Lift and Shift: The Effect of Fundraising Interventions in Charity Space and Time

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Appendix A:

Further information on CAF account data and comparison with *UK Giving*

CAF accounts work like dedicated checking accounts for making donations to charities. Anyone could use a regular bank account to make the same donations, but there are a least two reasons why someone might want to set up a CAF account – first, it facilitates tax-effective giving and second, it may act as a commitment device to encourage giving. We discuss each of these in turn.

The UK system of tax relief for charitable donations, known as *Gift Aid*, differs to the US system. Unlike the US income tax rebate through itemized charitable deductions, the UK government operates an effective match system, allowing the charity to claim tax relief on donations at the basic rate of tax, currently 20 per cent. However, for the charity to claim the relief, the donor must fill out a *Gift Aid* declaration for each donation made. Since CAF is itself a charity, the donor has to do this only once when the account is opened, and not each time thereafter when a donation is made from the account to a charity. There is also a second element to Gift Aid: Taxpayers whose marginal tax rate is higher than the basic 20 per cent can reclaim an additional rebate equal to the difference between their marginal rate of tax and the basic 20 percent; using a CAF account provides a record of donations for this reclaiming.

The second reason for having a CAF account is that it can help individuals manage their giving and commit them to making a certain level of donations to charity. Money paid in cannot be withdrawn (any unspent funds are allocated by CAF), committing the account holder to donate the funds to charity.

<u>Comparison with *UK Giving*</u>

From the above we can infer from the fact that people set up a CAF account that they have a high level of interest in giving, and also in giving tax-effectively. Other than that, we have no demographic information about CAF account-holders upon which we can base comparisons with other people in the UK. Therefore we compare donations made by CAF account holders with donations made by donors in a random sample from the UK population. The benchmark is the NCVO/CAF survey of individual giving, *UK Giving*, which collects information about charitable donations from adults aged 16 and over in the United Kingdom.

In 2010 *UK Giving* ran three times during the year (in June, October and February) as a module in the Omnibus survey conducted by the Office for National Statistics (ONS). This is a multipurpose, random-probability survey carried out face-to-face in people's homes, using Computer-Assisted Personal Interviewing (CAPI). Those interviewed are asked whether they have given to charity in the last four weeks by any of nine methods shown on a card.[1] For each of the reported methods, they are asked which of fifteen types of causes they have donated to. Then for each cause donated to by each method, they are asked how much they gave. This information is aggregated into a figure for total donations.

We compare monthly donations from *UK Giving* 2010 to donations per account-holder, per month by CAF account holders in 2010. In short, the comparison indicates that the CAF data capture both a small number of donors who give very large amounts but are not picked up at all *UK Giving*, and a large number of donors who represent the top two deciles in *UK Giving*. The details follow.

Table A1 shows that CAF account holders give much more than the donors in *UK Giving* – mean monthly donations in column II are £278.94 among CAF account holders compared to £33.42 in *UK Giving* (column I). Some of this difference is attributable to a small number of donors in the CAF sample who give very large donations, who are rarely picked up in random population surveys. Among CAF account holders, the largest monthly donation was £1.5 million, compared to £1,330 in *UK Giving*. In this respect, looking at CAF account-holders captures the behaviour of an important group of donors, who make very large donations, but who are typically missing in general household surveys. Indeed, in the CAF data, donations greater than £1,330 per month account for 43 per cent of all donations made by account holders.

However, this small group of large donors cannot explain all the difference in mean monthly donation size. As shown in column III, excluding monthly donations greater than £1,300 reduces mean monthly donations among CAF account holders to

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[1] Buying goods (eg charity shop, charity catalogue purchase, Big Issue); Credit/debit card or cheque; Cash gifts (eg collection at work, school, street, pub or place of worship, or sponsoring someone by cash); Direct Debit, standing order or covenant; Fundraising event (eg jumble sale, fetes, charity dinners); Buying a raffle or lottery ticket (not the National Lottery); Payroll giving/regular deduction direct from salary; Membership fees and subscriptions paid to charity; Other methods.

£162.47, still nearly five times the £33.42 average in *UK Giving*. In other words, even setting the very large donors aside, people in the CAF data give disproportionally much more than the random sample in *UK Giving*.

Table A1: Comparison of monthly donations (£), CAF account holders and donors in a random population survey (UK Giving)

	I UK Giving	II CAF	III CAF (<£1,330)
Mean	33.42	278.94	162.47
1%	0.5	5	5
5%	1	10	10
10%	2	20	20
25%	5	35	35
Median	13	90	82.5
75%	32	200	200
90%	76	500	420
95%	121	900	615
99%	335	2,760	1066.25
Largest	1,330	1,500,000	1,330
N	1,715	327,077	318,346

Table A2 maps the distribution of giving in the CAF data to the distribution of giving in *UK Giving*. We form decile boundaries using *UK Giving* (column I) and then place the CAF account holders into those bins (column II). Column III shows the percentage of total *UK Giving* done by the *UK Giving* people in each of the bins. The bottom row shows that just over half of the CAF account-holders would be placed in the top decile of *UK Giving* donors, and those top decile *UK Giving* donors give 53.6 per cent of total donations in *UK Giving*. Similarly, 70.9 per cent of the CAF sample would be placed in the top two deciles, and those two deciles give 69.8 per cent of total donations in *UK Giving*. Assuming that the giving behaviour of CAF account holders is typical of that of *UK Giving* donors in the top deciles, then they represent a group of donors whose behaviour drives a large share of total donations in the UK for most charities.[2]

Table A2: Comparison of CAF account holders against the distribution of donors in the UK Giving sample

Decile of UK Giving sample	I Percentage of UK Giving sample	II Percentage of CAF account-holders	III Percentage of total donations, UK Giving sample
1	10.0%	0.0%	0.4%
2	10.0%	0.1%	0.9%
3	10.0%	1.3%	1.5%
4	10.0%	1.7%	2.5%
5	10.0%	3.2%	3.3%
6	10.0%	4.3%	5.0%
7	10.0%	8.9%	6.5%
8	10.0%	9.5%	9.9%
9	10.0%	19.1%	16.2%
10	10.0%	51.8%	53.6%

Overdrafts and Off-Account

There are two features of the accounts that might be thought to affect the results. First. individuals can only make donations out of funds that are in their account—there is no overdraft facility. Although top-ups are possible at any time, not being able to overdraft would seem likely to bias the findings towards shifting contributions across causes – more going to one cause would automatically result in less going to another cause in the absence of a top-up. This makes the observed behavioral response of lift and no shift, if anything, more striking. Second, we do not observe any off-account donations. However, survey evidence collected in 2009 showed that CAF account holders use their accounts for nearly all their contributions. Furthermore, in a sensitivity analysis presented in Appendix B, we confirm our main results for a sample of regular account users (who give in each of the six years, 2009-2014), who are more likely to use their charity account exclusively.

