Online Appendix for
"Experimenting with Career Concerns"

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This Online Appendix contains the proofs of the results stated in Section 3 of the paper.

Proof of Proposition 4

We begin by examining the agent's indifference conditions. We then describe the equilibrium construction and show existence under the different parameter conditions considered in the proposition. Finally, we prove uniqueness.

Indifference conditions. We examine the agent's indifference conditions that must be satisfied whenever the agent follows a random stopping policy. These conditions allow us to derive an expression for the market's belief $\widehat{\mu}_t^1$.

Consider the agent's indifference condition (12) for each $t \in [\underline{t}, \overline{t}]$. Writing $\mu_{t+dt} = \mu_t + dt\dot{\mu}_t = \mu_t - \mu_t(1-\mu_t)\lambda_G dt$, dividing by dt both sides of the equation, and taking dt to zero, this condition can be rewritten as

$$R\left(\widehat{\mu}_t^1 - \mu_t\right) + x + \mu_t\lambda_G\left(V_t - \frac{R}{r}\right) = 0, \tag{19}$$

where $V_t = v + R\int_t^\infty e^{-r(s-t)}\widehat{\mu}_s^1 ds$. Since (19) holds at each $t \in [\underline{t}, \overline{t}]$, the agent must be indifferent between stopping and continuing until $\overline{t}$ absent success. For $\overline{t} < \infty$, the agent stops at $\overline{t}$ if he has not succeeded and continues forever otherwise. Thus, for each $t \in [\underline{t}, \overline{t}]$,

$$\begin{aligned}\mu_t R \;\; = \;\; & (x + \mu_t\lambda_G)\left(1 - e^{-r(\overline{t}-t)}\right) + rR\int_t^{\overline{t}} e^{-r(s-t)}\widehat{\mu}_s^1 ds \\ & + e^{-r(\overline{t}-t)}\left[\mu_t\left(1 - e^{-\lambda_G(\overline{t}-t)}\right)(x + \lambda_G + R) + \left(\mu_t e^{-\lambda_G(\overline{t}-t)} + 1 - \mu_t\right)\mu_{\overline{t}}R\right]. \end{aligned} \tag{20}$$

The left-hand side is the agent's payoff from stopping at $t < \bar{t}$; the right-hand side is the agent's expected payoff from continuing until $\bar{t}$ and stopping at $\bar{t}$ if and only if he has not succeeded by then. Differentiating this condition yields

$$\begin{aligned}
\dot{\mu}_t R \;=\;& \dot{\mu}_t \lambda_G \left(1 - e^{-r(\bar{t}-t)}\right) - r\left(x + \mu_t \lambda_G\right) e^{-r(\bar{t}-t)} - rR\left[\widehat{\mu}_t^1 - r\int_t^{\bar{t}} e^{-r(s-t)}\widehat{\mu}_s^1 ds\right] \\
&+ re^{-r(\bar{t}-t)}\left[\mu_t\left(1 - e^{-\lambda_G(\bar{t}-t)}\right)(x + \lambda_G + R) + \left(\mu_t e^{-\lambda_G(\bar{t}-t)} + 1 - \mu_t\right)\mu_{\bar{t}} R\right] \\
&+ e^{-r(\bar{t}-t)}\left\{\begin{array}{c} \left[\dot{\mu}_t\left(1 - e^{-\lambda_G(\bar{t}-t)}\right) - \mu_t\lambda_G e^{-\lambda_G(\bar{t}-t)}\right](x + \lambda_G + R) \\ + \left[\mu_t \lambda_G e^{-\lambda_G(\bar{t}-t)} - \dot{\mu}_t\left(1 - e^{-\lambda_G(\bar{t}-t)}\right)\right]\mu_{\bar{t}} R \end{array}\right\}.
\end{aligned}$$

Substituting with equation (20) yields

$$\begin{aligned}
\dot{\mu}_t R \;=\;& \dot{\mu}_t \lambda_G \left(1 - e^{-r(\bar{t}-t)}\right) + r\left(\mu_t R - x - \mu_t \lambda_G\right) - rR\widehat{\mu}_t^1 \\
&+ e^{-r(\bar{t}-t)}\left\{\begin{array}{c} \left[\dot{\mu}_t\left(1 - e^{-\lambda_G(\bar{t}-t)}\right) - \mu_t\lambda_G e^{-\lambda_G(\bar{t}-t)}\right](x + \lambda_G + R) \\ + \left[\mu_t \lambda_G e^{-\lambda_G(\bar{t}-t)} - \dot{\mu}_t\left(1 - e^{-\lambda_G(\bar{t}-t)}\right)\right]\mu_{\bar{t}} R \end{array}\right\}.
\end{aligned}$$

Substituting with $\dot{\mu}_t = -\mu_t(1-\mu_t)\lambda_G$ and rearranging terms yields

$$\widehat{\mu}_t^1 = \frac{1}{rR}\left\{\begin{array}{c} r\left[-x - \mu_t(\lambda_G - R)\right] - \mu_t(1-\mu_t)\lambda_G\left[\lambda_G\left(1 - e^{-r(\bar{t}-t)}\right) - R\right] \\ -e^{-r(\bar{t}-t)}\mu_t\lambda_G\left[1 - \mu_t\left(1 - e^{-\lambda_G(\bar{t}-t)}\right)\right]\left[x + \lambda_G + R(1-\mu_{\bar{t}})\right] \end{array}\right\}. \tag{21}$$

