Online Appendix for "Unpacking Skill Bias: Automation and New Tasks."

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Appendix A. Model, Additional Results, and Proofs

This section of the Appendix provides the derivation of equation (1) and the proofs of generalized versions of Propositions 1 and 2. We also present additional results on the effects of standardization and skill upgrading on wages, inequality and productivity.

CHARACTERIZATION OF EQUILIBRIUM

We first provide a full characterization of the equilibrium for the model presented in the main text.

To simplify the notation and without loss of any generality, we assume that when indifferent between producing with labor or capital, firms produce with capital. Furthermore, when indifferent between producing with skilled and unskilled labor, firms produce with skilled labor. Cost minimization then implies that

$$\mathcal{T}_L = \left\{ x : \frac{w_L}{\psi_L(x)} < \frac{w_H}{\psi_H(x)}, \frac{w_L}{\psi_L(x)} < \frac{q(x)}{\psi_K(x)} \right\}$$
$$\mathcal{T}_H = \left\{ x : \frac{w_H}{\psi_H(x)} \leq \frac{w_L}{\psi_L(x)}, \frac{w_H}{\psi_H(x)} < \frac{q(x)}{\psi_K(x)} \right\},$$
$$\mathcal{T}_K = \left\{ x : \frac{q(x)}{\psi_K(x)} \leq \frac{w_L}{\psi_L(x)}, \frac{q(x)}{\psi_K(x)} \leq \frac{w_H}{\psi_H(x)} \right\}.$$

It also follows that the price of task x is given by

$$p(x) = \begin{cases} \frac{w_L}{\psi_L(x)} & \text{if } x \in \mathcal{T}_L \\ \frac{w_H}{\psi_H(x)} & \text{if } x \in \mathcal{T}_H \\ \frac{q(x)}{\psi_K(x)} & \text{if } x \in \mathcal{T}_K \end{cases}$$

Because the price of the final good is normalized to 1, we have that task prices satisfy the price-index condition

$$1 = \frac{1}{M} \int_{\mathcal{T}} p(x)^{1-\lambda} dx,$$

which can be written in terms of factor prices and the cost of producing capital as follows:

$$\text{(A.1)} \qquad 1 = \frac{1}{M} \int_{\mathcal{T}_L} \left(\frac{w_L}{\psi_L(x)} \right)^{1-\lambda} dx + \frac{1}{M} \int_{\mathcal{T}_H} \left(\frac{w_H}{\psi_H(x)} \right)^{1-\lambda} dx + \frac{1}{M} \int_{\mathcal{T}_K} \left(\frac{q(x)}{\psi_K(x)} \right)^{1-\lambda} dx.$$

The demand for task x is given by $y(x) = \frac{1}{M} \cdot Y \cdot p(x)^{-\lambda}$. Thus, the demand for unskilled labor from tasks in $\mathcal{T}_L$ satisfies

$$L^d = \int_{\mathcal{T}_L} \frac{y(x)}{\psi_L(x)} dx = \int_{\mathcal{T}_L} \frac{\frac{1}{M} Y \cdot p(x)^{-\lambda}}{\psi_L(x)} dx = Y \cdot w_L^{-\lambda} \cdot \frac{1}{M} \int_{\mathcal{T}_L} \psi_L(x)^{\lambda-1} dx,$$

and the demand for skilled labor from tasks in $\mathcal{T}_H$ satisfies

$$H^d = \int_{\mathcal{T}_H} \frac{y(x)}{\psi_H(x)} dx = \int_{\mathcal{T}_H} \frac{\frac{1}{M} Y \cdot p(x)^{-\lambda}}{\psi_H(x)} dx = Y \cdot w_H^{-\lambda} \cdot \frac{1}{M} \int_{\mathcal{T}_H} \psi_H(x)^{\lambda-1} dx.$$

Let $K = \int_x q(x)k(x)$ denote the total amount of capital used in the economy. The demand for capital from tasks in $\mathcal{T}_K$ is

$$K^d = \int_{\mathcal{T}_K} q(x) \cdot \frac{y(x)}{\psi_K(x)} dx = \int_{\mathcal{T}_K} \frac{\frac{1}{M} q(x) \cdot Y \cdot p(x)^{-\lambda}}{\psi_K(x)} dx = Y \cdot \frac{1}{M} \int_{\mathcal{T}_K} \left(\frac{\psi_K(x)}{q(x)} \right)^{\lambda - 1} dx.$$

Market clearing implies that $L^d = L$, $H^d = H$ and $K^d = K$. Using the expressions for factor demands above, we can express equilibrium wages as

$$w_L = \left(\frac{Y}{L} \right)^{\frac{1}{\lambda}} \cdot \left(\frac{1}{M} \int_{\mathcal{T}_L} \psi_L(x)^{\lambda - 1} dx \right)^{\frac{1}{\lambda}}$$
$$w_H = \left(\frac{Y}{H} \right)^{\frac{1}{\lambda}} \cdot \left(\frac{1}{M} \int_{\mathcal{T}_H} \psi_H(x)^{\lambda - 1} dx \right)^{\frac{1}{\lambda}}.$$

Substituting these expressions into (A.1) and solving for Y we obtain

$$Y = \left(\frac{\left(\frac{1}{M} \int_{\mathcal{T}_L} \psi_L(x)^{\lambda-1} dx \right)^{\frac{1}{\lambda}}}{1 - \frac{1}{M} \int_{\mathcal{T}_K} \left(\frac{q(x)}{\psi_K(x)} \right)^{1-\lambda} dx} \cdot L^{\frac{\lambda-1}{\lambda}} + \frac{\left(\frac{1}{M} \int_{\mathcal{T}_H} \psi_H(x)^{\lambda-1} dx \right)^{\frac{1}{\lambda}}}{1 - \frac{1}{M} \int_{\mathcal{T}_K} \left(\frac{q(x)}{\psi_K(x)} \right)^{1-\lambda} dx} \cdot H^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}}.$$

Combining this expression with the market cleaning condition for capital, we can write the equilibrium net output as

$$\begin{aligned} Y - K =& Y \cdot \left(1 - \frac{1}{M} \int_{\mathcal{T}_K} \left(\frac{\psi_K(x)}{q(x)} \right)^{\lambda-1} dx. \right) \\ =& \left(\left(\frac{\frac{1}{M} \int_{\mathcal{T}_L} \psi_L(x)^{\lambda-1} dx}{1 - \frac{1}{M} \int_{\mathcal{T}_K} \left(\frac{q(x)}{\psi_K(x)} \right)^{1-\lambda} dx} \right)^{\frac{1}{\lambda}} \cdot L^{\frac{\lambda-1}{\lambda}} + \left(\frac{\frac{1}{M} \int_{\mathcal{T}_H} \psi_H(x)^{\lambda-1} dx}{1 - \frac{1}{M} \int_{\mathcal{T}_K} \left(\frac{q(x)}{\psi_K(x)} \right)^{1-\lambda} dx} \right)^{\frac{1}{\lambda}} \cdot H^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}}, \end{aligned}$$

which coincides with the expression for net output in the main text.

Finally, the capital share in output is given by K/Y. Using the market-clearing condition for capital we obtain

$$s_K = \frac{K}{Y} = \frac{1}{M} \int_{\mathcal{T}_K} \left(\frac{\psi_K(x)}{q(x)} \right)^{\lambda-1} dx,$$

and the labor share is given by

$$s = 1 - \frac{1}{M} \int_{\mathcal{T}_K} \left(\frac{\psi_K(x)}{q(x)} \right)^{\lambda-1} dx. \tag{A.2}$$

The labor share can be decomposed into the share of unskilled labor in production

$$s_L = \frac{1}{M} \int_{\mathcal{T}_L} \left(\frac{w_L}{\psi_L(x)} \right)^{1-\lambda} dx,$$

and the share of skilled labor in production

$$s_H = \frac{1}{M} \int_{\mathcal{T}_H} \left(\frac{w_H}{\psi_H(x)} \right)^{1-\lambda} dx,$$

where $s = s_L + s_H$.

Derivation of Equation (1)

A proportional increase in $A_H H$ and $A_L L$ does not alter the allocation of tasks to factors, and hence has no impact on Γ_H / Γ_L. This is because such changes increase the effective supply of capital (since it is elastically supplied), skilled and unskilled labor proportionally, and so it keeps wages per efficiency unit of labor and prices per unit of capital unchanged.

We can therefore write Γ_H / Γ_L as a function of $A_H H / A_L L$ and other technologies, such as automation and new tasks:

$$\ln \left(\frac{\Gamma_H}{\Gamma_L} \right) = \Gamma \left(\frac{A_H H}{A_L L}, \theta \right),$$

where θ is a vector denoting the state of technology. We can then decompose changes in $\ln \left(\frac{\Gamma_H}{\Gamma_L} \right)$ as

$$d \ln \left(\frac{\Gamma_H}{\Gamma_L} \right) = \frac{\partial \ln(\Gamma_H / \Gamma_L)}{\partial \ln(A_H H / A_L L)} \cdot d \ln \left(\frac{A_H H}{A_L L} \right) + d \ln \left(\frac{\Gamma_H}{\Gamma_L} \right) \Big|_{\frac{A_H H}{A_L L}},$$

where $d \ln \left(\frac{\Gamma_H}{\Gamma_L} \right) \Big|_{\frac{A_H H}{A_L L}}$ denotes changes in Γ_H and Γ_L due to technology holding $\frac{A_H H}{A_L L}$ constant.