[2] No million-pound donors are captured in *UK Giving* and few in the CAF data. Million-pound donors gave a total of nearly £2 billion in 2016 (Coutts, 2017), but this amount goes to a relatively small number of charities (around 300). Hence, million-pound donations are not relevant for most charities.

Appendix B. Additional results and robustness checks

In the main specification, we model the dynamics of the response to appeals using a set of indicators for the two weeks before, and the twenty weeks after, the date of the disaster. The coefficients on these weekly indicators capture the difference in donations in the weeks before and after the appeal, compared to a baseline level of donations in all other weeks. For convenience, the main results focus on the averages of these coefficients over the distinct phases of the response period – aftermath, adjustment, settling and return. In this Appendix we present further analysis of whether the approach is sufficient to capture the dynamics of the response to the appeals. We also report additional regression results.

First, weekly indicators for weeks 20-24 after the date of the appeal are not statistically significant, indicating that a twenty-week appeal period (i.e. weeks 0-19) is long enough to capture deviations in donations from the baseline level.

Table B1, panel b reports results from estimating a specification that adds five additional weekly indicators, extending into weeks 20-24. The coefficients on these indicators are close to zero and insignificant. Adding the extra indicators changes the definition of the baseline period, but the estimates for weeks 0 – 4, 5 – 9, 10 – 14, and 15 – 19 are virtually identical to those in our main specification, reported again in panel a for comparison.

Table B1: Estimated response to DEC appeals

	Week 0–4	Week 5–9	Week 10–14	Week 15–19	Week 20–24
a. 20-week disaster period					
DEC-13	1.571	.429	.112	.035	
(£10,331)	(.060)	(.052)	(.050)	(.051)	
OTHER	.100	-.062	-.045	-.026	
(£157,836)	(.028)	(.032)	(.030)	(.028)	
b. 25-week disaster period					
DEC-13	1.575	.431	.116	.038	.030
(£10,769)	(.061)	(.053)	(.051)	(.051)	(.056)
OTHER	.104	-.061	-.041	-.022	.028
(£136,232)	(.028)	(.032)	(.030)	(.028)	(.034)

Notes. The table reports the average response (the mean of the estimated weekly coefficients) during different phases of the appeal period, compared to baseline. All regressions (estimated using OLS) include controls for systematic time effects (indicators for day of week, day of month, month, public holidays and major telethons and a linear trend). N = 1884. Robust standard errors in brackets.

Second, the baseline level of donations is the same after each of the appeals despite variation in the magnitude of the response. This supports the approach of treating the response to appeals as deviations from a (common) baseline level of donations.

Our test is based on comparing the level of donations across baseline periods after each disaster appeals. We drop all days during the twenty-week post-appeal periods. We define a set of indicators for separate baseline periods, one after each appeal, starting twenty weeks after the date of the appeal and finishing the day before the next appeal.

Table B2 reports the coefficients on five baseline indicator variables.[3] The coefficient estimates, ranging from .121 to -.178, are small in magnitude relative to the response to the DEC appeal in weeks 0 – 4 (1.571 and .429; from Table 2 row 1). These coefficients capture differences in donations during each of the subsequent baseline periods relative to an initial baseline which runs from the date the panel data begin (1st June 2009) to the day before the first appeal in the sample (3rd October 2009). None of the coefficients is significant, indicating that donations are the same in subsequent baseline periods as in the initial baseline period.

Table B2: Test for differences in donations across separate baseline periods.

Estimated coefficients

Baseline	DEC-13	Other
Initial baseline, pre-Sumatra appeal	-	-
Post Haiti appeal (β_1)	.107 (.211)	-.065 (.208)
Post Pakistan appeal (β_2)	-.178 (.154)	.026 (.152)
Post East Africa appeal (β_3)	-.086 (.135)	.087 (.133)
Post Syria appeal (β_4)	.121 (.182)	.025 (.180)
Post Philippines appeal (β_5)	-.097 (.171)	-.003 (.168)
N	1,082	1,082

Tests for equality (p-values)

	DECβ_2	DECβ_3	DECβ_4	DECβ_5	OTHβ_2	OTHβ_3	OTHβ_4	OTHβ_5
β_1	.130	.259	.939	.295	.751	.494	.686	.831
β_2		.433	.048	.260		.588	.887	.898
β_3			.107	.925			.768	.524
β_4				.170				.803

Notes to table.
OLS regression of donations on indicators for separate baseline periods:<u>Post Haiti</u>: from 4th June 2010 to 2nd August 2010; <u>Post Pakistan</u>: from December 22nd 2010 to 5th July 2011; <u>Post East Asia</u>: from November 24th 2011 to 19 March 2013; <u>Post Syria</u>: from August 8th 2013 to 10th November 2013; <u>Post Philippines</u>: from April 1st 2014 to 30th June 2014 The omitted initial baseline runs from 1st June 2009 to 3rd October 2009. Robust standard errors in brackets. N = 1078.

[3] There is no baseline period after the Sumatra appeal (4th October 2009) because of the short elapsed time before the Haiti appeal (14 January 2010)

However, the initial baseline period is an arbitrary benchmark. It represents the start of the data period, but it is itself the post-appeal period for an earlier DEC appeal. Also relevant therefore is the fact that donations are the same across different subsequent baseline periods. Tests for equality of the coefficients on the five baseline indicator variables show that almost all differences are insignificant: Donations return to the same level in the baseline period after Haiti as in the baseline period after Syria, despite the magnitude of the response to the two DEC appeals being very different.

Third, we test for – and reject – the presence of serial correlation in the residuals from the main specification. This indicates that the dynamic response indicators $\{\mathbf{W_n}\}_{\{n=-2\}}^{20}$ are sufficient to capture the dynamics in the disaster periods.

Table B3 presents the serial correlation test. We first estimate the specification of log donations on the dynamic response indicators $\{W_n\}_{\{n=-2\}}^{20}$ and the systematic time-based controls v^i_t, form the residuals $\hat{u}^i_t$; and then estimate an auxiliary regression that is the same as the first specification, but with the additional term $\rho\, \hat{u}^i_{t-1}$. The coefficients in column 3 are the $\hat{\rho}$ coefficients, their standard errors are in parentheses; p-values for the test of $\rho = 0$ are in square brackets.

