Information and Communication in Organizations
Online Appendix

Inga Deimen and Dezső Szalay*

Preliminaries:

Let $Y = (Y_1, Y_2)$ be an n-dimensional Normal random variable with

$$\mu = (\mu_1, \mu_2), \quad \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix},$$

where the dimensions of Y_1, μ_1 and Σ_{11} are m, m, and $m \times m$. The conditional distribution of $(Y_1 | Y_2 = y_2)$ is Normal with conditional mean vector

$$\mathbb{E}[Y_1 | Y_2 = y_2] = \mu_1 + (y_2 - \mu_2) \Sigma_{22}^{-1} \Sigma_{21} \tag{A1}$$

and conditional covariance matrix satisfying

$$\Sigma^* = \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}. \tag{A2}$$

Applying equation $(A1)$, the conditional expectations are

$$\mathbb{E}[\eta | s_\omega, s_\eta] = \alpha^s s_\omega + \beta^s s_\eta \tag{A3}$$

and

$$\mathbb{E}[\omega | s_\omega, s_\eta] = \alpha^r s_\omega + \beta^r s_\eta, \tag{A4}$$

where the weights in the sender's ideal choice are

$$\alpha^s = \sigma_{\varepsilon_\eta}^2 \frac{\rho\sigma^2}{(\sigma^2 + \sigma_{\varepsilon_\omega}^2)(\sigma^2 + \sigma_{\varepsilon_\eta}^2) - (\rho\sigma^2)^2}$$

*Inga Deimen: University of Arizona, Eller College of Management, 1130 E Helen St, Tucson, AZ 85721, ideimen@email.arizona.edu. Dezső Szalay: University of Bonn, Institute for Microeconomics, Adenauerallee 24-42, 53113 Bonn, Germany, szalay@uni-bonn.de.

and

$$\beta^s = \sigma^2 \frac{\sigma^2_{\varepsilon_\omega} - \sigma^2\rho^2 + \sigma^2}{(\sigma^2 + \sigma^2_{\varepsilon_\omega})(\sigma^2 + \sigma^2_{\varepsilon_\eta}) - (\rho\sigma^2)^2},$$

and the weights in the receiver's ideal choice are

$$\alpha^r = \sigma^2 \frac{\sigma^2_{\varepsilon_\eta} + \sigma^2 - \sigma^2\rho^2}{(\sigma^2 + \sigma^2_{\varepsilon_\omega})(\sigma^2 + \sigma^2_{\varepsilon_\eta}) - (\rho\sigma^2)^2}$$

and

$$\beta^r = \sigma^2_{\varepsilon_\omega} \frac{\sigma^2\rho}{(\sigma^2 + \sigma^2_{\varepsilon_\omega})(\sigma^2 + \sigma^2_{\varepsilon_\eta}) - (\rho\sigma^2)^2}.$$

Proof of Lemma 1. The second moments of the random variable θ can be calculated as

$$C = \alpha^s\sigma^2 + \beta^s\rho\sigma^2 = \rho\sigma^2 \frac{\frac{\sigma^2_{\varepsilon_\omega}}{\sigma^2} + \frac{\sigma^2_{\varepsilon_\eta}}{\sigma^2} + 1 - \rho^2}{\left(1 + \frac{\sigma^2_{\varepsilon_\omega}}{\sigma^2}\right)\left(1 + \frac{\sigma^2_{\varepsilon_\eta}}{\sigma^2}\right) - \rho^2},$$