From the expression for net output given in the main text, that the skill premium can be written as

$$\ln \left(\frac{w_H}{w_L} \right) = \frac{1}{\lambda} \ln \left(\frac{\Gamma_H}{\Gamma_L} \right) + \frac{\lambda - 1}{\lambda} \ln \left(\frac{A_H}{A_L} \right) - \frac{1}{\lambda} \ln \left(\frac{H}{L} \right).$$

Taking a total differential of this equation, we obtain

$$\begin{aligned} d \ln \left(\frac{w_H}{w_L} \right) = & \frac{1}{\lambda} \left(\frac{\partial \ln(\Gamma_H / \Gamma_L)}{\partial \ln(A_H H / A_L L)} \cdot d \ln \left(\frac{A_H H}{A_L L} \right) + d \ln \left(\frac{\Gamma_H}{\Gamma_L} \right) \Big|_{\frac{A_H H}{A_L L}} \right) \\ & + \frac{\lambda - 1}{\lambda} d \ln \left(\frac{A_H}{A_L} \right) - \frac{1}{\lambda} d \ln \left(\frac{H}{L} \right). \end{aligned}$$

Regrouping terms, we obtain

$$\begin{aligned} d \ln \left(\frac{w_H}{w_L} \right) = & - \left(\frac{1}{\lambda} - \frac{1}{\lambda} \frac{\partial \ln(\Gamma_H / \Gamma_L)}{\partial \ln(A_H H / A_L L)} \right) d \ln \left(\frac{H}{L} \right) \\ & + \left(1 - \frac{1}{\lambda} + \frac{1}{\lambda} \frac{\partial \ln(\Gamma_H / \Gamma_L)}{\partial \ln(A_H H / A_L L)} \right) d \ln \left(\frac{A_H}{A_L} \right) + \frac{1}{\lambda} d \ln \left(\frac{\Gamma_H}{\Gamma_L} \right) \Big|_{\frac{A_H H}{A_L L}}, \end{aligned}$$

which coincides with equation (1) in the main text with $\sigma = \lambda \Big/ \left(1 - \frac{\partial \ln(\Gamma_H / \Gamma_L)}{\partial \ln(A_H H / A_L L)} \right)$.

Additional Results and Proofs

This section of the Appendix provides general statements and proofs for the propositions in the main text. We first present a lemma that provides sufficient conditions for all tasks that *can be* produced by capital *to be* produced by capital in equilibrium. We then state and prove an additional lemma that will be used for computing the productivity gains from different types of technology. Finally, we present five propositions characterizing the effects of different types of technologies on wages, skill premium and productivity. The first three of those are generalizations of Propositions 1 and 2 in the text. The next two study the implications of skill upgrading (technologies that allow skilled workers to perform more efficiently/cheaply some of the tasks that were previously allocated to unskilled labor) and standardization (technologies that simplify tasks and increase the relative productivity of unskilled labor in tasks reviously performed by skilled workers).

LEMMA A.1: *Suppose that $\gamma_K(x)$ is bounded away from zero in the set of tasks for which $\gamma_K(x) > 0$ and that $\gamma_L(x)$ and $\gamma_H(x)$ are bounded above. Then there exists a threshold $\underline{q}$ such that, if $q(x) < \underline{q}$ for all tasks, all tasks for which $\gamma_K(x) > 0$ are produced by capital.*

PROOF:

Consider an allocation in which

$$\begin{aligned} \mathcal{T}_L &= \left\{x : \frac{w_L}{\psi_L(x)} < \frac{w_H}{\psi_H(x)}, \gamma_k(x) = 0\right\} \\ \mathcal{T}_H &= \left\{x : \frac{w_H}{\psi_H(x)} \le \frac{w_L}{\psi_L(x)}, \gamma_K(x) = 0\right\}, \\ \mathcal{T}_K &= \{x : \gamma_K(x) > 0\}. \end{aligned}$$

We prove that there exists a $\underline{q}$ such that, if $q(x) < \underline{q}$ for all tasks, this is the equilibrium allocation. This is equivalent to showing that

$$w_j = \Gamma_L^{\frac{1}{\lambda}} A_L^{\frac{\lambda-1}{\lambda}} \left(\frac{NY}{L}\right)^{\frac{1}{\lambda}} > \frac{A_j}{A_K} \cdot q(x) \cdot \frac{\gamma_j(x)}{\gamma_K(x)} \text{ for } j \in \{L, H\} \text{ and } x \in \mathcal{T}_k.$$

A sufficient condition for this inequality to hold is that

$$w_j = \Gamma_L^{\frac{1}{\lambda}} A_L^{\frac{\lambda-1}{\lambda}} \left(\frac{NY}{L}\right)^{\frac{1}{\lambda}} > \frac{A_j}{A_K} \cdot \underline{q} \cdot \frac{\overline{\gamma}_j}{\underline{\gamma}_K} \text{ for } j \in \{L, H\}, \tag{A.3}$$

where $\overline{\gamma}_j$ is an upper bound for $\gamma_j(x)$ and $\underline{\gamma}_K$ is a lower bound for $\gamma_K(x)$ in $\mathcal{T}_K$.

As $\underline{q}$ declines, the left-hand side of this equation (weakly) increases. To see this, note that we can rewrite the left-hand side as

$$\begin{aligned} w_j =& \left(1 - \frac{1}{M}\int_{\mathcal{T}_K}\left(\frac{q(x)}{\psi_K(x)}\right)^{1-\lambda}\right)^{\frac{1}{1-\lambda}} \\ &\cdot \left(\left(\frac{1}{M}\int_{\mathcal{T}_L}\psi_L(x)^{\lambda-1}dx\right)^{\frac{1}{\lambda}} \cdot L^{\frac{\lambda-1}{\lambda}} + \left(\frac{1}{M}\int_{\mathcal{T}_H}\psi_H(x)^{\lambda-1}dx\right)^{\frac{1}{\lambda}} \cdot H^{\frac{\lambda-1}{\lambda}}\right)^{\frac{1}{\lambda-1}} \cdot \frac{A_L^{\frac{\lambda-1}{\lambda}}}{L^{\frac{1}{\lambda}}}, \end{aligned}$$

which increases as $q(x)$ falls.

Instead, as $\underline{q}$ declines towards zero, the right-hand side of equation (A.3) converges to zero. Thus, there exists $\underline{q} > 0$ such that the sufficient condition in equation (A.3) holds, as claimed. □

We now provide an additional lemma that we use repeatedly in the proof of the main propositions.

LEMMA A.2: *Consider any improvement in technology increasing TFP by $d \ln TFP > 0$. Then*

$$d \ln TFP = s_L \cdot d \ln w_L + s_H \cdot d \ln w_H.$$

PROOF:

Because of constant returns to scale and the fact that we have competitive markets,

$$Y = w_L \cdot L + w_H \cdot H + K.$$

Following an improvement in technology, both sides of this equation change by

$$\frac{\partial \ln Y}{\partial \ln K} d \ln K + d \ln TFP = s_L \cdot d \ln w_L + s_H \cdot d \ln w_H + s_K d \ln K,$$

where $d \ln TFP = d \ln Y|_{L,H,K}$ denotes the expansion in output holding inputs constant. The lemma follows from the fact that in a competitive equilibrium $\frac{\partial \ln Y}{\partial \ln K} = s_K$. $\square$

We now turn to general statements of Propositions 1 and 2 and their proofs.

