Online Appendix to "Fees, Reputation and Information Production in the Credit Rating Industry" by Jacopo Bizzotto and Adrien Vigier

Online Appendix A

In this appendix we check the robustness of our analysis by relaxing the assumption on the perfect observability of project returns in case the agency chooses to become informed. We show that, as in the baseline model:

- for some parameter values contingent fees improve expected social welfare relative to upfront fees;
- upfront fees improve expected social welfare relative to contingent fees as long as the cost of information acquisition, c, is sufficiently small.

The Model. The agency lives for two periods, $t = 1, 2$.[1] For concreteness, we distinguish now between a project's quality, $q_t \in \{-1, 1\}$, and a project's return, Q_t. By acquiring information in period t, the agency observes a noisy signal $s_t \in \{-1, 1\}$ of the project quality q_t, such that $\mathbb{P}(s_t = q_t | q_t) = 1 - \epsilon$, with $\epsilon \in (0, \frac{1}{2})$. A project's return is related to its quality by $Q_t = \frac{q_t}{1-2\epsilon}$.[2] As in the baseline model, the sequence $\{q_t\}$ is independent and identically distributed according to $\mathbb{P}(q_t = 1) = \frac{1}{2}$. The baseline model therefore corresponds to $\epsilon = 0$.

Applying Bayes' rule gives

$$\mathbb{E}[q_t | r_t = 1, \rho_t, \hat{e}_t] = \Phi(\rho_t, \hat{e}_t) := \frac{1 - (1-\rho_t)(1-\hat{e}_t)}{1 + (1-\rho_t)(1-\hat{e}_t)},$$

and

$$\mathbb{P}(r_t = 1 | \rho_t, \hat{e}_t)\mathbb{E}[q_t | r_t = 1, \rho_t, \hat{e}_t] = \Upsilon(\rho_t, \hat{e}_t) := \frac{1}{2}\big[1 - (1-\rho_t)(1-\hat{e}_t)\big].$$

Next, given $e : [0,1] \to [0,1]$, define

$$\rho^+ := \begin{cases} 0 & \text{if } \rho = 0 \\ \frac{\rho}{\rho + (1-\rho)e(\rho)} & \text{if } \rho > 0 \end{cases}; \qquad \rho^g := \begin{cases} 0 & \text{if } \rho = 0 \\ \frac{\rho(1-\epsilon)}{1-\epsilon+\epsilon(1-\rho)(1-e(\rho))} & \text{if } \rho > 0 \end{cases}; \qquad \text{and}$$

$$\rho^b := \begin{cases} 0 & \text{if } \rho = 0 \\ \frac{\rho\epsilon}{\epsilon + (1-\epsilon)(1-\rho)(1-e(\rho))} & \text{if } \rho > 0 \end{cases}.$$

The next definition is immediately adapted from the equilibrium concept of Section 2.

[1] We set here $\beta = 1$ to reduce notation.

[2] We scale up project returns by a factor equal to $\frac{1}{1-2\epsilon}$ to keep the expected return conditional on observing s_t precisely equal to s_t.

Definition 1. *An equilibrium with contingent fees comprises functions* $e_t : [0,1] \to [0,1]$ *specifying the probabilities* $e_t(\rho_t)$ *that the strategic agency acquires information in period* t *given reputation* ρ_t*, for* $t = 1, 2$*, such that each period:*

(i) the choice(s) implied by $e_t(\rho_t)$ *maximize the agency's expected intertemporal profit given*

$$\pi_t^{co} = \Phi(\rho_t, \hat{e}_t)\mathbf{1}_{\{r_t=1\}} - c\mathbf{1}_{\{information\ acquired\ in\ period\ t\}}, \tag{OA.1}$$

and

$$\rho_2 = \begin{cases} \rho_1^g & \text{if } q_1 = 1 = r_1; \\ \rho_1^+ & \text{if } q_1 = -1 = r_1; \\ \rho_1^b & \text{if } q_1 = -1 = -r_1. \end{cases}$$

(ii) firms and investors' beliefs satisfy $\hat{e}_t = e_t(\rho_t)$*.*

The definition of an equilibrium with upfront fees is obtained by replacing (OA.1) with

$$\pi_t^{up} = \Upsilon(\rho_t, \hat{e}_t) - c\mathbf{1}_{\{\text{information acquired in period } t\}}. \tag{OA.2}$$

The Equilibria. We next characterize the equilibria with, respectively, contingent fees and upfront fees. With 2 periods the strategic agency always shirks in period 2. So the focus is on information acquisition at $t = 1$. Proofs of the propositions are relegated to the end of this appendix.

Proposition OA.1. *With contingent fees, in equilibrium* $e_1(\rho_1) > 0$ *if and only if* $\delta > 2c$ *and* $\rho_1 < \rho_{co}(\delta)$*, where* $\rho_{co}(\delta)$ *is defined implicitly by*

$$\delta = \left(c + \frac{\rho_{co}}{2(2-\rho_{co})}\right)\left(\frac{2-(1+\epsilon)\rho_{co}}{2-(1+2\epsilon(1-\epsilon))\rho_{co}}\right)\left(\frac{2-(2-\epsilon)\rho_{co}}{1-\rho_{co}}\right).$$

Moreover $\lim_{c\to 0} e_1(\rho_1) < 1$ *for any* ρ_1 *and* δ*.*

Proposition OA.2. *With upfront fees, in equilibrium* $e_1(\rho_1) > 0$ *if and only if* $\delta > 4c$ *and* $\rho_1 < \rho_{up}(\delta)$*, where* $\rho_{up}(\delta)$ *is defined implicitly by*

$$\delta = 4c\left(\frac{1}{1-2\epsilon(1-\epsilon)\rho_{up}} + \frac{\epsilon(1-\epsilon)\rho_{up}^2}{(1-\rho_{up})(1-2\epsilon(1-\epsilon)\rho_{up})}\right).$$

Moreover $\lim_{c\to 0} e_1(\rho_1) = 1$ *for any* ρ_1 *and* δ*.*

We illustrate the propositions in Figure OA.1. At $t = 1$ the strategic agency acquires information, with some probability, for all (δ, ρ_1) lying to the right of the curve ρ_{co} when fees are contingent and to the right of the curve ρ_{up} when fees are upfront.

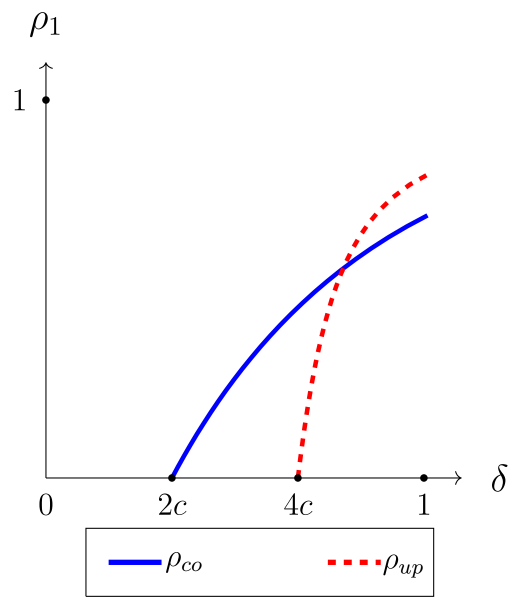

FIGURE OA.1: NOISY SIGNALS OF PROJECT RETURNS

Welfare Comparison. As in equilibrium the strategic agency shirks at $t = 2$ irrespective of the fee structure, comparing expected social welfare in equilibrium under different fee structures reduces to comparing the probability that a strategic agency acquires information at $t = 1$. When fees are contingent, for $\delta \in (2c, 4c)$ the strategic agency acquires information at $t = 1$ with positive probability provided its reputation is not too high (Proposition OA.1). By contrast, for $\delta \in (2c, 4c)$ the strategic agency shirks with probability 1 when fees are upfront (Proposition OA.2). For $\delta \in (2c, 4c)$ and sufficiently low reputation expected social welfare is therefore higher under contingent fees than under upfront fees. Moreover for any δ and ρ_1, $\lim_{c\to 0} e_1(\rho_1) = 1$ only in the case of upfront fees. Thus upfront fees increase expected social welfare if the cost of information acquisition is sufficiently low.

Proof of Proposition OA.1: First, note that the function Φ is continuous, weakly increasing in each of its arguments, and $\Phi(1,\cdot) = \Phi(\cdot,1) = 1$. Moreover, notice that in any equilibrium the strategic agency must shirk at $t = 2$ with probability 1 giving $\pi_2 = \Phi(\rho_2, 0) = \frac{\rho_2}{2-\rho_2}$. Define $V(\rho) := \frac{\rho}{2-\rho}$. At $t = 1$, the agency's expected intertemporal profit from shirking and announcing $r_t = 1$ is equal to $\Phi(\rho_1, \hat{e}_1) + \frac{\delta}{2}\left[V(\rho_1^g) + V(\rho_1^b)\right]$. The intertemporal profit from acquiring information is equal to $\frac{1}{2}\Phi(\rho_1, \hat{e}_1) - c + \frac{\delta}{2}\left[V(\rho_1^+) + (1-\epsilon)V(\rho_1^g) + \epsilon V(\rho_1^b)\right]$.

Fix $\rho_1 \in (0,1)$. An equilibrium in which $e_1(\rho_1) = 1$ exists if and only if

$$\frac{1}{2}\Phi(\rho_1, 1) + \frac{\delta}{2}\left[\epsilon V(\rho_1) + (1-\epsilon)V(\rho_1)\right] \leq \frac{\delta}{2}V(\rho_1) - c.$$

This condition is always violated, thus in equilibrium $e_1(\rho_1) < 1$. An equilibrium in which $e_1(\rho_1) = 0$ in turn exists if and only if

$$\frac{1}{2}\Phi(\rho_1, 0) + \frac{\delta}{2}\left[\epsilon V\left(\frac{\rho_1(1-\epsilon)}{1-\epsilon\rho_1}\right) + (1-\epsilon)V\left(\frac{\rho_1\epsilon}{1-(1-\epsilon)\rho_1}\right)\right] \geq \frac{\delta}{2}V(1) - c.$$

Substituting for $\Phi(\cdot)$ and $V(\cdot)$ and simplifying yields

$$\delta \leq \tilde{\delta}(\rho_1) := \left(c + \frac{\rho_1}{2(2-\rho_1)}\right)\left(\frac{2-(1+\epsilon)\rho_1}{2-(1+2\epsilon(1-\epsilon))\rho_1}\right)\left(\frac{2-(2-\epsilon)\rho_1}{1-\rho_1}\right).$$

Notice that $\tilde{\delta}(\rho_1)$ is continuous and increasing in ρ_1 for $\rho_1 \in (0,1)$ as it is the product of 3 terms, each of which is continuous and increasing in ρ_1 for $\rho_1 \in (0,1)$. Hence $\delta < \tilde{\delta}(\rho_1)$ is equivalent to $\rho_1 > \rho_{co}(\delta)$; moreover as $\tilde{\delta}(0) = 2c$, then $e_1(\rho_1) > 0$ only if $\delta > 2c$. Furthermore, an equilibrium in which $e_1(\rho_1) \in (0,1)$ exists if and only if

$$\frac{\delta}{2}\left[V\left(\frac{\rho_1}{\rho_1+(1-\rho_1)e(\rho_1)}\right) - \epsilon V\left(\frac{\rho_1(1-\epsilon)}{1-\epsilon+\epsilon(1-\rho_1)(1-e(\rho_1))}\right) - (1-\epsilon)V\left(\frac{\rho_1\epsilon}{\epsilon+(1-\epsilon)(1-\rho_1)(1-e(\rho_1))}\right)\right] = c + \frac{1}{2}\Phi(\rho_1, e(\rho_1)). \tag{OA.3}$$

The left-hand side of (OA.3) is strictly decreasing in $e(\rho_1)$, while the right-hand side is strictly increasing in $e(\rho_1)$, thus there is at most one $e(\rho_1)$ that satisfies the equality. Moreover, for $e(\rho_1) = 0$ this equality reduces to $\delta = \tilde{\delta}(\rho_1)$. As the left-hand side of (OA.3) is increasing in δ and the right-hand side does not depend on δ, then $\delta > \tilde{\delta}(\rho_1)$, which is equivalent to $\rho_1 < \rho_{co}(\delta)$, is necessary for an equilibrium in which $e_1(\rho_1) \in (0,1)$. That this condition is also sufficient follows from standard arguments.

Pick a pair δ and ρ_1. For sufficiently small c, $\delta > \tilde{\delta}(\rho_1)$, thus in equilibrium $e_1(\rho_1)$ satisfies (OA.3). Note also that for $c = 0$ the unique $e(\rho_1)$ that satisfies (OA.3) belongs to the interval $(0, 1)$. As both $\Phi(\cdot)$ and $V(\cdot)$ are continuous in each of their arguments, then in equilibrium $\lim_{c\to 0} e_1(\rho_1) < 1$. ■

Proof of Proposition OA.2: First notice that Υ is continuous, weakly increasing in each of its arguments, and $\Upsilon(1, \cdot) = \Upsilon(\cdot, 1) = \frac{1}{2}$. Moreover, note that in any equilibrium the strategic agency must shirk at $t = 2$ with probability 1, giving $\pi_2 = \Upsilon(\rho_2, 0) = \frac{\rho_2}{2}$. Define $V(\rho) := \frac{\rho}{2}$. At $t = 1$, the agency's expected intertemporal profit from shirking and announcing $r_t = 1$ is equal to $\Upsilon(\rho_1, \hat{e}_1) + \frac{\delta}{2}\left[V(\rho_1^g) + V(\rho_1^b)\right]$. The intertemporal profit from acquiring information is equal to $\Upsilon(\rho_1, \hat{e}_1) - c + \frac{\delta}{2}\left[V(\rho_1^+) + (1-\epsilon)V(\rho_1^g) + \epsilon V(\rho_1^b)\right]$. Fix $\rho_1 \in (0, 1)$. An equilibrium in which $e_1(\rho_1) = 1$ exists if and only if

$$\frac{\delta}{2}\left[\epsilon V(\rho_1) + (1-\epsilon)V(\rho_1)\right] \leq \frac{\delta}{2}V(\rho_1) - c.$$

This condition is always violated. An equilibrium in which $e_1(\rho_1) = 0$ exists if and only if

$$\frac{\delta}{2}\left[\epsilon V\left(\frac{\rho_1(1-\epsilon)}{1-\epsilon\rho_1}\right) + (1-\epsilon)V\left(\frac{\rho_1\epsilon}{1-(1-\epsilon)\rho_1}\right)\right] \geq \frac{\delta}{2}V(1) - c.$$

Substituting for $V(\cdot)$ and simplifying yields

$$\delta \leq \bar{\delta}(\rho_1) := 4c\left(\frac{1}{1-2\epsilon(1-\epsilon)\rho_1} + \frac{\epsilon(1-\epsilon)\rho_1^2}{(1-\rho_1)(1-2\epsilon(1-\epsilon)\rho_1)}\right).$$

As $\bar{\delta}(\rho_1)$ is continuous and increasing in ρ_1, then $\delta < \bar{\delta}(\rho_1)$ is equivalent to $\rho_1 > \rho_{up}(\delta)$; moreover, as $\bar{\delta}(0) = 4c$, then $e_1(\rho_1) > 0$ only if $\delta > 4c$. Furthermore, an equilibrium in which $e_1(\rho_1) \in (0, 1)$ exists if and only if

$$\frac{\delta}{2}\Bigg[V\left(\frac{\rho_1}{\rho_1 + (1-\rho_1)e(\rho_1)}\right) - \epsilon V\left(\frac{\rho_1(1-\epsilon)}{1-\epsilon+\epsilon(1-\rho_1)(1-e(\rho_1))}\right) - (1-\epsilon)V\left(\frac{\rho_1\epsilon}{\epsilon+(1-\epsilon)(1-\rho_1)(1-e(\rho_1))}\right)\Bigg] = c. \tag{OA.4}$$

The left-hand side of (OA.4) is strictly decreasing in $e(\rho_1)$, and the right-hand side is strictly increasing in $e(\rho_1)$, thus there is at most one $e(\rho_1)$ that satisfies the equality. Moreover, for

$e(\rho_1) = 0$ this equality reduces to $\delta = \overline{\delta}(\rho_1)$. As the left-hand side is increasing in δ and the left-hand side does not depend on δ, $\delta > \overline{\delta}(\rho_1)$ is necessary for an equilibrium in which $e_1(\rho_1) \in (0, 1)$. That this condition is also sufficient follows from standard arguments.

Next, fix δ and ρ_1. For sufficiently small c, $\delta > \overline{\delta}(\rho_1)$, thus in equilibrium $e_1(\rho_1)$ satisfies (OA.4). Note also that, for $c = 0$, $e(\rho_1) = 1$ satisfies (OA.4). Continuity of Φ thus ensures that in equilibrium $\lim_{c\to 0} e_1(\rho_1) = 1$. ■

Online Appendix B

In this appendix we check the robustness of our analysis by allowing the agency to announce $r_t = -1$ whether or not in period t the agency chooses to acquire information. We show that, as in the baseline model:

- for some parameter values contingent fees improve expected social welfare relative to upfront fees;
- upfront fees improve expected social welfare relative to contingent fees as long as the cost of information acquisition, c, is sufficiently small.

The Model. The agency lives for two periods, $t = 1, 2$.[3] We let $\hat{z}_t$ denote the beginning-of-period-t belief that the strategic agency will announce $r_t = -1$ in case it shirks.[4] Applying Bayes' rule,

$$\mathbb{E}[q_t | r_t = 1, \rho_t, \hat{e}_t, \hat{z}_t] = \Phi(\rho_t, \hat{e}_t, \hat{z}_t) := \frac{1 - (1-\rho_t)(1-\hat{e}_t)}{1 + (1-\rho_t)(1-\hat{e}_t)(1-2\hat{z}_t)},$$

while

$$\mathbb{E}[q_t | r_t = -1, \rho_t, \hat{e}_t, \hat{z}_t] = \frac{(-1)(1-(1-\rho_t)(1-\hat{e}_t))}{\rho_t + (1-\rho_t)\hat{e}_t + 2\hat{z}_t(1-\rho_t)(1-\hat{e}_t)}.$$

Note that $\mathbb{E}[q_t | r_t = 1, \rho_t, \hat{e}_t, \hat{z}_t] \geq 0 \geq \mathbb{E}[q_t | r_t = -1, \rho_t, \hat{e}_t, \hat{z}_t]$ for all ρ_t, $\hat{e}_t$ and $\hat{z}_t$. So with contingent fees the period-t profit of the agency is

$$\pi_t = \Phi(\rho_t, \hat{e}_t, \hat{z}_t)\mathbf{1}_{\{r_t=1\}} - c\mathbf{1}_{\{\text{information acquired in period } t\}}. \tag{OB.1}$$

Applying Bayes' rule again,

$$\mathbb{P}(r_t = 1 | \rho_t, \hat{e}_t, \hat{z}_t)\mathbb{E}[q_t | r_t = 1, \rho_t, \hat{e}_t, \hat{z}_t] = \Upsilon(\rho_t, \hat{e}_t) := \frac{1}{2}\big[1 - (1-\rho_t)(1-\hat{e}_t)\big].$$

With upfront fees, the period-t profit of the agency is

$$\pi_t = \Upsilon(\rho_t, \hat{e}_t) - c\mathbf{1}_{\{\text{information acquired in period } t\}}. \tag{OB.2}$$

[3] We set here $\beta = 1$ to reduce notation.

