sum on the r.h.s. of equation (75), we obtain

$$\frac{\Delta_{zt}}{P^\theta_{zt}} = (P^\star_{z,t,t})^{-\theta}\left\{\delta_z + (1-\alpha_z)\sum_{s=1}^{\infty}\delta_z(1-\delta_z)^s\left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}}\right)^{\theta-1}\right\} + \alpha_z(1-\delta_z)\frac{q_{zt}}{g_{zt}}\frac{\Delta_{zt-1}}{P^\theta_{zt-1}},$$

where the term in curly brackets is the same as the term in curly brackets in equation (63). Accordingly, rearranging the previous equation yields the recursive representation

$$\Delta_{zt} = (p^\star_{zt})^{-\theta}\left\{\alpha_z\delta_z + (1-\alpha_z)(\Delta^e_{zt})^{1-\theta}\right\} + \alpha_z(1-\delta_z)(\Pi_{zt})^{\theta}\left(g_{zt}/q_{zt}\right)^{-1}\Delta_{zt-1},$$

where $\Pi_{zt} = P_{zt}/P_{zt-1}$. The stationary variable Δ^e_{zt} evolves as described in equation (66) and $p^\star_{zt}$ is defined in equation (67). The previous equation shows that Δ_{zt} is constant in the balanced growth path, because $p^\star_{zt}$ is constant in this path due to equation (68).

C Efficient Allocation and Efficient Growth Trends

As a reference point and to better understand the distortions emerging in the decentralized economy, this section derives the first-best allocation. This involves deriving the allocation of factor inputs across products with different levels of product quality and productivity at the level of each expenditure item z, in addition to the allocation of factor inputs across items z with different average quality and productivity. It also requires determining the optimal intertemporal paths of aggregate variables. This appendix also derives the growth trend of variables in the efficient allocation. Using the efficient trends we drive expressions for the efficient allocation in terms of detrended variables. Throughout the appendix, variables carrying the superscript 'e' denote efficient quantities.

C.1 Efficient Allocation at the Item-Level

Consider a setting where it is efficient to allocate L^e_{zt} units of labor and K^e_{zt} units of capital to the production of products in item z. The optimal allocation of capital and labor across products j in item z maximizes then (quality-adjusted) item-level output/consumption in equation (5), subject to the production function (6) and the feasibility constraints $L^e_{zt} = \int_z L^e_{jzt}\,\mathrm{dj}$ and $K^e_{zt} = \int_z K^e_{jzt}\,\mathrm{dj}$. This allocation problem yields the efficient item-level output

$$Y^e_{zt} = \frac{A_{zt}Q_{zt}}{\Delta^e_{zt}}(K^e_{zt})^{1-\frac{1}{\phi}}(L^e_{zt})^{\frac{1}{\phi}}, \tag{78}$$

where the efficient productivity parameters $1/\Delta^e_{zt}$ is defined as

$$1/\Delta^e_{zt} \equiv \left(\int_0^1 (G_{jzt}Q_{jzt}/Q_{zt})^{\theta-1}\,\mathrm{dj}\right)^{\frac{1}{\theta-1}}. \tag{79}$$

To derive a recursive representation for $1/\Delta^e_{zt}$, we rearrange the previous equation to obtain

$$(\Delta^e_{zt})^{1-\theta} = \delta_z \sum_{s=0}^{\infty}(1-\delta_z)^s \frac{1}{\delta_z(1-\delta_z)^s}\int_{J_{t-s,t}} (G_{jzt}Q_{jzt}/Q_{zt})^{\theta-1}\,\mathrm{dj}, \tag{80}$$

where $J_{t-s,t}$ denotes the set of products with age $s \geq 0$ in period t. The integrals appearing on the r.h.s. of the infinite sum in the previous equation can bet expressed as

$$\frac{1}{\delta_z(1-\delta_z)^s}\int_{J_{t-s,t}} (G_{jzt}Q_{jzt}/Q_{zt})^{\theta-1}\,\mathrm{dj} = \left(\frac{\prod_{k=0}^{s-1} g_{zt-k}}{\prod_{k=0}^{s-1} q_{zt-k}}\right)^{\theta-1},$$

since $E[(\epsilon^Q_{jzt})^{\theta-1}] = 1$ and $E[(\epsilon^G_{jzt})^{\theta-1}] = 1$ and ϵ^Q_{jzt} and ϵ^G_{jzt} are independent. Plugging the previous equation into equation (80) yields equation (64) which as is shown in appendix B.4, has the recursive representation described in equation (66).

C.2 Efficient Allocation Across Items

The optimal allocation of capital and labor between items maximizes (quality-adjusted) aggregate output/consumption in equation (4), subject to the efficient item-level production function (78) and the feasibility conditions $L_t^e = \sum_z L_{zt}^e$ and $K_t^e = \sum_z K_{zt}^e$, for given levels of L_t^e and K_t^e. Solving this allocation problem delivers the aggregate economy-wide efficient production function

$$Y_t^e = \frac{(\Gamma_t^e)^{1/\phi}}{\Delta_t^e}(K_t^e)^{1-\frac{1}{\phi}}(L_t^e)^{\frac{1}{\phi}}, \tag{81}$$

where the efficient productivity level $1/\Delta_t^e$ is defined as

$$\frac{1}{\Delta_t^e} \equiv (\Gamma_t^e)^{-\frac{1}{\phi}}\left(\prod_{z=1}^{Z_t}\psi_{zt}^{\psi_{zt}}\left(\frac{A_{zt}Q_{zt}}{\Delta_{zt}^e}\right)^{\psi_{zt}}\right), \tag{82}$$

and Γ_t^e denotes the aggregate growth rate defined in Appendix C.4 and ensures that Δ_t^e a stationary variable.

C.3 Efficient Intertemporal Allocation

The intertemporal allocation maximizes expected discounted utility of the representative household, equation (2), subject to the intertemporal feasibility condition

$$C_t^e + K_{t+1}^e = (1-d)K_t^e + Y_t^e \tag{83}$$

and the aggregate economy-wide production function (81). The first order conditions to this problem comprise the feasibility condition (83) and

$$Y_{Lt}^e = -\frac{U_{Lt}^e}{U_{Ct}^e}, \tag{84}$$

$$1 = \beta E_t\left[\frac{U_{Ct+1}^e}{U_{Ct}^e}\left(Y_{Kt+1}^e + 1 - d\right)\right], \tag{85}$$

where U_{Ct} denotes the marginal utility of consumption in t, U_{Lt} the marginal disutility from labor, Y_{Kt}^e the marginal product of capital and Y_{Lt}^e the marginal product of labor.

C.4 Efficient Item-Level and Aggregate Growth Trends

This section determined the efficient growth for the balanced growth path equilibrium in which aggregate hours worked L_t^e and item-level hours worked L_{zt}^e are stationary for all z. The variables C_t^e, K_t^e and Y_t^e all display the same growth trend, which we denote by Γ_t^e. Since the captial-to-labor ratio is constant across products, it then follows the item-level capital stocks K_{zt} have the same growth trend Γ_t^e for all z.

We can then derive the item-level output growth trend by rewriting equation (78) as

$$Y_{zt}^e = \frac{A_{zt}Q_{zt}}{\Delta_{zt}^e}(\Gamma_t^e)^{1-\frac{1}{\phi}}\left(\frac{K_{zt}^e}{\Gamma_t^e}\right)^{1-\frac{1}{\phi}}(L_{zt}^e)^{\frac{1}{\phi}},$$

which shows that Y_{zt}^e grows at the same rate as $\frac{A_{zt}Q_{zt}}{\Delta_{zt}^e}(\Gamma_t^e)^{1-\frac{1}{\phi}}$ because all other variables are stationary. We can thus define the item-level growth trend as

$$\Gamma_{zt}^e \equiv \frac{A_{zt}Q_{zt}}{\Delta_{zt}^e}(\Gamma_t^e)^{1-\frac{1}{\phi}}. \tag{86}$$

To derive the aggregate growth trend Γ_t^e, we substitute equilibrium output for equilibrium consumption in equation (4) and detrend all output variables in the resulting equation by their respective growth trends, which yields

$$\frac{Y_t^e}{\Gamma_t^e} = \left[\frac{\prod_{z=1}^{Z_t}(\Gamma_{zt}^e)^{\psi_{zt}}}{\Gamma_t^e}\right]\prod_{z=1}^{Z_t}\left(\frac{Y_{zt}^e}{\Gamma_{zt}^e}\right)^{\psi_{zt}}.$$

Since Y_{zt}^e/Γ_{zt}^e is stationary, the the aggregate growth trend is given by

$$\Gamma_t^e \equiv \prod_{z=1}^{Z_t}(\Gamma_{zt}^e)^{\psi_{zt}}. \tag{87}$$

Using definition (86) to substitute for Γ_{zt}^e in the previous equation and solving for Γ_t^e yields

$$\Gamma_t^e = \prod_{z=1}^{Z_t}\left(\frac{A_{zt}Q_{zt}}{\Delta_{zt}^e}\right)^{\phi\psi_{zt}}, \tag{88}$$

which determines the aggregate growth trend in terms of model primitives. Substituting the previous equation for Γ_t^e into equation (86) shows that the item-level growth trend relative to the aggregate growth trend is independent of the parameter ϕ and given by

$$\frac{\Gamma_{zt}^e}{\Gamma_t^e} = \frac{\left(\frac{A_{zt}Q_{zt}}{\Delta_{zt}^e}\right)}{\prod_{z=1}^{Z_t}\left(\frac{A_{zt}Q_{zt}}{\Delta_{zt}^e}\right)^{\psi_{zt}}}. \tag{89}$$

We also define the aggregate growth rate as

$$\gamma_t^e \equiv \Gamma_t^e/\Gamma_{t-1}^e. \tag{90}$$

Using equation (88) to substitute for Γ_t^e and Γ_{t-1}^e we obtain:

$$\gamma^e = \prod_{z=1}^{Z}(a_z q_z)^{\psi_z\phi} \tag{91}$$

in the steady state. Furthermore, we define the item-level growth rate as

$$\gamma_{zt}^e \equiv \Gamma_{zt}^e/\Gamma_{zt-1}^e, \tag{92}$$

and using equation (89), we obtain that in steady state,

$$\frac{\gamma_z^e}{\gamma^e} = \frac{a_z q_z}{\prod_{z=1}^{Z}(a_z q_z)^{\psi_z}}.$$

C.5 Efficient Production in Terms of Detrended Variables

We now express the item-level and aggregate production functions in the planned economy in terms of detrended output and capital variables. Letting lower case letters denote stationary variables, we can define$y_t^e \equiv Y_t^e/\Gamma_t^e$, $k_t^e \equiv K_t^e/\Gamma_t^e$, $k_{zt}^e \equiv K_{zt}^e/\Gamma_t^e$ and $y_{zt}^e \equiv Y_{zt}^e/\Gamma_{zt}^e$. To obtain the production function in item z in terms of detrended variables, we divide equation (78) by equation (86) and use the definitions of item-level detrended variables. This yields

$$y_{zt}^{e} = \left(k_{zt}^{e}\right)^{1-\frac{1}{\phi}}\left(L_{zt}^{e}\right)^{\frac{1}{\phi}}. \tag{93}$$

To obtain the aggregate production function in terms of detrended variables, we divide equation (81) by Γ_t^e and use the definitions of aggregate detrended variables, which yields

$$y_t^e = \frac{1}{\Delta_t^e}\left(k_t^e\right)^{1-\frac{1}{\phi}}\left(L_t^e\right)^{\frac{1}{\phi}}. \tag{94}$$

Here, $1/\Delta_t^e$ is defined in equation (82), and this definition simplifies to

$$\frac{1}{\Delta_t^e} = \prod_{z=1}^{Z_t} \psi_{zt}^{\psi_{zt}}, \tag{95}$$

after substituting the equation (88) for Γ_t^e into the definition.

D The Decentralized Economy and its Distortions

We now express the prices and allocations in the decentralized economy in terms of detrended variables, using the efficient growth trends derived in the previous appendix to detrend quantities. We then relate the allocation in the decentralized economy to the first-best allocation derived in the previous section using two key distortions (or wedges), namely a mark-up distortion and a relative-price distortion.

Appendices D.1 and D.2 start by deriving the growth trends of relative prices and express optimal reset prices in terms of detrended variables. Appendix D.3 introduces the mark-up distortion and uses it to rewrite various first-order conditions of households and firms. Appendix D.4 derives the item-level and aggregate production functions for the decentralized economy and relates them to the efficient allocation by introducing a relative-price distortion term. Appendix D.5 summarizes the equations characterizing the decentralized economy in detrended variables.

D.1 Relative Price Trends and Relative Inflation Rates

To detrend the relative price of item z, P_{zt}/P_t, we multiply the demand function (19) by the (inverse of the) relative growth factor Γ_{zt}^e/Γ_t^e, which yields

$$y_{zt}/y_t = \psi_{zt} p_{zt}^{-1}, \tag{96}$$

where we have defined

$$p_{zt} \equiv \left(P_{zt}/P_t\right)\left(\Gamma_{zt}^e/\Gamma_t^e\right), \tag{97}$$

which is constant in steady state. The demand function (19) also implies

$$\frac{\Pi_{zt}}{\Pi_t} = \left(\frac{\psi_{zt}}{\psi_{zt-1}}\right)\left(\frac{\gamma_{zt}^e y_{zt}/y_{zt-1}}{\gamma_t^e y_t/y_{t-1}}\right)^{-1},$$

which shows that items with stronger price increases face stronger output declines, which is a result of Cobb-Douglas aggregation across expenditure items.