PROPOSITION A.1: *Suppose that $q(x) < \underline{q}$, with $\underline{q}$ as defined in Lemma A.1. Consider an improvement in automation technologies such that the productivity of capital in a small set of tasks in $\mathcal{A} \subset \mathcal{T}_L$ increases to $\psi_K(x) > 0$. Then:*

- *the skill premium changes by*

$$d \ln \left(\frac{w_H}{w_L}\right) = \frac{1}{\sigma} \frac{\int_{\mathcal{A}} \gamma_L(x)^{\lambda-1} dx}{\int_{\mathcal{T}_L} \gamma_L(x)^{\lambda-1} dx}; \tag{A.4}$$

- *TFP increases by*

$$d \ln TFP_{\mathcal{A}} = \frac{1}{M} \int_{\mathcal{A}} \frac{\left(\frac{w_L}{\psi_L(x)}\right)^{1-\lambda} - \left(\frac{q(x)}{\psi_K(x)}\right)^{1-\lambda}}{1-\lambda} dx > 0; \tag{A.5}$$

- *the labor share declines by*

$$ds = -\frac{1}{M} \int_{\mathcal{A}} \left(\frac{\psi_K(x)}{q(x)}\right)^{\lambda-1} dx$$

- *w_H increases while the effect on w_L is ambiguous.*

PROOF:

Define the function

$$\tilde{\Gamma}(w_H/w_L; \theta) = \frac{\displaystyle\int_{w_H/\psi_H(x) \le w_L/\psi_L(x), \gamma_K(x)=0} \psi_H(x)^{\lambda-1} dx}{\displaystyle\int_{w_H/\psi_H(x) > w_L/\psi_L(x), \gamma_K(x)=0} \psi_L(x)^{\lambda-1} dx}.$$

Because $q(x) < \underline{q}$, we have that in equilibrium $\tilde{\Gamma}(w_H/w_L; \theta) = \Gamma(A_H H / A_L L; \theta)$. Thus, the skill premium satisfies the implicit equation

$$\frac{w_H}{w_L} = \tilde{\Gamma}(w_H/w_L; \theta)^{\frac{1}{\lambda}} \cdot \left(\frac{A_H}{A_L}\right)^{\frac{\lambda-1}{\lambda}} \left(\frac{H}{L}\right)^{-\frac{1}{\lambda}}. \tag{A.6}$$

The definition of the derived elasticity of substitution implies that a change in $\ln H/L$ reduces the skill premium by

$$\frac{\partial \ln w_H/w_L}{\partial \ln H/L} = -\frac{1}{\sigma}.$$

Using equation (A.6), we can expand this expression as

$$\frac{\partial \ln w_H/w_L}{\partial \ln H/L} = \frac{1}{\lambda}\frac{\partial \ln \tilde{\Gamma}}{\partial \ln w_H/w_L}\frac{\partial \ln w_H/w_L}{\partial \ln H/L} - \frac{1}{\lambda},$$

and consequently,

$$\frac{\partial \ln w_H/w_L}{\partial \ln H/L} = -\frac{\frac{1}{\lambda}}{1-\frac{1}{\lambda}\frac{\partial \ln \tilde{\Gamma}}{\partial \ln w_H/w_L}}.$$

Therefore, the function $\tilde{\Gamma}$ satisfies the equation

$$\frac{1}{\sigma} = \frac{\frac{1}{\lambda}}{1-\frac{1}{\lambda}\frac{\partial \ln \tilde{\Gamma}}{\partial \ln w_H/w_L}}. \tag{A.7}$$

To obtain the effect of automation on the skill premium, we can take a log differential of (A.6):

$$d\ln\left(\frac{w_H}{w_L}\right) = \frac{1}{\lambda}\frac{\partial \ln \tilde{\Gamma}}{\partial \ln w_H/w_L} d\ln\ln\left(\frac{w_H}{w_L}\right) + \frac{1}{\lambda}\frac{\int_{\mathcal{A}} \gamma_L(x)^{\lambda-1}dx}{\int_{\mathcal{T}_L} \gamma_L(x)^{\lambda-1}dx}.$$

Solving for $d\ln\left(\frac{w_H}{w_L}\right)$ yields

$$d\ln\left(\frac{w_H}{w_L}\right) = \frac{\frac{1}{\lambda}}{1-\frac{1}{\lambda}\frac{\partial \ln \tilde{\Gamma}}{\partial \ln w_H/w_L}}\frac{\int_{\mathcal{A}} \gamma_L(x)^{\lambda-1}dx}{\int_{\mathcal{T}_L} \gamma_L(x)^{\lambda-1}dx} = \frac{1}{\sigma}\frac{\int_{\mathcal{A}} \gamma_L(x)^{\lambda-1}dx}{\int_{\mathcal{T}_L} \gamma_L(x)^{\lambda-1}dx},$$

where the last step follows by substituting σ from (A.7).

To derive the expression for the change in TFP, we start by differentiating equation (A.1):

$$0 = s_L \cdot (1-\lambda) \cdot d\ln w_L + s_H \cdot (1-\lambda) \cdot d\ln w_H + \frac{1}{M}\int_{\mathcal{A}}\left[\left(\frac{q(x)}{\psi_K(x)}\right)^{1-\lambda} - \left(\frac{w_L}{\psi_L(x)}\right)^{1-\lambda}\right]dx.$$

Note that, because the cost of producing a task with different factors is equated at marginal tasks, additional changes in the allocation of tasks to factors are second order and do not contribute to this expression. Hence, we can rewrite this equation as

$$s_L \cdot d\ln w_L + s_H \cdot d\ln w_H = \frac{1}{M}\int_{\mathcal{A}}\frac{\left(\frac{w_L}{\psi_L(x)}\right)^{1-\lambda} - \left(\frac{q(x)}{\psi_K(x)}\right)^{1-\lambda}}{1-\lambda}dx.$$

Lemma A.2 then implies that the left-hand side of the above equation equals $d\ln TFP_{\mathcal{A}}$, as claimed. Furthermore, because $q(x) < \underline{q}$, we have that $w_L/\psi_L(x) > q(x)/\psi_K(x)$ for tasks in $\mathcal{A}$, and therefore the right-hand side of the above equation is positive, as stated in the proposition.

The expression for the decline in the labor share follows from differentiating equation (A.2).

Finally, w_H increases because the skill premium increases and Lemma A.2 implies that $s_L \cdot d\ln w_L + s_H \cdot d\ln w_H = d\ln TFP_{\mathcal{A}} > 0$. That the effect on w_L is ambiguous follows from the fact that

$$w_L = \Gamma_L^{\frac{1}{\lambda}} \cdot A_L^{\frac{\lambda-1}{\lambda}}\left(\frac{NY}{L}\right)^{\frac{1}{\lambda}}.$$

On the one hand, an improvement in automation reduces Γ_L (in particular, equation (A.1) implies $A_L^{\lambda-1}\Gamma_L + A_H^{\lambda-1}\Gamma_H = 1$, and $\Gamma_H/\Gamma_L = \tilde{\Gamma}$ increases with automation, which implies that Γ_L

must decrease and Γ_H must increase). On the other hand, NY increases by $d\ln TFP_{\mathcal{A}}/(1-s_K)$. Consequently, automation reduces unskilled wages when it generates small productivity gains, but increases unskilled wages when productivity gains from automation are large. □

PROPOSITION A.2: *Suppose that $q(x) < \underline{q}$, with $\underline{q}$ as defined in Lemma A.1. Consider the introduction of a small set of tasks $\mathcal{N}$ that expand M such that: i. $w_H/\psi_H(x) < w_L/\psi_L(x)$, ii. $w_H/\psi_H(x) < 1$, and iii. $\gamma_K(x) = 0$ for all tasks in $\mathcal{N}$. These new tasks will be produced by skilled labor, and:*

- *the skill premium changes by*

$$d\ln\left(\frac{w_H}{w_L}\right) = \frac{1}{\sigma}\frac{\int_{\mathcal{N}}\gamma_H(x)^{\lambda-1}dx}{\int_{\mathcal{T}_H}\gamma_H(x)^{\lambda-1}dx};$$

- *TFP increases by*

$$d\ln TFP_{\mathcal{N}} = \frac{1}{M}\int_{\mathcal{N}}\frac{1-\left(\frac{w_h}{\psi_h(x)}\right)^{1-\lambda}}{1-\lambda}dx > 0;$$

- *and the labor share increases by*

$$ds = \frac{|\mathcal{N}|}{M^2}\int_{\mathcal{T}_K}\left(\frac{\psi_K(x)}{q(x)}\right)^{\lambda-1}dx.$$

PROOF:

By assumption, the most cost effective way of producing the new tasks is with skilled labor. Thus, new tasks expand the set $\mathcal{T}_H$ and the mass of tasks M increases to $M + |\mathcal{N}|$.

To obtain the effect of new tasks on the skill premium, we can take a log differential of (A.6):

$$d\ln\left(\frac{w_H}{w_L}\right) = \frac{1}{\lambda}\frac{\partial\ln\tilde{\Gamma}}{\partial\ln w_H/w_L}d\ln\left(\frac{w_H}{w_L}\right) + \frac{1}{\lambda}\frac{\int_{\mathcal{N}}\gamma_H(x)^{\lambda-1}dx}{\int_{\mathcal{T}_L}\gamma_H(x)^{\lambda-1}dx},$$

which implies

$$d\ln\left(\frac{w_H}{w_L}\right) = \frac{\frac{1}{\lambda}}{1-\frac{1}{\lambda}\frac{\partial\ln\tilde{\Gamma}}{\partial\ln w_H/w_L}}\frac{\int_{\mathcal{N}}\gamma_H^{\lambda-1}dx}{\int_{\mathcal{T}_L}\gamma_H^{\lambda-1}dx} = \frac{1}{\sigma}\frac{\int_{\mathcal{N}}\gamma_H(x)^{\lambda-1}dx}{\int_{\mathcal{T}_L}\gamma_H(x)^{\lambda-1}dx},$$

where the last step follows from (A.7).