[4] As noted in the baseline model, upon acquiring information the strategic agency would report truthfully even if it had the option to misreport. Thus we assume here, as in the baseline model, that upon acquiring information the agency reports truthfully.

Given functions $e_t(\cdot)$ and $z_t(\cdot)$ from $[0,1]$ to $[0,1]$, define

$$\rho_t^+ := \begin{cases} 0 & \text{if } \rho_t = 0; \\ \frac{\rho_t}{\rho_t + (1-\rho_t)[e_t(\rho_t) + (1-e_t(\rho_t))(1-z_t(\rho_t))]} & \text{if } \rho_t > 0; \end{cases}$$

and

$$\rho_t^{++} := \begin{cases} 0 & \text{if } \rho_t = 0; \\ \frac{\rho_t}{\rho_t + (1-\rho_t)[e(\rho_t) + (1-e(\rho_t))z_t(\rho_t)]} & \text{if } \rho_t > 0. \end{cases}$$

The next definition is immediately adapted from the equilibrium concept of Section 2.

Definition 2. *An equilibrium with contingent fees comprises functions $e_t : [0,1] \to [0,1]$ and $z_t : [0,1] \to [0,1]$, for $t = 1, 2$, specifying the probabilities $e_t(\rho_t)$ that the strategic agency acquires information and $z_t(\rho_t)$ of announcing $r_t = -1$ conditional on shirking, given reputation ρ_t, such that each period:*

(i) the choices implied by $e_t(\rho_t)$ and $z_t(\rho_t)$ maximize the agency's expected intertemporal profit given (OB.1) *and*

$$\rho_2 = \begin{cases} \rho_1^+ & \textit{if } q_1 = r_1 = 1; \\ \rho_1^{++} & \textit{if } q_1 = r_1 = -1; \\ 0 & \textit{if } q_1 = -r_1. \end{cases}$$

(ii) firms and investors' beliefs satisfy $\hat{e}_t = e_t(\rho_t)$ and $\hat{z}_t = z_t(\rho_t)$.

The definition of an equilibrium with upfront fees is obtained replacing (OB.1) with (OB.2).

The Equilibria. We next characterize the equilibria with, respectively, contingent fees and upfront fees. With 2 periods the strategic agency always shirks in period 2. So the focus is on information acquisition at $t = 1$. Proofs of the propositions are relegated to the end of this appendix.

Proposition OB.1. *Let fees be contingent. In equilibrium $e_1(\rho_1) < 1$ for all $\rho_1 \in (0,1)$. Moreover, $e_1(\rho_1) \in (0,1)$ for $\delta > \frac{2c-1+\sqrt{1+12c+4c^2}}{2}$ and $\rho_1 \in \left(\frac{2c(2+\delta)}{1+2\delta+\delta^2}, \frac{4c-2\delta}{2c-1-\delta}\right)$.*

Proposition OB.2. *Let fees be upfront. In equilibrium, if* $\delta \leq 4c$ *then* $e_1(\rho_1) = 0$ *for all* $\rho_1 \in (0,1)$. *If instead* $\delta > 4c$ *then*

$$\begin{cases} \textit{if} \quad \rho_1 \in [\frac{4c}{\delta}, 1) & \textit{then} \quad e(\rho_1) = 1; \\ \textit{if} \quad \rho_1 \in (\frac{2c}{\delta - 2c}, \frac{4c}{\delta}) & \textit{then} \quad e(\rho_1) \in (0,1); \\ \textit{if} \quad \rho_1 \in (0, \frac{2c}{\delta-2c}] & \textit{then} \quad e(\rho_1) = 0. \end{cases}$$

We illustrate in Figure OB.1 the regions in (δ, ρ_1)-space identified in the two propositions. When fees are contingent, at $t = 1$ the strategic agency acquires information, with some probability, for all (δ, ρ_1) lying to the right of the solid curves. When fees are upfront, the strategic agency acquires information, with some probability, for all (δ, ρ_1) in between the dashed lines, and acquires information with probability 1 for all (δ, ρ_1) to the right of both dashed lines.

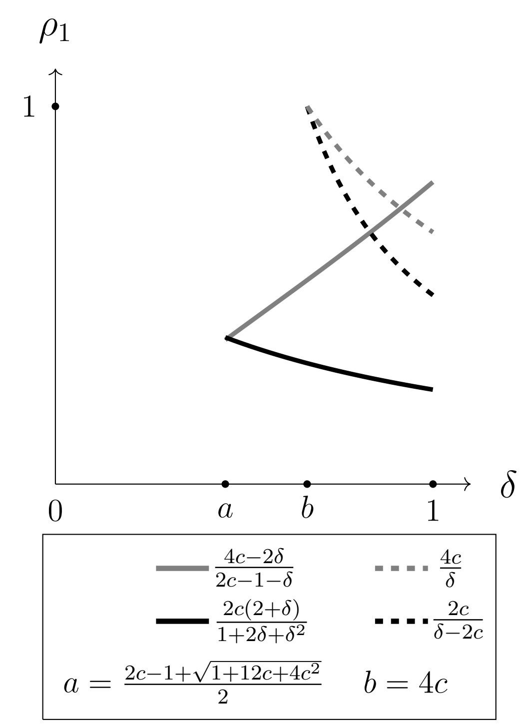

FIGURE OB.1: DEFLATED RATINGS

Welfare Comparison. Since the strategic agency shirks at $t = 2$ irrespective of the fee structure, and, for the symmetrical payoffs we consider, welfare does not depend on the agency's choice to inflate or deflate a rating in case it shirks, social welfare is therefore uniquely determined by the probability that the agency acquires information at $t = 1$. Note that in the parameter region defined by $\delta \in (\frac{2c-1+\sqrt{1+12c+4c^2}}{2}, 4c)$ and $\rho_1 \in \left(\frac{2c(2+\delta)}{1+2\delta+\delta^2}, \frac{4c-2\delta}{2c-1-\delta}\right)$ the agency shirks at $t = 1$ with upfront fees (Proposition OB.2) but acquires information with positive probability with contingent fees (Proposition OB.1), thus contingent fees improve expected social welfare. Note also that for c sufficiently close to 0 any ρ_1 and δ satisfy $\delta > 4c$ and $\rho_1 > \frac{4c}{\delta}$, hence ensuring $e_1(\rho_1) = 1$ with upfront fees (Proposition OB.2). As $e_1(\rho_1) < 1$ everywhere with contingent fees (Proposition OB.1), the previous remarks establish that, for any ρ_1 and δ, upfront fees improve expected social welfare relative to contingent fees as long as c is sufficiently small.

Proof of Proposition OB.1: The function Φ is continuous, weakly increasing in each of its arguments, and $\Phi(1,\cdot,\cdot) = \Phi(\cdot,1,\cdot) = 1$. Notice that in any equilibrium the strategic agency must shirk and announce $r_2 = 1$ at $t = 2$ with probability 1 giving $\pi_2 = \Phi(\rho_2, 0, 0) = \frac{\rho_2}{2-\rho_2}$. Define $V(\rho) := \frac{\rho}{2-\rho}$. Let $\rho_1^+(e,z)$ (respectively $\rho_1^{++}(e,z)$) denote the value of ρ_1^+ (respectively ρ_1^{++}) for $e_1(\rho_1) = e$ and $z_1(\rho_1) = z$. At $t = 1$, the agency's expected intertemporal profit from shirking and announcing $r_t = 1$ is equal to $\Phi(\rho_1, \hat{e}_1, \hat{z}_1) + \frac{\delta}{2}V(\rho_1^+(\hat{e}_1, \hat{z}_1))$, the intertemporal profit from shirking and announcing $r_t = -1$ is equal to $\frac{\delta}{2}V(\rho_1^{++}(\hat{e}_1, \hat{z}_1))$, and the intertemporal profit from acquiring information is equal to $\frac{1}{2}\Phi(\rho_1, \hat{e}_1, \hat{z}_1) - c + \frac{\delta}{2}\left[V(\rho_1^+(\hat{e}_1, \hat{z}_1)) + V(\rho_1^{++}(\hat{e}_1, \hat{z}_1))\right]$. The rest of the proof contains 3 steps. Step 1 establishes than in equilibrium $e_1(\rho_1) < 1$. Step 2 computes the probability with which the strategic agency must announce $r_1 = -1$ if in equilibrium $e_1(\rho_1) = 0$. Step 3 characterizes a parameter region in which $e_1(\rho_1) > 0$.

Step 1: An equilibrium with $e_1(\rho_1) = 1$ requires

$$\frac{\delta V(\rho_1)}{2} - c \geq \frac{\Phi(\rho_1, 1, z_1)}{2}, \tag{OB.3}$$

for some z_1. Note that, for any z_1 and any ρ_1: $\Phi(\rho_1, 1, z_1) = 1 > V(\rho_1)$. Thus (OB.3) holds only if $\frac{\delta-1}{2} > c$. This condition is violated as $c > 0 > \frac{\delta-1}{2}$. Hence in equilibrium $e(\rho_1) < 1$.

Step 2: Define $\tilde{z}(\rho_1)$ implicitly by

$$\Phi(\rho_1, 0, \tilde{z}(\rho_1)) = \frac{\delta}{2}\left[V(\rho_1^{++}(0, \tilde{z}(\rho_1))) - V(\rho_1^+(0, \tilde{z}(\rho_1)))\right],$$

which is equivalent to

$$\frac{\rho_1}{1+(1-\rho_1)(1-2\tilde{z})} = \frac{\delta}{2}\left(\frac{\rho_1}{\rho_1+2(1-\rho_1)\tilde{z}} - \frac{\rho_1}{1+(1-\rho_1)(1-2\tilde{z})}\right).$$

Simplifying to solve for $\tilde{z}$ gives $\tilde{z}(\rho_1) = \frac{\delta(1-\rho_1)-\rho_1}{2(1-\rho_1)(\delta+1)}$. Note that $\tilde{z}(\rho_1) < 1$ for all $\rho_1 \in (0,1)$, while $\tilde{z}(\rho_1) > 0$ if and only if $\rho_1 < \frac{\delta}{1+\delta}$.

Now, in equilibrium, if $e_1(\rho_1) = 0$ then $\Phi(\rho_1, 0, z_1(\rho_1)) > \frac{\delta}{2}\big[V(\rho_1^{++}) - V(\rho_1^{+})\big]$ implies $z_1(\rho_1) = 0$ and $\Phi(\rho_1, 0, z(\rho_1)) < \frac{\delta}{2}\big[V(\rho_1^{++}) - V(\rho_1^{+})\big]$ implies $z(\rho_1) = 1$. Thus, by construction of $\tilde{z}(\rho_1)$, in equilibrium, $e_1(\rho_1) = 0$ implies $z(\rho_1) = \tilde{z}(\rho_1)$ if $\rho_1 \leq \frac{\delta}{1+\delta}$ and $z(\rho_1) = 0$ if $\rho_1 > \frac{\delta}{1+\delta}$.

Step 3: Let $\overline{\rho}^{co}(\delta) := \frac{4c-2\delta}{2c-1-\delta}$ and $\underline{\rho}^{co}(\delta) := \frac{2c(2+\delta)}{1+2\delta+\delta^2}$. Note that $\overline{\rho}^{co}(\delta) \in (\frac{\delta}{1+\delta}, 1) \Leftrightarrow \underline{\rho}^{co}(\delta) \in (0, \frac{\delta}{1+\delta}) \Leftrightarrow \delta \in (\frac{2c-1+\sqrt{1+12c+4c^2}}{2}, 1)$.

Fix $\delta \in (\frac{2c-1+\sqrt{1+12c+4c^2}}{2}, 1)$. Consider a $\rho_1 \geq \frac{\delta}{1+\delta}$. Step 2 ensures that an equilibrium with $e_1(\rho_1) = 0$ exists if and only if: $\frac{\Phi(\rho_1,0,0)}{2} + c \geq \frac{\delta V(1)}{2}$. This condition is equivalent to: $\rho_1 \geq \overline{\rho}^{co}(\delta)$. Now consider a $\rho_1 \leq \frac{\delta}{1+\delta}$. Step 2 ensures that an equilibrium with $e_1(\rho_1) = 0$ exists if and only if: $\frac{\Phi(\rho_1,0,\tilde{z}(\rho_1))}{2} + c \geq \frac{\delta V(\rho_1^{++}(0,\tilde{z}(\rho_1)))}{2}$. This condition is equivalent to: $\rho_1 \leq \underline{\rho}^{co}(\delta)$. Thus, in light of step 1, we conclude that for $\rho_1 \in (\underline{\rho}^{co}(\delta), \overline{\rho}^{co}(\delta))$ in equilibrium $e_1(\rho_1) \in (0,1)$. ■

Proof of Proposition OB.2: Note that Υ is continuous, weakly increasing in each of its arguments, and $\Upsilon(1,\cdot) = \Upsilon(\cdot,1) = \frac{1}{2}$. Define $V(\rho) := \frac{\rho}{2}$. At $t = 1$, the agency's expected intertemporal profit from shirking and announcing $r_t = 1$ is equal to $\Upsilon(\rho_1, \hat{e}_1) + \frac{\delta}{2}V(\rho_1^{+})$, the intertemporal profit from shirking and announcing $r_t = -1$ is equal to $\Upsilon(\rho_1, \hat{e}_1) + \frac{\delta}{2}V(\rho_1^{++})$, and the intertemporal profit from acquiring information is equal to $\Upsilon(\rho_1, \hat{e}_1) + \frac{\delta}{2}\big[V(\rho_1^{+}) + V(\rho_1^{++})\big] - c$. Observe that in any equilibrium $e_1(\rho_1) < 1$ implies $z_1(\rho_1) = \frac{1}{2}$, since $z_1(\rho_1) > \frac{1}{2}$ (resp. $z_1(\rho_1) < \frac{1}{2}$) implies $\rho_1^{+} > \rho_1^{++}$ (resp. $\rho_1^{+} < \rho_1^{++}$) and thus $V(\rho_1^{++}) < V(\rho_1^{+})$ (resp. $V(\rho_1^{+}) < V(\rho_1^{++})$). Moreover, $e_1(\rho_1) = 1$ implies $\rho_1^{+} = \rho_1^{++} = \rho_1$. We thus obtain $\rho_1^{+} = \rho_1^{++} = f(\rho_1, e_1(\rho_1))$ in any equilibrium, where

$$f(\rho_1, e_1(\rho_1)) := \frac{\rho_1}{\rho_1 + (1-\rho_1)\big(e_1(\rho_1) + \frac{1}{2}(1-e_1(\rho_1))\big)}.$$

Next, an equilibrium in which $e_1(\rho_1) = 0$ exists if and only if $\frac{\delta}{2}V\left(f(\rho_1, 0)\right) \leq c$ that is, if and only if either $\delta \leq 4c$, or else $\delta > 4c$ and

$$\rho_1 \leq \frac{2c}{\delta - 2c}.$$

Similarly, an equilibrium in which $e_1(\rho_1) = 1$ exists if and only if $\frac{\delta}{2} V\left(f(\rho_1, 1)\right) \geq c$ that is, if and only if

$$\rho_1 \geq \frac{4c}{\delta}.$$

Note that $\frac{4c}{\delta} < 1 \Leftrightarrow \delta > 4c$.

An equilibrium with $e_1(\rho_1) \in (0,1)$ requires $\frac{\delta}{2} V\left(f(\rho_1, e_1(\rho_1))\right) = c$. As $\frac{\partial f(\cdot)}{\partial e_1(\rho_1)} < 0$, a (unique) equilibrium with $e_1(\rho_1) \in (0,1)$ exists if and only if $\delta > 4c$ and $\rho_1 \in (\frac{2c}{\delta - 2c}, \frac{4c}{\delta})$. ■

Online Appendix C

We show in this appendix that our main results do not depend on the assumption that the agency obtains a fraction β of all proceeds from selling projects to investors. Specifically, we generalize the baseline model by letting $\phi_t^{co}(r_t)$ satisfy

$$\phi_t^{co}(r_t) = \begin{cases} f(\mathbb{E}[q_t | r_t = 1, \rho_t, \hat{e}_t]) & \text{if } r_t = 1; \\ 0 & \text{if } r_t = -1, \end{cases}$$

where $f(\cdot) : [0,1] \to [0,1]$ denotes a strictly increasing continuous function satisfying $f(x) \leq x$ for all $x \in [0,1]$. We assume in line with the baseline model that $c < \frac{f(1)}{2}$. In what follows we first state the main results, and then provide all the proofs.

Proposition OC.1. *An equilibrium exists and is unique. In equilibrium, $e(0) = 0$; for $\rho > 0$ the equilibrium is characterized by cutoffs $\underline{\rho}$ and $\overline{\rho}$, $\underline{\rho} \leq \overline{\rho}$, such that*

$$\begin{cases} \textit{if} \quad \rho \in [\overline{\rho}, 1] & \textit{then} \quad e(\rho) = 0, \\ \textit{if} \quad \rho \in (\underline{\rho}, \overline{\rho}) & \textit{then} \quad e(\rho) \in (0,1), \\ \textit{if} \quad \rho \in (0, \underline{\rho}] & \textit{then} \quad e(\rho) = 1. \end{cases}$$

Moreover, the equilibrium fee is a non-decreasing function of the agency's reputation.

The model with upfront fees is generalized by letting

$$\phi_t^{up}(1) = \phi_t^{up}(-1) = f(\mathbb{P}(r_t = 1 | \rho_t, \hat{e}_t)\mathbb{E}[q_t | r_t = 1, \rho_t, \hat{e}_t]).$$

Proposition OC.2. *If $\delta < \frac{2c}{f(\frac{1}{2})+c}$ then $e(\rho) = 0$ for all $\rho \in [0,1]$ is the unique equilibrium with upfront fees. If $\delta > \frac{2c}{f(\frac{1}{2})+c}$ the unique equilibrium is*

$$e(\rho) = \begin{cases} 1 \ \textit{if} \ \rho > 0; \\ 0 \ \textit{if} \ \rho = 0. \end{cases}$$

Combining Propositions OC.1 and OC.2 yields the next theorem.