which for $\sigma^2_{\varepsilon_\omega} = 0$ amounts to

$$C = \rho\sigma^2.$$

Similarly,

$$\begin{aligned} V &= (\alpha^s)^2 Var(s_\omega) + (\beta^s)^2 Var(s_\eta) + 2\alpha^s\beta^s Cov(s_\omega, s_\eta) \\ &= (\alpha^s)^2 \left(\sigma^2 + \sigma^2_{\varepsilon_\omega}\right) + (\beta^s)\, 2\left(\sigma^2 + \sigma^2_{\varepsilon_\eta}\right) + 2\alpha^s\beta^s\rho\sigma^2 = \sigma^2 \frac{\frac{\sigma^2_{\varepsilon_\omega}}{\sigma^2} + \frac{\sigma^2_{\varepsilon_\eta}}{\sigma^2}\rho^2 + 1 - \rho^2}{\left(1 + \frac{\sigma^2_{\varepsilon_\omega}}{\sigma^2}\right)\left(1 + \frac{\sigma^2_{\varepsilon_\eta}}{\sigma^2}\right) - \rho^2}, \end{aligned}$$

which for $\sigma^2_{\varepsilon_\omega} = 0$ amounts to

$$V = \sigma^2 \frac{\frac{\sigma^2_{\varepsilon_\eta}}{\sigma^2}\rho^2 + 1 - \rho^2}{\frac{\sigma^2_{\varepsilon_\eta}}{\sigma^2} + 1 - \rho^2}.$$

For $\sigma^2_{\varepsilon_\eta} \to 0$, we get $V = \sigma^2$. Applying l'Hospital, for $\sigma^2_{\varepsilon_\eta} \to \infty$ we get $V = \rho^2\sigma^2$. $\blacksquare$

Lemma A1 *Expected losses are minimized for the receiver and for the sender for an action* $x^* = \mathbb{E}\left[y | \, s_\omega, s_\eta \right]$ *for* $y = \omega, \eta$, *respectively.*

Proof of Lemma A1. Let $u^s(\cdot) = u^r(\cdot) \equiv u(\cdot) = -\ell(\cdot)$ and $y = \omega, \eta$. Consider the problem

$$\max_x \int_{-\infty}^{\infty} u(x-y) f(y| \, s_\omega, s_\eta) \, dy,$$

where $f(y| \, s_\omega, s_\eta)$ is the conditional density of $y = \omega, \eta$ given the signals. Since the utility depends only on the distance between x and y we have $u'(x-y) > 0$ for $y > x$, $u'(x-y) = 0$ for $x = y$, and $u'(x-y) < 0$ for $y < x$.

Consider the candidate solution $x^* = \mu_y \equiv \mathbb{E}\left[y | \, s_\omega, s_\eta \right]$. The first-order condition can be written as

$$\int_{-\infty}^{\infty} u'(x^* - y) f(y| \, s_\omega, s_\eta) \, dy = \int_{-\infty}^{\infty} u'\left(\mu_y - y\right) f(y| \, s_\omega, s_\eta) \, dy = 0.$$

Consider two points $y_1 = \mu_y - \Delta$ and $y_2 = \mu_y + \Delta$ for arbitrary $\Delta > 0$. By symmetry of u around its bliss point and symmetry of the distribution around μ_y, we have

$$u'(\Delta) f\left(\mu_y - \Delta \middle| \, s_\omega, s_\eta\right) = -u'(-\Delta) f\left(\mu_y + \Delta \middle| \, s_\omega, s_\eta\right).$$

Since this holds point-wise for each Δ, it also holds if we integrate over Δ. Thus, the first-order condition is satisfied at $x^* = \mu_y$. By concavity of u in x, only one value of x satisfies the first-order condition. ■

Proof of Proposition 1. Recall that $u(\cdot) = -\ell(\cdot)$. An optimal information structure solves:

$$\max_{V \in [\rho^2\sigma^2, \sigma^2]} \int u\left(z \left(\sigma^2 - \frac{(\rho\sigma^2)^2}{V} \right)^{\frac{1}{2}} \right) \phi(z) \, dz + \int u\left(\left(\frac{(\rho\sigma^2)^2}{V} - 2\left(\rho\sigma^2\right) + \sigma^2 \right)^{\frac{1}{2}} t \right) \phi(t) \, dt.$$