To derive the expression for the change in TFP, we start by taking a differential of equation (A.1):

$$0 = s_L\cdot(1-\lambda)\cdot d\ln w_L + s_H\cdot(1-\lambda)\cdot d\ln w_H + \frac{1}{M}\int_{\mathcal{N}}\left(\frac{w_H}{\psi_H(x)}\right)^{1-\lambda}dx - \frac{|\mathcal{N}|}{M}$$

We can rewrite this equation as

$$s_L\cdot d\ln w_L + s_H\cdot d\ln w_H = \frac{1}{M}\int_{\mathcal{A}}\frac{1-\left(\frac{w_H}{\psi_H(x)}\right)^{1-\lambda}}{1-\lambda}dx.$$

Lemma A.2 then implies that the left-hand side of the above equation equals $d\ln TFP_{\mathcal{N}}$. Moreover, the assumptions made in the proposition ensure that $w_H/\psi_H(x) < 1$ for tasks in $\mathcal{N}$, and therefore the right-hand side of the above equation is positive, as stated in the proposition.

The expression for the increase in the labor share follows from differentiating equation (A.2). □

PROPOSITION A.3: *Suppose that $q(x) < \underline{q}$, with $\underline{q}$ as defined in Lemma A.1. Consider the introduction of a small set of tasks $\mathcal{N}$ that expand $\overline{M}$ such that: i. $w_L/\psi_L(x) < w_H/\psi_H(x)$, ii. $w_L/\psi_L(x) < 1$, and iii. $\gamma_K(x) = 0$ for all tasks in $\mathcal{N}$. These new tasks will be produced by unskilled labor, and:*

- *the skill premium falls by*

$$d\ln\left(\frac{w_H}{w_L}\right) = -\frac{1}{\sigma}\frac{\int_{\mathcal{N}}\gamma_L(x)^{\lambda-1}dx}{\int_{\mathcal{T}_L}\gamma_L(x)^{\lambda-1}dx};$$

- *TFP increases by*

$$d\ln TFP_{\mathcal{N}} = \frac{1}{M}\int_{\mathcal{N}}\frac{1-\left(\frac{w_L}{\psi_L(x)}\right)^{1-\lambda}}{1-\lambda}dx > 0;$$

- *and the labor share increases by*

$$ds = \frac{|\mathcal{N}|}{M^2}\int_{\mathcal{T}_K}\left(\frac{\psi_K(x)}{q(x)}\right)^{\lambda-1}dx.$$

PROOF:

The proof is analogous to that of Proposition A.2 and is omitted. □

Propositions 1 and 2 in the main text follow as corollaries from Propositions A.1–A.3. We now provide two additional propositions characterizing the effect of skill upgrading and standardization.

PROPOSITION A.4: *Suppose that $q(x) < \underline{q}$, with $\underline{q}$ as defined in Lemma A.1. Suppose that the productivity of skilled labor rises in a small set of tasks $\mathcal{U} \subset \mathcal{T}_L$ in such a way that $w_H/\psi_H(x) < w_L/\psi_L(x)$ for all $x \in \mathcal{U}$ at the new productivity levels. Then:*

- *the skill premium changes by*

$$d\ln\left(\frac{w_H}{w_L}\right) = \frac{1}{\sigma}\frac{\int_{\mathcal{U}}\gamma_H(x)^{\lambda-1}dx}{\int_{\mathcal{T}_H}\gamma_H(x)^{\lambda-1}dx} + \frac{1}{\sigma}\frac{\int_{\mathcal{U}}\gamma_L(x)^{\lambda-1}dx}{\int_{\mathcal{T}_L}\gamma_L(x)^{\lambda-1}dx} > 0;$$

- *TFP increases by*

$$d\ln TFP_{\mathcal{U}} = \frac{1}{M}\int_{\mathcal{A}}\frac{\left(\frac{w_L}{\psi_L(x)}\right)^{1-\lambda} - \left(\frac{w_H}{\psi_H(x)}\right)^{1-\lambda}}{1-\lambda}dx > 0;$$

- *the labor share remains unchanged;*
- *w_H increases while the effect on w_L is ambiguous.*

PROOF:

To obtain the effect of skill upgrading on the skill premium, we can take a log differential of (A.6), which yields

$$d\ln\left(\frac{w_H}{w_L}\right) = \frac{1}{\lambda}\frac{\partial\ln\tilde{\Gamma}}{\partial\ln w_H/w_L}d\ln\left(\frac{w_H}{w_L}\right) + \frac{1}{\lambda}\frac{\int_{\mathcal{U}}\gamma_H(x)^{\lambda-1}dx}{\int_{\mathcal{T}_H}\gamma_H(x)^{\lambda-1}dx} + \frac{1}{\lambda}\frac{\int_{\mathcal{U}}\gamma_L(x)^{\lambda-1}dx}{\int_{\mathcal{T}_L}\gamma_L(x)^{\lambda-1}dx}.$$

This expression can be solved for $d\ln\left(\frac{w_H}{w_L}\right)$:

$$\begin{aligned} d\ln\left(\frac{w_H}{w_L}\right) &= \frac{\frac{1}{\lambda}}{1-\frac{1}{\lambda}\frac{\partial \ln \tilde{\Gamma}}{\partial \ln w_H/w_L}}\left(\frac{\int_{\mathcal{U}}\gamma_H(x)^{\lambda-1}dx}{\int_{\mathcal{T}_H}\gamma_H(x)^{\lambda-1}dx}+\frac{\int_{\mathcal{U}}\gamma_L(x)^{\lambda-1}dx}{\int_{\mathcal{T}_L}\gamma_L(x)^{\lambda-1}dx}\right) \\ &= \frac{1}{\sigma}\frac{\int_{\mathcal{U}}\gamma_H(x)^{\lambda-1}dx}{\int_{\mathcal{T}_H}\gamma_H(x)^{\lambda-1}dx}+\frac{1}{\sigma}\frac{\int_{\mathcal{U}}\gamma_L(x)^{\lambda-1}dx}{\int_{\mathcal{T}_L}\gamma_L(x)^{\lambda-1}dx} > 0, \end{aligned}$$

where the second equation follows from (A.7), and the overall expression is positive because both terms are positive.

To derive the expression for the change in TFP, we start by taking a differential of equation (A.1):

$$0 = s_L \cdot (1-\lambda)\cdot d\ln w_L + s_H \cdot (1-\lambda)\cdot d\ln w_H + \frac{1}{M}\int_{\mathcal{U}}\left[\left(\frac{w_H}{\psi_H(x)}\right)^{1-\lambda} - \left(\frac{w_L}{\psi_L(x)}\right)^{1-\lambda}\right]dx.$$

We can rewrite this equation as

$$s_L \cdot d\ln w_L + s_H \cdot d\ln w_H = \frac{1}{M}\int_{\mathcal{U}}\frac{\left(\frac{w_L}{\psi_L(x)}\right)^{1-\lambda} - \left(\frac{w_H}{\psi_H(x)}\right)^{1-\lambda}}{1-\lambda}dx.$$

Lemma A.2 then implies that the left-hand side of this equation equals $d\ln TFP_{\mathcal{U}}$. Also, note that because $w_H/\psi_H(x) < w_L/\psi_L(x)$ for all tasks in $\mathcal{U}$, we have that the right-hand side of the same equation is positive, as stated in the proposition.

The fact that the labor share remains unchanged follows from equation (A.2).

Finally, w_H increases because the skill premium increases and Lemma A.2 implies that $s_L \cdot d\ln w_L + s_H \cdot d\ln w_H = d\ln TFP_{\mathcal{U}} > 0$. The effect on w_L is ambiguous because

$$w_L = \Gamma_L^{\frac{1}{\lambda}} \cdot A_L^{\frac{\lambda-1}{\lambda}}\left(\frac{NY}{L}\right)^{\frac{1}{\lambda}},$$

and skill upgrading reduces Γ_L, while NY increases by $d\ln TFP_{\mathcal{U}}/(1-s_K)$. Consequently, skill upgrading reduces unskilled wages when the productivity gains from this technology are small, but increases unskilled wages when the productivity gains are large. □

One interesting implication of this proposition is that skill upgrading, though it increases inequality between skilled and unskilled labor, leaves the labor share unchanged. This highlights that recent developments in the US labor market, involving both greater inequality between skilled and unskilled labor and lower labor share (at least in manufacturing, see Acemoglu and Restrepo, 2019), cannot just be explained by skill upgrading and likely entail some reallocation of tasks previously performed by workers to capital.

Finally, we turn to the implications of standardization.