Table B3: Serial correlation tests

		(1)	(2)	(3)
Periods	Controls for response dynamics $\{W_n\}_{\{n=-2\}}^{20}$		Donations	
		DEC-13	Other	Total
Baseline only (N_{days} = 1,082)	n.a.	−.023 (.032) [.472]	.006 (.032) [.863]	.002 (.031) [.962]
Post-appeal only (N_{days} = 800)	no	.508 (.032) [.000]	.037 (.038) [.334]	.173 (.037) [.000]
All periods (N_{days} = 1,883)	yes	.037 (.024) [.133]	−.005 (.025) [.851]	.016 (.025) [.519]

Notes: Estimated coefficients on the lagged residuals added to a specification of log donations on the full set of controls for systematic effects (indicators for day of week, day of month, month, public holidays and major telethons and a linear trend) and, where indicated, the set of coefficients $\{\beta^i_n\}_{\{n=-2\}}^{20}$ that we use to capture the response dynamics. Robust standard errors in parentheses; p-value [in square brackets] for the test of no serial correlation.

Row 1 focuses on just the baseline periods. The $\hat{\rho}$ = .002 coefficient [p-value = .962] in column (3) indicates that, after allowing for systematic time-based controls (ν^{i}_{t}), there are no first-order dynamics in daily total donations during the baseline periods. Columns (1) and (2) confirm no first-order dynamics in donations to DEC-13 and Other charities.

Row 2 focuses on the post-appeal periods. The systematic time-based controls ν^{i}_{t} are included, but the dynamic response indicators $\{W_n\}_{\{n=-2\}}^{20}$ are not. There is strong evidence of serial correlation in log total donations in the weeks that follow the appeals (column 3). Columns (1) and (2) indicate that this dynamic is driven by donations to DEC-13.

Row 3 combines post-appeal and baseline periods and includes the dynamic response indicators, $\{W_n\}_{\{n=-2\}}^{20}$. The $\hat{\rho}$ = .016 coefficient [p-value = .519] indicates that, after inclusion of the dynamic response indicators, there is no remaining dynamic in total donations. The coefficients of .037 [p -value = .133] and −.005 [p -value = .851] in columns (1) and (2) show the same for donations to DEC-13 and Other charities. These results indicate that the usual OLS standard errors are correctly specified.

Fourth, we confirm that the main findings are robust to aggregating the data to the week-level, five-week level and twenty-week level.

The main analysis is based on daily data. This allows the definition of week-before and week-after indicators relative to the exact date of the appeal and also to control flexibly for systematic time effects, including those that operate at the daily level, such as public holidays. The test for serial correlation confirms that there are no remaining first-order dynamics in the residuals from this specification, but there may be a concern about lower-frequency serial correlation (e.g. at the weekly level). We therefore re-run our analysis at a higher-level of aggregation to confirm that there is little, if any, distortion in the results from the daily-specification. Table B4, Panel a. replicates results from daily data for comparison. The remaining Panels (b. – d.) shows results from aggregating data to the week-, five-week- and twenty-week level.

Table B4, Panel b. presents results from running the regression on weekly-averaged data. As in the main specification, we define week-before and week-after indicators relative to the exact date of the appeal and average across seven-day periods. This leaves creates “incomplete weeks” (i.e. periods of less than seven days) that arise because appeals are launched on different days of the week and we drop these from the analysis. The resulting change in the underlying sample results in small changes in the coefficients moving from the daily to the weekly specification, but the overall findings are unaffected.

Table B4, Panel c. presents results from running the regression on data averaged across five-week periods, corresponding to the length of the phases. Again, we define periods by the exact date of the appeals and drop "incomplete five-week periods" which changes the underlying sample. However, again the key findings are the same. Finally Table B4, Panel d confirms the results for the average response across the entire twenty-week disaster period. This analysis of aggregated data confirms that the key findings are robust to mis-specification of the standard errors, e.g. from lower frequency serial correlation that may be present in the daily specification.

Table B4: Estimated response to DEC appeals, aggregated data

	Phase 1 Week 0–4	Phase 2 Week 5–9	Phase 3 Week 10–14	Phase 4 Week 15–19	Week 0–19
a. Main specification, daily data (N = 1884 days)					
DEC-13	1.571	.429	.112	.035	.537
(£10,769)	(.060)	(.052)	(.050)	(.051)	(.032)
OTHER	.100	-.062	-.045	-.026	-.008
(£136,232)	(.028)	(.032)	(.030)	(.028)	(.017)
b. Weekly averaged data (N = 265 weeks)					
DEC-13	1.595	.431	.112	.033	.543
(£10,769)	(.083)	(.060)	(.056)	(.051)	(.035)
OTHER	.113	-.063	-.039	-.026	-.004
(£136,232)	(.032)	(.024)	(.027)	(.024)	(.016)
c. Five-week averaged data (N = 50 five-week periods)					
DEC-13	1.584	.381	.129	.053	.536
(£10,769)	(.176)	(.121)	(.110)	(.096)	(.065)
OTHER	.083	-.079	-.060	-.016	-.018
(£136,232)	(.025)	(.026)	(.041)	(.037)	(.018)
d. Twenty-week averaged data (N = 9 twenty-week periods)					
DEC-13					.624
(£10,769)					(.104)
OTHER					.009
(£136,232)					(.072)

Notes. The table reports the average response during different phases of the appeal period, compared to baseline.Panel a. reports the average of the weekly coefficients, estimated on daily data. Panel b. reports the average of the weekly coefficients, estimated on weekly-averaged data. Panel c. reports coefficients on indicators for the five-week phases, based on data averaged across five-week periods. Panel d. reports the coefficient on an indicator for the twenty-week post-disaster period, based on data averaged across twenty-week periods. All regressions (estimated using OLS) include additional controls for systematic time effects (indicators for month, day of month, public holidays and major telethons that are averaged at the week, five-week and twenty-week level and a linear trend). Robust standard errors in brackets.