D.2 Optimal Price in Terms of Detrended Variables

To express the optimal reset price in equation (25) in terms of detrended variables, we multiply the equation by the relative sectoral growth trend, Γ^e_{zt}/Γ^e_t (see Appendix C) and divide by item price level P_{zt}. This yields

$$\frac{P^\star_{jzt}}{P_{zt}}\left(\frac{Q_{jzt}G_{jzt}}{Q_{zt}}\right)p_{zt}=\left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\right)\frac{N_{zt}}{D_{zt}}\left(\frac{\Gamma^e_{zt}}{\Gamma^e_t}\right), \tag{98}$$

where p_{zt} is defined in equation (97). Since D_{zt} is stationary, see equation (54), we can define

$$d_{zt}\equiv D_{zt}. \tag{99}$$

The variable N_{zt} in equation (98) grows over time, but the variable

$$n_{zt}\equiv N_{zt}\left(\frac{\Gamma^e_{zt}}{\Gamma^e_t}\right) \tag{100}$$

is again stationary, as we show below. Using these definitions, we can thus write equation (98) in terms of stationary variables according to

$$\frac{P^\star_{jzt}}{P_{zt}}\left(\frac{Q_{jzt}G_{jzt}}{Q_{zt}}\right)p_{zt}=\left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\right)\frac{n_{zt}}{d_{zt}}. \tag{101}$$

It remains to prove the stationarity of n_{zt}. Using the definition of n_{zt} and equation (53) delivers

$$\begin{aligned}n_{zt}=&\left(\frac{MC_t}{P_tA_{zt}Q_{zt}}\right)\left(\frac{\Gamma^e_{zt}}{\Gamma^e_t}\right)\\&+\alpha_z(1-\delta_z)E_t\left[\Omega_{t,t+1}\left(\frac{P_{zt+1}}{P_{zt}}\right)^{\theta-1}\left(\frac{P_{t+1}}{P_t}\right)\left(\frac{Y_{t+1}}{Y_t}\right)\left(\frac{q_{zt+1}}{g_{zt+1}}\right)\left(\frac{\Gamma^e_{zt}}{\Gamma^e_t}\right)\left(\frac{\Gamma^e_{zt+1}}{\Gamma^e_{t+1}}\right)^{-1}n_{zt+1}\right]\end{aligned}$$

or equivalently

$$\begin{aligned}n_{zt}=&\left(\frac{MC_t}{P_tA_{zt}Q_{zt}}\right)\left(\frac{\Gamma^e_{zt}}{\Gamma^e_t}\right)\\&+\alpha_z(1-\delta_z)E_t\left[\Omega_{t,t+1}\Pi^{\theta-1}_{zt+1}\Pi_{t+1}\left(y_{t+1}/y_t\right)\gamma^e_{t+1}\left(\frac{q_{zt+1}}{g_{zt+1}}\right)\left(\frac{\gamma^e_{t+1}}{\gamma^e_{zt+1}}\right)n_{zt+1}\right].\end{aligned} \tag{102}$$

We can rewrite equation (86) to obtain

$$\frac{\Gamma^e_{zt}}{\Gamma^e_t}=(\Gamma^e_t)^{-\frac{1}{\phi}}\left(\frac{A_{zt}Q_{zt}}{\Delta^e_{zt}}\right),$$

and use this equation to rearrange the term involving marginal costs in equation (102) according to

$$\left(\frac{MC_t}{P_tA_{zt}Q_{zt}}\right)\left(\frac{\Gamma^e_{zt}}{\Gamma^e_t}\right)=\left(\frac{MC_t}{P_tA_{zt}Q_{zt}}\right)(\Gamma^e_t)^{-\frac{1}{\phi}}\left(\frac{A_{zt}Q_{zt}}{\Delta^e_{zt}}\right)=\left(\frac{MC_t}{P_t(\Gamma^e_t)^{1/\phi}}\right)\left(\frac{1}{\Delta^e_{zt}}\right).$$

We then define real detrended marginal costs as

$$mc_t\equiv\frac{MC_t}{P_t(\Gamma^e_t)^{1/\phi}}, \tag{103}$$

where MC_t is defined in equation (21). Substituting the previous equation into equation (102) yields

$$n_{zt} = \frac{mc_t}{\Delta_{zt}^e} + \alpha_z(1-\delta_z)E_t\left[\Omega_{t,t+1}\Pi_{zt+1}^{\theta-1}\Pi_{t+1}\left(y_{t+1}/y_t\right)\gamma_{t+1}^e\left(\frac{q_{zt+1}}{g_{zt+1}}\right)\left(\frac{\gamma_{t+1}^e}{\gamma_{zt+1}^e}\right)n_{zt+1}\right],$$

which contains only stationary variables. From equation (54) and the definition of d_{zt} we likewise obtain

$$d_{zt} = 1 + \alpha_z(1-\delta_z)E_t\left[\Omega_{t,t+1}\Pi_{zt+1}^{\theta-1}\left(y_{t+1}/y_t\right)\gamma_{t+1}^e d_{zt+1}\right].$$

To obtain a detrended expression for the average optimal price of new products, we integrate equation (101) over the set of newly entering products in t, normalize the resulting equation and use the assumptions $E[(\epsilon_{jzt}^G)^{\theta-1}] = 1$ and $E[(\epsilon_{jzt}^Q)^{\theta-1}] = 1$ and independence of ϵ_{jzt}^G and ϵ_{jzt}^Q. This yields

$$p_{zt}^{\star}p_{zt} = \left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\right)\frac{n_{zt}}{d_{zt}}, \tag{104}$$

where we have also used the definition (67).

D.3 Aggregate Mark-Up Distortions

We define the average markup μ_{zt} at the item level as the relative price of item z over real marginal costs (all in detrended terms),

$$\mu_{zt} \equiv \frac{p_{zt}}{mc_t}, \tag{105}$$

and the aggregate markup as

$$\mu_t \equiv \prod_{z=1}^{Z_t}\mu_{zt}^{\psi_{zt}}. \tag{106}$$

Substituting equation (106) for μ_{zt} into the previous equation, we obtain

$$\mu_t = mc_t^{-1}\prod_{z=1}^{Z_t}p_{zt}^{\psi_{zt}}.$$

Expressing the aggregate price in equation (16) in terms of detrended relative prices and also using equation (95), we obtain from the previous equation

$$\mu_t = \frac{1}{mc_t\Delta_t^e}. \tag{107}$$

Using the definition (103) and equation (21), we obtain

$$mc_t = \left(\frac{k_t}{L_t}\right)^{\frac{1}{\phi}}\left(\frac{r_t}{1-1/\phi}\right),$$

where we have also used equation (48) determining the optimal input mix. Substituting into the previous equation the expression for the markup and rearranging yields

$$r_t = \mu_t^{-1}\left(1-\frac{1}{\phi}\right)\frac{1}{\Delta_t^e}\left(\frac{k_t}{L_t}\right)^{-\frac{1}{\phi}}. \tag{108}$$

Analogous steps deliver

$$w_t = \mu_t^{-1} \left(\frac{1}{\phi}\right) \frac{1}{\Delta_t^e} \left(\frac{k_t}{L_t}\right)^{1-\frac{1}{\phi}}. \tag{109}$$

The previous two equations show how the capital-to-labor ratio gets distorted by the aggregate markup μ_t.

D.4 Relative Price Distortions

We define detrended variables according to $y_t \equiv Y_t/\Gamma_t^e$, $k_t \equiv K_t/\Gamma_t^e$, $k_{zt} \equiv K_{zt}/\Gamma_t^e$ and $y_{zt} \equiv Y_{zt}/\Gamma_{zt}^e$. To obtain the production function in item z in terms of detrended variables, we rewrite equation (69) as

$$\frac{Y_{zt}}{\Gamma_{zt}^e} = \left[\frac{(\Gamma_t^e)^{1-\frac{1}{\phi}}}{\Gamma_{zt}^e} \frac{A_{zt}Q_{zt}}{\Delta_{zt}}\right] \left(\frac{K_{zt}}{\Gamma_t^e}\right)^{1-\frac{1}{\phi}} L_{zt}^{\frac{1}{\phi}}.$$

Using the definitions for detrended variables and the definition of the item-level growth trend in equation (86), we obtain a production function in detrended variables:

$$y_{zt} = \left(\frac{\Delta_{zt}^e}{\Delta_{zt}}\right) k_{zt}^{1-\frac{1}{\phi}} L_{zt}^{\frac{1}{\phi}}. \tag{110}$$

In a situation in which relative prices in the decentralized economy are efficient, we have

$$\Delta_{zt} = \Delta_{zt}^e,$$

such that equation (110) becomes equal to the efficient production function in the planner solution, see equation (93). Item-level distortions arising from inefficient price dispersion can thus be captured by the item-level distortion factor

$$\rho_{zt} \equiv \Delta_{zt}^e/\Delta_{zt} \leq 1 \tag{111}$$

We obtain the aggregate production function in detrended variables for the decentralized economy by dividing equation (71) by Γ_t^e and using the definitions of aggregate detrended variables:

$$y_t = \left(\frac{\Delta_t^e}{\Delta_t}\right) \left(\frac{1}{\Delta_t^e}\right) k_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}}. \tag{112}$$

We can then define an aggregate distortion factor capturing inefficiencies associated with relative price distortions across all items:

$$\rho_t \equiv \Delta_t^e/\Delta_t \leq 1. \tag{113}$$

When relative prices are efficient, we have $\rho_t = 1$, so that the aggregate production function in the decentralized economy (112) becomes equal to the aggregate production function in the planner allocation (94).

We take the inverse of equation (72) and multiply it by Δ_t^e. We simplify the resulting equation by substituting for $(\Gamma_t^e)^{1/\phi}$ using equation (86) and using the definition of p_{zt} in equation (97). This yields

$$\frac{\Delta_t^e}{\Delta_t} = \Delta_t^e \left(\sum_{z=1}^{Z_t} \psi_{zt} p_{zt}^{-1} \left(\Delta_{zt}/\Delta_{zt}^e\right)\right)^{-1},$$

and shows that the relative price distortion at the aggregate level is a weighted sum over item-level relative price distortions with weights equal to the item's relative output (recall $y_{zt}/y_t = \psi_{zt} p_{zt}^{-1}$ from equation (96)). We can rearrange the previous equation by using the definition (106) to substitute for p_{zt} and equation (107) to substitute for mc_t in this definition. This yields

$$(\rho_t \mu_t)^{-1} = \sum_{z=1}^{Z_t} \psi_{zt} (\mu_{zt} \rho_{zt})^{-1} \tag{114}$$

and shows that the product of (inverse) aggregate distortion corresponds to the weighted sum of the product of (inverse) item-level distortions.

D.5 Summary of Equations Characterizing the Decentralized Economy

At the aggregate level, the decentralized and detrended economy is summarized by the following four equations:

$$y_t = \left(\frac{\rho_t}{\Delta_t^e}\right) k_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} \tag{115}$$

$$\mu_t^{-1} \left(\frac{1}{\phi}\right) \frac{1}{\Delta_t^e} \left(\frac{k_t}{L_t}\right)^{1-\frac{1}{\phi}} = -c_t \left(\frac{\partial V(L_t)/\partial L_t}{V(L_t)}\right) \tag{116}$$

$$1 = E_t \left[\Omega_{t,t+1} \left\{ \mu_{t+1}^{-1} \left(1 - \frac{1}{\phi}\right) \frac{1}{\Delta_{t+1}^e} \left(\frac{k_{t+1}}{L_{t+1}}\right)^{-\frac{1}{\phi}} + 1 - d \right\}\right] \tag{117}$$

$$\gamma_{t+1}^e k_{t+1} = (1-d) k_t + y_t - c_t. \tag{118}$$

Equation (115) follows from substituting the definition of the relative price distortion (113) into the aggregate production function (112). Equation (116) follows from substituting equation (109) for the wage into the first-order condition (44). Equation (117) follows from substituting equation (108) for the real rate into the household's first-order condition (47). Equations (116) and (117) show how the markup distorts the intra- and inter-temporal optimal household choices compared to the first-best allocation, see equations (84) and (85). Equation (118) is derived from consolidating the budget constraints of the representative household and the government and expressing the resulting equation in terms of detrended variables.

Equations (115)–(118) determine the variables y_t, k_t, L_t and c_t given values for the aggregate distortions ρ_t and μ_t, which depend on the inflation rate, aggregate growth γ_t^e, the productivity parameter Δ_t^e determined by equation (95) and given the equation for the discount factor

$$\Omega_{t,t+1} = \beta \left(\frac{\gamma_{t+1}^e c_{t+1}}{c_t}\right)^{-\sigma} \left(\frac{V(L_{t+1})}{V(L_t)}\right)^{1-\sigma}.$$

Furthermore, we previously determined in equation (114) and definition (106) that the aggregate markup and relative price distortions are functions of the item-level markup and relative price distortions. These equations are repeated here, jointly with the definitions of item-level markup and relative price distortions (106) and (111), respectively:

$$(\rho_t\mu_t)^{-1} = \sum_{z=1}^{Z_t} \psi_{zt}(\mu_{zt}\rho_{zt})^{-1}$$
$$\mu_t = \prod_{z=1}^{Z_t} \mu_{zt}^{\psi_{zt}}$$
$$\rho_{zt} = \Delta_{zt}^e/\Delta_{zt}$$
$$\mu_{zt} = p_{zt}/mc_t.$$

Note that the distortions depend on the inflation rate.

The item-level outcomes are described by the following set of equations:

$$1 = \left\{\alpha_z\delta_z + (1-\alpha_z)(\Delta_{zt}^e)^{1-\theta}\right\}(p_{zt}^\star)^{1-\theta} + \alpha_z(1-\delta_z)(\Pi_{zt})^{\theta-1} \tag{119}$$
$$p_{zt}^\star p_{zt} = \left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\right)\frac{n_{zt}}{d_{zt}} \tag{120}$$
$$n_{zt} = \frac{mc_t}{\Delta_{zt}^e} + \alpha_z(1-\delta_z)E_t\left[\Omega_{t,t+1}\Pi_{zt+1}^{\theta-1}\Pi_{t+1}\left(y_{t+1}/y_t\right)\gamma_{t+1}^e\left(\frac{q_{zt+1}}{g_{zt+1}}\right)\left(\frac{\gamma_{t+1}^e}{\gamma_{zt+1}^e}\right)n_{zt+1}\right] \tag{121}$$
$$d_{zt} = 1 + \alpha_z(1-\delta_z)E_t\left[\Omega_{t,t+1}\Pi_{zt+1}^{\theta-1}\left(y_{t+1}/y_t\right)\gamma_{t+1}^e d_{zt+1}\right] \tag{122}$$
$$\left(\frac{\gamma_{zt}^e}{\gamma_t^e}\right)\Pi_{zt} = \left(\frac{\psi_{zt}}{\psi_{zt-1}}\frac{p_{zt}}{p_{zt-1}}\right)\Pi_t \tag{123}$$
$$\Delta_{zt} = (p_{zt}^\star)^{-\theta}\left\{\alpha_z\delta_z + (1-\alpha_z)(\Delta_{zt}^e)^{1-\theta}\right\} + \alpha_z(1-\delta_z)(\Pi_{zt})^{\theta}\left(g_{zt}/q_{zt}\right)^{-1}\Delta_{zt-1} \tag{124}$$
$$(\Delta_{zt}^e)^{1-\theta} = \delta_z + (1-\delta_z)\left(\Delta_{zt-1}^e q_{zt}/g_{zt}\right)^{1-\theta} \tag{125}$$
$$mc_t = \left(\frac{w_t}{1/\phi}\right)^{\frac{1}{\phi}}\left(\frac{r_t}{1-1/\phi}\right)^{1-\frac{1}{\phi}} \tag{126}$$
$$r_t k_t = (\phi-1)w_t L_t \tag{127}$$
$$\gamma_{zt}^e = (\gamma_t^e)^{1-\frac{1}{\phi}}\left(a_{zt}q_{zt}\Delta_{zt-1}^e/\Delta_{zt}^e\right), \tag{128}$$

where inflation Π_t is defined in equation (17) and the aggregate price level in equation (16). Furthermore, the aggregate growth rate γ_t^e is defined in equation (90) and the aggregate growth trend is determined by equation (88).