Theorem 1. *There exists $\tilde{\rho}$ such that, if $\delta \in (\frac{2c}{f(1)+c}, \frac{2c}{f(\frac{1}{2})+c})$ then, for $\rho_1 \in (0, \tilde{\rho})$, contingent fees improve expected social welfare relative to upfront fees. Moreover if $\frac{2c}{f(\frac{1}{2})+c} < \frac{2f(1)+4c}{3f(1)+2c}$ then* *for* $\delta \in (\frac{2c}{f(\frac{1}{2})+c}, \frac{2f(1)+4c}{3f(1)+2c})$ *upfront fees improve expected social welfare relative to contingent fees. In all other cases, expected social welfare is the same whether fees are upfront or contingent.*

We prove in the rest of this appendix all of the previous results. Define $\Phi(\cdot,\cdot) : [0,1] \times [0,1] \to \mathbb{R}$ by

$$\Phi(\rho, e) := \frac{1-(1-\rho)(1-e)}{1+(1-\rho)(1-e)}.$$

Given a function $e : [0,1] \to [0,1]$, define

$$\rho^+ := \begin{cases} 0 & \text{if } \rho = 0, \\ \frac{\rho}{\rho+(1-\rho)e(\rho)} & \text{if } \rho > 0. \end{cases}$$

Lemma OC.1. *In any equilibrium,* $e(0) = 0$ *and* $V(0) = 0$*, where* $V(\cdot)$ *denotes the equilibrium value function. For all* $\rho > 0$*,*

$$\begin{cases} \textit{if} \quad \frac{\delta}{2}V(\rho^+) > \frac{1}{2}f(\Phi(\rho, e(\rho))) + c \quad \textit{then} \quad e(\rho) = 1, \\ \textit{if} \quad \frac{\delta}{2}V(\rho^+) < \frac{1}{2}f(\Phi(\rho, e(\rho))) + c \quad \textit{then} \quad e(\rho) = 0. \end{cases}$$

Proof: By virtue of Bellman's Principle of Optimality,

$$V(\rho) = \max\left\{f(\Phi(\rho,e(\rho)))+\delta\Big(\frac{1}{2}V(\rho)+\frac{1}{2}V(0)\Big)\ ,\ \frac{1}{2}f(\Phi(\rho,e(\rho)))-c+\delta\Big(\frac{1}{2}V(\rho)+\frac{1}{2}V(\rho^+)\Big)\right\}, \tag{OC.1}$$

for all $\rho \in [0,1]$, and the choice implied by $e(\rho)$ maximizes the right-hand side of the expression above. That is:

$$\begin{cases} e(\rho) = 1 \quad \text{if} \quad \frac{1}{2}f(\Phi(\rho,e(\rho))) - c + \delta\Big(\frac{1}{2}V(\rho) + \frac{1}{2}V(\rho^+)\Big) > f(\Phi(\rho,e(\rho))) + \delta\Big(\frac{1}{2}V(\rho) + \frac{1}{2}V(0)\Big), \\ e(\rho) = 0 \quad \text{if} \quad \frac{1}{2}f(\Phi(\rho,e(\rho))) - c + \delta\Big(\frac{1}{2}V(\rho) + \frac{1}{2}V(\rho^+)\Big) < f(\Phi(\rho,e(\rho))) + \delta\Big(\frac{1}{2}V(\rho) + \frac{1}{2}V(0)\Big). \end{cases}$$

We are only left to show that $e(0) = 0$ and $V(0) = 0$. That $e(0) = 0$ follows from the above, noting that $\rho^+ = 0$ if $\rho = 0$. Substituting back into (OC.1) then yields $V(0) = f(\Phi(0,0)) + \delta V(0) = \delta V(0)$. Hence $V(0) = 0$. ■

Lemma OC.2. *In any equilibrium,*

$$\begin{cases} \textit{if} \quad e(\rho) = 1 \quad \textit{then} \quad V(\rho) = \frac{\frac{1}{2}f(\Phi(\rho,1))-c}{1-\delta} = \max\left\{\frac{\frac{1}{2}f(\Phi(\rho,1))-c}{1-\delta}\ ,\ \frac{f(\Phi(\rho,1))}{1-\frac{\delta}{2}}\right\}, \\ \textit{if} \quad e(\rho) < 1 \quad \textit{then} \quad V(\rho) = \frac{f(\Phi(\rho,e(\rho)))}{1-\frac{\delta}{2}}. \end{cases}$$

Furthermore, $V(1) = \max\left\{\frac{\frac{1}{2}f(\Phi(1,e(1)))-c}{1-\delta}\ ,\ \frac{f(\Phi(1,e(1)))}{1-\frac{\delta}{2}}\right\} \geq V(\rho)$, *for all* $\rho \in [0,1]$.

Proof: The lemma follows from Bellman's Principle of Optimality together with the observations that (a) $e(\rho) = 1$ implies $\rho^+ = \rho$, (b) $\rho = 1$ implies $\rho^+ = 1$, (c) $\Phi(1, e) = 1$ for all $e \in [0,1]$, and (d) $\Phi(\cdot,\cdot)$ is weakly increasing in both variables. ■

Proposition OC.3. *If* $\delta \geq \overline{\delta} := \frac{2f(1)+4c}{3(1)+2c}$ *then*

$$e(\rho) = \begin{cases} 1 \ \textit{if}\ \rho > 0, \\ 0 \ \textit{if}\ \rho = 0. \end{cases} \tag{OC.2}$$

is an equilibrium. If $\delta < \overline{\delta}$, *in any equilibrium:* $e(\rho) < 1$ *for all* $\rho \in [0,1]$.

Proof: By Lemma OC.1, $e(0) = 0$ for all δ. Next, consider $\rho > 0$. If in equilibrium $e(\rho) = 1$ then by Lemma OC.1 and the observation that $\rho^+ = \rho$:

$$\frac{\delta}{2}V(\rho) \geq \frac{1}{2}f(\Phi(\rho,1)) + c.$$

Applying Lemma OC.2 now yields

$$\frac{\delta}{2}\left(\frac{\frac{1}{2}f(\Phi(\rho,1)) - c}{1-\delta}\right) \geq \frac{1}{2}f(\Phi(\rho,1)) + c,$$

or, equivalently, $\delta \geq \overline{\delta}$ once we note that $\Phi(\rho, 1) = 1$. The condition $\delta \geq \overline{\delta}$ is thus necessary for $e(\rho) = 1$. Sufficiency follows from the one-shot deviation principle. ■

Lemma OC.3. *The following are equivalent:*

$$\delta \geq \overline{\delta} \tag{OC.3}$$

$$\frac{1}{2}f(\Phi(\rho,1))+c \leq \frac{\delta}{2}\left(\frac{\frac{1}{2}f(\Phi(\rho,1))-c}{1-\delta}\right) \tag{OC.4}$$

$$\frac{1}{2}f(\Phi(\rho,1))+c \leq \frac{\delta}{2}\left(\frac{f(\Phi(\rho,1))}{1-\frac{\delta}{2}}\right) \tag{OC.5}$$

$$\min\left\{\frac{\frac{1}{2}f(\Phi(\rho,1))-c}{1-\delta}\ ,\ \frac{f(\Phi(\rho,1))}{1-\frac{\delta}{2}}\right\} = \frac{f(\Phi(\rho,1))}{1-\frac{\delta}{2}} \tag{OC.6}$$

Moreover, the equivalence between (OC.3)-(OC.5) *continues to hold with strict inequalities instead of weak inequalities.*

Proof: Equivalence is easily checked using $\Phi(\rho,1)=1$. ■

Proposition OC.4. *If $\delta > \overline{\delta}$ then (OC.2) is the unique equilibrium.*

Proof: By Lemma OC.1, $e(0)=0$ in any equilibrium. So we are only left to show that, in any equilibrium, $e(\rho)=1$ for all $\rho>0$.

Suppose that an equilibrium exists such that $e(\hat{\rho})<1$ for some $\hat{\rho}>0$. Applying first Lemma OC.1 then Lemma OC.2:

$$\frac{1}{2}f(\Phi(\hat{\rho},e(\hat{\rho})))+c \geq \frac{\delta}{2}V(\hat{\rho}^{+}) \geq \frac{\delta}{2}\left(\frac{f(\Phi(\hat{\rho}^{+},e(\hat{\rho}^{+})))}{1-\frac{\delta}{2}}\right).$$

We thus obtain, using the equivalence between (OC.3) and (OC.5) (with strict inequalities), the following sequence of inequalities:

$$\frac{\delta}{2}\left(\frac{f(\Phi(\hat{\rho},1))}{1-\frac{\delta}{2}}\right) > \frac{1}{2}f(\Phi(\hat{\rho},1))+c \geq \frac{1}{2}f(\Phi(\hat{\rho},e(\hat{\rho})))+c \geq \frac{\delta}{2}\left(\frac{f(\Phi(\hat{\rho}^{+},e(\hat{\rho}^{+})))}{1-\frac{\delta}{2}}\right),$$

from which we infer that $e(\hat{\rho}^{+})<1$. We can thus repeat the steps above with $\hat{\rho}^{+}$ instead of $\hat{\rho}$, and so on. This process determines a sequence $\{\rho_n\}$ such that, for all n:

(i) $e(\rho_n)<1$,

(ii) $\rho_{n+1}=\frac{\rho_n}{\rho_n+(1-\rho_n)e(\rho_n)}>\rho_n$,

(iii) $\frac{1}{2}f(\Phi(\rho_n,e(\rho_n)))+c \geq \frac{\delta}{2}\left(\frac{f(\Phi(\rho_{n+1},e(\rho_{n+1})))}{1-\frac{\delta}{2}}\right)$.

By (i)-(ii), either $e(\rho_n) \to 1$ or $\rho_n \to 1$. Hence, taking limits in (iii) yields (using continuity of $\Phi(\cdot,\cdot)$, continuity of $f(\cdot)$ and the fact that $\Phi(1,e) = \Phi(\rho,1) = 1$ for all e and ρ in $[0,1]$):

$$\frac{1}{2} f(\Phi(1,1)) + c \geq \frac{\delta}{2}\left(\frac{f(\Phi(1,1))}{1-\frac{\delta}{2}}\right). \tag{OC.7}$$

The equivalence between (OC.3) and (OC.5) (with strict inequalities) establishes a contradiction between (OC.7) and $\delta > \overline{\delta}$. ■

Proposition OC.5. *If $\delta \leq \underline{\delta} := \frac{2c}{f(1)+c}$ then $e(\rho) = 0$ for all $\rho \in [0,1]$ is the unique equilibrium.*

Proof: Note first that $\delta \leq \underline{\delta}$ if and only if

$$c \geq \frac{\delta}{2}\left(\frac{f(\Phi(1,e(1)))}{1-\frac{\delta}{2}}\right). \tag{OC.8}$$

Next, the assumption $f(1) > 2c > 0$ implies $\underline{\delta} < \overline{\delta}$; combining Lemmas OC.2 and OC.3 thus shows that, in any equilibrium,

$$V(1) = \frac{f(\Phi(1,e(1)))}{1-\frac{\delta}{2}}, \tag{OC.9}$$

whenever $\delta < \underline{\delta}$. Combining (OC.8), (OC.9) and Lemma OC.2 now yields $c > \frac{\delta}{2}V(\rho)$, for all $\rho \in [0,1)$. Hence, by Lemma OC.1, $e(\rho) = 0$, for all $\rho \in [0,1]$.

That $e(\rho) = 0$ for all $\rho \in [0,1]$ is an equilibrium is immediate from (OC.8), (OC.9), and the one-shot deviation principle. ■

Proposition OC.6. *Let $\delta \in (\underline{\delta}, \overline{\delta})$. There exists a unique equilibrium. In equilibrium,*

$$\begin{cases} e(\rho) = 0 & \textit{if } \rho \in \{0\} \cup [\tilde{\rho}, 1] \\ e(\rho) \in (0, \tilde{e}] & \textit{if } \rho \in (0, \tilde{\rho}) \end{cases} \tag{OC.10}$$

where $\tilde{\rho} \in (0,1)$ and $\tilde{e} \in (0,1)$ are defined implicitly by

$$\frac{1}{2} f(\Phi(\tilde{\rho},0)) + c = \frac{\delta}{2}\left(\frac{f(\Phi(1,0))}{1-\frac{\delta}{2}}\right), \tag{OC.11}$$

and

$$\frac{1}{2}f(\Phi(0,\tilde{e}))+c=\frac{\delta}{2}\bigg(\frac{f(\Phi(0,1))}{1-\frac{\delta}{2}}\bigg), \tag{OC.12}$$

respectively.

Proof: Applying Lemma OC.3,

$$\delta<\overline{\delta} \quad \Leftrightarrow \quad \frac{1}{2}f(\Phi(\rho,1))+c>\frac{\delta}{2}\bigg(\frac{f(\Phi(\rho,1))}{1-\frac{\delta}{2}}\bigg).$$

Moreover, by (OC.8),

$$\delta>\underline{\delta} \quad \Leftrightarrow \quad c<\frac{\delta}{2}\bigg(\frac{f(\Phi(\rho,1))}{1-\frac{\delta}{2}}\bigg).$$

Thus $\tilde{\rho}\in(0,1)$ and $\tilde{e}\in(0,1)$.

We now prove the rest of the proposition. We will first proceed by induction to show that there can exist at most one equilibrium. We will then argue that the inductive procedure yields an equilibrium. As a preliminary step, observe that by Lemma OC.2 and the equivalence between (OC.3) and (OC.6), an equilibrium must satisfy:

$$V(\rho)=\frac{f(\Phi(\rho,e(\rho)))}{1-\frac{\delta}{2}}, \tag{OC.13}$$

for all $\rho\in[0,1]$.

The inductive procedure starts as follows. Combining (OC.11) and (OC.13), any equilibrium must be such that, for all $\rho>\tilde{\rho}$:

$$\frac{1}{2}f(\Phi(\rho,0))+c>\frac{\delta}{2}V(\rho^{+}).$$

Thus, by Lemma OC.1, if an equilibrium exists it must satisfy $e(\rho)=0$ for all $\rho>\tilde{\rho}$. A similar argument shows that in fact the same must be true for $\rho=\tilde{\rho}$.

By contrast, consider $\rho\in(0,\tilde{\rho})$. The combination of (OC.11), (OC.13), and Lemma OC.1 shows that $e(\rho)=0$ is impossible in equilibrium. Similarly, the combination of (OC.12), (OC.13) and Lemma OC.1 shows that $e(\rho)>\tilde{e}$ is impossible in equilibrium. Thus, any equilibrium must satisfy (OC.10). By virtue of Lemma OC.1 this in turn implies that the indifference condition

$$\frac{\delta}{2}V(\rho^{+})=\frac{1}{2}f(\Phi(\rho,e(\rho)))+c \tag{OC.14}$$

must hold for all $\rho \in (0, \tilde{\rho})$.

Next define $\rho_1 < \tilde{\rho}$ such that

$$\tilde{\rho} = \frac{\rho_1}{\rho_1 + (1-\rho_1)\tilde{e}}.$$

By construction of ρ_1 and property (OC.10), in any equilibrium: $\rho^+ \geq \tilde{\rho}$ for all $\rho \in [\rho_1, \tilde{\rho})$. (OC.14), (OC.10) and (OC.13) now pin down a unique candidate equilibrium $e(\rho)$ for each $\rho \in [\rho_1, \tilde{\rho})$ (which moreover is continuous in ρ). Repeating the step above with ρ_1 instead of $\tilde{\rho}$ yields $\rho_2 < \rho_1$ and a unique candidate equilibrium $e(\rho)$ for each $\rho \in [\rho_2, \rho_1)$, and so on. This defines a sequence $\{\rho_n\}$ where, for all n, $\tilde{\rho}_n = \frac{\rho_{n+1}}{\rho_{n+1}+(1-\rho_{n+1})\tilde{e}}$. As $\tilde{e} < 1$, $\rho_n \to 0$. This inductive procedure therefore pins down a unique candidate equilibrium. That this candidate equilibrium is in fact an equilibrium is a consequence of the one-shot deviation principle. ■

Proof of Proposition OC.1: Follows from Propositions OC.3-OC.6. ■

Define $\Upsilon(\cdot,\cdot) : [0,1] \times [0,1] \to \mathbb{R}$ by

$$\Upsilon(\rho, e) := \frac{1}{2}(1 - (1-\rho)(1-e)).$$

Lemma OC.4. *In any equilibrium with upfront fees, $e(0) = 0$ and $V(0) = 0$, where $V(\cdot)$ denotes the equilibrium value function. For all $\rho > 0$,*

$$\begin{cases} \textit{if} \quad \frac{\delta}{2}V(\rho^+) > c \quad \textit{then} \quad e(\rho) = 1, \\ \textit{if} \quad \frac{\delta}{2}V(\rho^+) < c \quad \textit{then} \quad e(\rho) = 0. \end{cases}$$

Proof: By virtue of Bellman's Principle of Optimality,

$$V(\rho) = \max\left\{ f(\Upsilon(\rho, e(\rho))) + \delta\Big(\frac{1}{2}V(\rho) + \frac{1}{2}V(0)\Big)\ ,\ f(\Upsilon(\rho, e(\rho))) - c + \delta\Big(\frac{1}{2}V(\rho) + \frac{1}{2}V(\rho^+)\Big)\right\}, \tag{OC.15}$$

for all $\rho \in [0,1]$, and the choice implied by $e(\rho)$ maximizes the right-hand side of (OC.15). That is:

$$\begin{cases} e(\rho) = 1 \quad \text{if} \quad f(\Upsilon(\rho, e(\rho))) - c + \delta\Big(\frac{1}{2}V(\rho) + \frac{1}{2}V(\rho^+)\Big) > f(\Upsilon(\rho, e(\rho))) + \delta\Big(\frac{1}{2}V(\rho) + \frac{1}{2}V(0)\Big), \\ e(\rho) = 0 \quad \text{if} \quad f(\Upsilon(\rho, e(\rho))) - c + \delta\Big(\frac{1}{2}V(\rho) + \frac{1}{2}V(\rho^+)\Big) < f(\Upsilon(\rho, e(\rho))) + \delta\Big(\frac{1}{2}V(\rho) + \frac{1}{2}V(0)\Big). \end{cases}$$

