Online Appendix:

Flow Origins of Labor Force Participation Fluctuations

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Derivation of equation (7)

For clarity, we include the equations from the main text in this appendix. We define the vector of labor market states as

$$\boldsymbol{s}_t = \begin{bmatrix} e_t & u_t \end{bmatrix}' \tag{A1}$$

The dynamics of this vector satisfies the system of two dynamic equations

$$\Delta \boldsymbol{s}_t = \boldsymbol{d}_t + \boldsymbol{P}_t \boldsymbol{s}_{t-1}, \tag{A2}$$

where

$$\boldsymbol{d}_t = \begin{bmatrix} p_{n,e,t} \\ p_{n,u,t} \end{bmatrix}, \text{ and } \boldsymbol{P}_t = \begin{bmatrix} -p_{e,n,t} - p_{e,u,t} - p_{n,e,t} & p_{u,e} - p_{n,e} \\ p_{e,u} - p_{n,u} & -p_{u,e} - p_{u,n} - p_{n,u} \end{bmatrix} \tag{A3}$$

For given matrices $\boldsymbol{d}_t$ and $\boldsymbol{P}_t$, this system has the following steady state

$$\bar{\boldsymbol{s}}_t = -\boldsymbol{P}_t^{-1} \boldsymbol{d}_t \tag{A4}$$

This allows us to split the change in $\boldsymbol{s}_t$ into two parts. The first is the transitional dynamics due to $\boldsymbol{s}_{t-1}$ deviating from the previous period's flow steady state. The second

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is because of the change in the flow steady state. That is

$$\Delta \boldsymbol{s}_t = \boldsymbol{P}_t \left(\boldsymbol{s}_{t-1} - \bar{\boldsymbol{s}}_t\right) = \boldsymbol{P}_t \left(\boldsymbol{s}_{t-1} - \bar{\boldsymbol{s}}_{t-1}\right) - \boldsymbol{P}_t \left(\bar{\boldsymbol{s}}_t - \bar{\boldsymbol{s}}_{t-1}\right) \tag{A5}$$

Moreover, we can write

$$\begin{aligned} \left(\boldsymbol{s}_t - \bar{\boldsymbol{s}}_t\right) &= \left(\boldsymbol{I} + \boldsymbol{P}_t\right)\left(\boldsymbol{s}_{t-1} - \bar{\boldsymbol{s}}_{t-1}\right) - \left(\boldsymbol{I} + \boldsymbol{P}_t\right)\left(\bar{\boldsymbol{s}}_t - \bar{\boldsymbol{s}}_{t-1}\right) \\ &= \left(\boldsymbol{I} + \boldsymbol{P}_t\right) \boldsymbol{P}_t^{-1} \Delta \boldsymbol{s}_t \end{aligned} \tag{A6}$$

This allows us to write the current change in the state as a function of the transitional dynamics through the past change in the state of the changes in the steady state. That is

$$\Delta \boldsymbol{s}_t = \boldsymbol{P}_t \left(\boldsymbol{I} + \boldsymbol{P}_{t-1}\right) \boldsymbol{P}_{t-1}^{-1} \Delta \boldsymbol{s}_{t-1} - \boldsymbol{P}_t \Delta \bar{\boldsymbol{s}}_t \tag{A7}$$

The final step is to attribute the changes in the steady state, i.e. $\Delta \bar{\boldsymbol{s}}_t$ to changes in the different matrices made up of transition probabilities. For this, we basically apply a shift-share analysis to the change in the steady state. That is, we use that we can write

$$\Delta \boldsymbol{d}_t = -\frac{1}{2} \Delta \boldsymbol{P}_t \left(\bar{\boldsymbol{s}}_t + \bar{\boldsymbol{s}}_{t-1}\right) - \frac{1}{2} \left(\boldsymbol{P}_t + \boldsymbol{P}_{t-1}\right) \Delta \bar{\boldsymbol{s}}_t \tag{A8}$$