E Derivation of the Steady State Equations in Section 5

In the steady state, the one-period discount factor in equation (45) is

$$\Omega = \beta(\gamma^e)^{-\sigma}.$$

Using this, equations (115)–(118) simplify to the equations (26)–(29) in the steady state. Furthermore, in the steady state, the aggregate markup in equation (106) and the relative price distortion in equation (114) simplify to equations (137) and (139), respectively. These aggregate distortions are functions of the item-level distortions, which are functions of the aggregate inflation rate. We now derive the steady-state expressions for the item-level distortions μ_z in equation (138) and ρ_z in equation (140).

E.1 Item-Level Relative Price Distortion

To express ρ_z as function of inflation, we consider the equations (119) and (124) in the steady state. This yields

$$\begin{aligned} 1-\alpha_z(1-\delta_z)\Pi_z^{\theta-1} &= \left\{\alpha_z\delta_z+(1-\alpha_z)(\Delta_z^e)^{1-\theta}\right\}(p_z^\star)^{1-\theta} \\ \left(1-\alpha_z(1-\delta_z)\Pi_z^{\theta}(g_z/q_z)^{-1}\right)\Delta_z &= \left\{\alpha_z\delta_z+(1-\alpha_z)(\Delta_z^e)^{1-\theta}\right\}(p_z^\star)^{-\theta}\,. \end{aligned} \tag{129}$$

Dividing both equations by each other yields

$$p_z^\star=\Delta_z^{-1}\left(\frac{1-\alpha_z(1-\delta_z)\Pi_z^{\theta-1}}{1-\alpha_z(1-\delta_z)\Pi_z^{\theta}(g_z/q_z)^{-1}}\right). \tag{130}$$

Substituting this expression for $p_z^\star$ into equation (129) yields

$$\left(\frac{\Delta_z}{\Delta_z^e}\right)^{1-\theta}=\left(\frac{\alpha_z\delta_z(\Delta_z^e)^{\theta-1}+(1-\alpha_z)}{1-\alpha_z(1-\delta_z)\Pi_z^{\theta}(g_z/q_z)^{-1}}\right)\left(\frac{1-\alpha_z(1-\delta_z)\Pi_z^{\theta-1}}{1-\alpha_z(1-\delta_z)\Pi_z^{\theta}(g_z/q_z)^{-1}}\right)^{-\theta}.$$

We substitute for Δ_z^e on the r.h.s. of the previous equation using the steady-state version of equation (125), which yields

$$\frac{\Delta_z}{\Delta_z^e}=\left(\frac{1-\alpha_z(1-\delta_z)\,(g_z/q_z)^{\theta-1}}{1-\alpha_z(1-\delta_z)\Pi_z^{\theta}(g_z/q_z)^{-1}}\right)^{\frac{1}{1-\theta}}\left(\frac{1-\alpha_z(1-\delta_z)\Pi_z^{\theta-1}}{1-\alpha_z(1-\delta_z)\Pi_z^{\theta}(g_z/q_z)^{-1}}\right)^{\frac{\theta}{\theta-1}}.$$

Simplifying the previous equation, using the definition (111) and substituting for Π_z using equation (123) in the steady state yields

$$\rho_z(\Pi)^{-1}=\left(\frac{1-\alpha_z(1-\delta_z)\,(g_z/q_z)^{\theta-1}}{1-\alpha_z(1-\delta_z)[(\gamma^e/\gamma_z^e)\Pi]^{\theta}(g_z/q_z)^{-1}}\right)\left(\frac{1-\alpha_z(1-\delta_z)[(\gamma^e/\gamma_z^e)\Pi]^{\theta-1}}{1-\alpha_z(1-\delta_z)\,(g_z/q_z)^{\theta-1}}\right)^{\frac{\theta}{\theta-1}}, \tag{131}$$

which shows that the item-level relative price distortion can be expressed as function of Π only. Rearranging the previous equation yields equation (140).

E.2 Item-Level Markup Distortion

To express μ_z as function of inflation, we consider the pricing equation (120) in the steady state and substitute for n and d using the equations (121) and (122) in the steady state. This yields

$$\frac{p_z}{mc}=\left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\right)\frac{1}{p_z^\star\Delta_z^e}\left(\frac{1-\alpha_z(1-\delta_z)\beta(\gamma^e)^{1-\sigma}[(\gamma^e/\gamma_z^e)\Pi]^{\theta-1}}{1-\alpha_z(1-\delta_z)\beta(\gamma^e)^{1-\sigma}[(\gamma^e/\gamma_z^e)\Pi]^{\theta}(g_z/q_z)^{-1}}\right), \tag{132}$$

where we have also substituted for Π_z using equation (123) in the steady state. Using equation (130), the definition (111) and equation (123) to substitute for Π_z, we obtain

$$\frac{1}{p_z^\star \Delta_z^e} = \rho_z(\Pi)^{-1} \left(\frac{1 - \alpha_z(1-\delta_z)[(\gamma^e/\gamma_z^e)\Pi]^{\theta-1}}{1 - \alpha_z(1-\delta_z)[(\gamma^e/\gamma_z^e)\Pi]^{\theta}(g_z/q_z)^{-1}} \right)^{-1}.$$

Using the previous equation to substitute for $(p_z^\star \Delta_z^e)^{-1}$ on the r.h.s. in equation (132) yields

$$\begin{aligned} \mu_z(\Pi) = & \left(\frac{1}{1+\tau} \frac{\theta}{\theta-1} \right) \rho_z(\Pi)^{-1} \left(\frac{1 - \alpha_z(1-\delta_z)[(\gamma^e/\gamma_z^e)\Pi]^{\theta-1}}{1 - \alpha_z(1-\delta_z)[(\gamma^e/\gamma_z^e)\Pi]^{\theta}(g_z/q_z)^{-1}} \right)^{-1} \\ & \times \left(\frac{1 - \alpha_z(1-\delta_z)\beta(\gamma^e)^{1-\sigma}[(\gamma^e/\gamma_z^e)\Pi]^{\theta-1}}{1 - \alpha_z(1-\delta_z)\beta(\gamma^e)^{1-\sigma}[(\gamma^e/\gamma_z^e)\Pi]^{\theta}(g_z/q_z)^{-1}} \right). \end{aligned} \tag{133}$$

Using equation (131) to substitute for $\rho_z(\Pi)^{-1}$ and the definition (105) to substitute for p_z/mc in the previous equation yields equation (138) determining the item-level markup as function of inflation.

E.3 Steady State: Existence Conditions

We now derive the existence conditions for a steady state (or deterministic balanced growth path). First, we need to impose

$$1 > (1-\delta_z)\,(g_z/q_z)^{\theta-1}, \tag{134}$$

for all z, so that $1/\Delta_z^e$, which measures quality-adjusted productivity in the efficient economy, see equation (125), has a well-defined steady-state value:

$$\left(\frac{1}{\Delta_z^e}\right)^{\theta-1} = \frac{\delta_z}{1-(1-\delta_z)\,(g_z/q_z)^{\theta-1}},$$

Given the substantial amount of product turnover ($\delta_z \gg 0$), see panel A of Figure 5, and the relatively low rates of relative price decline (g_z/q_z), see figure 3, condition (134) is likely to be fulfilled for reasonable values for the demand elasticity parameter θ.

To insure that the item-level distortions $\rho_z(\Pi)$ and $\mu_z(\Pi)$ in equations (140) and (138) have well-defined steady state values, we furthermore impose

$$1 > \alpha_z(1-\delta_z)[(\gamma^e/\gamma_z^e)\,\Pi]^{\theta}(g_z/q_z)^{-1} \tag{135}$$

$$1 > \alpha_z(1-\delta_z)[(\gamma^e/\gamma_z^e)\,\Pi]^{\theta-1}, \tag{136}$$

for all z. Since $\alpha_z \ll 1$ and $\delta_z \gg 0$, it follows from the fact that γ^e/γ_z^e and g_z/q_z take on values fairly close to one, that these conditions are easily fulfilled for reasonable values for the demand elasticity parameter θ and plausible (gross) steady-state inflation rates Π.

E.4 Aggregate Mark-up and Price Distortion

Using the results from the preceding sections, we can write the aggregate mark-up distortion $\mu(\Pi)$ as

$$\mu(\Pi) = \prod_{z=1}^{Z} \mu_z(\Pi)^{\psi_z}, \tag{137}$$

where the item-level distortions are given by

$$\mu_z(\Pi) \equiv \left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\right) M_z \left(\frac{1-\alpha_z(1-\delta_z)\beta(\gamma^e)^{1-\sigma}[(\gamma^e/\gamma_z^e)\,\Pi]^{\theta-1}}{1-\alpha_z(1-\delta_z)\beta(\gamma^e)^{1-\sigma}[(\gamma^e/\gamma_z^e)\,\Pi]^{\theta}(g_z/q_z)^{-1}}\right), \tag{138}$$

for all $z = 1, \ldots Z$, with

$$M_z \equiv \left(\frac{1-\alpha_z(1-\delta_z)[(\gamma^e/\gamma_z^e)\,\Pi]^{\theta-1}}{1-\alpha_z(1-\delta_z)(g_z/q_z)^{\theta-1}}\right)^{\frac{1}{\theta-1}},$$

and

$$\gamma_z^e \equiv (a_z q_z)(\gamma^e)^{1-\frac{1}{\phi}}.$$

Similarly, the relative price distortion $\rho(\Pi)$ is given by

$$(\rho(\Pi)\mu(\Pi))^{-1} = \sum_{z=1}^{Z} \psi_z(\mu_z(\Pi)\rho_z(\Pi))^{-1}, \tag{139}$$

where for all $z = 1, \ldots Z$ the item-level relative price distortions $\rho_z(\Pi)$ are given by

$$\rho_z(\Pi)^{-1} = M_z^{\theta}\left(\frac{1-\alpha_z(1-\delta_z)(g_z/q_z)^{\theta-1}}{1-\alpha_z(1-\delta_z)[(\gamma^e/\gamma_z^e)\,\Pi]^{\theta}(g_z/q_z)^{-1}}\right). \tag{140}$$

As is easy to see, for the limiting case without price stickiness ($\alpha_z \to 0$ for all z), we have

$$\mu = \left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\right)$$
$$\rho = 1,$$

independently of Π.

F Proofs

F.1 Proof of Lemma 1

For the limiting case $\beta\,(\gamma^e)^{1-\sigma} \to 1$, we have from item-level distortions in equations (138) and (140) that

$$\mu_z(\Pi) = \left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\right)\rho_z(\Pi)^{-1}. \tag{141}$$

Multiplying the previous equation by $\rho_z(\Pi)$ and substituting the result into equation (139) yields

$$(\rho(\Pi)\mu(\Pi))^{-1} = \left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\right)^{-1},$$

so that

$$\mu(\Pi) = \left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\right)\rho(\Pi)^{-1}.$$

F.2 Proof of Proposition 1

The proof proceed as follows: section F.2.1 derives a convenient formulation for the steady-state solution for general values of $\beta(\gamma^e)^{1-\sigma} < 1$; section 1 considers this formulation for the limiting case $\beta(\gamma^e)^{1-\sigma} \to 1$ and shows that labor is independent of the inflation rate, whereas consumption depends on the inflation rate only via the aggregate markup distortion; section F.2.3 derives the inflation rate that minimizes the aggregate markup distortion and thus maximizes consumption.

F.2.1 Steady State Solution

We rewrite equations (26) to (29) by expressing the variables y, c and k relative to hours worked L, which yields

$$\frac{y}{L} = \left(\frac{\rho(\Pi)}{\Delta^e}\right)\left(\frac{k}{L}\right)^{1-\frac{1}{\phi}} \tag{142}$$

$$\frac{c}{L} = \frac{1}{\mu(\Pi)}\frac{1}{\Delta^e}\left(\frac{1}{\phi}\right)\left(\frac{k}{L}\right)^{1-\frac{1}{\phi}}\left(-\frac{V(L)}{L\,\partial V(L)/\partial L}\right) \tag{143}$$

$$\frac{k}{L} = \frac{1}{\mu(\Pi)}\frac{1}{\Delta^e}\left(1-\frac{1}{\phi}\right)\left(\frac{k}{L}\right)^{1-\frac{1}{\phi}}\left(\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d\right)^{-1} \tag{144}$$

$$\frac{y}{L} = \frac{c}{L} + (\gamma^e - 1 + d)\frac{k}{L}. \tag{145}$$

We now show that these four equations determine the four variables y, c, L, k, given a steady-state inflation rate Π. For given Π, one can solve for hours worked L by substituting the equations (142) to (144) into equation (145). This yields

$$\left(-\frac{V(L)}{L\,\partial V(L)/\partial L}\right) = \phi\mu(\Pi)\rho(\Pi) - (\phi-1)\left(\frac{\gamma^e - 1 + d}{\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d}\right). \tag{146}$$

Given Π and L, the solutions for $k, c,$ and y can then be recursively computed from the equations (142) to (144). These solutions are

$$k(\Pi) = \left(\frac{1}{\mu(\Pi)}\frac{1}{\Delta^e}\right)^{\phi}\left(1-\frac{1}{\phi}\right)^{\phi}\left(\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d\right)^{-\phi} L \tag{147}$$

$$c(\Pi) = \frac{1}{\mu(\Pi)}\frac{1}{\Delta^e}\left(\frac{1}{\phi}\right)\left(\frac{k}{L}\right)^{1-\frac{1}{\phi}}\left(-\frac{V(L)}{\partial V(L)/\partial L}\right) \tag{148}$$

$$y(\Pi) = c + (\gamma^e - 1 + d)k. \tag{149}$$

F.2.2 Steady-state solution for the limiting case in proposition 1:

We now consider the steady-state solution from the previous section for the limiting case $\beta(\gamma^e)^{1-\sigma} \to 1$. Using lemma 1 equation (146) simplifies to

$$\left(-\frac{V(L)}{L\,\partial V(L)/\partial L}\right) = \left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\right)\phi - (\phi-1)\,. \tag{150}$$

This shows that the steady state amount of labor does not dependent on Π. Next, rewrite equation (147) as

$$\left(\frac{k(\Pi)}{L}\right)^{1-\frac{1}{\phi}} = \left(\frac{1}{\mu(\Pi)}\frac{1}{\Delta^e}\right)^{\phi-1}\left(1-\frac{1}{\phi}\right)^{\phi-1}(\gamma^e-1+d)^{1-\phi}.$$

Substitute this equation and equation (150) into equation (148), this delivers

$$c(\Pi) = \left(\frac{1}{\mu(\Pi)}\right)^{\phi}\left\{L\left(\frac{1}{\Delta^e}\right)^{\phi}(\gamma^e-1+d)^{1-\phi}\left(\left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\right)\phi-(\phi-1)\right)\phi^{-\phi}(\phi-1)^{\phi-1}\right\},$$

where the term in parentheses depends is independent of inflation Π. We thus have

$$c(\Pi) \propto \left(\frac{1}{\mu(\Pi)}\right)^{\phi}. \tag{151}$$

The inflation rate that minimizes the aggregate markup distortion thus maximizes steady-state consumption and thereby welfare, given that labor is fixed.