Existence under $-\mathbf{x} < \mathbf{R} < -\mathbf{x}/\boldsymbol{\mu}_\mathbf{0}$. We construct an equilibrium as described in the proposition in which $\underline{t} \in (t^{FB}, \infty)$ and $\bar{t} = \infty$. Equations (20) and (21) reduce to

$$\begin{aligned}
\mu_t R \;&=\; x + \mu_t \lambda_G + rR\int_t^{\infty} e^{-r(s-t)}\widehat{\mu}_s^1 ds, && (22)\\
\widehat{\mu}_t^1 \;&=\; \frac{1}{rR}\Big\{r\left[-x - \mu_t(\lambda_G - R)\right] - \mu_t(1-\mu_t)\lambda_G\left(\lambda_G - R\right)\Big\}, && (23)
\end{aligned}$$

for each $t \geq \underline{t}$. Consider an equilibrium in which the agent starts the project at time 0, he continues with probability one at time t if he has succeeded by t or if $t < \underline{t}$, and otherwise he follows a random stopping policy from time $\underline{t}$ on such that the market's belief $\widehat{\mu}_t^1$ satisfies (23). (Note that by the monotonicity and continuity of Bayes' rule, such a random stopping policy exists and is unique.) Let the market's belief at t be μ_0 if the agent has not started the project or has started and not stopped by t and $t < \underline{t}$, $\widehat{\mu}_t^1$ satisfying (23) if the agent has started and not stopped by t and $t \geq \underline{t}$, and μ_s if the agent has started and stopped at $s \in (0, t]$. (Note that the belief upon observing that the agent stops at a time $t < \underline{t}$ is off the equilibrium path; we show existence when this belief satisfies $\widehat{\mu}_t^0 = \mu_t$ for all such t.)

The market's on-the-equilibrium-path beliefs are consistent and the off-the-equilibrium-path beliefs satisfy our belief monotonicity refinement. We now show that given the market's beliefs, the agent's stopping plan is optimal. By construction, the agent is indifferent and thus willing to follow a mixed strategy for $t \geq \underline{t}$ in the absence of success. (Note that this mixed strategy attaches a positive probability to the agent never stopping.) Additionally, an agent who has succeeded strictly prefers to continue with the project at time t if an agent who has not succeeded weakly prefers to continue. Thus, all is left to be shown is that the agent has incentives to start the project at time 0 and to continue with the project at $t < \underline{t}$ in the absence of success.

For the start decision, note that the agent's payoff if he does not start is $\mu_0 R/r$. By the martingale property of beliefs, this is also the agent's expected reputation payoff if he starts the project and follows the equilibrium strategy. Moreover, by Assumption 1, the agent receives a strictly positive expected project payoff from any strategy that starts the project and continues forever upon success. Hence, the agent strictly prefers to start and follow the equilibrium strategy compared to not starting the project.

To show that it is optimal for the agent to continue at $t < \underline{t}$ absent success, it is sufficient to show that the left-hand side of (22) is smaller than its right-hand side for $t < \underline{t}$, or, equivalently,

$$\Psi_t := x + \mu_t(\lambda_G - R) + rR \int_t^\infty e^{-r(s-t)} \widehat{\mu}_s^1 ds \geq 0. \tag{24}$$

Suppose by contradiction that $\Psi_t < 0$ for some $t < \underline{t}$. Note that by the previous claims, $\Psi_0 > 0$, and by definition of $\underline{t}$, $\Psi_{\underline{t}} = 0$. Therefore, if $\Psi_t < 0$ for $t < \underline{t}$, there exist $t' < t'' < \underline{t}$ such that $\Psi_{t'} = 0$, $\dot{\Psi}_{t'} < 0$, $\Psi_{t''} < 0$, and $\dot{\Psi}_{t''} = 0$. Differentiating (24) yields that for $t < \underline{t}$,

$$\dot{\Psi}_t = -\mu_t(1-\mu_t)\lambda_G(\lambda_G - R) - rR\mu_0 + r^2 R \int_t^\infty e^{-r(s-t)}\widehat{\mu}_s^1 ds. \tag{25}$$

Using (24) and (25), note that $\Psi_{t'} = 0$, $\dot{\Psi}_{t'} < 0$, $\Psi_{t''} < 0$, and $\dot{\Psi}_{t''} = 0$ imply

$$-\mu_{t'}(1-\mu_{t'})\lambda_G(\lambda_G - R) - rR\mu_0 - r[x + \mu_{t'}(\lambda_G - R)] < 0, \tag{26}$$

$$-\mu_{t''}(1-\mu_{t''})\lambda_G(\lambda_G - R) - rR\mu_0 - r[x + \mu_{t''}(\lambda_G - R)] > 0. \tag{27}$$

However, given Assumption 1 and $-x < R < -x/\mu_0$, there is a unique value $\mu_{\underline{t}} \in (0, \mu_0)$ that solves (23) at $\underline{t}$ given $\widehat{\mu}_{\underline{t}}^1 = \mu_0$, and this value is given by

$$\mu_{\underline{t}} = \frac{\lambda_G + r - \sqrt{\frac{\lambda_G^2(2r+\lambda_G-R)+\lambda_G r[4(\mu_0 R+x)+r-2R]-r^2R}{\lambda_G - R}}}{2\lambda_G}. \tag{28}$$

Hence, we cannot have $\mu_{t'}, \mu_{t''} \in (\mu_{\underline{t}}, \mu_0)$. Contradiction.

Finally, we show that the equilibrium threshold time $\underline{t}$ satisfies $\underline{t} > t^{FB}$. Note that $\underline{t}$ is uniquely pinned down by (9) and (28). To prove $\underline{t} > t^{FB}$, we verify that $\mu_{\underline{t}} < \mu^{FB}$. The latter inequality is immediate from comparing $\mu_{\underline{t}}$ given in (28) and μ^{FB} given in (11), taking into account that Assumption 1 and $R < -x/\mu_0$ imply $\lambda_G > R$.

Existence under $\mathbf{R < -x}$. We consider an equilibrium analogous to that above, except that the agent now stops with certainty by a finite time $\bar{t}$ if he has not succeeded by then. Consistently, we specify beliefs for the market as those above but with $\widehat{\mu}_t^1 = 1$ for all $t \geq \bar{t}$. Given this, the agent's indifference condition (19) at $\bar{t}$ implies

$$R(1-\mu_{\bar{t}}) + x + \mu_{\bar{t}}\lambda_G v = 0, \tag{29}$$

and hence[26]

$$\mu_{\bar{t}} = \frac{-(x+R)r}{r(\lambda_G - R) + \lambda_G(\lambda_G + x)}. \tag{30}$$

The market's on-the-equilibrium-path beliefs are consistent and the off-the-equilibrium-path beliefs satisfy our belief monotonicity refinement. (Note that the belief upon observing that the agent stops at a time $t < \underline{t}$ or $t > \bar{t}$ is off the equilibrium path; we show existence when this belief satisfies $\widehat{\mu}_t^0 = \mu_t$ for all such t.) We now verify that given the market's beliefs, the agent's stopping plan is optimal. Since the construction is analogous to that in the previous case, all we need to verify is that the agent has no incentives to continue beyond $\bar{t}$ absent success. Note that μ_t is decreasing over time and $\lambda_G > R$ (by Assumption 1 and $R < -x$). Thus, the left-hand side of (29) evaluated at $t > \bar{t}$ instead of $\bar{t}$ is strictly negative, which implies that the agent has strict incentives not to continue beyond time $\bar{t}$.