Additional regression results

In the body of the paper, we report the average response in ln(donations) during the four phases of the response period. In this Appendix, we present additional results as follows:

- Table B5 reports the full set of estimated coefficients on the weekly indicators (corresponding to Figure 1).
- Table B6 reports results shedding light on what lies behind the increase in total donations – the results correspond to the main specification but with ln(number donations) and ln(mean donation size) as the dependent variable. There is an increase in both the number of donations and mean donation size to *DEC-13* in response to the appeal. The time-shifting occurs in the number of donations to other charities, which are below baseline in the adjustment period, but there is an increase in the size of donations to other charities in the immediate aftermath.
- Table B7 reports results corresponding to the main specification for each DEC appeal (with the exception of Sumatra since the response period includes the Haiti appeal). The point estimates indicate a similar pattern of responses across the appeals and do not support that a single appeal drives the aggregate pattern. Of course, fewer results are statistically significant than in the pooled analysis.
- Table B8 reports results corresponding to the main specification for CAF donors who make a donation in each of the years, 2009-2014. This addresses any possible concerns that we may miss part of the response that occurs off-account, by focusing on a set of regular donors who are more likely to do all their giving via their CAF account.
- Table B9 reports results from the main specification but with the number of donors as the dependent variable. We run three regressions – (a) the total number of donors per day giving to *Other*, (b) the total number of donors per day who give to both *Other* and *DEC* on the same day ("bunchers") and (c) the total number of donors per day who give to *Other* but not to *DEC* on the same day, where (a) = (b) + (c). The aim is to see how many of the additional donations made to *Other* in the aftermath are made by people bunching their *Other* donation with a donation to *DEC*. We focus on a sample of potential

bunchers, ie donors who give to both DEC and Other at any time within the five-week aftermath period. The results indicate that around one-third of the additional donations are attributable to bunching (37 out of 101). The majority of additional donations to Other are not made on the same day as a donation to DEC.

As in the main specification, all regressions (estimated using OLS) include controls for systematic time effects (indicators for day of week, day of month, month, public holidays and major telethons and a linear trend). Robust standard errors in brackets. N = 1884.

Table B5: Estimated responses, weekly coefficients

Dependent variable = Ln(donations)

	DEC-13		Other	
Week -2	.092	(.101)	-.013	(.062)
Week -1	.202	(.114)	.050	(.064)
Week 0	2.092	(.161)	.185	(.058)
Week 1	1.658	(.130)	.116	(.057)
Week 2	1.532	(.115)	.199	(.048)
Week 3	1.490	(.106)	.007	(.073)
Week 4	1.082	(.119)	-.007	(.060)
Week 5	.809	(.118)	-.014	(.048)
Week 6	.525	(.110)	-.016	(.071)
Week 7	.328	(.096)	-.041	(.052)
Week 8	.171	(.113)	-.158	(.092)
Week 9	.312	(.097)	-.081	(.061)
Week 10	.092	(.102)	-.095	(.064)
Week 11	.251	(.099)	-.090	(.068)
Week 12	.153	(.108)	-.004	(.069)
Week 13	-.058	(.101)	-.053	(.061)
Week 14	.122	(.096)	.016	(.054)
Week 15	.016	(.137)	-.066	(.080)
Week 16	-.024	(.101)	.009	(.042)
Week 17	-.023	(.091)	-.128	(.059)
Week 18	.168	(.101)	.001	(.052)
Week 19	.037	(.092)	.055	(.058)

Table B6: Estimated responses, alternative outcomes

a. Dependent variable = Ln(number of donations)

	Aftermath Weeks 0–4	Adjustment Weeks 5–9	Settling Weeks 10–14	Return Weeks 15–19	Overall Weeks 0–19
DEC-13	1.080	.193	.020	.031	.331
	(.057)	(.046)	(.049)	(.047)	(.029)
OTHER	.056	-.081	-.060	-.018	-.026
	(.032)	(.034)	(.033)	(.030)	(.019)

b. Dependent variable = Ln(mean donation)

	Aftermath Weeks 0–4	Adjustment Weeks 5–9	Settling Weeks 10–14	Return Weeks 15–19	Overall Weeks 0–19
DEC-13	.500	.227	.087	.002	.205
	(.023)	(.029)	(.026)	(.024)	(.016)
OTHER	.040	.019	.014	-.008	.016
	(.013)	(.015)	(.015)	(.013)	(.008)

Table B7: Estimated responses, by appeal

Dependent variable = Ln(donations)

	Aftermath Weeks 0–4	Adjustment Weeks 5–9	Settling Weeks 10–14	Return Weeks 15–19	Overall Weeks 0–19
Haiti					
DEC-13	2.284	.356	.008	-.223	.606
	(.277)	(.253)	(.198)	(.244)	(.091)
OTHER	.492	.010	-.128	-.179	.049
	(.320)	(.254)	(.224)	(.251)	(.100)
Pakistan					
DEC-13	1.958	.831	.336	.289	.854
	(.162)	(.206)	(.201)	(.172)	(.032)
OTHER	.018	-.123	.196	.365	.113
	(.222)	(.234)	(.206)	(.184)	(.112)
East Africa					
DEC-13	1.986	.724	.061	.045	.704
	(.154)	(.236)	(.196)	(.187)	(.105)
OTHER	.238	-.216	-.076	.087	-.045
	(.223)	(.237)	(.230)	(.219)	(.119)
Syria					
DEC-13	1.018	-.011	-.020	.137	.281
	(.173)	(.211)	(.223)	(.201)	(.108)
OTHER	.004	-.181	-.010	-.002	-.047
	(.225)	(.237)	(.224)	(.207)	(.117)
Philippines					
DEC-13	1.826	.474	-.133	-.060	.527
	(.187)	(.208)	(.224)	(.209)	(.110)
OTHER	.430	.096	-.079	.115	.140
	(.183)	(.236)	(.231)	(.191)	(.112)

Table B8: Estimated responses, regular CAF donors
Dependent variable = Ln(donations)

	Aftermath Weeks 0–4	Adjustment Weeks 5–9	Settling Weeks 10–14	Return Weeks 15–19	Overall Weeks 0–19
DEC-13	1.725	.275	-.066	-.030	.476
	(.100)	(.119)	(.116)	(.099)	(.064)
OTHER	.053	-.097	-.028	-.014	-.021
	(.040)	(.049)	(.044)	(.039)	(.025)

Table B9: Estimated responses, number of donors
Sample of donors giving to DEC + Other at any point in the 5-week aftermath
Dependent variable = Number of donors per day