F.2.3 Minimizing The Aggregate Markup Distortion

From equation (137), minimizing the aggregate markup distortion in the steady state implies

$$\frac{\partial\mu(\Pi)}{\partial\Pi} = \sum_{z=1}^{Z}\psi_z\mu_z(\Pi)^{\psi_z-1}[\partial\mu_z(\Pi)/\partial\Pi]\left(\prod_{z^C}\mu_z(\Pi)^{\psi_z}\right) = 0,$$

where z^C to denote the set of all items except for item z. The equation holds if and only if

$$\sum_{z=1}^{Z}\psi_z\frac{\partial\mu_z(\Pi)/\partial\Pi}{\mu_z(\Pi)} = 0. \tag{152}$$

Using equation (141), the expression for $\rho_z(\pi)$ in equation (140) and the shorthand notation $\tilde{\alpha}_z = \alpha_z(1-\delta_z)(\gamma^e/\gamma_z^e)^{\theta-1}$, we obtain

$$\frac{\partial\mu_z(\Pi)/\partial\Pi}{\mu_z(\Pi)} = \frac{\theta\tilde{\alpha}_z\Pi^{\theta-2}\left(\frac{q_z\gamma^e}{g_z\gamma_z^e}\right)}{\left(1-\tilde{\alpha}_z\Pi^{\theta}\left(\frac{q_z\gamma^e}{g_z\gamma_z^e}\right)\right)(1-\tilde{\alpha}_z\Pi^{\theta-1})}\left[\Pi-\frac{g_z\gamma_z^e}{q_z\gamma^e}\right].$$

Plugging this expression into equation (152) and multiplying by Π^2 yields

$$\sum_{z=1}^{Z}\left\{\frac{\psi_z\theta\tilde{\alpha}_z\Pi^{\theta}\left(\frac{q_z\gamma^e}{g_z\gamma_z^e}\right)}{\left(1-\tilde{\alpha}_z\Pi^{\theta}\left(\frac{q_z\gamma^e}{g_z\gamma_z^e}\right)\right)(1-\tilde{\alpha}_z\Pi^{\theta-1})}\right\}\left[\Pi-\frac{g_z\gamma_z^e}{q_z\gamma^e}\right] = 0. \tag{153}$$

The expression in the parentheses is the weight $\tilde{\omega}_z$ in proposition 1. We normalize the weights so that they sums to unity over all $z = 1,\dots Z$. This yields normalized weights $\omega_z = \tilde{\omega}_z/\sum_{z=1}^{Z}\tilde{\omega}_z$, with $\sum_{z=1}^{Z}\omega_z = 1$. Using these, we can rewrite equation (153) according to

$$\sum_{z=1}^{Z}\omega_z\left[\Pi^\star-\frac{g_z\gamma_z^e}{q_z\gamma^e}\right] = 0, \tag{154}$$

where ω_z is given by the expression in the proposition and $\Pi^\star$ denotes the optimal solution. Solving equation (154) for $\Pi^\star$ yields the expression for the optimal inflation target in proposition 1.

F.3 Proof of Lemma 2

Defining $m_z = \frac{g_z \gamma_z^e}{q_z \gamma^e}$ one can express equation (153) as

$$\sum_{z=1}^{Z} \tilde{\omega}_z(\Pi, m_z)\left[\Pi - m_z\right] = 0, \tag{155}$$

where $\tilde{\omega}_z(\Pi, m_z) = \frac{\psi_z \theta \tilde{\alpha}_z \Pi^\theta / m_z}{\left(1-\tilde{\alpha}_z \Pi^\theta / m_z\right)\left(1-\tilde{\alpha}_z \Pi^{\theta-1}\right)}$ and $\tilde{\alpha}_z = \alpha_z(1-\delta_z)(\gamma^e/\gamma_z^e)^{\theta-1}$. We use the implicit function theorem to derive how the optimal inflation target varies with m_z. To this end, we linearize equation (155) at a point where $\bar{\Pi} = \bar{m}_z$ for all z. This yields

$$\sum_{z=1}^{Z} \tilde{\omega}_z(\bar{\Pi}, \bar{m}_z)\left[\Pi - m_z\right] = 0 + O(2),$$

where $O(2)$ denotes a second-order residual. Solving for Π and letting again $\Pi^\star$ denote the optimal solution, we obtain

$$\Pi^\star = \sum_{z=1}^{Z} \frac{\tilde{\omega}_z(\bar{\Pi}, \bar{m}_z)}{\sum_{z'=1}^{Z} \tilde{\omega}_{z'}(\bar{\Pi}, \bar{m}_{z'})}\; m_z + O(2), \tag{156}$$

which shows that $\Pi^\star$ is a weighted average of m_z's for all item categories z and with weights evaluated at the expansion point and normalized to unity. The normalized weight of item z evaluated at $\bar{\Pi} = \bar{m}_z$ is given by

$$\begin{aligned}\frac{\tilde{\omega}_z(\bar{\Pi}, \bar{m}_z)}{\sum_{z=1}^{Z} \tilde{\omega}_z(\bar{\Pi}, \bar{m}_z)} &= \psi_z \left[\frac{\theta \tilde{\alpha}_z \bar{\Pi}^{\theta-1}}{\left(1-\tilde{\alpha}_z \bar{\Pi}^{\theta-1}\right)^2}\right] \left(\sum_{z=1}^{Z} \psi_z \left[\frac{\theta \tilde{\alpha}_z \bar{\Pi}^{\theta-1}}{\left(1-\tilde{\alpha}_z \bar{\Pi}^{\theta-1}\right)^2}\right]\right)^{-1}, \\ &= \psi_z,\end{aligned}$$

where the second equality follows from the fact that $\tilde{\alpha}_z$ is constant across item categories $z = 1, \ldots Z$ and the fact that $\sum_{z=1}^{Z} \psi_z = 1$. Equation (156) can be rearranged to obtain

$$\Pi^\star = \sum_{z=1}^{Z} \psi_z m_z + O(2),$$

which is the equation stated in lemma 2, when using $m_z = \frac{g_z \gamma_z^e}{q_z \gamma^e}$.

F.4 Proof of Proposition 2

The proof proceeds in four steps. The first three steps derive the optimal inflation rate ignoring the fact that resource losses associated with menu costs may depend on the inflation rate. In particular, step 1 shows that welfare maximization is then again identical to consumption maximization and that consumption depends only via relative price distortions on inflation. Step 2 derives an auxiliary lemma showing how relative-price distortions depend on the price gap distribution, where the price gap is defined as the difference between the log relative price of the firm minus the efficient log relative price. Step 3 uses results about the price-gap distribution in the menu-cost model under alternative steady-state inflation rates from Alvarez et al. (2019), combines these with the results from the previous steps, and derives the optimal inflation rate. In step 4, we show that the optimal inflation rate thus derived either also minimizes the output losses from menu costs (condition (ii) in assumption 1) or that the resource losses associated with menu costs generate effects that are irrelevant for optimal inflation to first order (condition (i) in assumption 1).

Step 1: Equations (26)-(28) continue to hold in the menu cost, as they are derived for arbitrary price distributions. The aggregate markup distortions $\mu(\Pi)$ and the aggregate relative-price distortion $\rho(\Pi)$ continue to be defined by equations (139) and (137), respectively, but the item-level mark-ups μ_z and item-level relative-price distortions ρ_z are now the ones implied by menu-cost frictions. To account for the resource loss from menu costs, the resource constraint (29) needs to be modified to include the economy-wide menu costs $F^m(\{\kappa_z, \lambda_z\}_{z=1}^Z)$, which depend on the adjustment cost parameters κ_z and the price adjustment frequencies λ_z:

$$y = c + (\gamma^e - 1 + d)k + F^m(\{\kappa_z, \lambda_z\}_{z=1}^Z)$$

The resource cost will generally depend on the inflation rate because the price adjustment frequencies λ_z depend on inflation. Steps 1-3 of the proof ignore this dependency. It will be considered in step 4 of the proof.

From the proof of proposition 1 in appendix F.2 then follows that welfare maximization is again equivalent to consumption maximization. This is true because labor input continues to be independent of inflation as long as the mark-up distortions are inversely proportional to the relative price distortion. The latter is insured by the assumed output subsidies. Welfare then continues to be captured by equation (151), reproduced here for convenience:

$$c(\Pi) \propto \left(\frac{1}{\mu(\Pi)}\right)^{\phi}.$$

Using the definition of the aggregate markup in equation (137) and the inverse proportionality of the distortions, we have

$$c(\Pi) \propto \left(\prod_{z=1}^{Z} (\rho_z(\Pi))^{\psi_z}\right)^{\phi},$$

where the item-level relative price distortion is defined in equation (111). Steps 2 and 3 of the proof below determine the inflation that maximizes

$$c(\Pi) \propto \left(\prod_{z=1}^{Z} (\Delta_z^e/\Delta_z(\Pi))^{\psi_z}\right)^{\phi}. \tag{157}$$

Step 2 proves the following auxiliary result:

Lemma 3 *We have*

$$\ln \frac{\Delta_{zt}}{\Delta_{zt}^e} = \frac{1}{2}\theta \int_0^1 X_{jzt}(p_{jzt}^g)^2 \; dj + O(3) \tag{158}$$

where $O(3)$ denotes a third-order approximation error, p^g_{jzt} the log relative-price gap

$$p^g_{jzt} \equiv p_{jzt} - p^e_{jzt},$$

with $p_{jzt} \equiv \ln(P_{jzt}/P_{zt})$ denoting the log relative price charged by the firm and $p^e_{jzt} \equiv \ln(P^e_{jzt}/P^e_{zt})$ the efficient log relative price. The firm weights X_{jzt} are given by

$$X_{jzt} \equiv \left(\left(\frac{Q_{zt}}{G_{jzt}Q_{jzt}}\right)\frac{1}{\Delta^e_{zt}}\right)^{1-\theta}, \tag{159}$$

and satisfy

$$\int_0^1 X_{jzt}\ dj = 1.$$

Proof of lemma 3: Recall the definitions of Δ_{zt} and $1/\Delta^e_{zt}$ from equations (70) and (79), reproduced here for convenience:

$$\Delta_{zt} \equiv \int_0^1 \left(\frac{Q_{zt}}{Q_{jzt}G_{jzt}}\right)\left(\frac{P_{jzt}}{P_{zt}}\right)^{-\theta} \mathrm{dj} \tag{160}$$

$$1/\Delta^e_{zt} \equiv \left(\int_0^1 \left(\frac{Q_{jzt}G_{jzt}}{Q_{zt}}\right)^{\theta-1} \mathrm{dj}\right)^{\frac{1}{\theta-1}}, \tag{161}$$

The efficient relative price is given by[61]

$$\frac{P^e_{jzt}}{P^e_{zt}} = \left(\frac{Q_{zt}}{G_{jzt}Q_{jzt}}\right)\frac{1}{\Delta^e_{zt}}. \tag{162}$$

Using the previous equation to substitute $\frac{Q_{zt}}{Q_{jzt}G_{jzt}}$ in equation (160) delivers

$$\begin{aligned}
\frac{\Delta_{zt}}{\Delta^e_{zt}} &= \int_0^1 \frac{P^e_{jzt}}{P^e_{zt}}\left(\frac{P_{jzt}}{P_{zt}}\right)^{-\theta} \mathrm{dj} \\
&= \int_0^1 \left(\left(\frac{Q_{zt}}{G_{jzt}Q_{jzt}}\right)\frac{1}{\Delta^e_{zt}}\right)^{1-\theta}\left(\frac{P_{jzt}/P_{zt}}{P^e_{jzt}/P^e_{zt}}\right)^{-\theta} \mathrm{dj} \\
&= \int_0^1 \left(\left(\frac{Q_{zt}}{G_{jzt}Q_{jzt}}\right)\frac{1}{\Delta^e_{zt}}\right)^{1-\theta} \exp\left(-\theta[p_{jzt} - p^e_{jzt}]\right)\ \mathrm{dj} \\
&= \int_0^1 X_{jzt}\exp\left(-\theta p^g_{jzt}\right)\ \mathrm{dj}.
\end{aligned}$$

Approximating the previous equation to second order in p^g_{jzt} at the point $p^g_{jzt} = 0$, yields

$$\frac{\Delta_{zt}}{\Delta^e_{zt}} = 1 - \theta\int_0^1 X_{jzt}\exp\left(-\theta p^g_{jzt}\right)|_{p^g=0}\ p^g_{jzt}\ \mathrm{dj} + \frac{1}{2}\theta^2\int_0^1 X_{jzt}\exp\left(-\theta p^g_{jzt}\right)|_{p^g=0}\ (p^g_{jzt})^2\ \mathrm{dj} + O(3).$$

Evaluating the derivatives of the first and second-order terms in the previous equation delivers

$$\frac{\Delta_{zt}}{\Delta^e_{zt}} - 1 = -\theta\int_0^1 X_{jzt}p^g_{jzt}\ \mathrm{dj} + \frac{1}{2}\theta^2\int_0^1 X_{jzt}(p^g_{jzt})^2\ \mathrm{dj} + O(3) \tag{163}$$

[61] This can be seen be substituting the efficient price into equation (160). We then obtain $\Delta_{zt} = \Delta^e_{zt}$.