The derivative wrt V is

$$\begin{aligned}
&\frac{1}{2}\frac{\left(\rho\sigma^2\right)^2}{V^2}\int z\left(\sigma^2-\frac{\left(\rho\sigma^2\right)^2}{V}\right)^{-\frac{1}{2}}u'\left(z\left(\sigma^2-\frac{\left(\rho\sigma^2\right)^2}{V}\right)^{\frac{1}{2}}\right)\phi\left(z\right)dz\\
&-\frac{1}{2}\frac{\left(\rho\sigma^2\right)^2}{V^2}\int\left(\frac{\left(\rho\sigma^2\right)^2}{V}-2\left(\rho\sigma^2\right)+\sigma^2\right)^{-\frac{1}{2}}tu'\left(\left(\frac{\left(\rho\sigma^2\right)^2}{V}-2\left(\rho\sigma^2\right)+\sigma^2\right)^{\frac{1}{2}}t\right)\phi\left(t\right)dt.
\end{aligned}\tag{A5}$$

First, suppose $V=\rho\sigma^2$. Then, the derivative wrt V satisfies

$$\begin{aligned}
&\int\frac{1}{2}z\left(\sigma^2-\rho\sigma^2\right)^{-\frac{1}{2}}u'\left(z\left(\sigma^2-\rho\sigma^2\right)^{\frac{1}{2}}\right)\phi\left(z\right)dz\\
&-\int\frac{1}{2}\left(-\rho\sigma^2+\sigma^2\right)^{-\frac{1}{2}}tu'\left(\left(-\rho\sigma^2+\sigma^2\right)^{\frac{1}{2}}t\right)\phi\left(t\right)dt\\
=\;&0.
\end{aligned}$$

Now suppose $V\neq\rho\sigma^2$. Note that both integrands in (A5) have the common representation

$$\int\frac{1}{a}ku'\left(ak\right)\phi\left(k\right)dk.\tag{A6}$$

Differentiating wrt a, we observe that (A6) is monotone decreasing in a,

$$-\frac{1}{a^3}\int aku'\left(ak\right)\phi\left(k\right)dk+\frac{1}{a^3}\int a^2k^2u''\left(ak\right)\phi\left(k\right)dk\leq 0,$$

where the inequality follows from the curvature condition

$$q\frac{u''\left(q\right)}{u'\left(q\right)}=q\frac{\ell''\left(q\right)}{\ell'\left(q\right)}\geq 1.\tag{A7}$$

$V<\rho\sigma^2$ implies $\frac{\left(\rho\sigma^2\right)^2}{V}-2\rho\sigma^2+\sigma^2>\sigma^2-\frac{\left(\rho\sigma^2\right)^2}{V}$. The curvature condition (A7) implies monotonicity and therefore

$$\begin{aligned}
&\frac{1}{2}\frac{\left(\rho\sigma^2\right)^2}{V^2}\int z\left(\sigma^2-\frac{\left(\rho\sigma^2\right)^2}{V}\right)^{-\frac{1}{2}}u'\left(z\left(\sigma^2-\frac{\left(\rho\sigma^2\right)^2}{V}\right)^{\frac{1}{2}}\right)\phi\left(z\right)dz\\
\geq\;&\frac{1}{2}\frac{\left(\rho\sigma^2\right)^2}{V^2}\int\left(\frac{\left(\rho\sigma^2\right)^2}{V}-2\left(\rho\sigma^2\right)+\sigma^2\right)^{-\frac{1}{2}}tu'\left(\left(\frac{\left(\rho\sigma^2\right)^2}{V}-2\left(\rho\sigma^2\right)+\sigma^2\right)^{\frac{1}{2}}t\right)\phi\left(t\right)dt.
\end{aligned}$$

Hence the derivative is non-negative for $V < \rho\sigma^2$. By symmetry, the derivative is non-positive for $V > \rho\sigma^2$. These inequalities become strict for functions that satisfy the curvature condition (A7) with strict inequality. It follows that the problem is maximized in V for $V = \rho\sigma^2$. $\blacksquare$