PROPOSITION A.5: *Suppose that $q(x) < \underline{q}$, with $\underline{q}$ as defined in Lemma A.1. Suppose that the productivity of unskilled labor rises in a small set of tasks $\mathcal{S} \subset \mathcal{T}_H$ in such a way that $w_L/\psi_L(x) < w_H/\psi_H(x)$ for all $x \in \mathcal{S}$ at the new productivity levels. Then:*

- *the skill premium falls by*

$$d\ln\left(\frac{w_H}{w_L}\right) = -\frac{1}{\sigma}\frac{\int_{\mathcal{S}}\gamma_H(x)^{\lambda-1}dx}{\int_{\mathcal{T}_H}\gamma_H(x)^{\lambda-1}dx} - \frac{1}{\sigma}\frac{\int_{\mathcal{S}}\gamma_L(x)^{\lambda-1}dx}{\int_{\mathcal{T}_L}\gamma_L(x)^{\lambda-1}dx};$$

- *TFP increases by*

$$d \ln TFP_{\mathcal{S}} = \frac{1}{M} \int_{\mathcal{A}} \frac{\left(\frac{w_H}{\psi_H(x)}\right)^{1-\lambda} - \left(\frac{w_L}{\psi_L(x)}\right)^{1-\lambda}}{1-\lambda} dx > 0;$$

- *the labor share remains unchanged;*
- *w_L increases while the effect on w_H is ambiguous.*

PROOF:

To obtain the effect of automation on the skill premium, we can take a log differential of (A.6):

$$d \ln \left(\frac{w_H}{w_L}\right) = \frac{1}{\lambda} \frac{\partial \ln \tilde{\Gamma}}{\partial \ln w_H/w_L} d \ln \left(\frac{w_H}{w_L}\right) - \frac{1}{\lambda} \frac{\int_{\mathcal{S}} \gamma_H(x)^{\lambda-1} dx}{\int_{\mathcal{T}_H} \gamma_H(x)^{\lambda-1} dx} - \frac{1}{\lambda} \frac{\int_{\mathcal{S}} \gamma_L(x)^{\lambda-1} dx}{\int_{\mathcal{T}_L} \gamma_L(x)^{\lambda-1} dx}.$$

Solving for $d \ln \left(\frac{w_H}{w_L}\right)$ yields

$$\begin{aligned} d \ln \left(\frac{w_H}{w_L}\right) &= -\frac{\frac{1}{\lambda}}{1 - \frac{1}{\lambda} \frac{\partial \ln \tilde{\Gamma}}{\partial \ln w_H/w_L}} \left(\frac{\int_{\mathcal{S}} \gamma_H(x)^{\lambda-1} dx}{\int_{\mathcal{T}_H} \gamma_H(x)^{\lambda-1} dx} + \frac{\int_{\mathcal{S}} \gamma_L(x)^{\lambda-1} dx}{\int_{\mathcal{T}_L} \gamma_L(x)^{\lambda-1} dx} \right) \\ &= -\frac{1}{\sigma} \frac{\int_{\mathcal{S}} \gamma_H(x)^{\lambda-1} dx}{\int_{\mathcal{T}_H} \gamma_H(x)^{\lambda-1} dx} - \frac{1}{\sigma} \frac{\int_{\mathcal{S}} \gamma_L(x)^{\lambda-1} dx}{\int_{\mathcal{T}_L} \gamma_L(x)^{\lambda-1} dx}, \end{aligned}$$

where the last step follows from (A.7).

To derive the expression for the change in TFP, let us differentiate equation (A.1):

$$0 = s_L \cdot (1-\lambda) \cdot d \ln w_L + s_H \cdot (1-\lambda) \cdot d \ln w_H + \frac{1}{M} \int_{\mathcal{S}} \left[\left(\frac{w_L}{\psi_L(x)}\right)^{1-\lambda} - \left(\frac{w_H}{\psi_H(x)}\right)^{1-\lambda} \right] dx.$$

We can rewrite this equation as

$$s_L \cdot d \ln w_L + s_H \cdot d \ln w_H = \frac{1}{M} \int_{\mathcal{S}} \frac{\left(\frac{w_H}{\psi_H(x)}\right)^{1-\lambda} - \left(\frac{w_L}{\psi_L(x)}\right)^{1-\lambda}}{1-\lambda} dx.$$

Lemma A.2 then implies that the left-hand side of the above equation equals $d \ln TFP_{\mathcal{S}}$. Also, note that because $w_L/\psi_L(x) < w_H/\psi_H(x)$ for all tasks in $\mathcal{S}$, the right-hand side of the above equation is positive, as stated in the proposition.

That the labor share remains unchanged follows from equation (A.2).

Finally, w_L increases because the skill premium decreases and Lemma A.2 implies that $s_L \cdot d \ln w_L + s_H \cdot d \ln w_H = d \ln TFP_{\mathcal{S}}$. The fact that the effect on w_H is ambiguous follows from the same argument as before: we have

$$w_H = \Gamma_H^{\frac{1}{\lambda}} \cdot A_H^{\frac{\lambda-1}{\lambda}} \left(\frac{NY}{L}\right)^{\frac{1}{\lambda}},$$

and following a standardization of tasks, Γ_H decreases and NY increases by $d \ln TFP_{\mathcal{S}}/(1-s_K)$. □

Appendix B. Productivity Calculations

This section provides the details for productivity calculations provided in the introduction and in footnote 2. Throughout, we approximate changes over time using first-order expansions.

Productivity Implications of Skill-Biased Technological Change in the Canonical Model

We provide two complementary exercises to illustrate the implications for productivity of the canonical SBTC model. First, we use the estimates for the growth rate in A_H/A_L from Katz and Murphy (1992) and Acemoglu and Autor (2011) and compute the productivity gains that would result from such changes. We then estimate the growth in A_H that one would need to explain the observed shift in the relative demand for college workers, and also compute how real wages would respond to such changes.

Regarding the fist exercise, the resulting productivity gains from improvements in factor-augmenting technologies are approximately

$$\Delta \ln TFP_{SBTC} = s_H \Delta \ln A_H + s_L \Delta \ln A_L.$$

If there is no technological regress, then $\Delta \ln A_L \geq 0$, and thus

$$\Delta \ln TFP_{SBTC} \geq s_H \Delta \ln A_H/A_L. \tag{A.8}$$

Katz and Murphy (1992) estimate $\sigma = 1.41$ and a yearly growth rate for $\ln A_H/A_L$ of 11.34% during the 1963–1987 period. In addition, $s_H = 17\%$ at the beginning of their sample (skilled workers accounted for 25% of wages, and the labor share was roughly of 2/3, which gives $s_H = 25\% \cdot 2/3 = 17\%$). Using equation (A.8), their estimates imply a yearly increase in TFP of at least 1.9% per annum. If we use the average value of s_H between 1963 and 1987, we obtain an increase in TFP of at least 2.76% per annum.

Acemoglu and Autor (2011) estimate $\sigma = 1.63$ and a yearly growth rate for $\ln A_H/A_L$ of 7.22% during the 1963–1992 period and of 4.64% during the 1992–2008 period. In addition, $s_H = 17\%$ at the beginning of their sample, $s_H = 32\%$ around 1992 and $s_H = 38\%$ around 2008. Using equation (A.8), their estimates imply an annual increase in TFP of at least 1.2% per annum for 1963–1992 (1.76% if we use the midpoint of s_H during this period). Finally, their estimates imply a yearly increase in TFP of at least 1.48% per annum for 1992–2008 (1.62% if we use the midpoint of s_H during this period).

The canonical SBTC model also has strong implications for real wages. In particular, if there is no technological regress (and H/L increases as in the data), an increase in A_H/A_L changes wages by at least

$$\Delta \ln w_L \geq s_H \cdot \frac{1}{\sigma} \cdot \Delta \ln A_H/A_L. \tag{A.9}$$

Using the midpoint estimates for s_H, this formula implies a growth rate for unskilled wages of 1.95% for 1963–1987 using Katz and Murphy estimates for σ and the growth rate of A_H/A_L. Likewise, this formula implies a growth rate for unskilled wages of 1.08% for 1963–1992 and 0.98% for 1992–2008 using Acemoglu and Autor estimates for σ and the growth rate of A_H/A_L. In contrast, as noted in the text, real wages for men with no college reached a maximum in 1970 and declined since then at a rate of 0.2% per annum; whereas real wages for women with no college rose by 0.2% per annum (see Acemoglu and Autor, 2011).

Table A.1 summarizes the estimates from the literature and our calculations for different time periods. For comparison, Fernald's (2012) estimates of TFP are provided in the last column of the table. In particular, these estimates imply a 1.2% per annum increase in TFP for 1963–1987; 1.1% per annum for 1963–1992; and 1% per annum for 1992–2008, which are much smaller than the lower bounds implied by the canonical SBTC model.