Next, we show that the first-order Taylor term in equation (163) moves only to second order. The item price level definition (15) implies

$$1 = \int_0^1 \exp((1-\theta)p_{jzt})\mathrm{dj}.$$

Using equation (162) and the definition of the relative price gap, we can express the previous equation in terms of the relative price gap:

$$1 = \int_0^1 \exp((1-\theta)p^g_{jzt}) \left(\left(\frac{Q_{zt}}{G_{jzt}Q_{jzt}} \right) \frac{1}{\Delta^e_{zt}} \right)^{1-\theta} \mathrm{dj}.$$

Using definition (159), we obtain

$$1 = \int_0^1 X_{jzt} \exp((1-\theta)p^g_{jzt}) \ \mathrm{dj}.$$

Approximating the previous equation to second order yields

$$\int_0^1 X_{jzt} \ p^g_{jzt} \ \mathrm{dj} = \frac{1}{2}(\theta-1) \int_0^1 X_{jzt}(p^g_{jzt})^2 \ \mathrm{dj} + o(3).$$

Using the previous equation to replace the first-order Taylor term in equation (163) yields

$$\frac{\Delta_{zt}}{\Delta^e_{zt}} - 1 = \frac{1}{2}\theta \int_0^1 X_{jzt}(p^g_{jzt})^2 \ \mathrm{dj} + O(3). \tag{164}$$

To obtain an approximation in terms $\ln(\frac{\Delta_{zt}}{\Delta^e_{zt}})$, we approximate $\ln(\frac{\Delta_{zt}}{\Delta^e_{zt}})$ at the point $\frac{\Delta_{zt}}{\Delta^e_{zt}} = 1$ to second order, which delivers

$$\ln\left(\frac{\Delta_{zt}}{\Delta^e_{zt}}\right) = (\Delta_{zt}/\Delta^e_{zt} - 1) - \frac{1}{2}(\Delta_{zt}/\Delta^e_{zt} - 1)^2 + O(3),$$

From equation (164) follows that $(\Delta_{zt}/\Delta^e_{zt} - 1)^2 \sim O(4)$ and can thus be ignored for the purpose of deriving a second-order approximation. Substituting equation (164) for $\Delta_{zt}/\Delta^e_{zt} - 1$ in the previous equation yields

$$\ln\left(\frac{\Delta_{zt}}{\Delta^e_{zt}}\right) = \frac{1}{2}\theta \int_0^1 X_{jzt}(p^g_{jzt})^2 \ \mathrm{dj} + O(3),$$

which is equation (158) in lemma 3.

Step 3. Since firms' menu costs are proportional to flexible price profits, the firm problem is homogenous in firm-level technology. As a result, the price gap distribution is independent of the firm-level (relative-productivity) weights X_{jzt}. Since $\int_0^1 X_{jzt}$ dj$= 1$, equation (158) in lemma 3 simplifies in a menu-cost setting to

$$\ln \frac{\Delta_{zt}}{\Delta^e_{zt}} = \frac{1}{2}\theta \int_0^1 (p^g_{jzt})^2 \mathrm{dj} + O(3).$$

Letting $f_z(p^g)$ denote the steady-state price-gap distribution of the menu-cost model, one can rewrite the previous equation in steady state as:

$$\ln \frac{\Delta_z}{\Delta^e_z} = \frac{1}{2}\theta \int (p^g)^2 f_z(p^g) \mathrm{dj} + O(3). \tag{165}$$

We now define ζ_z as the rate at which an individual firm's relative-price gap is drifting in steady state, in the absence of idiosyncratic shocks hitting the firm and in the absence of price adjustments. This rate is given by

$$\begin{aligned}\zeta_z &\equiv \ln \Pi_z - \ln(g_z/q_z) \\ &= \ln(\Pi\gamma^e/\gamma_z^e) - \ln(g_z/q_z) \\ &= \ln \Pi - \ln\left(\frac{g_z\gamma_z^e}{q_z\gamma^e}\right),\end{aligned}$$

where the second equality uses the steady-state relationship between item-level and aggregate inflation obtained from the product demand function (19).

Proposition 1 in Alvarez et al. (2019) shows that the steady-state density of price gaps in the menu-cost model for quadratic profit functions takes the form $f_z(p^g) = f(p^g|\zeta_z, \sigma_z^2, \kappa_z)$. We can thus express equation (165) as

$$\ln \frac{\Delta_z}{\Delta_z^e} = \frac{1}{2}\theta \int (p^g)^2 f(p^g|\zeta_z, \sigma_z^2, \kappa_z) dp^g + O(3)$$

Taking the log of equation (157) and using the previous expression to substitute $\ln(\Delta_z^e/\Delta_z(\Pi))$ and taking the first-order condition with respect to the optimal inflation rate $\ln \Pi$ delivers

$$\sum_z \psi_z \frac{\partial \ln(\Delta_z^e/\Delta_z))}{\partial \zeta_z} \underbrace{\frac{\partial \zeta_z}{\partial \ln \Pi}}_{\equiv 1} = 0$$

$$\sum_z \psi_z \underbrace{\frac{\partial \ln(\Delta_z^e/\Delta_z))}{\partial \zeta_z}}_{\equiv F_z(\ln \Pi, \ln\left(\frac{g_z\gamma_z^e}{q_z\gamma^e}\right))} = 0. \tag{166}$$

We have $F_z(\ln(g_z\gamma_z^e/(q_z\gamma^e)), \ln(g_z\gamma_z^e/(q_z\gamma^e))) = 0$ in the menu cost model, due to the symmetry of $f(p^g|\zeta_z, \sigma_z^2, \kappa_z)$ in the sense that $f(p^g|\zeta_z, \sigma_z^2, \kappa_z) = f(-p^g|-\zeta_z, \sigma_z^2, \kappa_z)$, the symmetry of $(p^g)^2$ around zero, the assumed differentiability of $f(p^g|\zeta_z, \sigma_z^2, \kappa_z)$ at the point of approximation ($\zeta_z = 0$), and the assumption that we can work with a quadratic profit function, see proposition 1 in in Alvarez et al. (2019). This implies that equation (166) holds at the point $\ln \Pi = \ln\left(\frac{g_z\gamma_z^e}{q_z\gamma^e}\right) = \bar{m}$, i.e., at the point of approximation in proposition 2. We can thus use the implicit function theorem to approximate the optimal solution of (166) to first order around the point $\ln \Pi = \ln\left(\frac{g_z\gamma_z^e}{q_z\gamma^e}\right) = \bar{m}$. This delivers

$$\ln \Pi = \bar{m} - \sum_z \psi_z \underbrace{\left.\frac{\partial F_z(\ln \Pi, \ln\left(\frac{g_z\gamma_z^e}{q_z\gamma^e}\right))/\partial \ln\left(\frac{g_z\gamma_z^e}{q_z\gamma^e}\right)}{\sum_{\tilde{z}} \psi_{\tilde{z}} \partial F_{\tilde{z}}(\ln \Pi, \ln\left(\frac{g_{\tilde{z}}\gamma_{\tilde{z}}^e}{q_{\tilde{z}}\gamma^e}\right))/\partial \ln \Pi}\right|_{\ln \Pi = \ln\left(\frac{g_z\gamma_z^e}{q_z\gamma^e}\right) = \bar{m}}}_{\equiv \widetilde{F}_z} (\ln\left(\frac{g_z\gamma_z^e}{q_z\gamma^e}\right) - \bar{m}) + O(2),$$

which exploits the fact that $\partial F_z(\ln\Pi, \ln\left(\frac{g_z\gamma_z^e}{q_z\gamma^e}\right))/\partial x = 0$ for $x = \{\kappa_z, \sigma_z^2, \delta_z\}$, as $\partial \ln(\Delta_z^e/\Delta_z))/\partial\zeta_z = 0$ holds independently of the considered values for $(\kappa_z, \sigma_z^2, \delta_z)$. For this reason, we do not get first-order contributions from heterogeneity in $(\kappa_z, \sigma_z^2, \delta_z)$.

From the definition of $F_z(\ln\Pi, \ln(g_z\gamma_z^e/(q_z\gamma^e)))$ follows that $\widetilde{F}_z = -1$ at the point of approximation, because the derivatives

$$\frac{\partial F_z(\ln\Pi, \ln(g_z\gamma_z^e/q_z\gamma^e))}{\partial\ln(g_z\gamma_z^e/q_z\gamma^e)}$$

are identical for all z at the point of approximation and

$$\partial F_z(\ln\Pi, \ln\left(\frac{g_z\gamma_z^e}{q_z\gamma^e}\right))/\partial\ln\Pi = -\partial F_z(\ln\Pi, \ln\left(\frac{g_z\gamma_z^e}{q_z\gamma^e}\right))/\partial\ln\left(\frac{g_z\gamma_z^e}{q_z\gamma^e}\right).$$

We thus obtain

$$\ln\Pi = \sum_z \psi_z \ln\left(\frac{g_z\gamma_z^e}{q_z\gamma^e}\right) + O(2).$$

The previous equation is to first order equal to

$$\Pi = \sum_z \psi_z \left(\frac{g_z\gamma_z^e}{q_z\gamma^e}\right) + O(2), \tag{167}$$

which is the result stated in the proposition. It now remains to show that it continuos to hold once we also take into account the resource effects from menu costs.

Step 4: We now consider the additional effects arising from the dependency of the resource loss associated with menu costs on the inflation rate. When condition (i) in assumption 1 holds, then menu costs vary only to third order with inflation. This is so because menu costs themselves are of first order, but the adjustment frequency λ_z moves only to second order with inflation. This is so because $\partial\lambda_z/\partial\ln\Pi = 0$ at the point of approximation, see proposition 1 in Alvarez et al. (2019). Menu cost then do not matter for optimal inflation to first order, as only effects that move allocations to second or lower order are relevant. Result (167) thus continues to apply.

When condition (ii) in assumption 1 holds, then menu costs move allocations to second order. To see this, write the adjustment frequency as $\lambda_z(\zeta_z)$ where $\zeta_z = \ln\Pi - \ln\frac{g_z}{q_z}\frac{\gamma_z^e}{\gamma_z}$.[62] The second-order approximation of menu costs with respect to inflation around the point of approximation is given by

$$\begin{aligned}
F^m(\{\kappa_z,\lambda_z\}_{z=1}^Z) = F^m &+ \frac{1}{2}\sum_z \frac{\partial F^m(\{\kappa_z,\lambda_z\}_{z=1}^Z)}{\partial\lambda_z}\frac{\partial^2\lambda_z}{(\partial\ln\Pi)^2}(\ln\Pi - \overline{m})^2 \\
&+ \frac{1}{2}\sum_z \frac{\partial F^m(\{\kappa_z,\lambda_z\}_{z=1}^Z)}{\partial\lambda_z}\frac{\partial^2\lambda_z}{(\partial\ln\Pi)^2}(\ln\frac{g_z}{q_z}\frac{\gamma_z^e}{\gamma_z} - \overline{m})^2 \\
&- \sum_z \frac{\partial F^m(\{\kappa_z,\lambda_z\}_{z=1}^Z)}{\partial\lambda_z}\frac{\partial^2\lambda_z}{(\partial\ln\Pi)^2}(\ln\Pi - \overline{m})(\ln\frac{g_z}{q_z}\frac{\gamma_z^e}{\gamma_z} - \overline{m}) \\
&+ O(3),
\end{aligned}$$

[62]The adjustment frequency also depends on other parameters, i.e., $(\kappa_z, \sigma_z^2, \delta_z)$. We capture depencency on these parameters in nonlinear form through the subscript z in λ_z.

where we used once more $\partial\lambda_z/\partial \ln \Pi = 0$, which causes all first-order terms and some second-order terms to disappear, and the fact that $\partial\lambda_z/\partial \ln \Pi \equiv -\ \partial\lambda_z/\partial \ln(\frac{g_z}{q_z}\frac{\gamma_z^e}{\gamma_z})$. Using $\frac{\partial F^m(\{\kappa_z,\lambda_z\}_{z=1}^Z)}{\partial\lambda_z}\frac{\partial^2\lambda_z}{(\partial \ln \Pi)^2} \propto \psi_z$ and the fact that $\sum_z \psi_z = 1$, the first-order condition of the previous equation with respect to $\ln \Pi$ shows that adjustment costs are minimized for

$$\ln \Pi = \sum_z \psi_z \left(\ln \frac{g_z}{q_z}\frac{\gamma_z^e}{\gamma_z} \right) + O(2),$$

which is to first order equal to (167). The optimal inflation rate (167) thus not only maximizes consumption for a given amount of labor input, as shown in steps 1-3 of the proof, but also minimizes the resource loss from price adjustments and thus total hours worked for a given amount of consumption. Under condition (ii) in assumption 1, the inflation rate (167) thus maximizes steady-state utility with respect to consumption and labor.