	Baseline	Aftermath Weeks 0–4	Adjustment Weeks 5–9	Settling Weeks 10–14	Return Weeks 15–19
a. OTHER (any)	813.5	101.4	-21.1	-0.6	44.1
		(35.6)	(32.0)	(39.1)	(31.1)
b. OTHER + DEC	79.2	37.2	0.9	0.1	-0.5
(same day)		(10.2)	(7.4)	(10.2)	(8.9)
c. OTHER only	734.3	64.2	-22.0	-0.7	44.5
		(27.2)	(26.4)	(30.4)	(24.8)

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Online Appendix C: Identifying Warm Glow Substitutability/Complementarity from Qualitative Donation Responses to Fundraising

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1 Introduction

In this appendix we analyze models of donations, s and r, to two charities S and R that permit an analysis of substitution across charity space and time. Our goal is to provide a theoretical analysis sufficiently flexible to achieve three objectives: (1) have a model with which we can organize our intuition about the variety of qualitative donation response patterns that can follow a fundraising appeal, (2) link lift/shift patterns in observed donation responses to fundraising interventions to underlying substitution/complementarity patterns in warm glow preferences, (3) demonstrate that the empirical pattern of responses observed in donations to *DEC* and donations to *Other* organizations is in line with standard price theory, and moreover implies that the underlying warm glows are substitutes. Theoretical flexibility beyond what is needed to achieve those objectives is set aside for future work.

Several substantive implications follow from the analysis: First, there is not a one-to-one mapping between lift/shift donation patterns and substitution or complementarity in terms of underlying preference parameters, which is the standard way economists think about substitution. The reason is that a lift (an observed increase in aggregate donations) works to obscure identification of substitution/complementarity in the underlying preferences from patterns that reflect a shift in donations from one charity to another within a particular period of time–in the data, this would be an observed pattern of donation responses such that $s \uparrow r \downarrow$ or $s \downarrow r \uparrow$. Time-shifting of a charity's donations can also obscure identification in the immediate aftermath of a fundraising appeal. It follows that observing $s \uparrow r \uparrow$ following a fundraising appeal launched by charity S does <u>not</u> imply that the underlying preferences associated with the two charities are complements. In other words, the underlying warm glows may be substitutes. This fact has implications for empirical work, including experimental work, that attempts to identify substitution/complementarity in preferences from observed donation patterns. On the other hand, time-shifting of donations can facilitate identification of underlying preferences in the time periods after the immediate aftermath of the fundraising appeal.

Second, observing $s \uparrow r \uparrow$ in the time period immediately following the launch of a fundraising appeal by charity S is sufficient evidence to conclude that there is substitution in preferences. Along the same lines, seeing lift being accompanied by a shift in donations such that $s \downarrow r \uparrow$ in the time period immediately following a fundraising appeal by charity $\underline{S}$ is sufficient evidence to conclude that there is complementarity in preferences; that a shift involving an observed response of $s \downarrow$ is so perverse suggests that this is scenario is unlikely to have practical relevance.

Third, in cases where $s \uparrow r \uparrow$ immediately following charity S's fundraising appeal (so that substitution/complementarity cannot be identified, as discussed above), it however may be possible to achieve identification by measuring the donation response pattern for a longer period of time. For example, an observed pattern of donation responses such that $s \uparrow r \downarrow$ in a later time period than the one immediately following an appeal by charity S would be sufficient evidence to conclude that there is substitution in preferences, even if the pattern $s \uparrow r \uparrow$ is observed in the period immediately following the appeal. Indeed, that is what happened in our analysis of donations to *DEC* and *Other* charities. The point is that if the immediate observed donation response pattern cannot identify substitution/complementarity, identification may yet be achieved if the donation response pattern is measured continuously until the pattern returns to baseline.

2 Static model with one warm glow characteristic

Consider a model of warm glow preferences with quasilinear utility:

$$U = c + \theta\, g^{\eta}. \tag{1}$$

Following Cornes and Sandler (1984) g is a warm glow characteristic that can be produced by donating money £d to a charity: $g = \alpha_g \;\; d$. The parameter α_g can be thought of as a salience parameter: it represents the extent to which donations produce warm glow, e.g. because donations make people feel good, fulfil a sense of duty, or act in accordance to a social norm. Own consumption is c and $0 < \eta < 1$ and $\theta \geq 0$ are preference parameters. The budget constraint is

$c + d = y$ (y is income), and can be re-written in terms of the warm glow characteristic g:

$$(2) \qquad c \ + \ \frac{1}{\alpha_g} \ g \ = \ c \ + \ p_g \ g \ = \ y$$

The units of the effective hedonic price $p_g \equiv \frac{1}{\alpha_g}$ are dollars required to be donated to achieve one unit of warm glow.[1]

A fundraising appeal can be modeled as an intervention that raises the salience α_g with which donations d produce warm glow g.[2] Equivalently, the appeal lowers p_g. The optimal level of the warm glow characteristic is:

$$(3) \qquad g^* = \kappa \ \ p_g^{\frac{1}{\eta - 1}}$$

where $\kappa \equiv (\frac{1}{\eta\theta})^{\frac{1}{\eta-1}}$. The price elasticity of g with respect to p_g is $\frac{1}{\eta-1} \triangleq \gamma$. It follows that $0 < \eta < 1$ implies $\gamma < -1$, that the warm glow characteristic is price elastic. A price elastic warm glow characteristic is necessary to model the situation in which an appeal that lowers p_g increases expenditures $p_g \ g = \frac{1}{\alpha_g} \ g = d$, that is, increases total expenditure (i.e., donations in dollars) on the charity, i.e., lifts donations.

3 Static model with two warm glow characteristics

Next consider two sources of warm glow stemming from donations s and r respectively made to two charities, S and R. For instance, in our empirical work s represents donations to charities that provide assistance after a disaster strikes (e.g., *DEC*) and r represents donations to all *Other* charities. Re-define g so that it represents a CES aggregate warm glow characteristic; the aggregation

[1] Modeling salience, α_g, makes explicit what all warm glow models implicitly assume: that donations to charity generate utility. For example, one might feel duty bound to donate to health charities more so than to art charities; in this case the α_g for the health charities is larger than the α_g for arts charities. As another example, one might feel more guilty if she does not donate to basic needs charities, more guilty than, say, not donating to a local sports team; in this case donating to basic needs is more effective in avoiding guilt.

