Online Appendix for Revealing Naïveté and Sophistication from Procrastination and Preproperation

David J. Freeman

June 9, 2020

Limited Dataset Tests

In applications, the analyst will likely only observe a limited number of choices. The analysis below provides a testable characterizations of the class of Strotzian, sophisticated, and naïve representations that also apply to such limited datasets.[1]

First, introduce notation for partial datasets. Let $\mathscr{A}_{obs}$ denote a subset of $\mathscr{A}$, and let $c_{obs} : \mathscr{A}_{obs} \to \bar{A}$ denote a choice function on $\mathscr{A}_{obs}$. The function c_{obs} denotes the observed choice function, with its domain being the set of observed choices $\mathscr{A}_{obs}$. Next, consider testable conditions under which a given c_{obs} can be extended to a full-domain choice function $c : \mathscr{A} \to \bar{A}$ with a Strotzian representation.

Strotzian No-Cycle Condition. There exist $\{R_t\}_{t\in\bar{A}}, \{\hat{R}_{t'|t}\}_{t\in A, t'\in A_{>t}}$ with each relation antisymmetric on its domain that satsify the following:

if $t = c_{obs}(A)$, then for each $t' \in A$,

(i) there exists a chain $t_1, \dots, t_n$ with $t_n = \max A$, and $t' < t_1 < \cdots < t_n$ such that $t_1 R_{t'} t'$ and for each $k < n$, $t_k \hat{R}_{t_k|t'} t_{k+1}$ and $t_{k+1} \hat{R}_{t''|t'} t''$ for all $t'' \in A_{<t_{k+1}} \cap A_{>t_k}$,

[1] De Clippel and Rozen (2018) note that given a set of axioms that characterize a given theory given a complete choice function, it may be possible for a partial dataset to pass direct tests of each axiom while not being consistent with the given theory. They show that this can be particularly important when testing models that violate standard choice axioms, motivating the exercise here.

(ii) there exists a chain $t_1,\ldots,t_n$ with $t_n = \max A$, and $t < t_1 < \cdots < t_n$ such that $tR_t t_1$ and for each $k < n$, $t_k \hat{R}_{t_k|t} t_{k+1}$ and $t_{k+1} \hat{R}_{t''|t'} t''$ for all $t'' \in A_{<t_{k+1}} \cap A_{>t_k}$.

Proposition 1. *c_{obs} satisfies the Strotzian No-Cycle Condition if and only if there exists a choice function $c : \mathscr{A} \to \bar{A}$ with a Strotzian representation and $c_{obs} \subseteq c$.*

Proof. Necessity and sufficiency almost immediately follows from the axiom.

Suppose c has a partially naïve representation $\left(\mathscr{U}, \hat{\mathscr{U}}\right)$. Pick each R_t as the order implied by U_t and $\hat{R}_{t'|t}$ as the order implied by $\hat{U}_{t'|t}$. These orders are antisymmetric by construction. Now suppose $t = c_{obs}(A)$. Given any t', pick $t_1,\ldots,t_n$ to satisfy $t' < t_1 < \cdots < t_n = \max_{\tilde{t} \in \bar{A}} \tilde{t}$ and $t_k = s(t_k, A, \hat{U}_{t_k|t'}, \hat{\mathscr{U}}_{\cdot|t'})$ for each k. By the definition of a perception-perfect strategy and the choice of $\{R_{\tilde{t}}\}_{\tilde{t}}, \{\hat{R}_{\tilde{t}'|\tilde{t}}\}_{\tilde{t},\tilde{t}'}$, these sequences verify that parts (i) and (ii) hold in the Strotzian No-Cycle Condition.