F.5 Proof of Proposition 3

Taking the natural logarithm of the equation (101), which describes the optimal reset price, yields

$$\ln \frac{P^\star_{jzt}}{P_{zt}} = \ln\left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\right) - \ln\left(\frac{Q_{jzt}G_{jzt}}{Q_{zt}}\right) + \ln\left(\frac{n_{zt}}{p_{zt}d_{zt}}\right). \tag{168}$$

We rearrange the term $\ln(Q_{jzt}G_{jzt}/Q_{zt})$ in the previous equation for $s_{jzt} \geq 1$ as

$$\begin{aligned}
\ln\left(\frac{Q_{jzt}G_{jzt}}{Q_{zt}}\right) &= \ln(\epsilon^G_{jzt}\epsilon^Q_{jzt}) + \ln\left(\frac{Q_{zt-s}\overline{G}_{jzt}}{Q_{zt}}\right) \\
&= \ln(\epsilon^G_{jzt}\epsilon^Q_{jzt}) + \ln\left(\frac{\prod_{k=0}^{s_{jzt}-1} g_{zt-k}}{\prod_{k=0}^{s_{jzt}-1} q_{zt-k}}\right) \\
&= \ln(\epsilon^G_{jzt}\epsilon^Q_{jzt}) + \ln\left(\frac{g_z}{q_z}\right)\cdot s_{jzt} + \sum_{i=t-s_{jzt}+1}^{t} \left(\ln \epsilon^g_{zi} - \ln \epsilon^q_{zi}\right).
\end{aligned} \tag{169}$$

where the first equality follows from using equations (7) and (11), the second equality follows from using equations (9) and (12), the third equality follows from using equations (10) and (13), and and where $\ln(\epsilon^G_{jzt}\epsilon^Q_{jzt})$ denotes the product-fixed effect. For the case with $s_{jzt} = 0$, we obtain $\ln(Q_{jzt}G_{jzt}/Q_{zt}) = \ln(\epsilon^G_{jzt}\epsilon^Q_{jzt})$. Substituting the equation (169) into equation (168) yields equation (36) in the proposition, where we have defined

$$f^\star_{jz} \equiv \ln\left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\frac{n_z}{p_z d_z}\right) - \ln(\epsilon^G_{jzt}\epsilon^Q_{jzt}) \tag{170}$$

$$u^\star_{jzt} \equiv \ln\left(\frac{n_{zt}}{p_{zt}d_{zt}}\frac{p_z d_z}{n_z}\right) - \sum_{i=t-s_{jzt}+1}^{t} \left(\ln \epsilon^g_{zi} - \ln \epsilon^q_{zi}\right), \tag{171}$$

and $E[u^\star_{jzt}] = 0$ holds because by assumption $E \ln \epsilon^g_{zt} = 0$ and $E \ln \epsilon^q_{zt} = 0$ and $\ln\left(\frac{n_{zt}}{p_{zt}d_{zt}}\frac{p_z d_z}{n_z}\right)$ denotes the percentage deviation of stationary variables from their steady state values.

F.6 Relative Price Regression Using all Prices (Equation 38)

As proven below, the intercepts and residuals of regression (38) satisfy the following properties:

Proposition 5 *The evolution of the relative product price in all periods, including adjustment periods, is described by equation (38), where*

$$f_{jz} = f^{\star}_{jz} + \bar{u}_z,$$

with $f^{\star}_{jz}$ being defined in equation (170) and

$$\bar{u}_z = -\frac{\alpha_z}{1-\alpha_z}[E \ln \Pi_{zt} - \ln(g_z/q_z)]. \tag{172}$$

For products with age $s_{jzt} > 0$, we have

$$u_{jzt} = \begin{cases} u^{\star}_{jzt} - \bar{u}_z & \textit{in price adjustment periods,} \\ u_{jz,t-1} + \ln(g_z/q_z) - \ln \Pi_{zt} & \textit{otherwise,} \end{cases} \tag{173}$$

where $u^{\star}_{jzt}$ is defined in equation (171). For new products with $s_{jzt} = 0$, we have

$$u_{jzt} = u^{\star}_{jzt} - \bar{u}_z,$$

where

$$u^{\star}_{jzt} \equiv \ln\left(\frac{n_{zt}}{p_{zt}d_{zt}} \frac{p_z d_z}{n_z}\right).$$

Given the results in the previous proposition, we can compute the unconditional mean of u_{jzt}. Rewrite equation (173) as

$$u_{jzt} = \xi_{jzt}[u_{jz,t-1} + \ln(g_z/q_z) - \ln \Pi_{zt}] + (1-\xi_{jzt})(u^{\star}_{jzt} - \bar{u}_z),$$

where the product-specific, idiosyncratic, and independent Poisson process ξ_{jzt} captures the price adjustment process: ξ_{jzt} is equal to zero with probability $1-\alpha_z$ and equal to one otherwise. Given the independence of ξ_{jzt} from $u_{jz,t-1}$, Π_{zt} and $u^{\star}_{jzt}$, we obtain

$$\begin{aligned} E[u_{jzt}] &= E[\xi_{jzt}]E[u_{jz,t-1} + \ln(g_z/q_z) - \ln \Pi_{zt}] + E[u^{\star}_{jzt} - \bar{u}_z] - E[\xi_{jzt}]E[u^{\star}_{jzt} - \bar{u}_z] \\ &= \alpha_z \left(E[u_{jz,t-1}] + \ln(g_z/q_z) - E[\ln \Pi_{zt}]\right) + (1-\alpha_z)E[u^{\star}_{jzt} - \bar{u}_z]. \end{aligned}$$

Since u_{jzt} is a stationary process, we have $E[u_{jzt}] = E[u_{jz,t-1}]$. Since $E[u^{\star}_{jzt}] = 0$, see proposition 3, we obtain from the previous equation and equation (172) that

$$E[u_{jzt}] = -\frac{\alpha_z}{1-\alpha_z}[E \ln \Pi_{zt} - \ln(g_z/q_z)] - \bar{u}_z = 0,$$

as claimed in the text.

Proof. We start by deriving the evolution of the modified residual u_{jzt}. Let the sticky price in t be equal to the optimal price set $k \geq 0$ periods ago, $P_{jzt} = P^{\star}_{jz,t-k}$, where $k \leq s_{jzt}$. Then, we can rewrite equation (38) as

$$\ln \frac{P^{\star}_{jz,t-k}}{P_{z,t-k}} + \ln \frac{P_{z,t-k}}{P_{zt}} = f_{jz} - \ln\left(\frac{g_z}{q_z}\right) \cdot (k + s_{jz,t-k}) + u_{jzt},$$

or equivalently

$$\ln \frac{P^\star_{jz,t-k}}{P_{z,t-k}} + \ln \frac{P_{z,t-k}}{P_{zt}} = f_{jz} - \bar{u}_z - \ln\left(\frac{g_z}{q_z}\right) \cdot (k + s_{jz,t-k}) + u_{jzt} + \bar{u}_z.$$

Defining $f_{jz} - \bar{u}_z = f^\star_{jz}$, the previous equation is equal to the reset price equation (36) shifted k periods into the past, i.e.,

$$\ln \frac{P^\star_{jz,t-k}}{P_{z,t-k}} = f^\star_{jz} - \ln\left(\frac{g_z}{q_z}\right) \cdot s_{jz,t-k} + u^\star_{jz,t-k},$$

where u_{jzt} is given by

$$u_{jzt} = u^\star_{jz,t-k} - \bar{u}_z + \ln\left(\frac{g_z}{q_z}\right) \cdot k - \ln \frac{P_{zt}}{P_{z,t-k}}. \tag{174}$$

For $k = 0$, we have $u_{jzt} = u^\star_{jzt} - \bar{u}_z$. For $k \geq 1$, we can derive a recursive representation. Equation (174) then also holds in period $t-1$, where the age of the price is $k-1$, so that

$$\begin{aligned} u_{jz,t-1} &= u^\star_{jz,t-k} - \bar{u}_z + \ln\left(\frac{g_z}{q_z}\right) \cdot (k-1) - \ln \frac{P_{z,t-1}}{P_{z,t-k}} \\ &= u_{jzt} - \ln\left(\frac{g_z}{q_z}\right) - \ln \frac{P_{z,t-1}}{P_{zt}}. \end{aligned}$$

The last line follows from equation (174). Rewriting the previous equation yields the postulated recursive law of motion of the residual u_{jzt} for non-adjustment periods:

$$u_{jzt} = u_{jz,t-1} + \ln(g_z/q_z) - \ln \Pi_{zt}.$$

■

F.7 Derivation of Equation (40)

The not-quality adjusted price level of item z, defined in equation (39), can be decomposed as follows:

$$\widetilde{P}^{1-\theta}_{zt} = \delta_z(\widetilde{P}^\star_{z,t,t})^{1-\theta} + (1-\alpha_z)\delta_z \sum_{s=1}^{\infty}(1-\delta_z)^s(\widetilde{P}^\star_{z,t-s,t})^{1-\theta} + \alpha_z(1-\delta_z)(\widetilde{P}_{zt-1})^{1-\theta}, \tag{175}$$

where the average optimal (not-quality adjusted) price of new products entering in t is given by

$$\widetilde{P}^\star_{z,t,t} \equiv \left(\frac{1}{\delta_z}\int_{J^\star_{t,t}} (\widetilde{P}^\star_{jzt})^{1-\theta} \mathrm{dj}\right)^{\frac{1}{1-\theta}}, \tag{176}$$

and the average optimal (not-quality adjusted) price of continuing products with age $s \geq 1$ is given by

$$\widetilde{P}^\star_{z,t-s,t} \equiv \left(\frac{1}{(1-\alpha_z)\delta_z(1-\delta_z)^s}\int_{J^\star_{t-s,t}} (\widetilde{P}^\star_{jzt})^{1-\theta} \mathrm{dj}\right)^{\frac{1}{1-\theta}}. \tag{177}$$

To obtain a recursive representation of equation (175), we derive the equation corresponding to equation (60) for the case without quality adjustment. This yields

$$\widetilde{P}^\star_{z,t-s,t} = \left(\prod_{k=0}^{s-1} g_{zt-k}\right)^{-1} \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right) \frac{N_{zt}P_t}{D_{zt}} Q_{zt}. \tag{178}$$

For the special case $s = 0$, we have

$$\widetilde{P}^{\star}_{z,t,t} = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{N_{zt}P_t}{D_{zt}}Q_{zt}. \tag{179}$$

Dividing equation (178) by equation (179) yields

$$\widetilde{P}^{\star}_{z,t-s,t} = \widetilde{P}^{\star}_{z,t,t}\left(\prod_{k=0}^{s-1} g_{zt-k}\right)^{-1}. \tag{180}$$

Using the previous equation to substitute for $P^{\star}_{z,t-s,t}$ in equation (175) yields

$$\widetilde{P}^{1-\theta}_{zt} = (\widetilde{P}^{\star}_{z,t,t})^{1-\theta}\left\{\delta_z + (1-\alpha_z)\sum_{s=1}^{\infty}\delta_z(1-\delta_z)^s\left(\prod_{k=0}^{s-1} g_{zt-k}\right)^{\theta-1}\right\} + \alpha_z(1-\delta_z)(\widetilde{P}_{zt-1})^{1-\theta},$$

which can be rearranged to obtain

$$\widetilde{P}^{1-\theta}_{zt} = \left\{\alpha_z\delta_z + (1-\alpha_z)(\widetilde{\Delta}^e_{zt})^{1-\theta}\right\}(\widetilde{P}^{\star}_{z,t,t})^{1-\theta} + \alpha_z(1-\delta_z)(\widetilde{P}_{zt-1})^{1-\theta}, \tag{181}$$

where the stationary variable $\widetilde{\Delta}^e_{zt}$ is given by

$$(\widetilde{\Delta}^e_{zt})^{1-\theta} = \delta_z + (1-\delta_z)(\widetilde{\Delta}^e_{zt-1}/g_{zt})^{1-\theta}. \tag{182}$$

In order to relate P_{zt} in equation (65) to $\widetilde{P}_{zt}$ in equation (181), we derive the mapping between $P^{\star}_{z,t,t}$ and $\widetilde{P}^{\star}_{z,t,t}$. In particular, dividing equation (179) by equation (61) and taking growth rates yields

$$\frac{\widetilde{P}^{\star}_{z,t,t}}{\widetilde{P}^{\star}_{z,t-1,t-1}} = \frac{Q_{zt}}{Q_{z,t-1}}\frac{P^{\star}_{z,t,t}}{P^{\star}_{z,t-1,t-1}}, \tag{183}$$

which shows that in item z, the growth rates of the average optimal price of newly entering products with and without quality adjustment are related via quality growth.

The steady-state version of equation (181) can be rearranged to obtain

$$\begin{aligned}(\widetilde{\Pi}_z\widetilde{P}_{z,t-1})^{1-\theta} = &\left\{\alpha_z\delta_z + (1-\alpha_z)(\widetilde{\Delta}^e_z)^{1-\theta}\right\}\left(\frac{\widetilde{P}^{\star}_{z,t,t}}{\widetilde{P}^{\star}_{z,t-1,t-1}}\widetilde{P}^{\star}_{z,t-1,t-1}\right)^{1-\theta} \\ &+ \alpha_z(1-\delta_z)(\widetilde{\Pi}_z\widetilde{P}_{z,t-2})^{1-\theta},\end{aligned}$$

For equation (184) to be consistent with equation (181), it must hold that

$$\widetilde{\Pi}_z = \widetilde{P}^{\star}_{z,t,t}/\widetilde{P}^{\star}_{z,t-1,t-1}. \tag{184}$$

Similar reasoning for the item price level with quality adjustment yields

$$\Pi_z = P^{\star}_{z,t,t}/P^{\star}_{z,t-1,t-1}. \tag{185}$$

Using equations (184) and (185) to rewrite equation (183) in the steady state yields equation (40) in the main text.