[2] In like manner Ottoni-Wilhelm (2017) models an experimental manipulation as an intervention that raises the effectiveness with which donations produce warm glow.

is over the two distinct warm glow characteristics that can be increased, respectively, by donations s and r:

$$(4) \qquad g \equiv \left(\tfrac{1}{2}\,\omega_s^{\mu} + \tfrac{1}{2}\,\omega_r^{\mu}\right)^{1/\mu}$$

where ω_s and ω_r are the two warm glow characteristics:

$$(5) \qquad \omega_s = \alpha_s \; s$$

$$(6) \qquad \omega_r = \alpha_r \; r$$

and α_s and α_r are the respective salience parameters. The parameter μ in the aggregation in (4) determines the elasticity of substitution between the characteristics ω_s and ω_r: $\sigma = \frac{1}{1-\mu}$. The budget constraint $c + s + r = y$ can again be re-written in terms of the warm glow characteristics:

$$(7) \qquad c + p_s \; \omega_s + p_r \; \omega_r = y$$

where the prices are the reciprocals of the salience parameters: $p_s \equiv \frac{1}{\alpha_s}$ and $p_r \equiv \frac{1}{\alpha_r}$.

The price of the aggregate warm glow characteristic g now depends on the underlying prices p_s and p_r. Using the dual for the CES aggregation (Diewert, 2014; also see Keller, 1976):

$$(8) \qquad p_g = ({p_s}^{1-\sigma} + {p_r}^{1-\sigma})^{1/(1-\sigma)}$$

Optimal g in (3) depends on this price. Therefore total expenditures on g (i.e., the sum of the donations to charity S and charity R) are:

$$(9) \qquad d^* = g^* \;\; ({p_s}^{1-\sigma} + {p_r}^{1-\sigma})^{1/(1-\sigma)}.$$

Equation (9) is a cost function: the amount of expenditure needed to achieve a pre-specified sub-utility level g^*. Inverting it yields an indirect (sub-) utility function:

$$(10) \qquad g^* \;=\; d^* \;\; ({p_s}^{1-\sigma} + {p_r}^{1-\sigma})^{-1/(1-\sigma)}.$$

Applying Roy's identity yields the Marshallian demands for the warm glow characteristics:

$$(11) \qquad \omega_s^* \;=\; d^* \;\; \frac{p_s^{-\sigma}}{p_s^{1-\sigma} \;+\; p_r^{1-\sigma}}$$

$$(12) \qquad \omega_r^* \;=\; d^* \;\; \frac{p_r^{-\sigma}}{p_s^{1-\sigma} \;+\; p_r^{1-\sigma}}$$

and expenditures on (donations to) the respective charities:

$$(13) \qquad s^* \;=\; d^* \;\; \frac{p_s^{1-\sigma}}{p_s^{1-\sigma} \;+\; p_r^{1-\sigma}} \;\; = d^* \; \phi$$

$$(14) \qquad r^* \;=\; d^* \;\; \frac{p_r^{1-\sigma}}{p_s^{1-\sigma} \;+\; p_r^{1-\sigma}} = d^* \; (1-\phi)$$

where $p_s^{1-\sigma}/(p_s^{1-\sigma} \;+\; p_r^{1-\sigma}) \triangleq \phi$ is the share of donations d^* spent on charity S. Note that ϕ is a function of α_s, α_r and σ.

An (observed) increase in total donations d^* to <u>both</u> charities S and R in response to charity S's fundraising appeal, $\alpha_s \uparrow$, is "lift". A change in the donation share ϕ in response to S's fundraising appeal drives shift, which is an observed donation response pattern of either $s \uparrow r \downarrow$ or $s \downarrow r \uparrow$. The percentage change in donations to charity <u>R</u> in response to charity S's fundraising appeal is $\partial \log\, r^*/\partial \alpha_s \;=\; \partial \log\, d^*/\partial \alpha_s \;-\; \frac{\phi}{(1-\phi)} \; \partial \log\, \phi/\partial \alpha_s$: the percentage increase in total donations minus the percentage change in the share of those donations going to charity S (the percentage change in the share being weighted by $\phi/(1-\phi)$). Even if there is a drop in the share of total donations going to charity R, because warm glow characteristics ω_s and ω_r are substitutes, there can be a net increase in donations to charity R if the increase in total donations to <u>both</u> S and R (the lift) is sufficiently large.

The lift in total donations d^* is driven by the drop in the price of the aggregate warm glow characteristic:

$$\frac{\partial \log\, p_g}{\partial \log\, \alpha_s} \quad = \quad -\ \phi. \tag{15}$$

The percentage increase in total donations is:

$$\frac{\partial \log\, d^*}{\partial \log\, \alpha_s} \quad = \quad -\ \phi\,(\gamma+1). \tag{16}$$

Lift in total donations requires that the aggregate warm glow characteristic g be price elastic, as was the case in the previous section, but now the magnitude of the lift depends upon the donation share ϕ.

The adjustment in the share is:

$$\frac{\partial \log\, \phi}{\partial \log\, \alpha_s} \quad = \quad -\ (1-\phi)\ (1-\sigma). \tag{17}$$

If the warm glow characteristics ω_s and ω_r are substitutes, then $\alpha_s \uparrow$ produces a shift in expenditures (donations) toward charity S.

Whether or not we ultimately <u>observe</u> a donation pattern response of $s \uparrow r \uparrow$, $s \uparrow r \downarrow$, or $s \downarrow r \uparrow$ can be discussed in terms of the comparative statics of (14):

$$\begin{aligned}
\frac{\partial \log\, r^*}{\partial \log\, \alpha_s} \quad &= \quad \frac{\partial \log\, d^*}{\partial \log\, \alpha_s} \quad - \quad \frac{\phi}{(1-\phi)}\ \frac{\partial \log\, \phi}{\partial \log\, \alpha_s} \\
&= \quad -\,\phi\ \left[(\gamma+1)\ -\ (1-\sigma)\right] \\
&= \quad -\ \phi\ \left[\gamma\ +\ \sigma\right], \qquad (18)
\end{aligned}$$

which can be used to generate the following sign condition:

$$\frac{\partial \log\, r^*}{\partial \log\, \alpha_s} > 0 \qquad \text{iff} \qquad \sigma < |\gamma|. \tag{19}$$

$|\gamma|$ is the upper bound to substitution between ω_s and ω_r that would be consistent with $s \uparrow r \uparrow$ as an observed response to charity S's fundraising appeal—in other words, that substitution can be no larger than the lift in d^*. If the substitution between ω_s and ω_r is larger than $|\gamma|$, then donations to charity R will decrease—i.e., we will observe a lift in total donations that will be accompanied by a donation pattern response such that $s \uparrow r \downarrow$, i.e., donations are shifted from charity R to charity S.