Finally, we show that the above conditions pin down $\mu_{\underline{t}}$ and imply $\underline{t} > t^{FB}$. Since (30) uniquely pins down $\mu_{\bar{t}}$ and (together with (9)) $\bar{t}$, these can be substituted into equation (21) at $t = \underline{t}$ (with $\widehat{\mu}_{\underline{t}}^1 = \mu_0$) to solve for $\mu_{\underline{t}}$ and $\underline{t}$. Combining (9), (21), and (30) yields

$$\begin{aligned} 0 \;=\;& r\left[x + \mu_{\underline{t}}\lambda_G + R(\mu_0 - \mu_{\underline{t}})\right] + \mu_{\underline{t}}(1-\mu_{\underline{t}})\lambda_G(\lambda_G - R) \\ &+\mu_{\underline{t}}(1-\mu_{\underline{t}})\lambda_G\left(\frac{-(x+R)r}{(\lambda_G + x)(r+\lambda_G)}\frac{1-\mu_{\underline{t}}}{\mu_{\underline{t}}}\right)^{\frac{r}{\lambda_G}}\frac{\lambda_G(R+x)}{r+\lambda_G}. \end{aligned} \tag{31}$$

To show that $\underline{t} > t^{FB}$, or equivalently $\mu_{\underline{t}} < \mu^{FB}$, note that $\mu_{\underline{t}}$ is continuous in R for all $R \geq 0$, and $\mu_{\underline{t}} \to \mu^{FB}$ as $R \to 0$. Moreover, in the limit as $R \to -x$, $\mu_{\underline{t}}$ coincides with (28), which implies $\mu_{\underline{t}} < \mu^{FB}$ in this limit. Thus, suppose by contradiction that $\mu_{\underline{t}} \geq \mu^{FB}$ for some $R \in (0, -x)$. Then there must exist $R' > 0$ such that $\mu_{\underline{t}}(R') = \mu^{FB}$. More precisely, let $g(R)$ be the right-hand-side of (31) when $\mu_{\underline{t}} = \mu^{FB}$, as a function of R. Algebraic manipulations yield

$$g(R) := \frac{r\begin{pmatrix} -x^2\left(\lambda_G^2 + \lambda_G(\mu_0+1)R + 2rR\right) - Rx(\lambda_G + r)(2\lambda_G\mu_0 + \lambda_G + r) \\ -\lambda_G\mu_0 R(\lambda_G + r)^2 + \lambda_G x(\lambda_G + x)(R+x)\left(\frac{R+x}{x}\right)^{r/\lambda_G} - \lambda_G x^3 \end{pmatrix}}{\lambda_G(\lambda_G + r + x)^2}.$$

[26] Equation (30) can equivalently be derived from (21) at $\bar{t}$.

As noted, $g(0) = 0$, and by the contradiction assumption, there exists $R' > 0$ such that $g(R') = 0$. Furthermore, there must then exist $0 < R'' < R'$ such that $g'(R'') = 0$, where differentiating $g(R)$ gives

$$g'(R) = \frac{r\begin{pmatrix} -x^2(\lambda_G\mu_0 + \lambda_G + 2r) - x(\lambda_G + r)(2\lambda_G\mu_0 + \lambda_G + r) \\ -\lambda_G\mu_0(\lambda_G + r)^2 + x(\lambda_G + r)(\lambda_G + x)\left(\frac{R+x}{x}\right)^{r/\lambda_G} \end{pmatrix}}{\lambda_G(\lambda_G + r + x)^2}.$$

Note that given a set of parameters $\{\mu_0, \lambda_G, x, r\}$, there is a unique value R'' for which $g'(R'') = 0$, and this value must satisfy $g(R'') < 0$. Therefore, it must be that $g'(R') > 0$. However, one can verify that $g(R') = 0$ implies $g'(R') < 0$, yielding a contradiction.

Existence under $\mathbf{R > -x/\boldsymbol{\mu_0}}$. Consider an equilibrium in which the agent starts the project and never stops. Let the market's beliefs be $\widehat{\mu}_t^1 = \mu_0$ and $\widehat{\mu}_t^0 = \mu_t$ for all $t \geq 0$. (Note that the belief upon observing that the agent stops at a time $t > 0$ is off the equilibrium path; we show existence when this belief satisfies $\widehat{\mu}_t^0 = \mu_t$ for all such t.) The market's on-the-equilibrium-path beliefs are consistent and the off-the-equilibrium-path beliefs satisfy our belief monotonicity refinement. We now show that given these beliefs, the agent always has strict incentives to continue. That is, for all $t \geq 0$,

$$\mu_t R < x + \mu_t\lambda_G + \mu_0 R.$$

Given $R > -x/\mu_0$, it is immediate that this condition always holds if $R \leq \lambda_G$. Suppose instead that $R > \lambda_G$. Since μ_t is decreasing over time, it suffices to show in this case that this condition holds at time 0. By Assumption 1, this is indeed true.

Uniqueness. We show that the equilibrium is unique up to off-the-equilibrium-path beliefs by proving a number of claims. We first show that in any equilibrium, the agent starts the project at time 0 (Claim 1) and continues forever after succeeding (Claims 2 and 3). Moreover, the agent stops with strictly positive probability if and only if $R < -x/\mu_0$ (Claims 4 and 5) and stops with certainty by a finite time absent success if and only if $R < -x$ (Claims 6 and 9). Finally, we show that the support of the agent's strategy is an interval (Claim 10) and that the agent's stopping policy and the market's on-the equilibrium-path beliefs are uniquely pinned down (Claims 7, 8, and 11).

Claim 1: An equilibrium in which the agent does not start the project at time 0 does not exist.
Proof of Claim 1: The proof of this claim is analogous to that of Claim 1 in the proof of Proposition 1 and thus omitted.