F.8 Proof of Proposition 4

Consider a steady state and use equation (14) to replace in equation (101) the quality-adjusted reset price $P^{\star}_{jzt}$ by $\widetilde{P}^{\star}_{jzt}/Q_{jzt}$. This yields

$$\frac{\widetilde{P}^{\star}_{jzt}}{\widetilde{P}_{zt}}\frac{\widetilde{P}_{zt}}{P_{zt}}\frac{1}{Q_{jzt}}\left(\frac{Q_{jzt}G_{jzt}}{Q_{zt}}\right)=\left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\right)\frac{n_z}{p_z d_z}.$$

Taking the natural logarithm of the previous equation and using equation (169) to substitute for $\ln(Q_{jzt}G_{jzt}/Q_{zt})$ in the steady state yields

$$\ln\frac{\widetilde{P}^{\star}_{jzt}}{\widetilde{P}_{zt}}=\ln\left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\frac{n_z}{p_z d_z}\right)+\ln\left(\frac{Q_{jzt}}{\epsilon^G_{jzt}\epsilon^Q_{jzt}}\right)-\ln\left(\frac{g_z}{q_z}\right)\cdot s_{jzt}+\ln\left(\frac{P_{zt}}{\widetilde{P}_{zt}}\right). \tag{186}$$

Steady-state relative item price levels evolve as

$$\begin{aligned}\ln(P_{zt}/\widetilde{P}_{zt}) &= (t+1)\cdot\ln(\Pi_z/\widetilde{\Pi}_z)+\ln(P_{z,-1}/\widetilde{P}_{z,-1})\\ &= -(t+1)\cdot\ln(q_z)+\ln(P_{z,-1}/\widetilde{P}_{z,-1})\\ &= -\ln(q_z)\cdot s_{jzt}-(t-s_{jzt}+1)\cdot\ln(q_z),\end{aligned}$$

where the second equality follows from equation (40) and the third equality uses the initial condition $P_{z,-1}/\widetilde{P}_{z,-1}=1$, without loss of generality. Using the previous equation to substitute for the ratio of item price levels in equation (186) yields

$$\ln\frac{\widetilde{P}^{\star}_{jzt}}{\widetilde{P}_{zt}}=\ln\left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\frac{n_z}{p_z d_z}\right)+\ln\left(\frac{Q_{jzt}}{\epsilon^G_{jzt}\epsilon^Q_{jzt}}\right)-(t-s_{jzt}+1)\cdot\ln(q_z)-\ln\left(\frac{g_z}{q_z}\right)\cdot s_{jzt}-\ln(q_z)\cdot s_{jzt}. \tag{187}$$

Defining the product-fixed effect as[63]

$$\widetilde{f}^{\star}_{jz}\equiv\ln\left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\frac{n_z}{p_z d_z}\right)+\ln\left(\frac{Q_{jzt}}{\epsilon^G_{jzt}\epsilon^Q_{jzt}}\right)-(t-s_{jzt}+1)\cdot\ln(q_z)$$

shows that equation (187) is equivalent to equation (42) in the proposition.

F.9 Imperfect Quality Adjustment: Deriving Equations (41) and (43)

To derive equation (41), we define the price level for the case without quality adjustment as

$$\widetilde{P}_t=\prod_{z=1}^{Z_t}\left(\widetilde{P}_{zt}/\psi_{zt}\right)^{\psi_{zt}},$$

analogously to equation (16). Taking growth rates of the previous equation and using equation (40) to substitute for $\widetilde{\Pi}_z$ in the steady state yields

$$\widetilde{\Pi}=\prod\nolimits_{z=1}^{Z}(q_z\Pi_z)^{\psi_z}.$$

[63] Recall that $t-s_{jzt}$ is constant over the product lifetime.

Taking the natural logarithm of the previous equation and using $\ln \Pi = \sum_{z=1}^{Z} \psi_z \ln \Pi_z$, which follows from equation (16), yields equation (41).

To derive equation (43), we rewrite the equation in Lemma 2, which holds to first order at the approximation point $(\bar{\Pi}, \bar{m}_z)$, with $\bar{\Pi} = \bar{m}_z$ and $m_z = \frac{g_z \gamma_z^e}{q_z \gamma^e}$, using

$$\Pi^\star = \bar{\Pi} + \bar{\Pi}(\ln \Pi^\star - \ln \bar{\Pi}) + O(2)$$
$$m_z = \bar{m}_z + \bar{m}_z(\ln m_z - \ln \bar{m}_z) + O(2),$$

to substitute for $\Pi^\star$ and m_z, respectively. This yields

$$\ln \Pi^\star - \ln \bar{\Pi} = \sum_{z=1}^{Z} \psi_z \left(\ln m_z - \ln \bar{m}_z\right) + O(2),$$

which after simplifying is equivalent to equation (43).

G Data Appendix

G.1 ONS Methodology for Constructing Quality-Adjusted Item-Level Price Indices

ONS constructs quality-adjusted item price indices using a three step approach that we describe below. Quality adjustment is thereby implemented via adjustment of the so-called base price of the product. The base prices are part of our data set. In the absence of quality adjustment, the base price is simply the price at which the product sells in the base price period, i.e., in January of each year. Product price increases (within any year) are then computed with respect to the previous January price.

If ONS adjusts prices for quality, then the base price differs from the January price. In particular, the base price is then higher (lower) when the quality of the product is judged higher (lower). Such quality changes typically only happen at times of product substitution. When ONS can place a value on the quality difference between the previous product and the replacement product, it uses the so-called direct quality adjustment method to adjust the base price in proportion to the quality change. For example, when the package size of a product changes permanently, ONS price collectors find in each outlet the nearest equivalent new size of the product priced in this outlet and adjust the base price proportionally to reflect the changed product quantity. In other cases, base prices are compute via imputation methods or hedonistic regressions. Whatever is the actual method used to arrive at base prices, we can implement the same quality-adjustment as ONS because the base prices are part of our data set.

We now briefly describe each of the three steps of index construction. We refer to Office for National Statistics (2014) for a more detailed description.

In the first step, ONS uses internal plausibility and cross-checking procedures to flag price quotes it considers invalid and then removes these quotes from the data set before computing price indices. ONS removes, for example, price quotes which belong to a non-comparable substitution in the month in which the substitution occurs and in the subsequent month. Similarly, ONS removes price quotes with an invalid base price. We restrict our sample to validated price quotes (see table 1).[64]

In the second step, ONS computes one or more stratum indices in each item category. To this end, ONS stratifies valid price quotes into stratum cells according to the type of shop (shops with ten or more outlets versus shops with less than ten outlets) and/or the region from which price quotes were sampled (ONS considers thirteen regions). In a given month, a stratum index comprises all valid price quotes in the stratum cell. The stratum index $\widetilde{I}_{kzt}$ for stratum cell k in month t of item z is given by[65]

$$\widetilde{I}_{kzt} = \exp\left[\frac{1}{\sum_{j\in J_{kz}} w_{jkzt}}\left(\sum_{j\in J_{kz}} w_{jkzt}\ln\left(\frac{P_{jkzt}}{P_{jkzb}}\right)\right)\right], \tag{188}$$

where J_{kz} denotes the set of products belonging to stratum cell k in item z and w_{jkzt} the weight of product j in stratum cell k at date t. This weight is a so-called ‘replication factor’ that represents the relative number of times that a price relative P_{jkzt}/P_{jkzb} is meant to appear in the stratum index. Here, P_{jkzb} denotes the base price, i.e., the price of the product in the base month (January), unless ONS applies a quality adjustment, as discussed above.

In the third step, ONS computes the price index for the item category. In a given month of a year, the item index is equal to the weighted sum of stratum indices available in this month in this category. Specifically, the item-level price index $\widetilde{I}_{zt}$ of item z in month t is given by

$$\widetilde{I}_{zt} = \sum_{k=1}^{K}\left(\frac{w_{kzt}}{\sum_{k'} w_{k'zt}}\right)\widetilde{I}_{kzt}, \tag{189}$$

where K denotes the number of stratum cells[66] and w_{kzt} the expenditure weight attached to stratum cell k in month t. ONS updates the expenditure weights annually.

Since $\widetilde{I}_{zt}$ represents the index increase between January (the base month) and month t of the same year, the within year item indices $\widetilde{I}_{zt}$ need to be chained together to obtain a consistent multi-year index series I_{zt}.

[64] In addition, we erase 201 validated price quotes for which the base price is exactly equal to 0.0004 GBP. This base price is clearly implausible on a priori grounds. Furthermore and contrary to previous studies focusing on the price change distribution, we also keep the validated price quotes that contain the VAT changes in December 2008, January 2010 and January 2011. Dropping all price quotes in a January would make it infeasible to construct chained item price indices. We also keep validated price quotes in May 2005 in our baseline sample even though May 2005 is a month in which unusually many nominal price quotes are equal to their value in January 2005. Our results are robust to excluding price quotes in May 2005 from the analysis. Finally, we also keep the validated price quotes in January 1999 in our baseline sample, even though unusually large replication errors arise in this month for some of the item indices that we recompute.

[65] The stratum index is also multiplied by 100, which we abstract from here.

[66] The number of stratum cells K varies over time and items. The reason for the time variation is that stratification varies over time. For instance, products in item z may not be stratified initially but at some point in time may be stratified.

G.2 Item Indices Without Duplicate Price Quotes

As described in section 3, our analysis requires us to track individual products and their relative price trajectories over the product life. Some of the product identifiers we construct contain duplicate price quotes for the same month because ONS does not disclose all location information of a price quote.[67] For our analysis, we discard all price quotes belonging to the product identifiers with duplicate price quotes.

When then recompute item indices using official ONS methodology (see appendix G.1), discarding products with duplicate price quotes, and compare the recomputed item indices with the official ONS item indices.

We consider a recomputed item index as sufficiently accurate, whenever the root mean squared error (RMSE) of the log difference between the recomputed and the official index is below 2%,

$$RMSE_z = \sqrt{\frac{1}{T_z}\sum_{T_z}[\ln\left(\widetilde{I}^O_{zt}\right) - \ln\left(\widetilde{I}_{zt}\right)]^2} < 0.02,$$

where $\widetilde{I}^O_{zt}$ denotes the official ONS index of item category z in month t, $\widetilde{I}_{zt}$ the recomputed item index and T_z the sample period for which both indices display non-missing values. We also require that recomputed item indices do not display temporarily missing values. We find that 1093 of the 1233 item categories fulfill these requirements.[68] These 1093 item categories constitute our baseline sample.

Panel A in figure 11 depicts the distribution of RMSEs for all 1233 item categories. RMSEs are generally low: the median (mean) error is equal to 0.006 (0.0079). Pairwise correlations between recomputed and official ONS item indices in Panel B typically exceed 0.95 and the median (mean) correlation is equal to 0.984 (0.972).[69] Panel C in figure 11 depicts the RMSE (the upward-sloping line) and the correlations for all items with an RMSE<0.02. It shows that for the vast majority of items that satisfy RMSE<0.02, we have a high correlation (above 0.9). Only few items display a somewhat lower correlation.

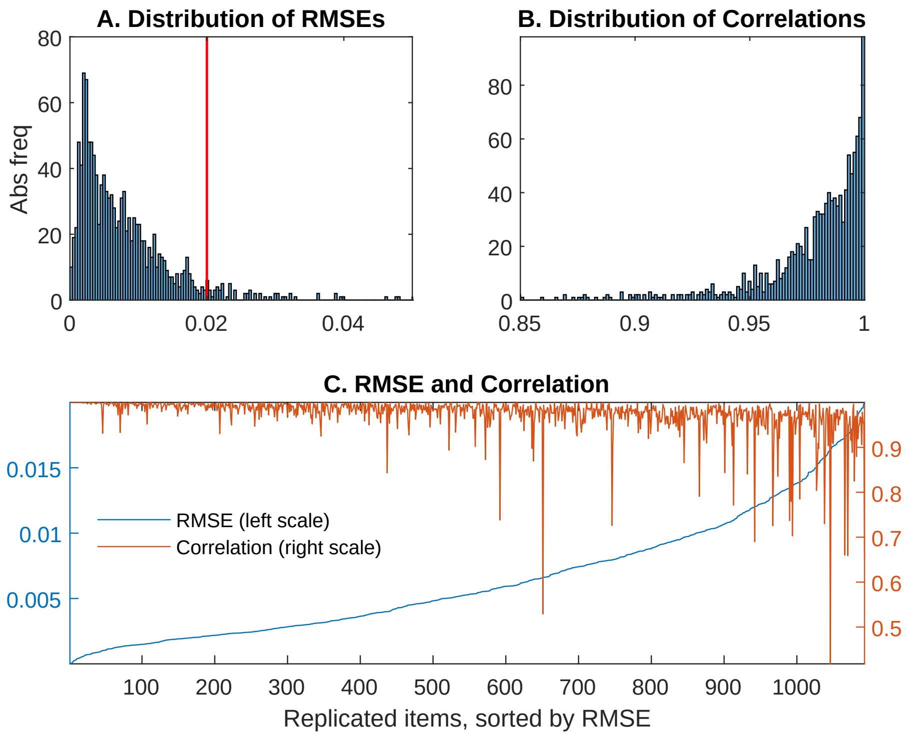
Figure 12 further illustrates the properties of the 1093 recomputed item indices in our baseline sample. Panel A shows that the numbers of recomputed and ONS item indices evolve in parallel and

[67] We construct the ONS product identifier as the tuple consisting of item ID, region, shop code, shop type, and stratum type. The "item ID" is a six digit reference number which can be used to allocate each price quote in a particular item category to its constituent COICOP classification. The "region" is equal to one of thirteen region classifications. The "shop code" denotes the outlet code from which the individual price quote was obtained. The "shop type" discriminates shops with ten or more outlets versus shops with less than ten outlets. The "stratum type" is equal to "not stratified", "stratified by region", "stratified by region and shop type" or "stratified by shope type". These variables are contained in the ONS meta data.

[68] In particular, 68 of the recomputed indices do not fulfill the RMSE criterion. Another 72 of the recomputed item indices fulfill the RMSE criterion but display temporarily missing values. We exclude these indices, which often refer to seasonal products for which prices are missing in certain months in each year, to avoid complications when chaining item indices with missing values in the month of January.

[69] Correlations are meaningful statistics because at this stage of the analysis, the base period of item indices corresponds to the month of January in the current year.

Figure 11: Recomputed and Official ONS Item Indices



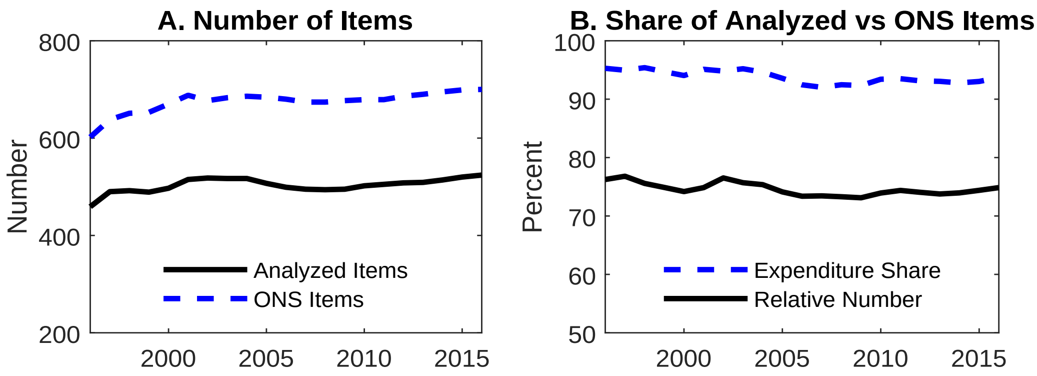
tend to both increase over the sample period. For the item categories in our baseline sample, the implied annual entry and exit rates are equal to 6.02% and 5.37%, respectively, which indicates fairly modest turnover at the item category level.[70] Furthermore, only about half of the 1093 recomputed item indices are present in the average year (503 out of 1093). The same pattern is present when considering all ONS item indices for which micro price data is available (675 out of 1233). Panel B reports the relative number and the expenditure share of items in our baseline sample relative to the full ONS sample. It shows that the baseline sample covers around 75% of the available items and 94% of the expenditure share.