We are only left to show that $e(0) = 0$ and $V(0) = 0$. That $e(0) = 0$ follows from the above, noting that $\rho^+ = 0$ if $\rho = 0$. Substituting back into (OC.15) then yields $V(0) = f(\Phi(0,0)) + \delta V(0) = \delta V(0)$. Hence $V(0) = 0$. ■

Lemma OC.5. *In any equilibrium with upfront fees,*

$$\begin{cases} \textit{if} \quad e(\rho) = 1 \quad \textit{then} \quad V(\rho) = \frac{f(\Upsilon(\rho,1))-c}{1-\delta} = \max\left\{ \frac{f(\Upsilon(\rho,1))-c}{1-\delta} \ , \ \frac{f(\Upsilon(\rho,1))}{1-\frac{\delta}{2}} \right\}, \\ \textit{if} \quad e(\rho) < 1 \quad \textit{then} \quad V(\rho) = \frac{f(\Upsilon(\rho,e(\rho)))}{1-\frac{\delta}{2}}. \end{cases}$$

Furthermore, $V(1) = \max\left\{ \frac{f(\Upsilon(1,e(1)))-c}{1-\delta} \ , \ \frac{f(\Upsilon(1,e(1)))}{1-\frac{\delta}{2}} \right\} \geq V(\rho)$*, for all* $\rho \in [0,1]$*.*

Proof: The lemma follows from Bellman's Principle of Optimality together with the observations that (a) $e(\rho) = 1$ implies $\rho^+ = \rho$, (b) $\rho = 1$ implies $\rho^+ = 1$, (c) $\Upsilon(1,e) = \frac{1}{2}$ for all $e \in [0,1]$, and (d) $\Upsilon(\cdot,\cdot)$ is weakly increasing in both variables. ■

Proposition OC.7. *If* $\delta > \frac{2c}{f(\frac{1}{2})+c}$ *then*

$$e(\rho) = \begin{cases} 1 \ \textit{if} \ \rho > 0, \\ 0 \ \textit{if} \ \rho = 0. \end{cases} \tag{OC.16}$$

is an equilibrium with upfront fees. If $\delta < \frac{c}{f(\frac{1}{2})+c}$*, in any equilibrium with upfront fees:* $e(\rho) < 1$ *for all* $\rho \in [0,1]$*.*

Proof: By Lemma OC.4, $e(0) = 0$ for all δ. Next, consider $\rho > 0$. If in equilibrium $e(\rho) = 1$ then Lemma OC.4 and the observation that $\rho^+ = \rho$ yield

$$\frac{\delta}{2} V(\rho) \geq c.$$

Applying Lemma OC.5 now gives

$$\frac{\delta}{2}\left(\frac{f(\Upsilon(\rho,1)) - c}{1-\delta} \right) \geq c,$$

or, equivalently, $\delta \geq \frac{2c}{f(\frac{1}{2})+c}$ once we note that $\Upsilon(\rho, 1) = \frac{1}{2}$. The condition $\delta \geq \frac{2c}{f(\frac{1}{2})+c}$ is thus necessary for $e(\rho) = 1$. Sufficiency follows from the one-shot deviation principle. ■

Lemma OC.6. *The following are equivalent:*

$$\delta \geq \frac{2c}{f(\frac{1}{2}) + c}, \tag{OC.17}$$

$$c \leq \frac{\delta}{2}\left(\frac{f(\Upsilon(\rho, 1)) - c}{1 - \delta}\right), \tag{OC.18}$$

$$c \leq \frac{\delta}{2}\left(\frac{f(\Upsilon(\rho, 1))}{1 - \frac{\delta}{2}}\right), \tag{OC.19}$$

$$\min\left\{\frac{f(\Upsilon(\rho, 1)) - c}{1 - \delta}, \frac{f(\Upsilon(\rho, 1))}{1 - \frac{\delta}{2}}\right\} = \frac{f(\Upsilon(\rho, 1))}{1 - \frac{\delta}{2}}. \tag{OC.20}$$

The equivalence between (OC.17)-(OC.19) *continues to hold with strict inequalities instead of weak inequalities.*

Proof: Equivalence is easily checked using $\Upsilon(\rho, 1) = \frac{1}{2}$. ■

Proposition OC.8. *If $\delta > \frac{2c}{f(\frac{1}{2})+c}$ then (OC.16) is the unique equilibrium.*

Proof: By Lemma OC.4, $e(0) = 0$ in any equilibrium. So we are only left to show that, in any equilibrium, $e(\rho) = 1$ for all $\rho > 0$.

Suppose an equilibrium exists such that $e(\hat{\rho}) < 1$ for some $\hat{\rho} > 0$. Applying first Lemma OC.4 then Lemma OC.5:

$$c \geq \frac{\delta}{2}V(\hat{\rho}^+) \geq \frac{\delta}{2}\left(\frac{f(\Upsilon(\hat{\rho}^+, e(\hat{\rho}^+)))}{1 - \frac{\delta}{2}}\right).$$

We thus obtain, using equivalence of (OC.17) and (OC.19) (with strict inequalities), the following sequence of inequalities:

$$\frac{\delta}{2}\left(\frac{f(\Upsilon(\hat{\rho}, 1))}{1 - \frac{\delta}{2}}\right) > c \geq \frac{\delta}{2}\left(\frac{f(\Upsilon(\hat{\rho}^+, e(\hat{\rho}^+)))}{1 - \frac{\delta}{2}}\right),$$

from which we infer that $e(\hat{\rho}^+) < 1$. We can thus repeat the steps above with $\hat{\rho}^+$ instead of $\hat{\rho}$, and so on. This process determines a sequence $\{\rho_n\}$ such that, for all n:

(i) $e(\rho_n) < 1$,

(ii) $\rho_{n+1} = \frac{\rho_n}{\rho_n + (1-\rho_n)e(\rho_n)} > \rho_n$,

(iii) $c \geq \frac{\delta}{2}\left(\frac{f(\Upsilon(\rho_{n+1}, e(\rho_{n+1})))}{1-\frac{\delta}{2}}\right)$.

By (i)-(ii), either $e(\rho_n) \to 1$ or $\rho_n \to 1$. Hence, taking limits in (iii) yields (using continuity of $\Upsilon(\cdot, \cdot)$ and the fact that $\Upsilon(1, e) = \Upsilon(\rho, 1) = \frac{1}{2}$ for all e and ρ in $[0, 1]$):

$$c \geq \frac{\delta}{2}\left(\frac{f(\Upsilon(1,1))}{1-\frac{\delta}{2}}\right). \tag{OC.21}$$

The equivalence between (OC.17) and (OC.19) (with strict inequalities) establishes a contradiction with (OC.21). ■

Proposition OC.9. *If $\delta < \frac{2c}{f(\frac{1}{2})+c}$ then $e(\rho) = 0$ for all $\rho \in [0, 1]$ is the unique equilibrium with upfront fees.*

Proof: By Lemma OC.6, we have $\delta < \frac{2c}{f(\frac{1}{2})+c}$ if and only if

$$c > \frac{\delta}{2}\left(\frac{f(\Upsilon(1, e(1)))}{1-\frac{\delta}{2}}\right). \tag{OC.22}$$

Next, combining Lemmas OC.5 and OC.6 yields

$$V(1) = \frac{f(\Upsilon(1, e(1)))}{1-\frac{\delta}{2}}. \tag{OC.23}$$

Combining (OC.22) and (OC.23) gives $c > \frac{\delta}{2}V(1)$; hence, by Lemma OC.5, $c > \frac{\delta}{2}V(\rho)$ for all $\rho \in [0, 1]$. Lemma OC.4 thus yields $e(\rho) = 0$, for all $\rho \in [0, 1]$.

That $e(\rho) = 0$, for all $\rho \in [0, 1]$ is an equilibrium is immediate from (OC.22), (OC.23), and the one-shot deviation principle. ■

Proof of Proposition OC.2: Follows from Propositions OC.7-OC.9. ■

Proof of Theorem 1: Proposition OC.1 characterizes the unique equilibrium with contingent fees, and Proposition OC.2 characterizes the unique equilibrium with upfront fees. The cutoff $\overline{\delta} = \frac{2f(1)+4c}{3f(1)+2c}$ is taken from Proposition OC.3. The cutoff $\underline{\delta} = \frac{2c}{f(1)+c}$ is taken from Proposition OC.5. With upfront fees, either the strategic agency shirks irrespective of ρ_t or the strategic agency acquires information with probability 1 irrespective of ρ_t. Hence, all that remains to show is that the expected period-t welfare is an increasing function of the probability with which the strategic agency chooses to acquire information. The expected period-t welfare is

$$\begin{aligned}\mathbb{P}(r_t = 1)\mathbb{E}[q_t|r_t = 1] - c\Big(\rho_t + (1-\rho_t)e(\rho_t)\Big) &= \Big(\frac{1}{2} + \frac{1}{2}(1-\rho_t)(1-e(\rho_t))\Big)\frac{1-(1-\rho_t)(1-e(\rho_t))}{1+(1-\rho_t)(1-e(\rho_t))} \\ &\quad - c\Big(\rho_t + (1-\rho_t)e(\rho_t)\Big) \\ &= \Big(\frac{1}{2} - c\Big)\Big(\rho_t + (1-\rho_t)e(\rho_t)\Big);\end{aligned}$$

the result follows since $c < \frac{1}{2}$. ■

Online Appendix D

In this appendix we generalize the model presented in Online Appendix B by letting the prior probability γ of $q_t = 1$ take any value in $(0,1)$ and show that all our main results continue to hold. We normalize a project's return so that either $q_t = 1$ or $q_t = \frac{-\gamma}{1-\gamma}$. Hence $\mathbb{E}[q_t] = 0$ irrespective of γ. The model described in Online Appendix B corresponds to the case $\gamma = \frac{1}{2}$. The definitions from Online Appendix B all apply here as well with the functions Φ and Υ now replaced, respectively, by

$$\Phi^{\dagger}(\rho_t, \hat{e}_t, \hat{z}_t) := \frac{1-(1-\rho_t)(1-\hat{e}_t)}{1+\frac{1-\gamma}{\gamma}(1-\rho_t)(1-\hat{e}_t)(1-\frac{\hat{z}_t}{1-\gamma})}$$

and

$$\Upsilon^{\dagger}(\rho_t, \hat{e}_t) := \gamma\big[1-(1-\rho_t)(1-\hat{e}_t)\big].$$

The Equilibria. We next characterize the equilibria with, respectively, contingent fees and upfront fees. With 2 periods the strategic agency always shirks in period 2. So the focus is on information acquisition at $t = 1$. Proofs of the propositions are relegated to the end of this appendix.

Proposition OD.1. *Let fees be contingent. In equilibrium $e_1(\rho_1) < 1$ for all $\rho_1 \in (0,1)$. Moreover, the equilibrium is characterized by cutoffs $\overline{\rho}^{\dagger}$, $\underline{\rho}^{\dagger}$ and $\underline{\delta}$ such that, for $\delta > \underline{\delta}$, $e_1(\rho_1) \in (0,1)$ for all $\rho_1 \in \big(\underline{\rho}^{\dagger}, \overline{\rho}^{\dagger}\big)$.*

Proposition OD.2. *Let fees be upfront. Let $x := min\{\gamma, 1-\gamma\}$. In equilibrium, if $\delta \leq \frac{c}{\gamma x}$ then $e_1(\rho_1) = 0$ for all $\rho_1 \in (0,1)$. If instead $\delta > \frac{c}{\gamma x}$ then*

$$\begin{cases} \rho_1 \in (\frac{c}{\delta\gamma x}, 1) & \textit{implies} \quad e(\rho_1) = 1; \\ \rho_1 \in (\frac{c}{\delta\gamma - c}, \frac{c}{\delta\gamma x}) & \textit{implies} \quad e(\rho_1) \in (0,1); \\ \rho_1 \in (0, \frac{c}{\delta\gamma - c}) & \textit{implies} \quad e(\rho_1) = 0. \end{cases}$$

Welfare Comparison. As in Online Appendix B, since the strategic agency shirks at $t = 2$ irrespective of the fee structure, and, for the payoffs we consider, welfare does not depend on the agency's choice to inflate or deflate a rating in case it shirks, social welfare is uniquely determined by the probability that the agency acquires information at $t = 1$. In the parameter region defined by $\delta \in (\underline{\delta}, \frac{c}{\gamma x})$ and $\rho_1 \in \left(\underline{\rho}^\dagger, \overline{\rho}^\dagger\right)$ the agency shirks at $t = 1$ with upfront fees (Proposition OD.2) but acquires information with positive probability with contingent fees (Proposition OD.1), thus contingent fees improve expected social welfare. Since $\underline{\delta} < \frac{c}{\gamma x}$, the aforementioned region is non-empty (we call this observation Remark 1 and prove it below). Note too that, for c sufficiently close to 0, any ρ_1 and δ satisfy $\delta > \frac{c}{\gamma x}$ and $\rho_1 > \frac{c}{\delta \gamma x}$, hence ensuring $e_1(\rho_1) = 1$ with upfront fees (Proposition OD.2). As $e_1(\rho_1) < 1$ everywhere with contingent fees (Proposition OD.1), then for any ρ_1 and δ upfront fees improve expected social welfare for sufficiently small c.

Proof of Proposition OD.1: The function $\Phi^\dagger$ is continuous, weakly increasing in each of its arguments, and $\Phi^\dagger(1, \cdot, \cdot) = \Phi^\dagger(\cdot, 1, \cdot) = 1$. Notice that in any equilibrium the strategic agency must shirk and announce $r_2 = 1$ at $t = 2$ with probability 1 giving $\pi_2 = \Phi^\dagger(\rho_2, 0, 0) = \frac{\gamma \rho_2}{1-(1-\gamma)\rho_2}$. Define $V(\rho) := \frac{\gamma \rho}{1-(1-\gamma)\rho}$. Let $\rho_1^+(e, z)$ (respectively $\rho_1^{++}(e, z)$) denote the value of ρ_1^+ (respectively ρ_1^{++}) for $e_1(\rho_1) = e$ and $z_1(\rho_1) = z$. At $t = 1$, the agency's expected intertemporal profit from shirking and announcing $r_t = 1$ is equal to $\Phi^\dagger(\rho_1, \hat{e}_1, \hat{z}_1) + \delta \gamma V(\rho_1^+(\hat{e}_1, \hat{z}_1))$, the intertemporal profit from shirking and announcing $r_t = -1$ is equal to $\delta(1-\gamma)V(\rho_1^{++}(\hat{e}_1, \hat{z}_1))$, and the intertemporal profit from acquiring information is equal to $\gamma \Phi^\dagger(\rho_1, \hat{e}_1, \hat{z}_1) - c + \delta\big[\gamma V(\rho_1^+(\hat{e}_1, \hat{z}_1)) + (1-\gamma)V(\rho_1^{++}(\hat{e}_1, \hat{z}_1))\big]$. The rest of the proof contains 3 steps. Step 1 establishes than in equilibrium $e_1(\rho_1) < 1$. Step 2 computes the probability with which the strategic agency must announce $r_1 = -1$ if in equilibrium $e_1(\rho_1) = 0$. Step 3 characterizes a parameter region in which $e_1(\rho_1) > 0$.

Step 1: An equilibrium with $e_1(\rho_1) = 1$ requires

$$\delta(1-\gamma)V(\rho_1) - c \geq (1-\gamma)\Phi^\dagger(\rho_1, 1, z_1), \tag{OD.1}$$

for some z_1. Note that, for any z_1 and any ρ_1: $\Phi(\rho_1, 1, z_1) = 1 > V(\rho_1)$. Thus (OD.1) is violated as $c > 0$ and $\delta < 1$. Hence in equilibrium $e(\rho_1) < 1$.

Step 2: Define $\tilde{z}(\rho_1)$ implicitly by

$$\Phi^\dagger(\rho_1, 0, \tilde{z}(\rho_1)) = \delta\big[(1-\gamma)V(\rho_1^{++}(0, \tilde{z}(\rho_1))) - \gamma V(\rho_1^+(0, \tilde{z}(\rho_1)))\big],$$

which is equivalent to

$$\frac{\rho_1\gamma}{1-\rho_1(1-\gamma)-(1-\rho_1)\tilde{z}} = \delta\left(\frac{\rho_1\gamma(1-\gamma)}{\rho_1+(1-\rho_1)\tilde{z}-(1-\gamma)\rho_1} - \frac{\gamma^2\rho_1}{1-\rho_1(1-\gamma)-(1-\rho_1)\tilde{z}}\right).$$

Simplifying to solve for $\tilde{z}$ gives $\tilde{z}(\rho_1) = \frac{\delta(1-\gamma)}{1+\delta} + \frac{\rho_1\gamma}{1-\rho_1}\frac{\delta(1-2\gamma)-1}{1+\delta}$. Note that $\tilde{z}(\rho_1) < 1$ if and only if $\rho_1 < \frac{1+\gamma\delta}{(1+\gamma\delta)-(\gamma(1-\delta)+2\delta\gamma^2)}$ and this condition is satisfied for all $\rho_1 \in (0,1)$; furthermore, $\tilde{z}(\rho_1) > 0$ if and only if $\rho_1 < \rho_1^\dagger(\delta)$ where $\rho_1^\dagger(\delta) := \frac{(1-\gamma)\delta}{(1-\gamma)\delta+(\gamma(1-\delta)+2\gamma^2\delta)}$. Note that $\rho_1^\dagger(\delta) \in (0,1)$. By construction of $\tilde{z}(\rho_1)$, in equilibrium, $e_1(\rho_1) = 0$ implies $z(\rho_1) = \tilde{z}(\rho_1)$ if $\rho_1 \le \rho_1^\dagger(\delta)$ and $z(\rho_1) = 0$ if $\rho_1 > \rho_1^\dagger(\delta)$.

Step 3: Define $\overline{\rho}^\dagger(\delta) = \frac{\delta(1-\gamma)-c}{\delta(1-\gamma)^2+(\gamma-c)(1-\gamma)}$, $\underline{\rho}^\dagger(\delta) := \frac{c(1+\delta\gamma)}{\delta^2\gamma^2-2c\delta\gamma^2+c\delta\gamma+2\delta\gamma^2-2c\gamma+\gamma^2+c}$. Consider first $\rho_1 \ge \rho_1^\dagger(\delta)$. Step 2 ensures that an equilibrium with $e_1(\rho_1) = 0$ exists if and only if: $(1-\gamma)\Phi^\dagger(\rho_1, 0, 0) \ge (1-\gamma)\delta V(1) - c$. This condition is equivalent to: $\rho_1 \le \overline{\rho}^\dagger(\delta)$. We conclude that if $\rho_1 \in [\rho_1^\dagger(\delta), \overline{\rho}^\dagger(\delta))$ in equilibrium $e_1 \in (0,1)$. Now consider a $\rho_1 \le \rho_1^\dagger(\delta)$. Step 2 ensures that an equilibrium with $e_1(\rho_1) = 0$ exists if and only if $(1-\gamma)\Phi^\dagger(\rho_1, 0, \tilde{z}(\rho_1)) + c \ge (1-\gamma)\delta V(\rho_1^{++}(0, \tilde{z}(\rho_1)))$. This condition is equivalent to: $\rho_1 \le \underline{\rho}^{co}(\delta)$. We conclude that if $\rho_1(\underline{\rho}^{co}(\delta), \rho_1^\dagger(\delta))$ in equilibrium $e_1 \in (0,1)$.