Turning to the second exercise, note that the total shift in the relative demand for college workers

TABLE A.1—PRODUCTIVITY IMPLICATIONS OF THE CANONICAL SBTC MODEL

	σ	Growth rate of A_H/A_L	Share of college labor in GDP (start of period)	Share of college labor in GDP (end of period)	TFP growth using start of period estimate for s_H	TFP growth using midpoint estimate for s_H	Implied growth of unskilled wages, w_L	Observed TFP growth (Fernald, 2012)
Katz and Murphy, 63–87	1.41	11.3%	16.7%	32.0%	1.89%	2.76%	1.95%	1.18%
Acemoglu and Autor, 63–92	1.63	7.2%	16.7%	32.0%	1.20%	1.76%	1.08%	1.11%
Acemoglu and Autor, 92–08	1.63	4.6%	32.0%	37.8%	1.48%	1.62%	0.98%	0.98%

is given by

$$\Delta \ln\left(\frac{w_H}{w_L}\right) + \frac{1}{\sigma}\Delta \ln\left(\frac{H}{L}\right).$$

Using the numbers from Acemoglu and Autor (2011), it follows that the relative demand for college workers increased by 3.3% per annum from 1963 to 1992 (1.3% from wages and 2% from the 90% increase in the relative supply of skills during this period), and then by 2.4% per annum from 1992 to 2008.

Equation (A.8) implies that, if shifts in the relative demand for college workers were driven by factor augmenting technologies, then:

$$\frac{\Delta \ln TFP_{SBTC}}{\Delta \ln w_H/w_L} = s_H \cdot \frac{\sigma}{\sigma - 1}.$$

The estimates from Katz and Murphy in Table A.1 then imply that, if the only source of technological change were improvements in A_H, a 1% increase in the relative demand for college workers would be associated with an increase in TFP of 0.83% for 1963–1987 (using the midpoint estimate for s_H). Likewise, The estimates from Acemoglu and Autor in Table A.1 imply that a 1% increase in the relative demand for college workers would be associated with an increase in TFP of 0.63% for 1963–1992 and 0.9% for 1992–2008 (using the midpoint estimate for s_H). Thus, the changes in A_H required to explain the total shift in the relative demand for college workers would generate productivity increases of at least 2.08% per annum for 1963–1992 ($= 0.63 \times 3.3$) and 2.16% per annum for 1992–2008 ($= 0.9 \times 2.4$).

Moreover, equation (A.9) implies that if all changes in inequality were driven by factor-augmenting technologies, we would expect an increase in unskilled wages of at least 1.27% per annum for 1963–1992 and of 1.32% per annum for 1992–2008.

PRODUCTIVITY IMPLICATIONS OF AUTOMATION

To illustrate the differences between the task framework and the canonical SBTC model, we now estimate the amount of automation that one would need to explain the observed shift in the relative demand for college workers, and also compute how real wages respond to such technological changes.

Suppose instead that technological changes are driven by automation. Then, the increases in TFP would be given by equation (A.5). Using a first-order Taylor expansion, these productivity gains can be approximated as

$$d \ln TFP_{\mathcal{A}} \approx \int_{\mathcal{A}} \left(\frac{w_L}{\psi_L(x)}\right)^{1-\lambda} \cdot \left(\ln\left(\frac{w_L}{\psi_L(x)}\right) - \ln\left(\frac{q(x)}{\psi_K(x)}\right)\right) dx.$$

This expression shows that the productivity gains from automating a task are given by its initial share in value added (the term $(w_L/\psi_L(x))^{1-\lambda}$), and the percent reduction in the unit cost of producing the task (the term $\ln(w_L/\psi_L(x)) - \ln(q(x)/\psi_K(x))$). We can also express the productivity gains from automation as

$$d \ln TFP_{\mathcal{A}} \approx \pi \cdot \int_{\mathcal{A}} \left(\frac{w_L}{\psi_L(x)} \right)^{1-\lambda} dx, \tag{A.10}$$

where $\pi > 0$ is the (weighted) average reduction in the cost of producing tasks due to automation and $\int_{\mathcal{A}} \left(\frac{w_L}{\psi_L(x)} \right)^{1-\lambda} dx$ gives the share of automated tasks in value added.

Using equations (A.4) and (A.10), it follows that if shifts in the relative demand for college workers were driven by automation, then:

$$\frac{\Delta \ln TFP_{\mathcal{A}}}{\Delta \ln w_H/w_L} = \sigma \cdot s_L \cdot \pi.$$

This equation shows that automation technologies that bring modest reductions in costs (in the extreme, $\pi \to 0$) can generate sizable changes in inequality accompanied by modest increases in TFP.

In particular, suppose $\pi = 30\%$, which is in line with estimates for industrial automation surveyed in Acemoglu and Restrepo (2020a). Using a value for σ of 1.63 and a midpoint estimate for s_L, we obtain that, if the only source of technological change were automation, a 1% increase in the relative demand for college workers would be associated with an increase in TFP of 0.21% for 1963–1992 and 0.14% for 1992–2008. Using a value for σ of 1.41 (as in Katz and Murphy, 1992) and a midpoint estimate for s_L, we obtain that a 1% increase in the relative demand for college workers would be associated with an increase in TFP of 0.18% for 1963–1987.

Thus, the changes in automation technology required to explain the total shift in the relative demand for skilled labor would generate productivity increases of as little as 0.54% per annum for 1963–1987 (using Katz and Murphy's estimates of 0.18 × 3.3); 0.66% per annum for 1963–1992 (= 0.21 × 3.3); and 0.34% per annum for 1992–2008 (= 0.14 × 2.4).

Moreover, automation technologies would change unskilled wages by

$$\Delta \ln w_L = \Delta \ln TFP_{\mathcal{A}} - s_H \Delta \ln \left(\frac{w_H}{w_L} \right) = (\sigma \cdot s_L \cdot \pi - s_H) \cdot \Delta \ln \left(\frac{w_H}{w_L} \right)$$

Thus, if all changes in inequality were driven by automation, we would expect a reduction of unskilled wages by 0.12% per annum for 1963–1992 and of 0.5% per annum for 1992–2008 (which contrasts with the predicted increase in unskilled wages under just factor-augmenting technologies as in the standard SBTC model).

Appendix C. Data Description and Additional Empirical Exercises

This part of the Appendix describes the data and provides additional empirical exercises.

Set of Industries Used in the Analysis

We use a set of 44 industries which we could track across different sources, including the Census, the BEA industry accounts, and NIPA. The crosswalks used are part of the replication package for this paper (see http://economics.mit.edu/faculty/acemoglu/data). Our sample excludes industries that are heavily dependent on commodity prices, in particular, oil and gas, mining, agriculture, and petroleum derivatives.

Measures of Relative Demand for Skills

Using the US Census and the American Community Survey (ACS), we compiled data on the college and high school wage bill and hours of work by industry for 1950, 1990, and 2016. We follow Acemoglu and Autor (2011) and define college workers as those with a college degree and half of those with some college. We then define high school workers as those with a high school degree or less and half of the workers with some college.

For the 44 industries in our sample, we study two separate periods. First, for the period from 1987–2016 we use the 1990 Census and 2016 ACS to construct measures of changes in the relative demand for skills across industries during this period. Second, for the period from 1947–1987, we use the 1950 and 1990 Censuses to construct measures of changes in the relative demand for skills across industries during this period.

Measures of Displacement and Reinstatement

The construction of these measures follows Acemoglu and Restrepo (2019). First, suppose that the model in the main text describes the production process of an industry. The labor share in that industry is given by

$$s = \frac{\int_{\mathcal{T}_L}\left(\frac{w_L}{\psi_L(x)}\right)^{1-\lambda}dx + \int_{\mathcal{T}_H}\left(\frac{w_H}{\psi_H(x)}\right)^{1-\lambda}dx}{\int_{\mathcal{T}_L}\left(\frac{w_L}{\psi_L(x)}\right)^{1-\lambda}dx + \int_{\mathcal{T}_H}\left(\frac{w_H}{\psi_H(x)}\right)^{1-\lambda}dx + \int_{\mathcal{T}_K}\left(\frac{q(x)}{\psi_K(x)}\right)^{1-\lambda}dx}.$$

We can then decompose changes in the labor share in two components. On the one hand, we have changes driven by factor prices and by technologies that do not change the allocation of tasks between capital and labor (including improvements in factor-augmenting technologies). On the other hand, we have the effect of technologies, like automation and new tasks, which directly change the allocation of tasks between capital and labor. As in Acemoglu and Restrepo (2019) we refer to these as *changes in the task content of production*. Specifically, we decompose changes in the labor share of an industry as follows (suppressing industry indices to simplify notation):

$$d\ln s = d\text{task content} + (1-\lambda)\cdot(1-s)\cdot(d\ln w - d\ln r + g), \tag{A.11}$$

where $d\ln w = (s_L/(s_L+s_H))\cdot d\ln w_L + (s_H/(s_L+s_H))\cdot d\ln w_H$ denotes the change in the average wage paid in the industry, $d\ln r = \frac{1}{s_K}\int_{\mathcal{T}_K}\left(\frac{q(x)}{\psi_K(x)}\right)^{1-\lambda} d\ln q(x)dx$ denotes the change in the average rental rate of capital used in the industry, and

$$\begin{aligned} g =& \frac{1}{s_L+s_H}\left(\int_{\mathcal{T}_L}\left(\frac{w_L}{\psi_L(x)}\right)^{1-\lambda} d\ln\psi_L(x)dx + \int_{\mathcal{T}_H}\left(\frac{w_H}{\psi_H(x)}\right)^{1-\lambda} d\ln\psi_H(x)dx\right) \\ &- \frac{1}{s_K}\int_{\mathcal{T}_K}\left(\frac{q(x)}{\psi_K(x)}\right)^{1-\lambda} d\ln\psi_K(x)dx \end{aligned}$$

denotes the increase in the productivity of labor relative to capital in the tasks that are currently allocated to labor. Note that g also incorporates the effect of changes in A_L, A_H and A_K through the ψ terms. Because $q(x) < \underline{q}$, these improvements in factor-augmenting technologies do not alter the allocation of tasks between capital and labor, and for the same reason λ, coincides with the elasticity of substitution between capital and labor.