Claim 2: Suppose there exists an equilibrium in which an agent who succeeds at $t > 0$ stops with strictly positive probability after t. Then the agent stops with certainty at a time $t' \geq t$.
Proof of Claim 2: The proof of this claim is analogous to that of Claim 1 in the proof of Lemma 1 and thus omitted.

Claim 3: There exists no equilibrium in which the agent stops with certainty at a time $t' > 0$ by which he has succeeded.
Proof of Claim 3: The proof of this claim is analogous to that of Claim 2 in the proof of Lemma 1 and thus omitted.

Claim 4: If $R > -x/\mu_0$, an equilibrium in which the agent stops with strictly positive probability does not exist.
Proof of Claim 4: Our proof of existence under $R > -x/\mu_0$ shows that the agent has strict incentives to continue at a time t if the market's beliefs satisfy $\widehat{\mu}_t^0 = \mu_t$ and $\widehat{\mu}_s^1 = \mu_0$ for all $s \geq t$. It follows that the agent also has strict incentives to continue at t if $\widehat{\mu}_t^0 \leq \mu_t$ and $\widehat{\mu}_s^1 \geq \mu_0$ for all $s \geq t$. By Claims 2 and 3 above, $\widehat{\mu}_s^1 \geq \mu_0$ for all $s \geq 0$. Hence, the agent can only be willing to stop at a time t if $\widehat{\mu}_t^0 > \mu_t$. However, by Claims 2 and 3 such a belief would not be consistent. The claim follows.

Claim 5: If $R < -x/\mu_0$, an equilibrium in which the agent never stops does not exist.
Proof of Claim 5: Suppose by contradiction that such an equilibrium exists. Then the market's belief conditional on the agent continuing is $\widehat{\mu}_t^1 = \mu_0$ for all $t \geq 0$, and the agent must be willing to continue rather than stop at all times.

However, since the agent's payoff from stopping is weakly positive and $\mu_t \to 0$ as $t \to \infty$, this requires $\mu_0 R + x \geq 0$. Contradiction.

Claim 6: If $R < -x$, an equilibrium in which the agent continues with the project with strictly positive probability absent success in the limit as $t \to \infty$ does not exist.

Proof of Claim 6: Suppose by contradiction that such an equilibrium exists. The agent must be willing to continue rather than stop absent success in the limit as $t \to \infty$. Since the agent's payoff from stopping is weakly positive and $\mu_t \to 0$ as $t \to \infty$, this requires that for some $\widehat{\mu}^1_\infty \leq 1$, $\widehat{\mu}^1_\infty R + x \geq 0$. This inequality however cannot be satisfied when $R < -x$.

Claim 7: In any equilibrium, the market's belief conditional on the agent not having stopped, $\widehat{\mu}^1_t$, must be continuous.

Proof of Claim 7: Suppose by contradiction that an equilibrium in which $\widehat{\mu}^1_t$ is discontinuous exists. Let $\hat{t}$ be the earliest time at which this belief jumps. By Claims 2 and 3, $\widehat{\mu}^1_t$ is weakly increasing and can only jump up. Suppose the belief jumps at $\hat{t}$ from $\widehat{\mu}^1_{\hat{t}-} = \widehat{\mu}^{1-}$ to $\widehat{\mu}^1_{\hat{t}+} = \widehat{\mu}^{1+} > \widehat{\mu}^{1-}$. This requires the agent stopping with strictly positive probability, and by consistency of beliefs and Claims 2 and 3, the market's belief must satisfy $\widehat{\mu}^0_{\hat{t}} = \mu_{\hat{t}}$. Observe also that the market's belief satisfies $\widehat{\mu}^0_t \geq \mu_t$ for all $t > 0$, on and off the equilibrium path. (That is, the most pessimistic belief at t corresponds to no success having arrived by t.)

Consider now the agent's incentives. In the absence of success, the agent must be willing to stop at $\hat{t}$ rather than continue for an arbitrarily small amount of time dt and stop at $\hat{t} + dt$ if no success is obtained over $[\hat{t}, \hat{t} + dt]$. Following similar steps to those used to derive (19), taking dt to 0, this condition is

$$R\left(\widehat{\mu}^{1+} - \mu_{\hat{t}}\right) + x + \mu_{\hat{t}} \lambda_G \left(V_{\hat{t}} - \frac{R}{r}\right) \leq 0. \tag{32}$$

In the absence of success, the agent must also be willing to continue working over $[\hat{t} - dt, \hat{t}]$ and stop at $\hat{t}$ if no success is obtained over $[\hat{t} - dt, \hat{t}]$ rather than stop at $\hat{t} - dt$. This condition can be written as

$$R\left(\widehat{\mu}^{1-} - \widehat{\mu}^{0}_{\hat{t}-}\right) + x + \mu_{\hat{t}}\lambda_G\left(V_{\hat{t}} - \frac{R}{r}\right) \geq 0. \tag{33}$$

However, $\widehat{\mu}^{1+} > \widehat{\mu}^{1-}$ and $\widehat{\mu}^{0}_{\hat{t}-} \geq \mu_{\hat{t}}$ imply that (32) and (33) cannot be simultaneously satisfied. Contradiction.

Claim 8: Suppose there exists an equilibrium in which, absent success, the agent stops with strictly positive probability by a time $\bar{t} < \infty$ *and with zero probability at all times* $t > \bar{t}$. *Then* $\widehat{\mu}^{1}_{t} = 1$ *for all* $t \geq \bar{t}$.