Finally, straightforward algebra shows that both $\overline{\rho}^{co}(\delta) > \rho^\dagger(\delta)$ and $\underline{\rho}^{co}(\delta) < \rho^\dagger(\delta)$ are equivalent to $c < \frac{\delta\gamma(1-\gamma)(1+\delta)}{1+\delta\gamma}$. This inequality is satisfied if $\delta \in (\underline{\delta}, 1)$ and violated if $\delta \in (0, \underline{\delta}]$, where $\underline{\delta} := \frac{c\gamma+\gamma^2-\gamma+\sqrt{c^2\gamma^2+2c\gamma^3+\gamma^4-6c\gamma^2-2\gamma^3+4c\gamma+\gamma^2}}{2\gamma(1-\gamma)}$. This concludes the proof. ■

Proof of Proposition OD.2: Note that $\Upsilon^\dagger$ is continuous, weakly increasing in each of its arguments, and $\Upsilon^\dagger(1,\cdot) = \Upsilon^\dagger(\cdot,1) = \gamma$. Define $V(\rho) := \gamma\rho$. Let $\rho_1^+(e,z)$ (respectively $\rho_1^{++}(e,z)$) denote the value of ρ_1^+ (respectively ρ_1^{++}) for $e_1(\rho_1) = e$ and $z_1(\rho_1) = z$. At $t = 1$, the agency's expected intertemporal profit from shirking and announcing $r_t = 1$ is equal to $\Upsilon^\dagger(\rho_1, \hat{e}_1) + \delta\gamma V(\rho_1^+(\hat{e}_1, \hat{z}_1))$, the intertemporal profit from shirking and announcing $r_t = -1$ is equal to $\Upsilon^\dagger(\rho_1, \hat{e}_1) + \delta(1-\gamma)V(\rho_1^{++}(\hat{e}_1, \hat{z}_1))$, and the intertemporal profit from acquiring information is equal to $\Upsilon^\dagger(\rho_1, \hat{e}_1) + \delta\big[\gamma V(\rho_1^+(\hat{e}_1, \hat{z}_1)) + (1-\gamma)V(\rho_1^{++}(\hat{e}_1, \hat{z}_1))\big] - c$.

We now consider two cases. These cases together prove the proposition.

Case 1. Let $\gamma \in (0, \frac{1}{2}]$. In equilibrium, if $\delta < \frac{c}{\gamma^2}$ then $e_1(\rho_1) = 0$ for all $\rho_1 \in (0,1)$. If *instead* $\delta > \frac{c}{\gamma^2}$ *then*

$$\begin{cases} \text{if} \quad \rho_1 \in (\frac{c}{\delta\gamma^2}, 1) \qquad \text{then} \quad e(\rho_1) = 1, \\ \text{if} \quad \rho_1 \in (\frac{c}{\delta\gamma - c}, \frac{c}{\delta\gamma^2}) \quad \text{then} \quad e(\rho_1) \in (0,1), \\ \text{if} \quad \rho_1 \in (0, \frac{c}{\delta\gamma - c}) \quad \text{then} \quad e(\rho_1) = 0. \end{cases}$$

Define $\tilde{z}(\rho_1)$ implicitly as follows:

$$\gamma V(\rho_1^{+}(0, \tilde{z}(\rho_1))) = (1-\gamma)V(\rho_1^{++}(0, \tilde{z}(\rho_1))), \tag{OD.2}$$

Solving (OD.2) we obtain $\tilde{z}(\rho_1) = 1 - \gamma + (1-2\gamma)\frac{\rho_1}{1-\rho_1}$.

As $\gamma \in (0, \frac{1}{2}]$, then $\tilde{z}(\rho_1) > 0$ for all ρ_1, while $\tilde{z}(\rho_1) < 1$ if and only if $\rho_1 < \frac{\gamma}{1-\gamma}$, where $\frac{\gamma}{1-\gamma} \in (0,1)$. By construction of $\tilde{z}(\rho_1)$, in equilibrium $e_1(\rho_1) = 0$ implies $z(\rho_1) = \tilde{z}(\rho_1)$ if $\rho_1 < \frac{\gamma}{1-\gamma}$, and $z(\rho_1) = 1$ if $\rho_1 \geq \frac{\gamma}{1-\gamma}$.

Fix first $\rho_1 \geq \frac{\gamma}{1-\gamma}$. An equilibrium with $e_1(\rho_1) = 0$ exists if and only if $\delta\gamma V(\rho_1^{+}(0,1)) \leq c$, which is equivalent to $\delta \leq \frac{c}{\gamma^2}$. Fix $\delta < \frac{c}{\gamma^2}$. As $V(\rho_1^{+}(0,1)) = V(1) \geq V(\rho_2)$ for all $\rho_2 \in [0,1]$, then for and any e_1 and z_1: $\delta\gamma V(\rho_1^{+}(e_1, z_1)) < c$. Hence in any equilibrium $e_1(\rho_1) = 0$ (and $z_1(\rho_1) = 1$).

Fix now $\rho_1 < \frac{\gamma}{1-\gamma}$. An equilibrium with $e_1(\rho_1) = 0$ exists if and only if $\delta\gamma V(\rho_1^{+}(0, \tilde{z}(\rho_1))) \leq c$, which is equivalent to $\rho_1 \leq \frac{c}{\delta\gamma - c}$. Fix $\rho_1 < \frac{c}{\delta\gamma - c}$. Clearly in this region if in equilibrium with $z_1(\rho_1) = \tilde{z}(\rho_1)$ then $e_1(\rho_1) = 0$. Suppose an equilibrium exists in which $z_1(\rho_1) > \tilde{z}(\rho_1)$; note that for any $\hat{e}_1$: $\delta(1-\gamma)V(\rho_1^{++}(\hat{e}_1, z_1(\rho_1))) \leq \delta(1-\gamma)V(\rho_1^{++}(0, z_1(\rho_1))) < \delta(1-\gamma)V(\rho_1^{++}(0, \tilde{z}_1(\rho_1))) = \delta\gamma V(\rho_1^{+}(0, \tilde{z}_1(\rho_1))) < c$, hence in such an equilibrium $e_1(\rho_1) = 0$. Suppose now that an equilibrium exists in which $z_1(\rho_1) < \tilde{z}(\rho_1)$; note that for any $\hat{e}_1$: $\delta\gamma V(\rho_1^{+}(\hat{e}_1, z_1(\rho_1))) \leq \delta\gamma V(\rho_1^{+}(0, z_1(\rho_1))) < \delta\gamma V(\rho_1^{+}(0, \tilde{z}_1(\rho_1))) < c$, hence in such an equilibrium $e_1(\rho_1) = 0$. Hence in any equilibrium $e_1(\rho_1) = 0$ (and $z_1(\rho_1) = \tilde{z}(\rho_1)$).

Note that, for $\delta < \frac{c}{\gamma^2}$, $\frac{c}{\delta\gamma - c} > 1 \geq \rho_1$ for all ρ_1. Hence if either $\delta < \frac{c}{\gamma^2}$ or $\delta > \frac{c}{\gamma^2}$ and $\rho_1 < \frac{c}{\delta\gamma - c}$ then in equilibrium $e_1(\rho_1) = 0$.

Consider now an equilibrium in which $e_1(\rho_1) = 1$. In such equilibrium $\rho_1^{++} = \rho_1^{+} = \rho_1$. As $\gamma \leq \frac{1}{2}$, in such equilibrium $\gamma V(\rho_1^{+}) < (1-\gamma)V(\rho_1^{++})$, hence $z_1(\rho_1) = 1$. Thus an equilibrium with $e_1(\rho_1) = 1$ exists if and only if $\delta\gamma V(\rho_1) \geq c$ that is, if and only if $\rho_1 \geq \frac{c}{\delta\gamma^2}$. Let $\rho_1 > \frac{c}{\delta\gamma^2}$. Note that for any e_1 and z_1, $\rho_1^{+}(e_1, z_1) \geq \rho_1$, hence $\delta\gamma V(\rho_1^{+}) > c$, i.e. there is no equilibrium in which the agency shirks and deflates with positive probability. Moreover, as $\gamma \leq \frac{1}{2}$: $\delta(1-\gamma)V(\rho_1^{+}) \geq \delta(1-\gamma)V(\rho_1) \geq \delta\gamma V(\rho_1) > c$, i.e. there is no equilibrium in which the agency shirks and inflates with positive probability. Thus for $\rho_1 > \frac{c}{\delta\gamma^2}$ in equilibrium $e_1(\rho) = 1$. Noting that an equilibrium exists for all parameter values concludes the proof of case 1.

Case 2. Let $\gamma \in (\frac{1}{2}, 1)$. In equilibrium, if $\delta < \frac{c}{\gamma(1-\gamma)}$ then $e_1(\rho_1) = 0$ for all $\rho_1 \in (0,1)$. If instead $\delta > \frac{c}{\gamma(1-\gamma)}$ then

$$\begin{cases} \textit{if} \quad \rho_1 \in (\frac{c}{\delta\gamma(1-\gamma)}, 1) & \textit{then} \quad e(\rho_1) = 1, \\ \textit{if} \quad \rho_1 \in (\frac{c}{\delta\gamma-c}, \frac{c}{\delta\gamma(1-\gamma)}) & \textit{then} \quad e(\rho_1) \in (0,1), \\ \textit{if} \quad \rho_1 \in (0, \frac{c}{\delta\gamma-c}) & \textit{then} \quad e(\rho_1) = 0. \end{cases}$$

Consider $\tilde{z}(\rho_1)$, as defined by (OD.2). As $\gamma \in (\frac{1}{2}, 1)$, then $\tilde{z}(\rho_1) < 1$ for all ρ_1, while $\tilde{z}(\rho_1) > 0$ if and only if $\rho_1 < \frac{1-\gamma}{\gamma}$, where $\frac{1-\gamma}{\gamma} \in (0,1)$. By construction of $\tilde{z}(\rho_1)$, in equilibrium $e_1(\rho_1) = 0$ implies $z(\rho_1) = \tilde{z}(\rho_1)$ if $\rho_1 < \frac{1-\gamma}{\gamma}$, and $z(\rho_1) = 0$ if $\rho_1 \geq \frac{1-\gamma}{\gamma}$.

So for $\rho_1 \geq \frac{1-\gamma}{\gamma}$ an equilibrium with $e_1(\rho_1) = 0$ exists if and only if $\delta(1-\gamma)V(\rho_1^{++}(0,0)) \leq c$, which is equivalent to $\delta \leq \frac{c}{\gamma(1-\gamma)}$. Consider $\delta < \frac{c}{\gamma(1-\gamma)}$. As $V(\rho_1^{++}(0,0)) = V(1) \geq V(\rho_2)$ for all ρ_2, then for any e_1 and z_1: $\delta(1-\gamma)V(\rho_1^{++}(e_1, z_1)) < c$. Thus in this region, in equilibrium, $e_1(\rho_1) = 0$.

For $\rho_1 < \frac{1-\gamma}{\gamma}$ instead, an equilibrium with $e_1(\rho_1) = 0$ exists if and only if $\delta\gamma V(\rho_1^{+}(0, \tilde{z}(\rho_1))) \leq c$, which is equivalent to $\rho_1 \leq \frac{c}{\delta\gamma-c}$. The proof that in equilibrium $e_1(\rho_1) = 0$ for $\rho_1 < \frac{c}{\delta\gamma-c}$ follows the same steps as the proof that $e_1(\rho_1) = 0$ for $\gamma \in (0, \frac{1}{2}]$ and $\rho_1 < \frac{c}{\delta\gamma-c}$ discussed above.

Consider now an equilibrium in which $e_1(\rho_1) = 1$. As $\gamma > \frac{1}{2}$, in such equilibrium $z_1(\rho_1) = 0$. Thus an equilibrium with $e_1(\rho_1) = 1$ exists if and only if $\delta(1-\gamma)V(\rho_1) \geq c$ that is, if and only if $\rho_1 \geq \frac{c}{\delta\gamma(1-\gamma)}$. Let $\rho_1 > \frac{c}{\delta\gamma(1-\gamma)}$. Note that for any e_1 and z_1, $\rho_1^{++}(e_1, z_1) \geq \rho_1$, hence $\delta(1-\gamma)V(\rho_1^{++}) > c$. Moreover, as $\gamma > \frac{1}{2}$: $\delta\gamma V(\rho_1^{+}) \geq \delta\gamma V(\rho_1) > \delta(1-\gamma)V(\rho_1) > c$. Thus for $\rho_1 > \frac{c}{\delta\gamma(1-\gamma)}$ in equilibrium $e_1(\rho) = 1$. Noting that an equilibrium exists for all parameter values concludes the proof of case 2. ■

Proof of Remark 1: As shown in the Proof of Proposition OD.1, $\delta > \underline{\delta}$ implies

$$c < \frac{\delta\gamma(1-\gamma)(1+\delta)}{1+\delta\gamma}. \tag{OD.3}$$

We prove the remark by showing that for $\delta = \frac{c}{\gamma x}$ condition (OD.3) holds. We consider the cases $\gamma \leq \frac{1}{2}$ and $\gamma > \frac{1}{2}$ separately.

Let $\gamma \leq \frac{1}{2}$, hence $x = \gamma$. For $\delta = \frac{c}{\gamma^2}$, (OD.3) reduces to:

$$c < \frac{\frac{c}{\gamma}(1-\gamma)(1+\frac{c}{\gamma^2})}{1+\frac{c}{\gamma}} \Leftrightarrow (2\gamma - 1)\gamma^2 < (1-\gamma-\gamma^2)c.$$

This last inequality holds as $(2\gamma - 1)\gamma^2 < 0 < (1-\gamma-\gamma^2)c$.

Let now $\gamma > \frac{1}{2}$, hence $x = 1-\gamma$. For $\delta = \frac{c}{\gamma(1-\gamma)}$, (OD.3) reduces to:

$$c < \frac{\frac{c}{\gamma(1-\gamma)}\gamma(1-\gamma)(1+\frac{c}{\gamma(1-\gamma)})}{1+\frac{c}{\gamma(1-\gamma)}\gamma}$$

This condition is equivalent to $\gamma < 1$, which clearly holds. ■

Online Appendix E

In this appendix we check the robustness of our analysis by assuming that investors and firms never observe q. We show that, as in the baseline model:

- for some parameter values contingent fees improve expected social welfare relative to upfront fees;

- upfront fees improve expected social welfare relative to contingent fees as long as the cost of information acquisition, c, is sufficiently small.

The Model. The agency lives for two periods. The model differs from the one discussed in Section III only in that investors and firms never observe q_t (and we set $\beta = 1$).

Given $e : [0,1] \to [0,1]$, define

$$\rho^+ := \begin{cases} 0 & \text{if } \rho = 0 \\ \frac{\rho}{\rho+(1-\rho)e(\rho)} & \text{if } \rho > 0 \end{cases} \quad ; \qquad \rho^- := \begin{cases} 0 & \text{if } \rho = 0 \\ \frac{\rho}{1+(1-\rho)(1-e)} & \text{if } \rho > 0 \end{cases}$$

The next definition is immediately adapted from the equilibrium concept of Section 2.

Definition 3. *An equilibrium with contingent fees comprises functions $e_t : [0,1] \to [0,1]$ specifying the probabilities $e_t(\rho_t)$ that the strategic agency acquires information in period t given reputation ρ_t, for $t = 1, 2$, such that each period:*

(i) the choice(s) implied by $e_t(\rho_t)$ maximize the agency's expected intertemporal profit given by (OA.1)*, and*

$$\rho_2 = \begin{cases} \rho_1^+ & \textit{if } r_1 = -1; \\ \rho_1^- & \textit{if } r_1 = 1; \end{cases}$$

(ii) firms and investors' beliefs satisfy $\hat{e}_t = e_t(\rho_t)$.

The definition of an equilibrium with upfront fees is obtained by replacing (OA.1) with (OA.2).

The Equilibria. We next characterize the equilibria with, respectively, contingent fees and upfront fees. Proofs of the propositions are relegated to the end of this appendix.

Proposition OE.1. *With contingent fees, in equilibrium* $e_1(\rho_1) > 0$ *if and only if* $\delta > 2c$ *and* $\rho_1 < \rho_{co}(\delta)$*, where* $\rho_{co}(\delta)$ *is defined implicitly by*

$$\delta = \left(\frac{4 - 3\rho_{co}}{2(1-\rho_{co})}\right)\left(\frac{\rho_{co}}{2-\rho_{co}} + c\right).$$

Moreover $\lim_{c \to 0} e_1(\rho_1) < 1$ *for any* ρ_1 *and* δ*.*

Proposition OE.2. *With upfront fees, in equilibrium* $e_1(\rho_1) > 0$ *if and only if* $\delta > 4c$ *and* $\rho_1 < \rho_{up}(\delta) := \frac{4c-\delta}{2-\delta}$*. Moreover* $\lim_{c \to 0} e_1(\rho_1) = 1$ *for any* ρ_1 *and* δ*.*

Welfare Comparison. When fees are contingent, for $\delta \in (2c, 4c)$ the strategic agency acquires information at $t = 1$ with positive probability provided its reputation is not too high (Proposition OE.1). By contrast, for $\delta \in (2c, 4c)$ the strategic agency shirks with probability 1 when fees are upfront (Proposition OE.2). For $\delta \in (2c, 4c)$ and sufficiently low reputation expected social welfare is therefore higher under contingent fees than under upfront fees. Moreover for any δ and ρ_1, $\lim_{c \to 0} e_1(\rho_1) = 1$ only in the case of upfront fees. Thus upfront fees increase expected social welfare if the cost of information acquisition is sufficiently low.

Proof of Proposition OE.1: Define $V(\rho) := \frac{\rho}{2-\rho}$. At $t = 1$, the agency's expected intertemporal profit from shirking and announcing $r_t = 1$ is equal to $\Phi(\rho_1, \hat{e}_1) + \delta V(\rho_1^-)$. The intertemporal profit from acquiring information is equal to $\frac{1}{2}\Phi(\rho_1, \hat{e}_1) - c + \frac{\delta}{2}\left[V(\rho_1^+) + V(\rho_1^-)\right]$.