Such that

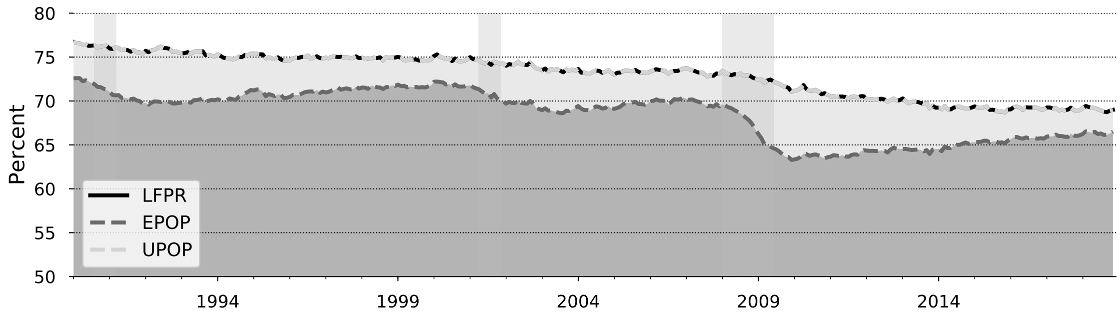
$$\Delta \bar{\boldsymbol{s}}_t = \left[\frac{1}{2} \left(\boldsymbol{P}_t + \boldsymbol{P}_{t-1}\right)\right]^{-1} \left[-\Delta \boldsymbol{d}_t - \frac{1}{2} \Delta \boldsymbol{P}_t \left(\bar{\boldsymbol{s}}_t + \bar{\boldsymbol{s}}_{t-1}\right)\right] \tag{A9}$$

Combining equations, we obtain the decomposition in the main text

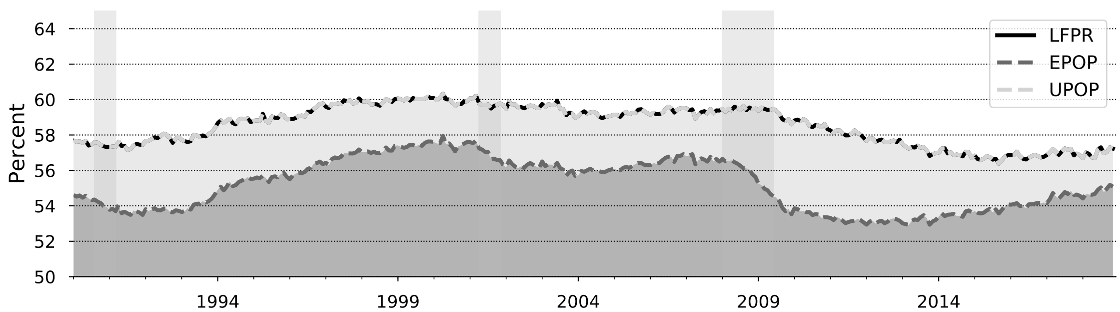
$$\Delta \boldsymbol{s}_t = \boldsymbol{P}_t \left(\boldsymbol{I} + \boldsymbol{P}_{t-1}\right) \boldsymbol{P}_{t-1}^{-1} \Delta \boldsymbol{s}_{t-1} + \boldsymbol{P}_t \left(\boldsymbol{P}_t + \boldsymbol{P}_{t-1}\right)^{-1} \left[2 \Delta \boldsymbol{d}_t + \Delta \boldsymbol{P}_t \left(\bar{\boldsymbol{s}}_t + \bar{\boldsymbol{s}}_{t-1}\right)\right]$$

This is the decomposition we use for our results

Additional results



(a) labor force participation rate (LFPR) and its components: Men



(b) LFPR and its components: Women

Figure B1. : Labor force participation rate and its components by gender.

Source: Bureau of Labor Statistics (BLS)