Proof of Claim 8: Suppose the claim is not true. Then there exists an equilibrium in which the agent quits with strictly positive probability absent success, he ceases quitting at a time $\bar{t} < \infty$, and the market's belief satisfies $\widehat{\mu}^{1}_{t} < 1$ for some $t \geq \bar{t}$. Since the agent continues with certainty after $\bar{t}$ if he has not stopped by then, the market's belief $\widehat{\mu}^{1}_{t}$ must be constant at some value, call it $\overline{\mu}$, for all $t \geq \bar{t}$. The agent's indifference condition (19) at $\bar{t}$ yields

$$R\overline{\mu} + x + \mu_{\bar{t}}\left(\lambda_G - R + \lambda_G\frac{\lambda_G + x}{r} - \lambda_G(1 - \overline{\mu})\frac{R}{r}\right) = 0. \tag{34}$$

Note that $\overline{\mu} < 1$ requires that an agent who has not succeeded by $\bar{t}$ be willing to continue beyond this time. Since μ_t is decreasing over time, equation (34) implies that the agent is willing to continue after $\bar{t}$ absent success if and only if the expression in parenthesis is negative. That is, rearranging terms, the equilibrium requires

$$\lambda_G - R + \frac{\lambda_G}{r}\left(\lambda_G - R + x + \overline{\mu}R\right) \leq 0.$$

By Claim 4, the agent stopping with strictly positive probability in equilibrium requires $R < -x/\mu_0$. Together with Assumption 1, this implies $\lambda_G > R$. Hence, the above inequality can hold only if $x + \overline{\mu}R < 0$. However, if the parenthesis in (34) is negative and $x + \overline{\mu}R < 0$, (34) cannot hold. Contradiction.

Claim 9: Suppose $R > -x$. *There is no equilibrium in which, absent success, the agent stops with strictly positive probability by a time* $\bar{t} < \infty$ *and with zero* *probability at all times $t > \bar{t}$.*

Proof of Claim 9: Suppose by contradiction that such an equilibrium exists. By Claim 4, the agent stopping with strictly positive probability in equilibrium requires $R < -x/\mu_0$. Moreover, as shown in Claim 8, if the agent's quitting ceases by a time $\bar{t} < \infty$, the market's belief must be $\widehat{\mu}_t^1 = 1$ for all $t \geq \bar{t}$, and hence equation (30) must hold at $\bar{t}$. However, if $R > -x$, this equation yields $\mu_{\bar{t}} < 0$ (recall $\lambda_G > R$ by Assumption 1 and $R < -x/\mu_0$), a contradiction.

Claim 10: Suppose there exists an equilibrium in which, absent success, the agent stops with strictly positive probability over $[t_1, t_2]$ and with zero probability over $[t_2, t_3]$, for some $0 < t_1 < t_2 < t_3$. Then the agent stops with zero probability at all $t \geq t_2$.

Proof of Claim 10: Suppose the claim is not true. Then there exists an equilibrium in which, absent success, the agent stops with strictly positive probability over $[t_1, t_2]$, with zero probability over $[t_2, t_3]$, and with strictly positive probability over $[t_3, t_4]$, for some $0 < t_1 < t_2 < t_3 < t_4$. Let $\bar{t} > 0$ be such that either $\bar{t} < \infty$ and the probability of stopping absent success is zero at all $t > \bar{t}$, or $\bar{t} = \infty$. By construction, around both times t_2 and t_3, the agent must be indifferent between stopping and continuing until $\bar{t}$ absent success. It follows that equation (21) must hold at t_2 and t_3, where note that if $\bar{t} < \infty$, then (30) uniquely pins down $\mu_{\bar{t}}$ and (together with (9)) $\bar{t}$. However, since the agent stops with zero probability between t_2 and t_3, we must have $\widehat{\mu}_{t_2}^1 = \widehat{\mu}_{t_3}^1 \geq \mu_0$. Given $\mu_{t_2} > \mu_{t_3}$, (21) cannot simultaneously hold at t_2 and t_3, yielding a contradiction.

Claim 11: Up to off-the-equilibrium-path beliefs, the equilibrium is unique.

Proof of Claim 11: This follows from the claims above, the fact that the solutions for the threshold times and $\widehat{\mu}_t^1$ shown in the proofs of existence are unique, and the fact that the random stopping policy that generates $\widehat{\mu}_t^1$ is also unique (as Bayes' rule is continuous and monotone).

Proof of Proposition 5

Preliminaries. We begin by deriving preliminary results that will be useful to evaluate the welfare effects of changes in μ_0 and information that refines μ_0.

Consider parameters with $-x/\mu_0 > R > -x$. As shown in Proposition 4, the equilibrium features $\underline{t} \in (t^{FB}, \infty)$ and $\overline{t} = \infty$, where $\underline{t}$ is given by (9) and (28), and equations (22) and (23) hold at each $t \geq \underline{t}$. The market's belief conditional on the agent not having stopped is $\widehat{\mu}_t^1 = \mu_0$ for $t < \underline{t}$, and, by equation (23), this belief can be written as a function of μ_t independent of μ_0 for $t \geq \underline{t}$. Note also that the posterior belief at which the agent starts quitting, $\mu_{\underline{t}}$, is decreasing in μ_0; this can be verified using (28).

The equilibrium therefore implies that, for any $\delta \geq 0$, $\widehat{\mu}^1_{t^{FB}(\mu_0)+\delta}$ is increasing and convex in μ_0. To see this, fix a prior μ_0' and a posterior belief $\mu' \leq \mu^{FB}$. For any prior $\mu_0 \geq \mu_0'$, consider the market's belief that corresponds to such a posterior, $\widehat{\mu}^1(\mu', \mu_0)$. The construction implies that if $\mu' > \mu_{\underline{t}(\mu_0')}$, $\widehat{\mu}^1(\mu', \mu_0)$ increases one-for-one as μ_0 increases from μ_0'. If $\mu' < \mu_{\underline{t}(\mu_0')}$, then as μ_0 increases from μ_0', the belief $\widehat{\mu}^1(\mu', \mu_0)$ is invariant to μ_0 up to $\mu_0 = \widehat{\mu}^1(\mu', \mu_0')$, and increases one-for-one with μ_0 for $\mu_0 > \widehat{\mu}^1(\mu', \mu_0')$. Figure 5 provides an illustration.