Building on equation (A.11), for each of the 44 industries in our sample, we compute its yearly changes in the task content of production as

$$\Delta\text{task content}_{it} = \Delta\ln s_{it} - (1-\sigma_K)\cdot(1-s_{it})\cdot(\Delta\ln w_{it} - \Delta\ln r_{it} - g_{it}).$$

We measure s_{it} using the industry payroll share, which we obtained from the BEA industry accounts (in some of our robustness checks, we also used a measure from the BEA and BLS KLEMS that adjusts the payroll share for self-employment). In addition, σ_K denotes the elasticity of substitution between capital and labor, which we set to 0.8 following Oberfield and Raval (2014). We obtained the

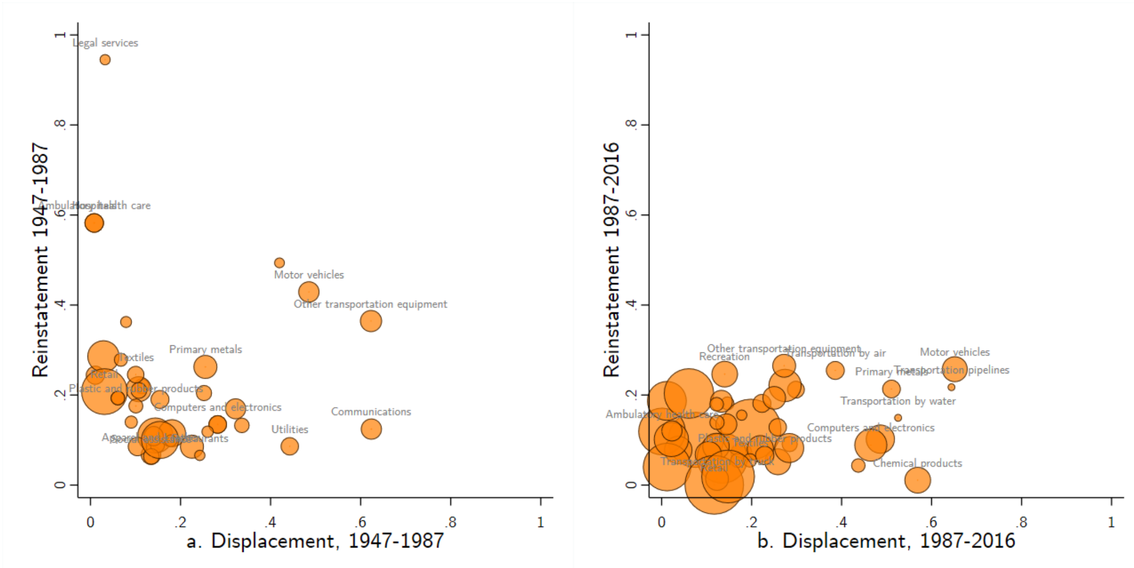

FIGURE A.1. MEASURES OF DISPLACEMENT AND REINSTATEMENT, 1947–1987 AND 1987–2016.

industry-specific wage and capital rental rate indices, w_{it} and r_{it}, from the BLS KLEMS accounts for 1987–2016. For the earlier period, we constructed these indices using data on the quantity of labor and capital used in each industry from NIPA. Finally, we follow Acemoglu and Restrepo (2019) and set g_{it}—improvements in labor productivity relative to capital productivity—to 2% per annum for 1947–1987 and 1.46% per annum for 1987–2016.

Increases in the (labor) task content of an industry are indicative of the reinstatement effect generated by new tasks, whereas reductions in the (labor) task content are indicative of the displacement effect brought by automation. To separate these two effects, we assume that over a five-year period, each industry either introduces new automation technologies or new tasks but not both. This assumption implies that we can compute the extent of displacement and reinstatement in a given year and industry as

$$\text{displacement}_{it} = \max\left\{0, -\frac{1}{5}\sum_{\tau=t-2}^{t+2} \Delta \text{task content}_{i\tau}\right\}$$

$$\text{reinstatement}_{it} = \max\left\{0, \frac{1}{5}\sum_{\tau=t-2}^{t+2} \Delta \text{task content}_{i\tau}\right\}.$$

(If there are simultaneously new automation technologies and new tasks within five-year periods in our data, then our estimates will be lower bounds on the extent of displacement and reinstatement).

Finally, in our regressions we use the cumulative extent of displacement and reinstatement during our period of analysis. These measures are given in percent changes over the entire period, so that a 0.1 displacement corresponds to a 10% decline in the labor share that is unexplained by changes in factor prices.

Figure A.1 shows the total displacement and reinstatement in each industry for 1947–1987 and 1987–2016. For 1947–1987, the average reinstatement across industries was 19.6% (0.49% per annum) and the average displacement was 17% (0.425% per annum). For 1987–2016, the average reinstatement was 10% (0.345% per annum) and the average displacement was 16% (0.55% per annum).

Regression Results

Tables A.2, A.3 and A.4 present various estimates of equation (2).

Table A.2 presents our main estimates. Panels A–C provide estimates for 1947–1987 and Panels D–F provide estimates for 1987–2016. In Panels A and D we use the wage bill of college workers relative to high school workers as our measure for the demand for skills in an industry. In Panels B and E we use the hours worked by college workers relative to high school workers as our measure for the demand for skills in an industry. In Panels C and F we use the number of college workers relative to high school workers as our measure for the demand for skills in an industry. Columns 1–3 present estimates of (2) for all workers, and columns 4–7 present estimates separately for men, women, and workers in different age groups.

Tables A.3 and A.4 provide estimates using alternative measures of changes in the task content of industries and the resulting measures of displacement and reinstatement. For this exercise, we use relative wage bill (columns 1–3) and relative hours (columns 4–6) as our measures of skill demand. Table A.3 focuses on the 1947–1987 period. Panel A provides results obtained by setting $\sigma_K = 1$ in our computation of the displacement and reinstatement effects. Panel B reverts to $\sigma_K = 0.8$ but we now use a 10-year moving average, rather than a 5-year moving average in our calculation of the displacement and reinstatement effects. Finally, in Panel C we implement both changes simultaneously.

Table A.4 focuses on the 1987–2016 period. Panel A provides results obtained by setting $\sigma_K = 1$ in our computation of the displacement and reinstatement effects. Panel B reverts to $\sigma_{KL} = 0.8$ but we now use a 10-year moving average, rather than a five-year moving average, in our calculation of the displacement and reinstatement effects. In Panel C we implement both changes simultaneously. In Panel D–F we repeat these exercises but now we use data from the BEA KLEMS accounts for 1987–2016. These data provide the labor share for each industry inclusive of self employment.

Overall, the results in Tables A.2, A.3 and A.4 confirm our summary in the text. Automation is associated with significant increases in the relative demand for skills in both periods, regardless of the specification or measure we use (and for different subgroups such as men, women and younger workers). Reinstatement between 1947 and 1987 is associated with lower relative demand for skills, whereas between 1987 and 2016, it is associated with higher relative demand for skills. This pattern is robust as well. One additional finding is worth noting: even between 1947 and 1987, reinstatement does not appear to increase the demand for unskilled men by much, likely reflecting the fact that less skilled women may have been the ones with comparative advantage in new tasks introduced during this period.

Additional References

Fernald, J.G. (2012) “A Quarterly, Utilization-Adjusted Series on Total Factor Productivity.” FRBSF Working Paper 2012–19 (data accessed on 12/25/2019).

Oberfield, E. and Raval, D. (2014) “Micro Data and Macro Technology,” MIMEO, Princeton University.

TABLE A.2—CHANGES IN TASK CONTENT AND RELATIVE DEMAND FOR SKILLS, 1947–1987 AND 1987–2016.