Fix $\rho_1 \in (0, 1)$. An equilibrium in which $e_1(\rho_1) = 1$ exists if and only if

$$\Phi(\rho_1, 1) + \delta V(\rho_1) \leq \frac{1}{2}\Phi(\rho_1, 1) - c + \delta V(\rho_1). \tag{OE.1}$$

This condition is always violated, thus in equilibrium $e_1(\rho_1) < 1$. An equilibrium in which $e_1(\rho_1) = 0$ in turn exists if and only if

$$\Phi(\rho_1, 0) + \delta V\left(\frac{\rho_1}{2-\rho_1}\right) \geq \frac{1}{2}\Phi(\rho_1, 0) + \frac{\delta}{2}\left(V(1) + V\left(\frac{\rho_1}{2-\rho_1}\right)\right) - c.$$

Substituting for $\Phi(\cdot)$ and $V(\cdot)$ and simplifying yields

$$\delta \leq \tilde{\delta}(\rho_1) := \left(\frac{4-3\rho_1}{2(1-\rho_1)}\right)\left(\frac{\rho_1}{2-\rho_1} + c\right).$$

Notice that $\tilde{\delta}(\rho_1)$ is continuous and increasing in ρ_1 for $\rho_1 \in (0,1)$ as it is the product of 2 terms, each of which is continuous and increasing in ρ_1 for $\rho_1 \in (0,1)$. Hence $\delta < \tilde{\delta}(\rho_1)$ is equivalent to $\rho_1 > \rho_{co}(\delta)$; moreover as $\tilde{\delta}(0) = 2c$, then $e_1(\rho_1) > 0$ only if $\delta > 2c$. The proof that $\delta > \tilde{\delta}(\rho_1)$ is necessary and sufficient for an equilibrium in which $e_1 \in (0,1)$ follows standard arguments.

Pick a pair δ and ρ_1. For sufficiently small c, $\delta > 2c$, thus in equilibrium $e_1(\rho_1) \in (0,1)$. Note also that for $c = 0$ and $e(\rho_1) = 1$ (OE.1) is violated. Thus, by continuity of V and Φ in equilibrium $\lim_{c\to 0} e_1(\rho_1) < 1$. ■

Proof of Proposition OE.2: Define $V(\rho) := \frac{\rho}{2}$. At $t = 1$, the agency's expected intertemporal profit from shirking and announcing $r_t = 1$ is equal to $\Upsilon(\rho_1, \hat{e}_1) + \delta V(\rho_1^-)$. The intertemporal profit from acquiring information is equal to $\Upsilon(\rho_1, \hat{e}_1) - c + \frac{\delta}{2}\left[V(\rho_1^+) + V(\rho_1^-)\right]$. Fix $\rho_1 \in (0,1)$. An equilibrium in which $e_1(\rho_1) = 1$ exists if and only if

$$\delta V\left(\rho_1\right) \leq \delta V(\rho_1) - c. \tag{OE.2}$$

This condition is always violated for $c > 0$. An equilibrium in which $e_1(\rho_1) = 0$ exists if and only if

$$\frac{\delta}{2} V\left(\frac{\rho_1}{2-\rho_1}\right) \geq \frac{\delta}{2} V(1) - c.$$

Substituting for $V(\cdot)$ and simplifying yields

$$\delta \leq \overline{\delta}(\rho_1) := \frac{2c(2-\rho_1)}{1-\rho_1}.$$

As $\overline{\delta}(\rho_1)$ is continuous and increasing in ρ_1, then $\delta < \overline{\delta}(\rho_1)$ is equivalent to $\rho_1 > \rho_{up}(\delta)$; moreover, as $\overline{\delta}(0) = 4c$, then $e_1(\rho_1) > 0$ only if $\delta > 4c$. The proof that $\delta > \overline{\delta}(\rho_1)$ is necessary and sufficient for an equilibrium in which $e_1 \in (0,1)$ follows standard arguments.

Next, fix δ and ρ_1. Note also that, for $c = 0$, $e(\rho_1) = 1$ satisfies (OE.2). Continuity of Υ and V thus ensure that in equilibrium $\lim_{c\to 0} e_1(\rho_1) = 1$. ■

Online Appendix F

In this appendix we present and analyze the model with observable information acquisition. The model and the main result of this appendix (Proposition OF.1) are presented in Subsection F.1. The analysis of an auxiliary game needed in the proof of Proposition OF.1 is carried out in Subsection F.2. Subsection F.3 contains the proof of Proposition OF.1.

F.1 The model

Fees. Each period, firms and investors form beliefs regarding the probability that the strategic agency will truthfully report what it observes. Let $\hat{a}_t$ denote the beginning-of-period-t belief that, conditional on acquiring information and observing $q_t = -1$, the strategic agency truthfully assigns the rating $r_t = -1$. We maintain the notation $\hat{e}_t$ for the beginning-of-period-t belief that the strategic agency will acquire information in period t, and set $\beta = 1$ to reduce notation. So the contingent fee is

$$\phi_t(r_t) = \begin{cases} \mathbb{E}[q_t | r_t = 1, \rho_t, \hat{e}_t, \hat{a}_t] & \text{if } r_t = 1; \\ 0 & \text{if } r_t = -1. \end{cases} \tag{OF.1}$$

while the upfront fee is $\phi_t(1) = \phi_t(-1) = \mathbb{P}(r_t = 1 | \rho_t, \hat{e}_t, \hat{a}_t)\mathbb{E}[q_t | r_t = 1, \rho_t, \hat{e}_t, \hat{a}_t]$.

Timing. The timing within period t is as follows. The agency first decides whether or not to acquire information. In case the agency shirks the game moves on to the next period. This captures the idea that a regulatory authority prevents the agency from rating firm t in case the agency is caught shirking. In case it chose to acquire information and observed $q_t = 1$ the agency publicly announces the rating $r_t = 1$. If it observed $q_t = -1$ the agency chooses whether to truthfully assign $r_t = -1$, or inflate the rating and assign $r_t = 1$. The agency then receives $\phi_t(r_t)$, all players observe q_t and the game moves on to the next period.

Strategies and Payoffs. A stationary strategy for the agency now comprises a pair $\big(e(\cdot), a(\cdot)\big)$, where $e : [0, 1] \to [0, 1]$ and $a : [0, 1] \to [0, 1]$, specifying respectively the probability of acquiring information and the probability of truthfully assigning the rating $r_t = -1$ conditional on observing $q_t = -1$, both expressed as a function of the agency's reputation ρ_t. The payoffs are as in Section **??**. The next definition adapts the equilibrium concept of that section.

Definition 4. *An equilibrium with observable information acquisition comprises a pair* $\big(e(\cdot), a(\cdot)\big)$ *such that:*

(i) in period t*, the strategic agency acquires information with probability* $e(\rho_t)$ *and, conditional on observing* $q_t = -1$*, truthfully assigns the rating* $r_t = -1$ *with probability* $a(\rho_t)$*;*

(ii) the strategy $\big(e(\cdot), a(\cdot)\big)$ *maximizes the agency's expected intertemporal profit given* $\rho_{t+1} = \Psi(\rho_t, r_t, q_t)$*, where*[5]

$$\Psi(\rho_t, r_t, q_t) := \begin{cases} \frac{\rho_t}{\rho_t + (1-\rho_t)\hat{e}_t} & \text{if } q_t = 1 = r_t \text{ and } \rho_t > 0; \\ \frac{\rho_t}{\rho_t + (1-\rho_t)\hat{e}_t\hat{a}_t} & \text{if } q_t = -1 = r_t \text{ and } \rho_t > 0; \\ 0 & \text{if } r_t = \emptyset, \text{ or } q_t = -1 = -r_t, \text{ or } \rho_t = 0; \end{cases}$$

(iii) firms and investors' beliefs satisfy $\hat{e}_t = e(\rho_t)$ *and* $\hat{a}_t = a(\rho_t)$*.*

In equilibrium investors correctly infer the probabilities with which the strategic agency chooses to acquire information and to truthfully assign $r_t = -1$ when observing $q_t = -1$, and these choices are optimal for the agency. Firms and investors' beliefs are updated using Bayes' rule whenever possible. The agency loses its reputation whenever it is caught shirking. If it acquires information and $q_t = 1$ then reputation is updated based on the belief $\hat{e}_t = e(\rho_t)$ alone, that is, reputation jumps up to

$$\rho_t^+ := \frac{\rho_t}{\rho_t + (1-\rho_t)e(\rho_t)}.$$

By contrast, two cases arise if the agency acquires information and $q_t = -1$: $r_t = 1$ reveals that the agency inflated the rating (and thus, that the agency is strategic), and $r_t = -1$ that the agency truthfully reported what it observed. In the latter case reputation is updated based both on the belief $\hat{e}_t = e(\rho_t)$ and on the belief $\hat{a}_t = a(\rho_t)$, that is, reputation jumps up to[6]

$$\rho_t^{++} := \frac{\rho_t}{\rho_t + (1-\rho_t)e(\rho_t)a(\rho_t)}.$$

We proceed to characterize the equilibrium behavior of the strategic agency. If the agency shirks, the agency is revealed to be strategic and the game moves on to the next period. So in

[5] We let $r_t = \emptyset$ denote the case in which the agency shirks in period t.

[6] As usual zero-probability events are dealt with by assuring that $\rho_t = 0$ is an absorbing state of the Markov process, and by ascribing any misreporting to the strategic agency.

order to obtain a positive payoff the agency is now forced to acquire information. Conditional on acquiring information, the agency can inflate the rating in case $q_t = -1$ or truthfully report what it observes. Inflating the rating guarantees the fee $\phi_t(1)$. The downside is that the agency could lose its reputation: either $q_t = 1$ in which case $\rho_{t+1} = \rho_t^+$, or $q_t = -1$ in which case $\rho_{t+1} = 0$. By contrast, truthfully reporting what the agency observes lowers the probability of receiving $\phi_t(1)$ to just $\frac{1}{2}$, but could induce a reputation boost: either $q_t = 1$ in which case $\rho_{t+1} = \rho_t^+$, or $q_t = -1$ in which case $\rho_{t+1} = \rho_t^{++}$. By virtue of Bellman's Principle of Optimality an equilibrium with value function $V(\cdot)$ therefore satisfies the Bellman equation

$$V(\rho) = \max\left\{\phi(\rho) - c + \delta\Big(\frac{1}{2}V(\rho^+) + \frac{1}{2}V(0)\Big)\ ,\ \frac{\phi(\rho)}{2} - c + \delta\Big(\frac{1}{2}V(\rho^+) + \frac{1}{2}V(\rho^{++})\Big)\right\}.$$

We show later that $V(0) = 0$. Hence, the strategic agency is either indifferent between inflating the rating and truthful reporting, or $\frac{\phi(\rho)}{2} > \frac{\delta}{2}V(\rho^{++})$ in which case inflating the rating is uniquely optimal, or, lastly, $\frac{\phi(\rho)}{2} < \frac{\delta}{2}V(\rho^{++})$ in which case truthful reporting is uniquely optimal.

The following proposition is the main result of this appendix.

Proposition OF.1. *With observable information acquisition:*

1. *if* $\delta < \frac{2}{3-2c}$ *then upfront fees improve expected social welfare relative to contingent fees;*
2. *if* $\delta > \frac{2}{3-2c}$ *then expected social welfare is the same whether fees are upfront or contingent.*

F.2 An Auxiliary Game

We analyze in this subsection the *auxiliary game* in which, by assumption, the strategic agency (i) shirks if $\rho_t = 0$, and (ii) acquires information if $\rho_t > 0$. Hence, $e(0) = 0$ and $e(\rho) = 1$ for all $\rho > 0$, and in this setting a stationary strategy for the agency is simply a mapping $a : (0,1] \to [0,1]$ specifying the probability of truthfully assigning the rating $r_t = -1$ when observing $q_t = -1$, as a function of the agency's reputation in period t. Our objective is to prove the following result:

Proposition OF.2. *Let* $\delta < \frac{2}{3-2c}$. *There exists a unique equilibrium of the auxiliary game. Its value function,* $\tilde{V}(\cdot)$, *is strictly increasing and continuous over* $(0,1]$, *and* $\tilde{V}(1) > 0$.

We start with two simple lemmas. Define, $\Xi(\cdot,\cdot) : (0,1] \times [0,1] \times [0,1] \to \mathbb{R}$ such that

$$\Xi(\rho, e, a) := \frac{\rho + (1-\rho)ea}{\rho + (1-\rho)e(2-a)}.$$

Note that $\mathbb{E}[q_t|r_t = 1, \rho_t, \hat{e}_t, \hat{a}_t] = \Xi(\rho_t, \hat{e}_t, \hat{a}_t)$ for all $\rho_t > 0$, and that $\Xi(\cdot,\cdot,\cdot)$ is continuous, weakly increasing in ρ and in a, weakly decreasing in e, $\Xi(\rho, e, a) > 0$, and $\Xi(1, e, a) = \Xi(\rho, e, 1) = \Xi(\rho, 0, a) = 1$.

Given a function $a : [0, 1] \to [0, 1]$, define

$$\rho^\dagger := \begin{cases} 0 & \text{if } \rho = 0, \\ \frac{\rho}{\rho+(1-\rho)a(\rho)} & \text{if } \rho > 0. \end{cases}$$

Lemma OF.1. *In any equilibrium of the auxiliary game with value function $\tilde{V}(\cdot)$, for all $\rho > 0$,*

$$\begin{cases} \textit{if} \quad \delta\tilde{V}(\rho^\dagger) > \Xi(\rho, 1, a(\rho)) \quad \textit{then} \quad a(\rho) = 1, \\ \textit{if} \quad \delta\tilde{V}(\rho^\dagger) < \Xi(\rho, 1, a(\rho)) \quad \textit{then} \quad a(\rho) = 0. \end{cases}$$

Proof: By virtue of Bellman's Principle of Optimality,

$$\tilde{V}(\rho) = \max\Big\{\Xi(\rho, 1, a(\rho)) - c + \frac{\delta}{2}\tilde{V}(\rho)\ ,\ \frac{1}{2}\Xi(\rho, 1, a(\rho)) - c + \delta\Big(\frac{1}{2}\tilde{V}(\rho) + \frac{1}{2}\tilde{V}(\rho^\dagger)\Big)\Big\} \quad \text{(OF.2)}$$

for all $\rho > 0$, and the choice implied by $a(\rho)$ maximizes the right-hand side of (OF.2). ■

Lemma OF.2. *In any equilibrium of the auxiliary game,*

$$\begin{cases} \textit{if} \quad a(\rho) = 1 \quad \textit{then} \quad \tilde{V}(\rho) = \frac{\frac{1}{2}\Xi(\rho,1,1)-c}{1-\delta} = \max\left\{\frac{\frac{1}{2}\Xi(\rho,1,1)-c}{1-\delta}\ ,\ \frac{\Xi(\rho,1,1)-c}{1-\frac{\delta}{2}}\right\}, \\ \textit{if} \quad a(\rho) < 1 \quad \textit{then} \quad \tilde{V}(\rho) = \frac{\Xi(\rho,1,a(\rho))-c}{1-\frac{\delta}{2}}. \end{cases}$$

Furthermore, $\tilde{V}(1) = \max\left\{\frac{\frac{1}{2}\Xi(1,1,1)-c}{1-\delta}\ ,\ \frac{\Xi(1,1,1)-c}{1-\frac{\delta}{2}}\right\} \geq \tilde{V}(\rho)$, for all $\rho \in [0, 1]$.

Proof: The lemma follows from Bellman's Principle of Optimality together with the observations that (a) $a(\rho) = 1$ implies $\rho^\dagger = \rho$, (b) $\rho = 1$ implies $\rho^\dagger = 1$, (c) $\Xi(1, 1, a) = 1$ for all $a \in [0, 1]$, and (d) $\Xi(\cdot, 1, \cdot)$ is weakly increasing in both variables. ■

Proof of Proposition OF.2: Define $\overline{\rho}$ and $\overline{a}$ implicitly by

$$\Xi(\overline{\rho},1,0)=\delta\Big(\frac{\Xi(1,1,1)-c}{1-\frac{\delta}{2}}\Big), \tag{OF.3}$$

and

$$\Xi(0,1,\overline{a})=\delta\Big(\frac{\Xi(1,1,1)-c}{1-\frac{\delta}{2}}\Big), \tag{OF.4}$$

respectively. As $\delta<\frac{2}{3-2c} \Leftrightarrow \Xi(1,1,1)>\delta\Big(\frac{\Xi(1,1,1)-c}{1-\frac{\delta}{2}}\Big)$, we have $\overline{\rho}<1$ and $\overline{a}<1$. Noting that (since $c<\frac{\beta}{2}=\frac{1}{2}$) the right-hand side in (OF.3) and (OF.4) is strictly positive whereas $\Xi(0,1,0)=0$ yields $\overline{\rho}>0$ and $\overline{a}>0$. Thus $\overline{\rho}\in(0,1)$ and $\overline{a}\in(0,1)$.

We now show that there is a unique equilibrium of the auxiliary game, and that the equilibrium satisfies

$$\begin{cases} a(\rho)=0 & \text{if } \rho\geq\overline{\rho} \\ a(\rho)\in(0,\overline{a}] & \text{if } \rho\in(0,\overline{\rho}). \end{cases} \tag{OF.5}$$

We will first proceed by induction to show that there can exist at most one equilibrium. We will then argue that the inductive procedure yields an equilibrium.

As a preliminary step observe that, as $\delta<\frac{2}{3-2c}$, then $\max\left\{\frac{\frac{1}{2}\Xi(1,1,1)-c}{1-\delta}\ ,\ \frac{\Xi(1,1,1)-c}{1-\frac{\delta}{2}}\right\}=\frac{\Xi(1,1,1)-c}{1-\frac{\delta}{2}}$. Hence, by Lemma OF.2,

$$\tilde{V}(\rho)=\frac{\Xi(\rho,1,a(\rho))-c}{1-\frac{\delta}{2}}, \tag{OF.6}$$

for all $\rho>0$.

The inductive procedure starts as follows. Combining (OF.3) and (OF.6) any equilibrium must be such that, for all $\rho>\overline{\rho}$:

$$\Xi(\rho,1,0)>\delta\tilde{V}(\rho^{\dagger}).$$

Thus, by Lemma OF.1, if an equilibrium exists it must satisfy $a(\rho)=0$ for all $\rho>\overline{\rho}$. A similar argument shows that in fact the same must be true for $\rho=\overline{\rho}$.