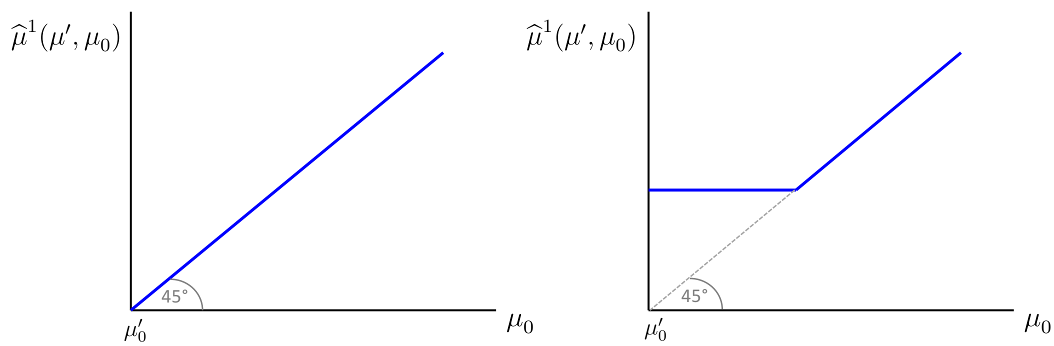

Figure 5: Market's belief conditional on the agent not having stopped, as a function of μ_0 for a fixed posterior belief μ'. The left graph corresponds to $\mu' > \mu_{\underline{t}(\mu_0')}$, where μ_0' is the lowest prior considered in the figure. The right graph corresponds to $\mu' < \mu_{\underline{t}(\mu_0')}$.

Hence, for any $\delta \geq 0$, we have

$$\frac{\partial \widehat{\mu}^1_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0} \geq 0, \quad \frac{\partial^2 \widehat{\mu}^1_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0^2} \geq 0. \tag{35}$$

Let $\eta_{t^{FB}(\mu_0)+\delta}$ denote the probability that the agent has not succeeded by time $t^{FB}(\mu_0) + \delta$ conditional on the agent continuing until this time:

$$\eta_{t^{FB}(\mu_0)+\delta} = \frac{\Pr(\text{did not succeed \& cont})_{t^{FB}(\mu_0)+\delta}}{\Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}.$$

The market's belief satisfies

$$\widehat{\mu}^1_{t^{FB}(\mu_0)+\delta} = 1 - \eta_{t^{FB}(\mu_0)+\delta} + \eta_{t^{FB}(\mu_0)+\delta}\ \mu_{t^{FB}(\mu_0)+\delta}. \tag{36}$$

Since $\mu_{t^{FB}(\mu_0)+\delta}$ is independent of μ_0, differentiating (36) yields

$$\begin{aligned} -\frac{\partial \widehat{\mu}^1_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0} &= \frac{\partial \eta_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0}(1 - \mu_{t^{FB}(\mu_0)+\delta}), \\ -\frac{\partial^2 \widehat{\mu}^1_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0^2} &= \frac{\partial^2 \eta_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0^2}(1 - \mu_{t^{FB}(\mu_0)+\delta}). \end{aligned}$$

Combining this with (35), we obtain that for any $\delta \geq 0$,

$$\frac{\partial \eta_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0} \leq 0, \quad \frac{\partial^2 \eta_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0^2} \leq 0. \tag{37}$$

Effects of μ_0 on welfare. We show that welfare is increasing in μ_0. Since first-best welfare is increasing in μ_0, it suffices to show that flow welfare at any time $t > t^{FB}(\mu_0)$ is increasing in μ_0. For any $\delta > 0$, welfare at time $t^{FB}(\mu_0) + \delta$ is

$$\begin{aligned} &\Pr(\text{succeeded \& cont})_{t^{FB}(\mu_0)+\delta}\ (\lambda_G + x) \\ &+ \Pr(\text{did not succeed \& cont})_{t^{FB}(\mu_0)+\delta}\ (\mu_{t^{FB}(\mu_0)+\delta}\lambda_G + x). \end{aligned} \tag{38}$$

Suppose for the purpose of contradiction that for some $\delta > 0$, (38) is decreasing in μ_0, that is (using the fact that $\mu_{t^{FB}(\mu_0)+\delta}$ is independent of μ_0),

$$\begin{aligned} &\frac{\partial \Pr(\text{succeeded \& cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0}\ (\lambda_G + x) \\ &+ \frac{\partial \Pr(\text{did not succeed \& cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0}\ (\mu_{t^{FB}(\mu_0)+\delta}\lambda_G + x) < 0. \end{aligned} \tag{39}$$

We can rewrite (39) as

$$\frac{\partial \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0}(\lambda_G + x) < \frac{\partial \Pr(\text{did not succeed \& cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0}\lambda_G(1-\mu_{t^{FB}(\mu_0)+\delta}). \tag{40}$$

Note that the derivative on the left-hand side is positive.[27] Moreover, Assumption 1 implies $(\lambda_G + x)/\lambda_G > 1 - \mu_0$. Hence, (40) implies

$$\frac{\partial \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0}(1-\mu_0) < \frac{\partial \Pr(\text{did not succeed \& cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0}(1-\mu_{t^{FB}(\mu_0)+\delta}).$$

Substituting with $1-\mu_{t^{FB}(\mu_0)+\delta} = \frac{1-\mu_0}{\mu_0 e^{-\lambda_G(t^{FB}(\mu_0)+\delta)}+1-\mu_0}$, this can be rewritten as

$$\frac{\partial \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0}\left(\mu_0 e^{-\lambda_G(t^{FB}(\mu_0)+\delta)} + 1 - \mu_0\right) < \frac{\partial \Pr(\text{did not succeed \& cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0}. \tag{41}$$

Finally, note that $\eta_{t^{FB}(\mu_0)+\delta} \leq \mu_0 e^{-\lambda_G(t^{FB}(\mu_0)+\delta)} + 1 - \mu_0$, as an agent who has succeeded does not stop. Thus, (41) implies

$$\frac{\partial \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0}\eta_{t^{FB}(\mu_0)+\delta} < \frac{\partial \Pr(\text{did not succeed \& cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0}. \tag{42}$$

We now show that (42) contradicts (37), namely the fact that $\eta_{t^{FB}(\mu_0)+\delta}$ is decreasing in μ_0. The derivative of $\eta_{t^{FB}(\mu_0)+\delta}$ with respect to μ_0 being negative implies

$$\begin{aligned}&\frac{\partial \Pr(\text{did not succeed \& cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0}\Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}\\ &-\frac{\partial \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0}\Pr(\text{did not succeed \& cont})_{t^{FB}(\mu_0)+\delta} \leq 0,\end{aligned}$$