Conversely, suppose c_{obs} satisfies the Strotzian No-Cycle Condition. Notice that for any t and $t' > t$, choice only pins down whether $U_t(t) \gtrless U_t(t')$; since each R_t is well defined, we can construct U_t to represent R_t for each t, and by a similar argument we can construct $\hat{U}_{t'|t}$ to represent $\hat{R}_{t'|t}$ for each $t, t' > t$. Then by construction and the Strotzian No-Cycle Condition, and the $c(A)$ is the perception-perfect equilibrium choice from A given $\left(\mathscr{U}, \hat{\mathscr{U}}\right)$ for each $A \in \mathscr{A}_{obs}$. □

Using the Strotzian No-Cycle Condition to test the Strotzian model requires checking $\sum_{t=1}^{|\mathscr{T}|} (t^2 - t)$ different binary relations. The large number of different binary relations reflects flexibility in the partially naïve model. Note, however, that the sophisticated and naïve models are restrictions of the Strotzian model in which each $\hat{U}_{t'|t}$ is tied to an element in $\mathscr{U}$ in a particular way — in the sophisticated model, $\hat{U}_{t'|t} = U_{t'}$ for each t and $t' > t$, while in the naïve model, $\hat{U}_{t'|t}$ is the restriction of U_t to $\bar{A}_{\geq t'}$. These restrictions correspond to analogous restrictions on the Strotzian No-Cycle Condition. These place additional restrictions on $\mathscr{U}$ that allow choice to identify whether $U_{t_1}(t_2) \gtrless U_{t_1}(t_3)$, even if $U_{t_1}(t_1)$ is higher than (or lower than) both of them. As a result, we must now ensure that each R_t has no cycles to ensure that each U_t can be constructed.

Naïve No-Cycle Condition. There exists $\{R_t\}_{t\in\mathscr{T}}$ with each complete, transitive, and antisymmetric on its domain that satsify the following:

if $t = c_{obs}(A)$, then for each $t' \in A$,

(i) if $t' \neq t$, there exists a chain $t_1,\dots,t_n$ with $t_n = \max \bar{A}$, and $t' < t_1 < \cdots < t_n$ such that $t_1 R_{t'} t'$ and for each $k < n$, $t_k R_{t'} t_{k+1}$ and $t_{k+1} R_{t'} t''$ for all $t'' \in A_{<t_{k+1}} \cap A_{>t_k}$.

(ii) there exists a chain $t_1,\dots,t_n$ with $t_n = \max \bar{A}$, and $t < t_1 < \cdots < t_n$ such that $t R_t t_1$ and for each $k < n$, $t_k R_t t_{k+1}$ and $t_{k+1} R_{t'} t''$ for all $t'' \in A_{<t_{k+1}} \cap A_{>t_k}$.

Sophisticated No Cycle Condition. There exists $\{R_t\}_{t\in\mathscr{T}}$ with each complete, transitive, and antisymmetric on its domain that satsify the following:

if $(a,t) = c_{obs}(A)$, then for each $t' \in A$,

(i) if $t' \neq t$, there exists a chain $t_1,\dots,t_n$ with $t_n = \max \bar{A}$, and $t' < t_1 < \cdots < t_n$ such that $t_1 R_{t'} t'$ and $t_k R_{t_k} t_{k+1}$ for each $k < n$,

(ii) there exists a chain $t_1,\dots,t_n$ with $t_n = \max \bar{A}$, and $t < t_1 < \cdots < t_n$ such that $t R_t t_1$ and $t_k R_{t_k} t_{k+1}$ for each $k < n$.

Notice that checking either of the Sophisticated and Naïve No-Cycle Conditions only requires checking for the existence of $\bar{A}$ transitive completions of binary relations.

Corollary 1. *(i) c_{obs} satisfies the Naïve No-Cycle Condition if and only if there exists a choice function $c : \mathscr{A} \to \bar{A}$ with a naïve representation and $c_{obs} \subseteq c$.*

(ii) c_{obs} satisfies the Sophisticated No-Cycle Condition if and only if there exists a choice function $c : \mathscr{A} \to \bar{A}$ with a sophisticated representation and $c_{obs} \subseteq c$.

References

De Clippel, Geoffroy, and Kareen Rozen. 2018. "Bounded rationality and limited datasets." Working Paper.