By contrast, consider $\rho\in(0,\overline{\rho})$. The combination of (OF.3), (OF.6), and Lemma OF.1 shows that $a(\rho)=0$ is impossible in equilibrium. Similarly, the combination of (OF.4), (OF.6) and Lemma OF.1 shows that $a(\rho) > \overline{a}$ is impossible in equilibrium. Thus, any equilibrium must satisfy (OF.5). By Lemma OF.1 this in turn implies that the indifference condition

$$\delta\tilde{V}(\rho^{\dagger}) = \Xi(\rho, 1, a(\rho)) \tag{OF.7}$$

must hold for all $\rho \in (0, \overline{\rho})$.

Next define ρ_1 such that

$$\overline{\rho} = \frac{\rho_1}{\rho_1 + (1-\rho_1)\overline{a}}.$$

Thus $\rho_1 < \overline{\rho}$. By construction of ρ_1 and property (OF.5), in any equilibrium: $\rho^{\dagger} \geq \overline{\rho}$ for all $\rho \in [\rho_1, \overline{\rho})$. (OF.7), (OF.5) and (OF.6) now pin down a unique candidate equilibrium $a(\rho)$ for each $\rho \in [\rho_1, \overline{\rho})$ (which moreover is continuous in ρ). Repeating the step above with ρ_1 instead of $\overline{\rho}$ yields $\rho_2 < \rho_1$ and a unique candidate equilibrium $a(\rho)$ for each $\rho \in [\rho_2, \rho_1)$, and so on. This defines a sequence $\{\rho_n\}$ where, for all n, $\rho_n = \frac{\rho_{n+1}}{\rho_{n+1}+(1-\rho_{n+1})\overline{a}}$. As $\overline{a} < 1$, $\rho_n \to 0$. This inductive procedure therefore pins down a unique candidate equilibrium, whose value function is continuous over $(0, 1]$ (see (OF.6)). That this candidate equilibrium is in fact an equilibrium is a consequence of the one-shot deviation principle.

It remains only to show that the value function $\tilde{V}$ of the unique equilibrium is strictly increasing over the interval $(0, 1]$. We proceed by induction. $\tilde{V}$ is trivially increasing over $[\overline{\rho}, 1]$. Next, suppose that we can find ρ_a and ρ_b with $\overline{\rho} > \rho_b > \rho_a \geq \rho_1$ and $\Xi(\rho_a, 1, a(\rho_a)) \geq \Xi(\rho_b, 1, a(\rho_b))$. Then we must also have $a(\rho_a) > a(\rho_b)$, which in turn implies $\rho_b^{\dagger} > \rho_a^{\dagger} \geq \overline{\rho}$ and $\tilde{V}(\rho_b^{\dagger}) > \tilde{V}(\rho_a^{\dagger})$. The latter inequality contradicts Lemma OF.1. Hence, $\Xi(\rho_a, 1, a(\rho_a)) < \Xi(\rho_b, 1, a(\rho_b))$. (OF.6) thus yields $\tilde{V}(\rho_a) < \tilde{V}(\rho_b)$ and establishes that $\tilde{V}$ is increasing over $[\rho_1, 1]$. Repeating the step above with ρ_1 instead of $\overline{\rho}$, and so on, establishes that $\tilde{V}$ is increasing over $(0, 1]$. ■

F.3 Proof of Proposition OF.1

Define $\Xi(\cdot,\cdot,\cdot) : (0, 1] \times [0, 1] \times [0, 1]\times \to \mathbb{R}$ such that

$$\Xi(\rho, e, a) := \frac{\rho + (1-\rho)ea}{\rho + (1-\rho)e(2-a)}.$$

Note that $\mathbb{E}[q_t | r_t = 1, \rho_t, \hat{e}_t, \hat{a}_t] = \Xi(\rho_t, \hat{e}_t, \hat{a}_t)$ for all $\rho_t > 0$, and that $\Xi(\cdot,\cdot,\cdot)$ is continuous, weakly increasing in ρ and in a, weakly decreasing in e, $\Xi(\rho, e, a) > 0$, and $\Xi(1, e, a) = \Xi(\rho, e, 1) = \Xi(\rho, 0, a) = 1$.

Given two functions $e : [0,1] \to [0,1]$ and $a : [0,1] \to [0,1]$, define

$$\rho^+ := \begin{cases} 0 & \text{if } \rho = 0, \\ \frac{\rho}{\rho + (1-\rho)e(\rho)} & \text{if } \rho > 0, \end{cases}$$

and

$$\rho^{++} := \begin{cases} 0 & \text{if } \rho = 0, \\ \frac{\rho}{\rho + (1-\rho)e(\rho)a(\rho)} & \text{if } \rho > 0. \end{cases}$$

Lemma OF.3. *In any equilibrium, with $V(\cdot)$ denoting the value function of the equilibrium:*

1. $e(0) = a(0) = V(0) = 0$,
2. $e(\rho) > 0$ *for all* $\rho > 0$,
3. *for all* $\rho > 0$*:*

$$V(\rho) = \max\left\{\Xi(\rho, e(\rho), a(\rho)) - c + \frac{\delta}{2}V(\rho^+)\ ,\ \frac{1}{2}\Xi(\rho, e(\rho), a(\rho)) - c + \delta\Big(\frac{1}{2}V(\rho^+) + \frac{1}{2}V(\rho^{++})\Big)\right\}. \tag{OF.8}$$

Proof: Consider an arbitrary equilibrium; by virtue of Bellman's Principle of Optimality, since $\rho = 0$ is an absorbing state, $a(0) > 0$ implies $0 \geq \frac{1}{2}\mathbb{E}[q_t | r_t = 1, 0, e(0), a(0)]$. However, if $a(0) > 0$ then $\mathbb{E}[q_t | r_t = 1, 0, e(0), a(0)] = \frac{a(0)}{2-a(0)} > 0$. Thus, by contradiction, $a(0) = 0$ and the fee of the agency with reputation $\rho_t = 0$ is 0. As $c > 0$, for $\rho_t = 0$ the agency's expected intertemporal profit from acquiring information in period t is strictly negative. This implies $e(0) = 0$ and $V(0) = 0$.

Next observe that, whichever e and a,

$$V(1) \geq \frac{\frac{1}{2}\Xi(1, e, a) - c}{1 - \delta} = \frac{\frac{1}{2} - c}{1 - \delta} > 0. \tag{OF.9}$$

Thus $e(1) = 1$, since each period the agency's payoff from shirking is 0. Suppose now that $e(\hat{\rho}) = 0$ for some $\hat{\rho} > 0$, and $\rho_t = \hat{\rho}$. Then by acquiring information in period t the agency would (i) command in period t the fee $\Xi(\hat{\rho}, 0, a(\hat{\rho})) = 1$ and (ii) guarantee itself $\rho_{t+1} = 1$. In other words, the agency's expected intertemporal profit from acquiring information in period t equals $V(1)$. But then acquiring information strictly dominates shirking, contradicting the initial assumption that $e(\hat{\rho}) = 0$. This shows that $e(\rho) > 0$ for all $\rho > 0$. We can thus compute the value function of the equilibrium by conditioning on the strategic agency acquiring information, yielding the Bellman equation (OF.8). The first expression on the right-hand side is the expected intertemporal profit conditional on acquiring information and lying about q_t in case $q_t = -1$. The second expression on the right-hand side is the expected intertemporal profit conditional on acquiring information and truthfully assigning $r_t = -1$ in case $q_t = -1$. ■

Proposition OF.3. *Let $\delta > \frac{2}{3-2c}$. There exists a unique equilibrium. In equilibrium:*

1. $e(0) = a(0) = 0$,
2. $e(\rho) = a(\rho) = 1$, *for all* $\rho > 0$.

Proof: Let $\hat{V}$ denote the value function corresponding to the strategy described in the statement of the proposition. Thus $\hat{V}(0) = 0$ and $\hat{V}(\rho) = \frac{\frac{1}{2}-c}{1-\delta} > 0$ for all, $\rho > 0$. By virtue of the one-shot deviation principle if no single deviation is profitable then the strategy considered is an equilibrium. It is easy to see that there is no profitable deviation if $\rho_t = 0$. It is also clear that if $\rho_t > 0$ then shirking is not a profitable deviation. So we only have to check that if $\rho_t > 0$ then lying about q_t when $q_t = -1$ is not profitable either. That is, we have to check that $\Xi(\rho, 1, 1) \leq \delta\left(\frac{\frac{1}{2}-c}{1-\delta}\right)$, which is equivalent to $\delta \geq \frac{2}{3-2c}$.

We proceed to show that the strategy described in the statement of the proposition is the unique equilibrium. If $a(\rho) = 1$ for all $\rho > 0$ then $V(\rho) \geq \frac{\frac{1}{2}-c}{1-\delta} > 0$ for all $\rho > 0$. Therefore $a(\rho) = 1$ for all $\rho > 0$ implies $e(\rho) = 1$ for all $\rho > 0$. Next, suppose that an equilibrium exists such that $a(\hat{\rho}) < 1$ for some $\hat{\rho} \in (0, 1)$. By virtue of (OF.8),

$$\Xi(\hat{\rho}, e(\hat{\rho}), a(\hat{\rho})) \geq \delta V(\hat{\rho}^{++}).$$

If $a(\hat{\rho}^{++})$ were equal to 1 we would then have (recall, from Lemma OF.3, $e(\hat{\rho}) > 0$),

$$1 > \Xi(\hat{\rho}, e(\hat{\rho}), a(\hat{\rho})) \geq \delta\left(\frac{\frac{1}{2}\Xi(\hat{\rho}^{++}, e(\hat{\rho}^{++}), 1) - c}{1-\delta}\right),$$

that is, $\delta < \frac{2}{3-2c}$. Hence by contradiction $a(\hat{\rho}^{++}) < 1$. We can thus repeat the steps above with $\hat{\rho}^{++}$ instead of $\hat{\rho}$, and so on. This process determines a sequence $\{\rho_n\}$ such that, for all n:

(i) $a(\rho_n) < 1$,

(ii) $\rho_{n+1} = \frac{\rho_n}{\rho_n + (1-\rho_n)e(\rho_n)a(\rho_n)} > \rho_n$,

(iii) $\Xi(\rho_n, e(\rho_n), a(\rho_n)) \geq \delta V(\rho_{n+1}) \geq \delta\Big(\frac{\Xi(\rho_{n+1}, e(\rho_{n+1}), a(\rho_{n+1})) - c}{1-\frac{\delta}{2}}\Big)$.

By (i)-(ii), either $a(\rho_n) \to 1$ or $\rho_n \to 1$. Hence, taking limits in (iii) yields (using continuity of $\Xi(\cdot,\cdot,\cdot)$ and the fact that $\Xi(1, e, a) = \Phi(\rho, e, 1) = 1$ for all ρ, e and a):

$$\Xi(1,1,1) \geq \delta\Big(\frac{\Xi(1,1,1) - c}{1-\frac{\delta}{2}}\Big). \tag{OF.10}$$

This simplifies to $\delta \leq \frac{2}{3-2c}$. ■

Lemma OF.4. *Let $\delta < \frac{2}{3-2c}$. There exists $\overline{\rho} \in (0,1)$ and $\overline{a} \in (0,1)$ such that, in any equilibrium:*

1. *$e(\rho) = 1$ for all $\rho \geq \overline{\rho}$,*

2. *$a(\rho) \leq \overline{a}$ for all $\rho \in [0,1]$.*

Proof: Since $\Xi(1, e, a) = 1$ for all e and a and $\Xi(\cdot,\cdot,\cdot)$ is continuous, (OF.8) yields $V(\rho) > 0$ for all ρ sufficiently close to 1. But $V(\rho) > 0$ implies $e(\rho) = 1$.

Next, define $\overline{a}$ implicitly by

$$\Xi(0,1,\overline{a}) = \delta\Big(\frac{\Xi(1,1,1) - c}{1-\frac{\delta}{2}}\Big). \tag{OF.11}$$

Observe that $\overline{a} < 1$ since $\delta < \frac{2}{3-2c}$, and $\overline{a} > 0$ since the right-hand side of (OF.11) is strictly positive.

Suppose that we can find an equilibrium with $a(\hat{\rho}) > \overline{a}$ for $\hat{\rho} > 0$. Then, by (OF.8) and Bellman's Principle of Optimality, $\delta V(\hat{\rho}^{++}) \geq \Xi(\hat{\rho}, e(\hat{\rho}), a(\hat{\rho}))$. Combined with (OF.11), this yields $V(\hat{\rho}^{++}) > \frac{\Xi(1,1,1)-c}{1-\frac{\delta}{2}}$. Yet, given $\delta < \frac{2}{3-2c}$, we have

$$V(1) = \max\left\{\frac{\Xi(1,1,1)-c}{1-\frac{\delta}{2}}\,,\ \frac{\frac{1}{2}\Xi(1,1,1)-c}{1-\delta}\right\} = \frac{\Xi(1,1,1)-c}{1-\frac{\delta}{2}},$$

yielding $V(\hat{\rho}^{++}) > V(1)$, which cannot be. ■

In what follows let $\tilde{a}(\cdot)$ denote the unique equilibrium of the auxiliary game analyzed in Subsection F.2, $\tilde{V}(\cdot)$ the corresponding value function, and for $\delta < \frac{2}{3-2c}$ define $\tilde{\rho}$ implicitly by

$$\begin{cases} \tilde{\rho} = 0 & \text{if } \tilde{V}(\rho) > 0 \text{ for all } \rho > 0, \\ \tilde{V}(\tilde{\rho}) = 0 & \text{otherwise.} \end{cases}$$

By virtue of Proposition OF.2, $\tilde{\rho}$ is well defined. Finally, given $\rho > 0$, let

$$\rho^{\dagger} := \frac{\rho}{\rho + (1-\rho)\tilde{a}(\rho)}.$$

We will now show that there can be at most one equilibrium for $\delta < \frac{2}{3-2c}$. The proof is somewhat complicated. To help the reader get the gist of the argument, we defer the main result and start with a slightly weaker version of the result, by focusing on the class of equilibria with a non-decreasing value function.

Lemma OF.5. *Let $\delta < \frac{2}{3-2c}$. In any equilibrium whose value function is non-decreasing:*

1. *if $\tilde{\rho} = 0$: $e(\rho) = 1$ for all $\rho > 0$,*
2. *if $\tilde{\rho} > 0$: $e(\rho) = 1$ for all $\rho \geq \tilde{\rho}$ and $e(\rho) \in (0,1)$ for all $\rho \in (0,\tilde{\rho})$.*

Proof: We first show the proof of the lemma for the case in which $\tilde{\rho} = 0$. The case $\tilde{\rho} > 0$ is considered at the end.

Define ρ_1 implicitly by

$$\overline{\rho} = \frac{\rho_1}{\rho_1 + (1-\rho_1)\overline{a}},$$

with $\overline{\rho}$ and $\overline{a}$ satisfying the conditions stated in Lemma OF.4. Thus $\rho_1 < \overline{\rho}$ (since $\overline{a} < 1$), and in any equilibrium $\rho^{++} \geq \overline{\rho}$ for all $\rho \geq \rho_1$.

The proof is by induction: we have $e(\rho) = 1$ for all $\rho \geq \overline{\rho}$ and we proceed to show that $e(\rho) = 1$ for all $\rho \geq \rho_1$.

Step 1: for all $\rho \geq \rho_1$, $\Xi(\rho, e(\rho), a(\rho)) \geq \Xi(\rho, 1, \tilde{a}(\rho))$. If $e(\rho) = 1$ then $a(\rho) = \tilde{a}(\rho)$ and so the result is trivial. Suppose now that $e(\rho) < 1$, and $\Xi(\rho, e(\rho), a(\rho)) < \Xi(\rho, 1, \tilde{a}(\rho))$. Then $a(\rho) < \tilde{a}(\rho)$, which in turn implies

$$\Xi(\rho, e(\rho), a(\rho)) \geq \delta V(\rho^{++}) = \delta\tilde{V}(\rho^{++}) > \delta\tilde{V}(\rho^{\dagger}) \geq \Xi(\rho, 1, \tilde{a}(\rho)).$$

The first inequality follows from (OF.8) and $a(\rho) < 1$. The subsequent equality follows from noting that $\rho^{++} \geq \overline{\rho}$ while $V(\rho) = \tilde{V}(\rho)$ for all $\rho \geq \overline{\rho}$. The second inequality is due to the fact that since $a(\rho) < \tilde{a}(\rho)$, $\rho^{++} > \rho^{\dagger}$, while $\tilde{V}$ is strictly increasing (Proposition OF.2). The last inequality follows from (OF.2) and $\tilde{a}(\rho) > 0$.

Step 2: for all $\rho \geq \rho_1$, $\tilde{V}(\rho^{++}) \geq \tilde{V}(\rho^{\dagger})$. First, suppose $a(\rho) = 0$. Then $\rho^{++} = 1$ and, since $\tilde{V}(\cdot)$ is increasing on $(0, 1]$ (Proposition OF.2), $\tilde{V}(\rho^{++}) \geq \tilde{V}(\rho^{\dagger})$.

Next, suppose $\tilde{a}(\rho) = 0$. Then $\rho^{\dagger} = 1$ and $\Xi(\rho, 1, \tilde{a}(\rho)) \geq \delta\tilde{V}(\rho^{\dagger}) = \delta\tilde{V}(1)$. Therefore, if we had $\tilde{V}(\rho^{++}) < \tilde{V}(\rho^{\dagger})$ we would have (by Step 1) $\Xi(\rho, e(\rho), a(\rho)) > \delta\tilde{V}(\rho^{++})$, and since $V(\rho) = \tilde{V}(\rho)$ for all $\rho \geq \overline{\rho}$, $\Xi(\rho, e(\rho), a(\rho)) > \delta V(\rho^{++})$. Given (OF.8), this would imply $a(\rho) = 0$, and $\rho^{++} = 1$. But then $\tilde{V}(\rho^{++}) = \tilde{V}(\rho^{\dagger})$.

Finally, suppose $a(\rho) > 0$ and $\tilde{a}(\rho) > 0$. We then obtain, in view of (OF.2), (OF.8), Step 1 and $V(\rho) = \tilde{V}(\rho)$ for all $\rho \geq \overline{\rho}$,

$$\delta\tilde{V}(\rho^{++}) = \delta V(\rho^{++}) = \Xi(\rho, e(\rho), a(\rho)) \geq \Xi(\rho, 1, \tilde{a}(\rho)) = \delta\tilde{V}(\rho^{\dagger}).$$

Step 3: for all $\rho \geq \rho_1$, $V(\rho) \geq \tilde{V}(\rho)$. Using $V(\rho) = \tilde{V}(\rho)$ for all $\rho \geq \overline{\rho}$, if $V(\cdot)$ is non-decreasing then (OF.8), Step 1 and Step 2 yield

$$V(\rho) \geq \max\left\{\Xi(\rho, 1, \tilde{a}(\rho)) - c + \frac{\delta}{2}V(\rho)\ ,\ \frac{1}{2}\Xi(\rho, 1, \tilde{a}(\rho)) - c + \delta\Big(\frac{1}{2}V(\rho) + \frac{1}{2}\tilde{V}(\rho^{\dagger})\Big)\right\}, \tag{OF.12}$$

for all $\rho \geq \rho_1$. Comparing (OF.12) with (OF.2) gives $V(\rho) \geq \tilde{V}(\rho)$, for all $\rho \geq \rho_1$.