Conversely, if ω_s and ω_r are very strong complements, then charity S's fundraising appeal can produce the counter-intuitive result that $s \downarrow$. A lower bound to σ that rules out this counter-intuitive situation can be derived from the comparative statics of (13):

$$\begin{aligned} \frac{\partial \log s^*}{\partial \log \alpha_s} &= \frac{\partial \log d^*}{\partial \log \alpha_s} + \frac{\partial \log \phi}{\partial \log \alpha_s} \\ &= -\left[\phi\,(\gamma+1) + (1-\phi)\,(1-\sigma)\right] \\ &= -(1-\phi)\left[1 + (\gamma+1)\,\frac{\phi}{1-\phi} - \sigma\right] \end{aligned} \tag{20}$$

from which it follows that:

$$\frac{\partial \log s^*}{\partial \log \alpha_s} > 0 \quad \text{iff} \quad 1 + (\gamma+1)\,\frac{\phi}{1-\phi} < \sigma. \tag{21}$$

The left-hand side of the above inequality is a lower bound on the elasticity of substitution between the warm glow characteristics consistent with donations to charity S increasing in response to a fundraising appeal. The term $(\gamma+1)\,\frac{\phi}{(1-\phi)}$ indicates how strong complementarity between ω_s and ω_r can be—how far σ can be below 1 (recall $(\gamma+1)$ must be negative for total donations d^* to increase)—and still have donations to charity S increase when α_s increases. If the complementarity were stronger than this, then preferences would require that ω_r^* increase in lock-step with ω_s^* so much so that expenditures on (donations to) charity S must fall to fund the increase in expenditures on (donations to) charity R required to maintain the complementarity between the two warm glow characteristics.

We call the left-hand side of the inequality in (21) the "complementarity lower bound". This lower bound depends on how elastic total donations are $(\gamma+1)$ and the ratio of the shares of total donations going to charities S and R. The lower bound approaches 1 as the share of donations spent on S goes to zero (i.e., as $\phi \to 0$); in this case $\sigma > 1$ indicating that ω_s and ω_r must be substitutes to rule out the counter-intuitive situation of charity S's fundraising appeal not increasing the donations it receives. At donation shares $\phi > 0$, the complementarity lower bound decreases below 1, allowing ω_s and ω_r to be somewhat complementary (and still the counterintuitive situation is ruled out), but not too strongly complementary.

The following proposition summarizes these results:

Proposition 1. In a static model with quasilinear utility and two warm glow characteristics in which the price elasticity of the aggregate of the two characteristics is $\gamma < -1$, the parameter space of the elasticity of substitution σ between the characteristics is partitioned into three sets that align with the possible effects of a fundraising appeal by charity S on donations to itself and to charity R:

(22) $s\uparrow$ and $r\downarrow$ iff $|\gamma| \leqslant \sigma < \infty$

(23) $s\uparrow$ and $r\uparrow$ iff $1 + (\gamma+1)\dfrac{\phi}{1-\phi} < \sigma < |\gamma|$

(24) $s\downarrow$ and $r\uparrow$ iff $0 < \sigma \leqslant 1 + (\gamma+1)\dfrac{\phi}{1-\phi}$

where ϕ is the share of total donations spent on charity S.

Remarks. The substantive take-away point is that observations of "lift and no-shift" or "lift and shift" donation response patterns, which the empirical literature has focused on and that charities obviously care about, do not line up one-to-one with the elasticity of substitution in preferences defined over warm glow characteristics. Furthermore, one might be tempted to conclude from a lift/no-shift empirical donation response pattern, $s\uparrow r\uparrow$, that warm glow from ω_s and ω_r are complements in preferences, but the proposition indicates that observing $s\uparrow r\uparrow$ is not sufficient evidence upon which to base that conclusion. The reason is that the lift in overall donations can obscure a preference-substitution relationship. Sufficient evidence to conclude that there is substitution between the two sources of warm glow in preferences is an observed lift/shift pattern of $s \uparrow r \downarrow$. In this case the shift response in donations from charity R to charity S is strong enough to make itself visible in the observed donation response pattern. Sufficient evidence to conclude complementarity in preferences is $s \downarrow r \uparrow$.

The pattern of donations we observe in the weeks immediately following the disaster appeal, $\alpha_s \uparrow$, is $\partial \log s/\partial\alpha_s > 0$ and $\partial \log r/\partial\alpha_s > 0$. Hence, (23) provides the lower and upper bounds on the elasticity of substitution between the two warm glow characteristics necessary and sufficient to generate the empirical donation pattern we observe, again, in the weeks immediately following the appeal.

The intuition of $\alpha_s \uparrow$ ($p_s \downarrow$) consistent with the pattern of donations we observe is straightforward. Consider the case where the g-sub-utility function (4) is Cobb-Douglas. At $\sigma = 1$ the increase in total donations d spent on the aggregate warm glow characteristic g is split on equal percentage increases in donations s and r to keep the expenditure ratio $s/r = (p_s\ \omega_s)/(p_r\ \omega_r)$ unchanged; obviously in this case we have $r \uparrow$. From this point, consider a limited amount of substitution between the characteristics ω_s and ω_r: then the expenditure ratio s/r shifts in favor of charity S, but r still increases as long as the percentage change, σ, in the expenditure ratio is less than the percentage increase $|\gamma|$ in donations spread across the two charities. Conversely, starting again from $\sigma = 1$, consider a limited amount of complementarity between ω_s and ω_r: then s/r shifts in favor of charity R, but expenditure on charity S still increases as long as the shift toward charity R is not too strong.

There are conceptual take-away points as well. First, when the elasticity of substitution between the warm glow characteristics derived from donations to two different charities is in the middle set, whether the underlying warm glows are substitutes or complements cannot be identified without also having measured the price elasticity of total donations. Indeed, this is the case in our study. This also would be the case for other studies, even experimental studies that use controlled manipulations to investigate donations intending to provide evidence about substitution/complementarity in preferences. Note that if there is zero lift in donations ($\gamma = -1$) then the three-set partition collapses to a two-set partition—the middle set disappears—and the donation pattern will necessarily be either $s \uparrow r \downarrow$ (if $\sigma > 1$) or $s \downarrow r \uparrow$ (if $\sigma < 1$): the lift/shift pattern of donation responses does reveal one-to-one whether the underlying warm glows are substitutes or complements.