	All employees			Men	Women	Ages 25–34	Ages 35–64
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Panel A. College wage bill relative to high school wage bill—1947–1987						
Automation	0.504		0.470	0.108	0.384	0.764	0.293
	(0.193)		(0.184)	(0.352)	(0.423)	(0.225)	(0.273)
Reinstatement		-0.585	-0.546	0.023	-0.639	-0.594	-0.544
		(0.306)	(0.278)	(0.482)	(0.501)	(0.430)	(0.261)
Observations	44	44	44	44	44	44	44
R-squared	0.06	0.06	0.12	0.00	0.05	0.08	0.07
	Panel B. College hours relative to high school hours—1947–1987						
Automation	0.686		0.644	0.315	0.458	0.738	0.608
	(0.219)		(0.165)	(0.301)	(0.401)	(0.252)	(0.194)
Reinstatement		-0.723	-0.670	-0.361	-0.630	-0.707	-0.633
		(0.343)	(0.304)	(0.431)	(0.434)	(0.463)	(0.234)
Observations	44	44	44	44	44	44	44
R-squared	0.09	0.08	0.16	0.04	0.07	0.11	0.15
	Panel C. College employees relative to high school employees—1947–1987						
Automation	0.873		0.834	0.587	0.536	0.941	0.769
	(0.204)		(0.158)	(0.323)	(0.337)	(0.224)	(0.206)
Reinstatement		-0.697	-0.629	-0.368	-0.575	-0.596	-0.644
		(0.352)	(0.292)	(0.363)	(0.415)	(0.422)	(0.256)
Observations	44	44	44	44	44	44	44
R-squared	0.15	0.07	0.21	0.09	0.07	0.15	0.17
	Panel D. College wage bill relative to high school wage bill—1987–2016						
Automation	0.800		0.764	1.053	1.061	0.353	0.947
	(0.152)		(0.159)	(0.288)	(0.247)	(0.209)	(0.186)
Reinstatement		0.707	0.483	0.299	0.299	0.850	0.390
		(0.348)	(0.340)	(0.401)	(0.506)	(0.391)	(0.384)
Observations	44	44	44	44	44	44	44
R-squared	0.31	0.06	0.34	0.34	0.40	0.16	0.37
	Panel E. College hours relative to high school hours—1987–2016						
Automation	0.558		0.520	0.754	0.778	0.185	0.697
	(0.137)		(0.141)	(0.220)	(0.227)	(0.179)	(0.169)
Reinstatement		0.658	0.506	0.196	0.404	0.768	0.431
		(0.310)	(0.317)	(0.329)	(0.431)	(0.349)	(0.371)
Observations	44	44	44	44	44	44	44
R-squared	0.19	0.07	0.22	0.29	0.33	0.12	0.25
	Panel F. College employees relative to high school employees—1987–2016						
Automation	0.546		0.514	0.696	0.793	0.257	0.657
	(0.134)		(0.135)	(0.195)	(0.214)	(0.154)	(0.166)
Reinstatement		0.582	0.431	0.100	0.345	0.540	0.450
		(0.326)	(0.325)	(0.335)	(0.409)	(0.323)	(0.376)
Observations	44	44	44	44	44	44	44
R-squared	0.19	0.05	0.22	0.29	0.34	0.11	0.24

Notes: the table provides regression estimates of changes in the relative demand for skills across industries on measures of displacement and reinstatement. The Appendix provides a description of the construction of these explanatory variables. Panels A–C provide estimates for 1947–1987. Panels D–F provide estimates for 1987–2016. Each panel uses a different measure of changes in the relative demand for skills across industries. Panels A and D use the change in the log of the college wage bill relative to the high school wage bill in each industry as outcome. Panels B and E use the change in the log of college hours relative to high school hours in each industry as outcome. Panels C and F use the change in the log of the number of college employees relative to high school employees in each industry as outcome. In columns 1–3, the measures of changes in relative demand for skills are computed for all employed in an industry; in column 4 only for men; in column 5 only for women; in column 6 for employees aged 25–34 years; and in column 7 for employees aged 35–64 years. Standard errors robust against heteroskedasticity are in parentheses.

TABLE A.3—ROBUSTNESS TO MEASURES OF TASK CONTENT, 1947–1987

	College wage bill relative to highschool wage bill			College hours relative to highschool hours		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. BEA data with $\sigma_K = 1$ and 5-year moving averages						
Automation	0.447		0.446	0.647		0.646
	(0.207)		(0.161)	(0.249)		(0.165)
Reinstatement		-0.484	-0.483		-0.580	-0.578
		(0.226)	(0.205)		(0.256)	(0.228)
Observations	44	44	44	44	44	44
R-squared	0.06	0.07	0.13	0.10	0.08	0.18
Panel B. BEA data with $\sigma_K = 0.8$ and 10-year moving averages						
Automation	0.536		0.410	0.774		0.624
	(0.224)		(0.219)	(0.220)		(0.183)
Reinstatement		-0.660	-0.595		-0.806	-0.708
		(0.265)	(0.262)		(0.303)	(0.294)
Observations	44	44	44	44	44	44
R-squared	0.04	0.09	0.11	0.07	0.11	0.16
Panel C. BEA data with $\sigma_K = 1$ and 10-year moving averages						
Automation	0.488		0.352	0.759		0.601
	(0.235)		(0.204)	(0.245)		(0.190)
Reinstatement		-0.577	-0.529		-0.698	-0.618
		(0.203)	(0.200)		(0.230)	(0.224)
Observations	44	44	44	44	44	44
R-squared	0.04	0.10	0.12	0.08	0.12	0.17

Notes: the table provides regression estimates of changes from 1947 to 1987 in the relative demand for skills across industries on measures of displacement and reinstatement. The Appendix provides a description of the construction of these explanatory variables. Columns 1–3 use the change in the log of the college wage bill relative to the high school wage bill in each industry as outcome. Columns 4–6 use the change in the log of college hours relative to high school hours in each industry as outcome. Each panel presents results for a different construction of the displacement and reinstatement measures, as explained in the Appendix. Standard errors robust against heteroskedasticity are in parentheses.

TABLE A.4—ROBUSTNESS TO MEASURES OF TASK CONTENT, 1987–2016

	College wage bill relative to highschool wage bill			College hours relative to highschool hours		
	(1)	(2)	(3)	(4)	(5)	(6)
			Panel A. BEA data with $\sigma_K = 1$ and 5-year moving averages			
Automation	0.620		0.535	0.412		0.335
	(0.138)		(0.154)	(0.120)		(0.136)
Reinstatement		0.931	0.606		0.755	0.551
		(0.333)	(0.350)		(0.301)	(0.329)
Observations	44	44	44	44	44	44
R-squared	0.23	0.12	0.27	0.12	0.10	0.17
			Panel B. BEA data with $\sigma_K = 0.8$ and 10-year moving averages			
Automation	0.773		0.928	0.500		0.645
	(0.153)		(0.210)	(0.131)		(0.188)
Reinstatement		0.122	0.873		0.296	0.818
		(0.516)	(0.522)		(0.424)	(0.466)
Observations	44	44	44	44	44	44
R-squared	0.22	0.00	0.27	0.11	0.01	0.17
			Panel C. BEA data with $\sigma_K = 1$ and 10-year moving averages			
Automation	0.630		0.807	0.385		0.537
	(0.149)		(0.195)	(0.130)		(0.169)
Reinstatement		0.593	1.195		0.627	1.028
		(0.557)	(0.563)		(0.486)	(0.512)
Observations	44	44	44	44	44	44
R-squared	0.16	0.03	0.27	0.07	0.04	0.17
			Panel D. KLEMS data with $\sigma_K = 0.8$ and 5-year moving averages			
Automation	0.520		0.550	0.366		0.379
	(0.143)		(0.140)	(0.117)		(0.118)
Reinstatement		0.024	0.321		-0.072	0.132
		(0.368)	(0.333)		(0.355)	(0.344)
Observations	44	44	44	44	44	44
R-squared	0.24	0.00	0.26	0.15	0.00	0.15
			Panel E. KLEMS data with $\sigma_K = 1$ and 5-year moving averages			
Automation	0.521		0.404	0.331		0.251
	(0.167)		(0.199)	(0.142)		(0.182)
Reinstatement		0.957	0.666		0.632	0.451
		(0.351)	(0.382)		(0.299)	(0.362)
Observations	44	44	44	44	44	44
R-squared	0.14	0.11	0.19	0.07	0.06	0.10
			Panel F. KLEMS data with $\sigma_K = 1$ and 10-year moving averages			
Automation	0.444		0.558	0.243		0.322
	(0.200)		(0.199)	(0.170)		(0.165)
Reinstatement		1.196	1.535		0.865	1.060
		(0.716)	(0.719)		(0.670)	(0.673)
Observations	44	44	44	44	44	44
R-squared	0.08	0.07	0.18	0.03	0.04	0.09

Notes: the table provides regression estimates of changes from 1987 to 2016 in the relative demand for skills across industries on measures of displacement and reinstatement. The Appendix provides a description of the construction of these explanatory variables. Columns 1–3 use the change in the log of the college wage bill relative to the high school wage bill in each industry as outcome. Columns 4–6 use the change in the log of college hours relative to high school hours in each industry as outcome. Each panel presents results for a different construction of the displacement and reinstatement measures, as explained in the Appendix. Standard errors robust against heteroskedasticity are in parentheses.