We conclude from Step 3 that $V(\rho) > 0$ for all $\rho \geq \rho_1$, which in turn implies that $e(\rho) = 1$ for all $\rho \geq \rho_1$. We can thus repeat Steps 1-3 with ρ_1 instead of $\overline{\rho}$, and so on. This process defines a sequence $\{\rho_n\}$ such that, for all n, $e(\rho) = 1$ for all $\rho \geq \rho_n$, and $\rho_n = \frac{\rho_{n+1}}{\rho_{n+1}+(1-\rho_{n+1})\overline{a}}$. As $\overline{a} < 1$, $\rho_n \to 0$. Thus $e(\rho) = 1$ for all $\rho > 0$.

We now show the proof of the lemma for the case in which $\tilde{\rho} > 0$. Reasoning as in the previous case shows that $V(\rho) > 0$ for all $\rho > \tilde{\rho}$, and therefore that $e(\rho) = 1$ and $V(\rho) = \tilde{V}(\rho)$ for all $\rho > \tilde{\rho}$. Furthermore $V(1) \geq \frac{\frac{1}{2}\Xi(1,1,1)-c}{1-\delta} > 0$, from which follows that $e(\rho) > 0$ for all $\rho > 0$; if this were not the case, acquiring information would yield the strategic agency $V(1) > 0$. Hence, we are only left to show that $e(\tilde{\rho}) = 1$. Suppose that $e(\tilde{\rho}) < 1$. Then $\tilde{\rho}^+ > \tilde{\rho}$, and so $\tilde{V}(\tilde{\rho}^+) > 0$. Hence, we obtain

$$\begin{aligned}
V(\tilde{\rho}) &\geq \frac{1}{2}\Xi(\tilde{\rho}, e(\tilde{\rho}), a(\tilde{\rho})) - c + \delta\Big(\frac{1}{2}V(\tilde{\rho}^+) + \frac{1}{2}V(\tilde{\rho}^{++})\Big) \\
&= \frac{1}{2}\Xi(\tilde{\rho}^+, 1, a(\tilde{\rho})) - c + \delta\Big(\frac{1}{2}V(\tilde{\rho}^+) + \frac{1}{2}V(\tilde{\rho}^{++})\Big) \\
&= \frac{1}{2}\Xi(\tilde{\rho}^+, 1, \tilde{a}(\tilde{\rho}^+)) - c + \delta\Big(\frac{1}{2}\tilde{V}(\tilde{\rho}^+) + \frac{1}{2}\tilde{V}(\tilde{\rho}^{++})\Big) \\
&= \tilde{V}(\tilde{\rho}^+) > 0.
\end{aligned}$$

The first line follows from (OF.8), and the second from noting that $\Xi(\tilde{\rho}, e(\tilde{\rho}), a(\tilde{\rho})) = \Xi(\tilde{\rho}^+, 1, \tilde{a}(\tilde{\rho}))$. The third line is obtained by noting that, since $e(\tilde{\rho}^+) = 1 = e(\tilde{\rho}^{++})$, the trade off between lying and telling the truth faced by the strategic agency with reputation $\tilde{\rho}$ is the same as the trade off faced by the strategic agency with reputation $\tilde{\rho}^+$ in the auxiliary game. Therefore, $a(\tilde{\rho}) = \tilde{a}(\tilde{\rho}^+)$. The sequence above yields $V(\tilde{\rho}) > 0$, and so $e(\tilde{\rho}) = 1$. ■

Lemma OF.6. *Let* $\delta < \frac{2}{3-2c}$. *In any equilibrium:*

1. *if* $\tilde{\rho} = 0$: $e(\rho) = 1$ *for all* $\rho > 0$,
2. *if* $\tilde{\rho} > 0$: $e(\rho) = 1$ *for all* $\rho \geq \tilde{\rho}$ *and* $e(\rho) \in (0, 1)$ *for all* $\rho \in (0, \tilde{\rho})$.

Proof: We will show the proof of the lemma for the case in which $\tilde{\rho} = 0$. We omit the proof of the case $\tilde{\rho} > 0$, which is very similar to the case we consider.

First, notice that $V(1) \geq \frac{\frac{1}{2}-c}{1-\delta} > 0$. It ensues that $e(\rho) > 0$ for all $\rho > 0$; if this were not the case, acquiring information would yield the strategic agency $V(1) > 0$. Since $a(\rho) < 1$ for all $\rho > 0$ (Lemma OF.4) Bellman's Principle of Optimality yields

$$V(\rho) = \Xi(\rho, e(\rho), a(\rho)) - c + \frac{\delta}{2}V(\rho^+). \qquad \text{(OF.13)}$$

Define ρ_1 implicitly by

$$\overline{\rho} = \frac{\rho_1}{\rho_1 + (1-\rho_1)\overline{a}},$$

with $\overline{\rho}$ and $\overline{a}$ satisfying the conditions stated in Lemma OF.4. Thus $\rho_1 < \overline{\rho}$ (since $\overline{a} < 1$), and in any equilibrium $\rho^{++} \geq \overline{\rho}$ for all $\rho \geq \rho_1$.

The proof is by induction: we have $e(\rho) = 1$ for all $\rho \geq \overline{\rho}$ and we proceed to show that $e(\rho) = 1$ for all $\rho \geq \rho_1$.

Suppose that we can find $\rho \geq \rho_1$ such that $e(\rho) < 1$. We claim that $e(\rho) < 1$ implies $e(\rho^+) < 1$. To see this, observe that if $e(\rho^+) = 1$ then

$$\begin{aligned} V(\rho) &\geq \frac{1}{2}\Xi(\rho, e(\rho), a(\rho)) - c + \delta\Big(\frac{1}{2}V(\rho^+) + \frac{1}{2}V(\rho^{++})\Big) \\ &= \frac{1}{2}\Xi(\rho^+, 1, a(\rho)) - c + \delta\Big(\frac{1}{2}V(\rho^+) + \frac{1}{2}V(\rho^{++})\Big) \\ &= \frac{1}{2}\Xi(\rho^+, 1, \tilde{a}(\rho^+)) - c + \delta\Big(\frac{1}{2}\tilde{V}(\rho^+) + \frac{1}{2}\tilde{V}(\rho^{++})\Big) \\ &= \tilde{V}(\rho^+). \end{aligned} \tag{OF.14}$$

The first line follows from (OF.8), and the second from noting that $\Xi(\rho, e(\rho), a(\rho)) = \Xi(\rho^+, 1, a(\rho))$. The third line is obtained by noting that, since $e(\rho^+) = 1 = e(\rho^{++})$, the trade-off between lying and telling the truth faced by the strategic agency with reputation ρ is the same as the trade-off faced by the strategic agency with reputation ρ^+ in the auxiliary game. Therefore, $a(\rho) = \tilde{a}(\rho^+)$. The sequence (OF.14) yields $V(\rho) \geq \tilde{V}(\rho^+) > 0$, and so $e(\rho) = 1$, contradicting our initial assumption.

Since we showed that $e(\rho) < 1$ implies $e(\rho^+) < 1$, if we can find $\rho \geq \rho_1$ such that $e(\rho) < 1$ then there exists a strictly increasing sequence $\{\rho_n\}$ with $e(\rho_n) < 1$ for all n, and $\rho_{n+1} = \rho_n^+$. Let $\hat{\rho} = \lim_{n\to\infty} \rho_n$; by Lemma OF.4, $\hat{\rho} < \overline{\rho} + z$ for all $z > 0$, as otherwise for n large enough we would get $e(\rho_n) = 1$. As $\overline{\rho} < 1$ this is turn implies that $\lim_{n\to\infty} e(\rho_n) = 1$.

We next claim that for all $\epsilon > 0$ there exists N such that, for all $n > N$:

$$\mid \Xi(\rho_n, e(\rho_n), a(\rho_n)) - \Xi(\hat{\rho}, 1, \tilde{a}(\hat{\rho})) \mid < \epsilon. \tag{OF.15}$$

To see this, suppose that $\tilde{a}(\hat{\rho}) > 0$ (the case $\tilde{a}(\hat{\rho}) = 0$ can be dealt with in a similar way). Hence,

$$\delta\tilde{V}(\hat{\rho}^\dagger) = \Xi(\hat{\rho}, 1, \tilde{a}(\hat{\rho})). \tag{OF.16}$$

We therefore have

$$\lim_{n\to\infty} \Xi(\rho_n, e(\rho_n), 0) = \Xi(\hat{\rho}, 1, 0) < \Xi(\hat{\rho}, 1, \tilde{a}(\hat{\rho})) = \delta\tilde{V}(\hat{\rho}^{\dagger}) \leq \delta\tilde{V}(1),$$

from which follows that, for n large enough, $a(\rho) > 0$. Hence $\delta V(\rho_n^{++}) = \Xi(\rho_n, e(\rho_n), a(\rho_n))$ for all n large enough. Since, $\rho_n^{++} \geq \overline{\rho}$ for all n, we obtain

$$\delta\tilde{V}(\rho_n^{++}) = \Xi(\rho_n, e(\rho_n), a(\rho_n)), \tag{OF.17}$$

for all n large enough. That (OF.15) holds for sufficiently large n now follows from (OF.16), (OF.17) and continuity of $\tilde{V}(\cdot)$ (see Proposition OF.2).

Now, by construction of $\{\rho_n\}$, (OF.13) yields

$$V(\rho_n) = \sum_{k=0}^{\infty} \left(\frac{\delta}{2}\right)^k \Big(\Xi(\rho_{n+k}, e(\rho_{n+k}), a(\rho_{n+k})) - c\Big)$$

for all n. Moreover, we showed above that for all $\epsilon > 0$ there exists N such that, for all $n > N$, (OF.15) holds. Thus, for all $\eta > 0$, choosing ϵ sufficiently small and n sufficiently large gives

$$\left| V(\rho_n) - \sum_{k=0}^{\infty} \left(\frac{\delta}{2}\right)^k \Big(\Xi(\hat{\rho}, 1, \tilde{a}(\hat{\rho})) - c\Big) \right| < \eta,$$

i.e., noting that $\sum_{k=0}^{\infty} \left(\frac{\delta}{2}\right)^k \Big(\Xi(\hat{\rho}, 1, \tilde{a}(\hat{\rho})) - c\Big) = \frac{\Xi(\hat{\rho},1,\tilde{a}(\hat{\rho}))-c}{1-\frac{\delta}{2}} = \tilde{V}(\hat{\rho})$,

$$\left| V(\rho_n) - \tilde{V}(\hat{\rho}) \right| < \eta.$$

As $\tilde{V}(\hat{\rho}) > 0$, we conclude that $V(\rho_n) > 0$, implying $e(\rho_n) = 1$. This contradicts the construction of the sequence $\{\rho_n\}$. Thus $e(\rho) = 1$ for all $\rho \geq \rho_1$.

We can now repeat the steps above with ρ_1 instead of $\overline{\rho}$, and so on. This process defines a sequence $\{\hat{\hat{\rho}}_n\}$ such that, for all n, $e(\rho) = 1$ for all $\rho \geq \hat{\hat{\rho}}_n$, and $\hat{\hat{\rho}}_n = \frac{\hat{\hat{\rho}}_{n+1}}{\hat{\hat{\rho}}_{n+1} + (1-\hat{\hat{\rho}}_{n+1})\overline{a}}$. As $\overline{a} < 1$, $\hat{\hat{\rho}}_n \to 0$. Thus $e(\rho) = 1$ for all $\rho > 0$. ∎

Proposition OF.4. *Let $\delta < \frac{2}{3-2c}$. There exists a unique equilibrium. This equilibrium is such that, for some cutoff ρ_c:*

1. $e(0) = a(0) = 0$;

2. $e(\rho) = 1$ *for all* $\rho \in (\tilde{\rho}, 1]$, *and* $e(\rho) \in (0, 1)$ *for all* $\rho \in (0, \tilde{\rho})$;

3. $a(\rho) = 0$ *for all* $\rho \in [\rho_c, 1]$, *and* $a(\rho) > 0$ *for all* $\rho \in (0, \rho_c)$.

Proof: We take up the case in which $\tilde{\rho} > 0$. The case in which $\tilde{\rho} = 0$ is similar and simpler, we therefore omit the proof.

By virtue of the one-shot deviation principle, the following strategy is an equilibrium:

$$\begin{cases} e(0) = a(0) = 0; & \\ e(\rho) = 1 & \text{for all } \rho \geq \tilde{\rho}; \\ \tilde{\rho} = \frac{\rho}{\rho + (1-\rho)e(\rho)} & \text{for all } \rho < \tilde{\rho}; \\ a(\rho) = \tilde{a}(\rho) & \text{for all } \rho \geq \tilde{\rho}; \\ a(\rho) = \tilde{a}(\tilde{\rho}) & \text{for all } \rho \in (0, \tilde{\rho}). \end{cases}$$

Uniqueness follows from Lemma OF.6. The existence of the cutoff ρ_{co} is a consequence of Proposition OF.2. ■

Proposition OF.5. *Let* $\delta \neq \frac{2}{3-2c}$. *There is a unique equilibrium with observable information acquisition and contingent fees. In this equilibrium, if* $\delta > \frac{2}{3-2c}$, *then* $e(\rho) = a(\rho) = 1$ *for all* $\rho > 0$. *If instead* $\delta < \frac{2}{3-2c}$ *then the equilibrium is characterized by cutoffs* $\rho_{c_1} < 1$ *and* $\rho_{c_2} < 1$ *such that:*

1. $e(\rho) = 1$ *if* $\rho > \rho_{c_1}$ *and* $e(\rho) \in (0, 1)$ *if* $\rho \in (0, \rho_{c_1})$;

2. $a(\rho) = 0$ *if* $\rho > \rho_{c_2}$ *and* $a(\rho) > 0$ *if* $\rho \in (0, \rho_{c_2})$.

Furthermore, $e(0) = a(0) = 0$ *for all* δ.

Proof: Follows from Propositions OF.3 and OF.4. ■

Proposition OF.6. *There is a unique equilibrium with observable information acquisition and upfront fees. In this equilibrium,* $e(\rho) = a(\rho) = 1$ *for all* $\rho > 0$.

Proof: Consider an equilibrium of the game with observable information acquisition and fees received upfront by the agency such that $e(0) = a(0) = 0$. Let V denote the value function. Thus $V(0) = 0$, while $V(1) \geq \frac{\frac{1}{2}\Xi(1,1,1)-c}{1-\delta} > 0$. Hence $e(1) = 1$. It immediately follows that $e(\rho) > 0$ for all $\rho > 0$; if this were not the case, acquiring information would yield the strategic agency $V(1) > 0$. This gives

$$\begin{aligned} &V(\rho) = \\ &\max\Big\{\mathbb{P}(r_t = 1|\rho, e(\rho), a(\rho))\Xi(\rho, e(\rho), a(\rho)) - c + \frac{\delta}{2}V(\rho^+)\ , \\ &\quad \mathbb{P}(r_t = 1|\rho, e(\rho), a(\rho))\Xi(\rho, e(\rho), a(\rho)) - c + \delta\Big(\frac{1}{2}V(\rho^+) + \frac{1}{2}V(\rho^{++})\Big)\Big\}, \end{aligned} \tag{OF.18}$$

for all $\rho > 0$. Moreover, no matter $e(\rho)$, we have $\mathbb{P}(r_t = 1|\rho, e(\rho), 1)\Xi(\rho, e(\rho), 1) = 1$. This implies that we can find $\overline{a} < 1$ such that, either (i) $V(\rho) > 0$ or (ii) $a(\rho) < \overline{a}$ and $V(\rho^{++}) = 0$. Case (ii) is however impossible as it implies the existence of a sequence $\{\rho_n\}$ tending to 1 as $n \to \infty$ and such that $V(\rho_n) = 0$ for all n. Therefore, $V(\rho) > 0$ for all $\rho > 0$. It ensues that $e(\rho) = a(\rho) = 1$ for all $\rho > 0$. ■

Proof of Proposition OF.1: Follows from Propositions OF.5 and OF.6. ■

Online Appendix G

In this appendix we characterize the set of socially optimal fee structures. Consider a fee structure such that: if $\rho_t = \rho_1$ then $\phi_t(1) = a$ and $\phi_t(-1) = b$ while if $\rho_t = 0$ then $\phi_t(1) = y$ and $\phi_t(-1) = z$. The fee structure is socially optimal if: firms prefer to get a rating as long as $\rho_t = \rho_1$ and their beliefs satisfy $\hat{e}_t = 1$ (call this condition 1) and there exist an equilibrium of the game with such a fee structure in which $e(\rho_1) = 1$ (call this condition 2). Condition 1 boils down to:

$$\frac{a}{2} + \frac{b}{2} \leq (\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 0),$$

where the left-hand side is the firm's expected payment to the agency and the right-hand side is the firm's expected revenue from investors if firm t decides to obtain a rating for $\rho_t = \rho_1$ and $\hat{e}_t = 1$. Condition 1 is equivalent to:

$$a + b \leq 1.$$

Condition 2 boils down to:

$$\frac{a-b}{2} + c \leq \frac{\delta}{2}\left(\frac{1}{1-\delta}\left(\frac{a+b}{2} - c\right) - \max\left\{\frac{y}{1-\delta}, \frac{y+2}{2(1-\delta)} - c\right\}\right).$$

where the left-hand side the CRA's short-run incentive to shirk and the right-hand side is the CRA's long-run incentive to acquire information for $\rho_t = \rho_1$ and $\hat{e}_t = 1$. Rearranging terms, this condition can be written as:

$$\frac{b-a}{2} + \frac{a\delta}{2-\delta} \geq c + \max\left\{\frac{y}{1-\delta}, \frac{y+2}{2(1-\delta)} - c\right\}\frac{2(1-\delta)}{2-\delta}.$$

The largest set of a and b for which this condition holds is obtained for $y = z = 0$, in which case the condition reduces to:

$$\frac{b-a}{2} + \frac{a\delta}{2-\delta} \geq c.$$