[27]To see why, take $\mu_0' > \mu_0''$. It is clear that $\Pr(\text{cont})_{t^{FB}(\mu_0')+\delta} \geq \Pr(\text{cont})_{t^{FB}(\mu_0'')+\delta}$ for $t^{FB}(\mu_0') + \delta \leq \underline{t}(\mu_0')$, as $\mu_{\underline{t}}$ is decreasing in μ_0. Moreover, since $\eta_{t^{FB}(\mu_0')+\delta} = \eta_{t^{FB}(\mu_0'')+\delta}$ for $t^{FB}(\mu_0') + \delta \geq \underline{t}(\mu_0')$, the percentage change over time in $\Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}$ must be the same under μ_0' and μ_0'' at all $t^{FB}(\mu_0') + \delta \geq \underline{t}(\mu_0')$, and hence we also obtain $\Pr(\text{cont})_{t^{FB}(\mu_0')+\delta} \geq \Pr(\text{cont})_{t^{FB}(\mu_0'')+\delta}$ for all those times.

or, equivalently,

$$\frac{\partial \Pr(\text{did not succeed \& cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0} \leq \frac{\partial \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0} \eta_{t^{FB}(\mu_0)+\delta}.$$

This inequality is in contradiction with (42).

Effects of information on welfare. We show that the welfare effects of information are positive. Consider a public signal at time 0 that refines μ_0 while satisfying $-x/\mu_0 > R$ and Assumption 1 for all of its realizations. Since first-best welfare increases with information, it suffices to show that flow welfare at any time $t > t^{FB}(\mu_0)$ is convex in μ_0. Note that

$$\frac{\partial \eta_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0} = \left\{ \frac{\partial \Pr(\text{did not succeed \& cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0} - \frac{\partial \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0} \eta_{t^{FB}(\mu_0)+\delta} \right\} \frac{1}{\Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}.$$

By (37), $\frac{\partial^2 \eta_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0^2} \leq 0$. Hence,

$$0 \geq \left\{ \frac{\partial^2 \Pr(\text{did not succeed \& cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0^2} - \frac{\partial^2 \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0^2} \eta_{t^{FB}(\mu_0)+\delta} - \frac{\partial \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0} \frac{\partial \eta_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0} \right\} \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta} - \frac{\partial \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0} \frac{\partial \eta_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0} \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}.$$

Equivalently,

$$2\frac{\partial \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0} \frac{\partial \eta_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0} \geq \left\{ \frac{\partial^2 \Pr(\text{did not succeed \& cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0^2} - \frac{\partial^2 \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0^2} \eta_{t^{FB}(\mu_0)+\delta} \right\}.$$

Since $\frac{\partial \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0} \geq 0$ and $\frac{\partial \eta_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0} \leq 0$, the left-hand side is negative, which implies

$$\frac{\partial^2 \Pr(\text{did not succeed \& cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0^2} \leq \frac{\partial^2 \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0^2} \eta_{t^{FB}(\mu_0)+\delta}. \quad (43)$$

If the derivative on the left-hand side of (43) is negative, the distortion relative to first best is concave in μ_0 and thus welfare is convex in μ_0.

Suppose instead that the derivative on the left-hand side of (43) is strictly positive. Then this equation implies $\frac{\partial^2 \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0^2} > 0$. Suppose for the purpose of contradiction that welfare is concave in μ_0, that is:

$$\begin{aligned} &\frac{\partial^2 \Pr(\text{succeeded \& cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0^2} (\lambda_G + x) \\ &+ \frac{\partial^2 \Pr(\text{did not succeed \& cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0^2} (\mu_{t^{FB}(\mu_0)+\delta} \lambda_G + x) < 0. \end{aligned} \quad (44)$$

We can rewrite (44) as

$$\frac{\partial^2 \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0^2}(\lambda_G + x) < \frac{\partial^2 \Pr(\text{did not succeed \& cont})_{t^{FB}(\mu_0)+\delta}}{\partial^2 \mu_0} \lambda_G (1 - \mu_{t^{FB}(\mu_0)+\delta}). \quad (45)$$

Recall that we are considering the case in which the derivative on the left-hand side is strictly positive. Hence, we can follow analogous steps to those in (40)-(42) to show that (45) implies

$$\frac{\partial^2 \Pr(\text{cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0^2} \eta_{t^{FB}(\mu_0)+\delta} < \frac{\partial^2 \Pr(\text{did not succeed \& cont})_{t^{FB}(\mu_0)+\delta}}{\partial \mu_0^2}. \quad (46)$$

This inequality is in contradiction with (43).

Finally, consider a fully informative signal that reveals at time 0 whether the project is good or bad. Arguments analogous to those in the proof of Proposition 3 imply that this signal eliminates distortions and increases welfare.

Effects of μ_0 and information on distortion relative to first best. We show by example that an increase in the prior μ_0 and imperfect information that refines μ_0 can reduce the distortion relative to first best. To do this, we compute numerically the equilibrium for different prior beliefs.

We approximate the continuous time outcome by taking a discrete time model with periods of small length. Specifically, discretize time in periods of dt length, so $t \in \{0, dt, 2dt, ...\}$, and assume t^{FB} and $\underline{t}$ are on the grid (i.e., t^{FB}/dt and $\underline{t}/dt$ are integers). The probability that a good project succeeds over a period of length dt is $\lambda_G dt$. The probability that a good project succeeds before time $\underline{t}$ is $1-(1-\lambda_G dt)^{\frac{\underline{t}}{dt}}$.