G.3 Further Evidence on the Tails of the Relative Price Trends Distribution

Table 6 presents information on the tails of the relative price trend distribution from figure 3. It lists the 15 items with the most positive and most negative relative price trends that have at least an expenditure weight of 0.15%. The table shows that the largest rates of price declines are recorded for products that display a certain news value, i.e., fashion and entertainment products, as well as consumer electronics, see also Ueda et al. (2019) for an in-depth discussion of this issue. For most

[70]The entry rate is the share of item categories newly introduced in the current year, relative to all item categories present in this year. The exit rate is the share of item categories present in the previous year but no longer present in this year. Item turnover primarily reflects decisions taken at ONS, which are often determined by methodological changes or data production requirements such as keeping the number of items in the basket roughly steady over time.

Figure 12: Number, Share and Spell Duration of Analyzed Items



of the items displaying positive relative price trends, the relative price increase remains well below 1% per year. The most positive relative price trend is observed for a luxury product.

G.4 The Quality-Adjusted Item Price Level

This appendix describes how we compute the quality-adjusted item price levels P_{zt} used in regression (1) from the micro price data.

Since we cannot use all price observations underlying the official item-price index (due to problems with duplicates and other issues discussed in section 3.1), we compute item price levels using only the micro price observations that we actually use in the regressions. We show how this price level can be computed such that it is both consistent with the theory and consistent with the way ONS computes the price level (to a first-order approximation).

Using the theory equations (14), (18) and (20), we can write the item price level in equation (15) as

$$P_{zt} = \int_0^1 \frac{Y_{jzt}}{Y_{zt}} P_{jzt} \, \mathrm{dj} \; .$$

Dividing the previous equation by P_{zb}, which is the item price level in the base period b, and augmenting the integrand, we obtain

$$\frac{P_{zt}}{P_{zb}} = \int_0^1 w_{jzb} \, \frac{Y_{jzt}Y_{zb}}{Y_{zt}Y_{jzb}} \, \frac{P_{jzt}}{P_{jzb}} \mathrm{dj}, \tag{190}$$

where P_{jzb} denotes the price of product j in base period b, which also reflects quality adjustments made by ONS (see Appendix G.1), and $w_{jzb} \equiv \frac{P_{jzb}Y_{jzb}}{P_{zb}Y_{zb}}$ denotes the expenditure weight of product j in the base period, with weights satisfying $\int w_{jzb} \, \mathrm{dj} \; = 1$. The product demand function in equation (18) implies

$$\frac{Y_{jzt}Y_{zb}}{Y_{zt}Y_{jzb}} = \left(\frac{P_{jzt}P_{zb}}{P_{zt}P_{jzb}}\right)^{-\theta} .$$

Substituting the previous equation into (190) yields

$$\frac{P_{zt}}{P_{zb}} = \left(\int_0^1 w_{jzb} \left(\frac{P_{jzt}}{P_{jzb}}\right)^{1-\theta} \mathrm{dj}\right)^{\frac{1}{1-\theta}} . \tag{191}$$

Table 6: Top and Bottom Rates of Relative Price Change

Item Description	Relative Price Change (in % per year)	Exp. Weight (in %)
	Relative Price Increase	
HIFI - 2007	3.28	0.15
WIDESCREEN TV - 2005	2.55	0.31
CAMCORDER-8MM OR VHS-C	2.34	0.16
WASHING MACHINE - 2008	1.82	0.16
WASHING MACH NO DRYER MAX 1800	1.48	0.17
LEISURE CENTRE ANNUAL MSHIP	1.34	0.16
COOKED HAM PREPACKED/SLICED	0.84	0.17
PRIV RENTD UNFURNISHD PROPERTY	0.41	1.02
AUTOMATIC WASHING MACHINE 2009	0.35	0.16
MILK SEMI-PER 2 PINTS/1.136 L	0.34	0.26
CIGARETTES 5	0.33	0.25
VEGETARIAN MAIN COURSE	0.24	0.17
DOMESTIC CLEANER HOURLY RATE	0.22	0.23
HOME REMOVAL- 1 VAN	0.17	0.18
STAFF RESTAURANT SANDWICH	0.17	0.20
	Relative Price Decline	
NEWSPAPER AD NON TRADE 20 WORD	-3.66	0.19
COFFEE TABLE -2	-3.68	0.16
FLAT PANEL TV 33" +	-3.84	0.16
KITCHEN WALL UNIT SELF ASSMBLY	-3.94	0.16
FLAT PANEL TV 26" - 42"	-4.26	0.29
WIDESCREEN TV (24-32 INCH)	-4.50	0.19
AUTOMATIC WASHING MACHINE	-4.76	0.18
WOMENS TROUSERS-FORMAL	-7.12	0.17
MENS SHOES TRAINERS	-7.84	0.18
PRE-RECORDED DVD TOP 20	-8.14	0.23
WOMENS SUIT	-8.95	0.17
LADYS SCARF	-20.19	0.17
COMPUTER GAME TOP 20 CHART	-21.69	0.31
WOMENS DRESS-CASUAL 1	-25.55	0.17
PRE-RECORDED DVD (FILM)	-35.03	0.16

Notes: The table reports the fifteen top and bottom rates of relative price change for items with expenditure weight greater than 0.15%. Weights are average expenditure weights for the full sample period.

Linearizing the previous equation around $P_{jzt}/P_{jzb} = 1$ delivers

$$\frac{P_{zt}}{P_{zb}} = \int_0^1 w_{jzb} \; \frac{P_{jzt}}{P_{jzb}} \;\mathrm{dj} \;\; + O(2). \tag{192}$$

The advantage of the linearized model-consistent equation (192) is that it does not depend on the demand elasticity θ showing up in the non-linear expression (191).

Linearizing the ONS stratum price index in equation (188) around $P_{jkzt}/P_{jkzb} = 1$ delivers

$$\widetilde{I}_{kzt} = \sum_{j \in J_{kz}} \widetilde{w}_{jkzt} \frac{P_{jkzt}}{P_{jkzb}} + O(2),$$

where $\widetilde{w}_{jkzt} \equiv w_{jkzt} / \left(\sum_{j' \in J_{kz}} w_{j'kzt} \right)$. Using the ONS approach to aggretate stratum indices to item indices, see equation (189), we obtain from the previous equation

$$\widetilde{I}_{zt} = \sum_{k=1}^{K} \left(\left(\frac{w_{kzt}}{\sum_k w_{kzt}} \right) \sum_{j \in J_{kz}} \widetilde{w}_{jkzt} \frac{P_{jkzt}}{P_{jkzb}} \right) + O(2). \tag{193}$$

This shows that the ONS approach (193) and the theory-consistent approach (192) deliver to first order the same price index, provided we set the product weight in equation (192) equal to

$$w_{jzb} = \left(\frac{w_{kzt}}{\sum_{k'} w_{k'zt}} \right) \widetilde{w}_{jkzt},$$

where k denotes the stratum to which product j belongs. Using the previous weights we compute the quality-adjusted item price level. Following ONS, we then chain the index growth rates across years to get the multi-year series for the price index at the item level.

G.5 Measurement of Relative Growth Weights, Price Change Frequencies and Product Turnover Rates

We measure relative growth weights using the model-implied relationship $\gamma_z^e/\gamma^e = \Pi/\Pi_z$. Based on this relationship, we estimate Π_z as the sample mean of item-level price indices. Specifically, we set Π_z equal to the average month-to-month change of price indices for item z using the same price indices that we also use to compute relative product prices in regression equation (38). We trim Π_z estimates larger than 20% per year in absolute value, which is the case for less than 1% of the items we consider. Furthermore, we compute implied aggregate inflation Π according to

$$\Pi = \prod_{z=1}^{Z} \Pi_z^{\psi_z},$$

which follows from taking growth rates of equation (16) and considering a steady state. In the previous equation, ψ_z denotes the expenditure share of item z and Z denotes the relevant set of expenditure items. With these estimates of Π_z and Π, we compute monthly relative growth weights according to $\gamma_z^e/\gamma^e = \Pi/\Pi_z$.

We measure the frequency of nominal price changes and the rates of product turnover at the item level from the sample of micro prices that excludes all price observations which ONS flags as sale. This sample corresponds to the one labeled "baseline w/o sales prices" in figure 13. In fact, a series of papers argues that monetary non-neutrality depends mostly on the frequency of changes in regular prices, e.g., Midrigan (2011), Eichenbaum et al. (2011), Guimaraes and Sheedy (2011), and Kehoe and Midrigan (2015).

For this sample, the item-level frequency of price changes α_z is equal to the ratio of the number of nonzero price changes in item z over the number of all price changes in this item. The item-level product turnover rate is equal to the unweighted average of item-level product entry rate and item-level product exit rate. The product entry rate for item z is equal to the average per-period entry rate, i.e., the number of products with age zero in month t over the number of all products in the same month. We exclude the first month of each item, in which the entry rate is 100% by construction. The product exit rate for item z is computed correspondingly.

H Alternative Treatment of Sales Prices

An important feature of micro price data is that it features many short-lived price changes that are subsequently reversed. These typically take the form of temporary price reductions (sales), but also occasionally the form of temporary price increases. The sticky price model outlined in the previous sections does not allow for such temporary price changes. We show below that the model can be augmented, following the lines of Kehoe and Midrigan (2015), and that doing so leaves our empirical estimation approach unchanged. We furthermore explore the quantitative effects of alternative treatments of sales prices for our results.

Consider for a moment the following augmented sticky-price setup featuring also temporary prices. Firms choose a regular list price P^L_{jzt}, which is subject to the same price adjustment frictions as the prices in the pure Calvo model presented before. After learning about the adjustment opportunity for the list price, a share $\alpha^T_z \in [0, 1)$ of producers gets to choose freely a temporary price P^T_{jzt} at which they can sell the product in the current period. The temporary price is valid for one period only and does not affect the list price. Furthermore, absent further temporary price adjustment opportunities in the next period, prices revert to the list price in the next period. With this setup, the optimal temporary price $P^{T\star}_{jzt}$ is equal to the static optimal price in the period, i.e., equal to the flexible price P^f_{jzt}. It follows from equation (37) that the relative price trend of temporary (or flexible) prices is no different from that of all prices, so that the inclusion of temporary prices in the relative price trend regressions should make no difference for our results.

Nevertheless, sales prices can make a difference for the estimated relative price trends due to a number of reasons. Sales prices might, for instance, not be evenly distributed over the product life cycle, unlike assumed in the augmented theoretical setup sketched in the previous paragraph. Sales may happen, for instance, predominantly at the beginning (or at the end) of the product lifetime. If this were the case, then our baseline regressions would probably underestimate (overestimate) relative price declines and thereby underestimate (overestimate) the optimal inflation target. In light of this, it appears of interest to investigate the robustness of our baseline results towards using alternative approaches for treating sales prices in the data.

Figure 13: Optimal inflation target for alternative treatments of sales prices

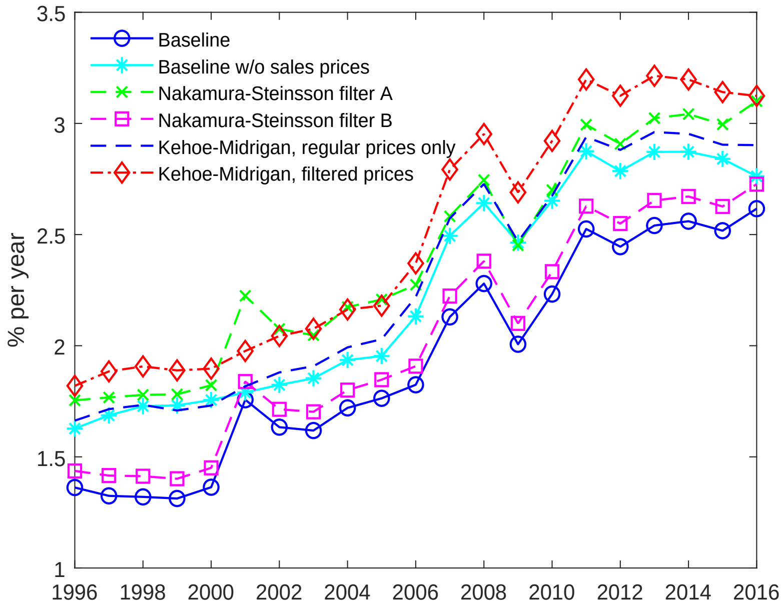


Figure 13 displays the baseline estimate of the optimal inflation target together with various alternative estimates for the optimal inflation target. A first approach (baseline w/o sales prices) uses the ONS sales flag to exclude all sales prices from regression (38).[71] The figure shows that the optimal inflation target increases by around 0.3% per year as a result. A quantitatively similar result is obtained, if only the so-called "regular prices" are used in the regression (Kehoe-Midrigan, regular prices only), where regular prices are defined according to the regular price filter of Kehoe and Midrigan (2015).

Instead of simply excluding sales prices from the regression, one can adjust sales prices based on various adjustment techniques and continue using them in the estimation. Figure 13 reports the outcome when making adjustments using the sales filters A and B from Nakamura and Steinsson (2008) and the regular price filter of Kehoe-Midrigan (2015) (Kehoe-Midrigan, filtered prices). The outcomes across these filtering approaches vary quite substantially. While the Nakamura-Steinsson filter B leads to only small adjustments relative to the baseline estimation, filter A leads to adjustments of the same order of magnitude as when dropping sales prices from the regression. The largest upward revision of the inflation target is observed for the regular price filter of Kehoe and Midrigan: the inflation target is then on average about 0.5% higher than the baseline estimate.

[71] A sales flag is an indicator variable that the price collector records, whenever she/he finds the product to be on sale. In this and subsequent robustness checks, we always recompute the item price levels after excluding or adjusting sales prices.

Overall, we can conclude that a different treatment of sales prices can lead to considerably higher optimal inflation targets than the ones obtained via our baseline approach.