Recall that the agent continues with certainty until time $\underline{t}$, and we can compute the market's belief $\widehat{\mu}_t^1$ for each time $t \geq \underline{t}$ using equation (23). Using $\widehat{\mu}_t^1$, we can then solve for the probability with which the agent continues at each time. Call $\gamma_{\underline{t}} dt$ the probability that the agent stops over $[\underline{t}, \underline{t}+dt]$ absent success. Then

$$\widehat{\mu}_{\underline{t}+dt}^1 = \frac{\mu_0 \left[1-(1-\lambda_G dt)^{\frac{\underline{t}}{dt}}\right] + \mu_0 (1-\lambda_G dt)^{\frac{\underline{t}}{dt}} (1-\gamma_{\underline{t}} dt)}{\mu_0 \left[1-(1-\lambda_G dt)^{\frac{\underline{t}}{dt}}\right] + (\mu_0 (1-\lambda_G dt)^{\frac{\underline{t}}{dt}} + 1 - \mu_0)(1-\gamma_{\underline{t}} dt)}.$$

Similarly, call $\gamma_{\underline{t}+dt} dt$ the probability that the agent stops over $[\underline{t}+dt, \underline{t}+2dt]$ absent success. The probability that an agent who has a bad project will stay until time $\underline{t}+2dt$ is $(1-\gamma_{\underline{t}} dt)(1-\gamma_{\underline{t}+dt} dt)$. The probability that an agent who has a good project and had not succeeded by time $\underline{t}$ will stay until $\underline{t}+2dt$ is $(1-\gamma_{\underline{t}} dt)[\lambda_G dt + (1-\lambda_G dt)(1-\gamma_{\underline{t}+dt} dt)]$. Thus,

$$\widehat{\mu}_{\underline{t}+2dt}^1 = \frac{\mu_0 \left[1-(1-\lambda_G dt)^{\frac{\underline{t}}{dt}}\right] + \mu_0 (1-\lambda_G dt)^{\frac{\underline{t}}{dt}} (1-\gamma_{\underline{t}} dt)[\lambda_G dt + (1-\lambda_G dt)(1-\gamma_{\underline{t}+dt} dt)]}{\left[\begin{array}{c} \mu_0 \left[1-(1-\lambda_G dt)^{\frac{\underline{t}}{dt}}\right] + \mu_0 (1-\lambda_G dt)^{\frac{\underline{t}}{dt}} (1-\gamma_1 dt)[\lambda_G dt + (1-\lambda_G dt)(1-\gamma_{\underline{t}+dt} dt)] \\ +(1-\mu_0)(1-\gamma_{\underline{t}} dt)(1-\gamma_{\underline{t}+dt} dt) \end{array}\right]}.$$

We perform analogous computations for $\underline{t}+3dt$, $\underline{t}+4dt$, and so on. Equilibrium welfare can then be written as

$$\mu_0 \sum_{t=0}^{\underline{t}-dt} (1-rdt)^{\frac{t}{dt}} (1-\lambda_G dt)^{\frac{t}{dt}} (xdt + \lambda_G dt \ v) + (1-\mu_0) \sum_{t=0}^{\underline{t}-dt} (1-rdt)^{\frac{t}{dt}} xdt$$
$$+\mu_0 \sum_{t=\underline{t}}^{\infty} (1-rdt)^{\frac{t}{dt}} (1-\lambda_G dt)^{\frac{t}{dt}} \Pi_{s=\underline{t}}^{t} (1-\gamma_s dt)(xdt + \lambda_G dt \ v)$$
$$+(1-\mu_0) \sum_{t=\underline{t}}^{\infty} (1-rdt)^{\frac{t}{dt}} \Pi_{s=\underline{t}}^{t} (1-\gamma_s dt) xdt,$$

where $\Pi_{s=\underline{t}}^{t}(1-\gamma_s dt) = (1-\gamma_{\underline{t}} dt)(1-\gamma_{\underline{t}+dt} dt)(1-\gamma_{\underline{t}+2dt} dt)\ldots(1-\gamma_{\underline{t}+ndt} dt)$ for $\underline{t}+ndt = t$. We take a large time T (on the grid) such that γ_t is virtually zero for $t > T$ and approximate welfare by computing

$$\begin{aligned} S \;=\; & \mu_0 \sum_{t=0}^{\underline{t}-dt} (1-rdt)^{\frac{t}{dt}} (1-\lambda_G dt)^{\frac{t}{dt}} (xdt + \lambda_G dt \ v) + (1-\mu_0) \sum_{t=0}^{\underline{t}-dt} (1-rdt)^{\frac{t}{dt}} xdt \\ & +\mu_0 \sum_{t=\underline{t}}^{T-dt} (1-rdt)^{\frac{t}{dt}} (1-\lambda_G dt)^{\frac{t}{dt}} \Pi_{s=\underline{t}}^{t} (1-\gamma_s dt)(xdt + \lambda_G dt \ v) \\ & +(1-\mu_0) \sum_{t=\underline{t}}^{T-dt} (1-rdt)^{\frac{t}{dt}} \Pi_{s=\underline{t}}^{t} (1-\gamma_s dt) xdt \\ & +(1-rdt)^{\frac{T}{dt}} \Pi_{s=\underline{t}}^{T} (1-\gamma_s dt) \left[\mu_0 (1-\lambda_G dt)^{\frac{T}{dt}} \frac{(x+\lambda_G v)}{r+\lambda_G - r\lambda_G dt} + (1-\mu_0)\frac{x}{r} \right]. \end{aligned}$$

Finally, we compute first-best welfare,

$$S^{FB} = \mu_0 \sum_{t=0}^{t^{FB}} (1-rdt)^{\frac{t}{dt}} (1-\lambda_G dt)^{\frac{t}{dt}} (xdt + \lambda_G dt \ v) + (1-\mu_0) \sum_{t=0}^{t^{FB}} (1-rdt)^{\frac{t}{dt}} xdt,$$

where $v = 1 + \frac{x+\lambda_G}{r}$, and we compute the distortion, $D = S^{FB} - S$.

Consider the parameters reported in the example of Figure 3, with a prior $\mu_0 = 0.5$, and take periods of length $dt = 0.001$. We verify that γ_t becomes virtually zero after a large enough number of periods; accordingly, we compute equilibrium welfare S above for $T = 3,000$. Let $\mu_0' = 0.75$ and $\mu_0'' = 0.25$, and denote by $D(\mu_0)$ the distortion given a prior belief μ_0. We obtain $D(\mu_0) =$

0.0426, $D(\mu_0') = 0.0409$, and $D(\mu_0'') = 0.0286$. Hence, an increase in the prior from μ_0 to μ_0' reduces the distortion relative to first best. Furthermore, take a binary public signal that increases the prior to μ_0' when the realization is high and decreases the prior to μ_0'' when the realization is low, with each realization occurring with equal probability. Since $D(\mu_0) > 0.5D(\mu_0') + 0.5D(\mu_0'')$, releasing this public signal at time 0 reduces the distortion relative to first best.