Second, because in a static model the only dimension of shift is in the charity-space, the partition set of the parameter space consistent with any observed donation response pattern to a fundraising appeal is one-dimensional. In a two-period model shift is possible in a second dimension: time. Consequently, the partition set of the parameter space consistent with observed donation response patterns across time will be two-dimensional. Moreover, observing $s \uparrow r \downarrow$ in the second time period would be sufficient evidence to conclude that the two sources of warm glow are substitutes in preferences. We now turn to a demonstration of this in a two-period extension of the model.

4 Two-period model with two warm glow characteristics

In this section we expand the warm glow characteristics model from Section 3 to allow for two time periods. We define $\tilde{g}$ to represent a <u>two-period</u> aggregate warm glow characteristic:

(25)
$$\tilde{g} \equiv \left(\tfrac{1}{2}\, g_1^{\delta} + \tfrac{1}{2}\, g_2^{\delta}\right)^{1/\delta}$$

where the (single-period) aggregate warm glow characteristics are a time-indexed version of (4):

(26)
$$g_t = \left(\tfrac{1}{2}\, \omega_{s_t}^{\mu} + \tfrac{1}{2}\, \omega_{r_t}^{\mu}\right)^{1/\mu}, \quad t = 1, 2$$

and $\rho = \frac{1}{1-\delta}$ is the intertemporal elasticity of substitution in the aggregate warm glow characteristics.

Time-indexed versions of (5) and (6) describe how warm glow characteristics are increased by donations s_t and r_t:

$$\omega_{s_t} = \alpha_{s_t} \; s_t \tag{27}$$

$$\omega_{r_t} = \alpha_{r_t} \; r_t. \tag{28}$$

Note that the salience parameters, such as α_{s_t}, and consequently the hedonic prices, such as p_{s_t}, are allowed to be different according to time period. This allows us to model a fundraising appeal dissipating in its salience over time: $\alpha_{s_1} \uparrow\uparrow \; \rightarrow \; \alpha_{s_2} \uparrow$.

Aggregate donations in time-periods $t = 1$ and $t = 2$ are the sums of the donations to both charities in the relevant time period:

$$d_1 = s_1 + r_1 \tag{29}$$

$$d_2 = s_2 + r_2. \tag{30}$$

We will use tilde-variables to denote two-period aggregates: $\tilde{d} = d_1 + d_2$ (and $\tilde{y} = y_1 + y_2$, $\tilde{c} = c_1 + c_2$), just as $\tilde{g}$ is the two-period aggregate of g_1 and g_2.

To simplify the exposition, we assume unconstrained lending and borrowing at no interest so that the two-period budget constraint is:

$$c_1 + c_2 + s_1 + s_2 + r_1 + r_2 = y_1 + y_2. \tag{31}$$

These assumptions not only simplify the interpretation of the results to come, but are reasonable because time differences between our $t = 1$ and $t = 2$—the immediate aftermath of the appeal and the adjustment/settling—are measured in weeks. The budget constraint can re-written in terms of the warm glow characteristics:

$$\tilde{c} \ + \ p_{s_1} \ \omega_{s_1} \ + \ p_{s_2} \ \omega_{s_2} \ + \ p_{r_1} \ \omega_{r_1} \ + \ p_{r_2} \ \omega_{r_2} \ = \ \tilde{y} \tag{32}$$

where as before the prices are reciprocals of the salience parameters:

$$p_{s_t} \ = \ \frac{1}{\alpha_{s_t}} \tag{33}$$

$$p_{r_t} \ = \ \frac{1}{\alpha_{r_t}} \tag{34}$$

both for $t = 1, 2$

The intertemporal optimization problem is then to choose warm glow characteristics ω_{s_t} and ω_{r_t}, $t = 1, 2$ to maximize utility, the combination of (1), (25) and (26), with respect to (32). The CES aggregation leads to a tractable solution of this problem using the dual approach from Section 3 in two nested levels. First, given two-period aggregate expenditures (donations) $\tilde{d}^*$ on two-period aggregate warm glow $\tilde{g}^*$, use the dual to determine the split of $\tilde{d}^*$ across the time period 1 and time period 2 warm glow aggregates (g_1^* and g_2^*); the respective expenditures on them are d_1^* and d_2^*. Then second, given expenditures (donations) d_1^* in time period $t = 1$, use the dual to determine the split of d_1^* across the warm glow characteristics in time period 1 ($\omega_{s_1}^*$ and $\omega_{r_1}^*$); the respective expenditures on them are s_1^* and r_1^*; do likewise for time period 2.

From the dual for the CES aggregation, the prices of the aggregate warm glow characteristics g_1 and g_2 depend on the underlying prices (33) and (34):

$$p_{g_t} = ({p_{s_t}}^{1-\sigma} + {p_{r_t}}^{1-\sigma})^{1/(1-\sigma)}, \qquad t = 1, 2 \tag{35}$$

and in turn the price of the two-period aggregate warm glow characteristic $\tilde{g}$ is:

$$p_{\tilde{g}} = ({p_{g_1}}^{1-\rho} + {p_{g_2}}^{1-\rho})^{1/(1-\rho)} \tag{36}$$

Equations (33), (34), (35) and (36) can be used to write $p_{\tilde{g}}$ in terms of the salience parameters α_{s_t} and α_{r_t}, the fundamentals that can be influenced by a fundraising appeal.

Optimal two-period aggregate warm glow $\tilde{g}^*$ is as in (3), $\tilde{g}^* = \kappa \;\; p_{\tilde{g}}^{\gamma}$, where $p_{\tilde{g}}$ is from (36). At that specific level of $\tilde{g}^*$, expenditures on $\tilde{g}^*$ are:

$$\tilde{d}^* \;\; \equiv \;\; p_{\tilde{g}} \;\; \tilde{g}^* \;\; = \;\; \tilde{g}^* \;\; ({p_{g_1}}^{1-\rho} + {p_{g_2}}^{1-\rho})^{1/(1-\rho)}. \tag{37}$$

Inverting (37) (to get the two-period aggregate sub-utility function, $\tilde{g}^*$) and applying Roy's identity yields Marshallian demands for the time period 1 and 2 warm glow aggregate characteristics:

$$g_1^* \;\; = \;\; \tilde{d}^* \;\; \frac{p_{g_1}^{-\rho}}{p_{g_1}^{1-\rho} \;\; + \;\; p_{g_2}^{1-\rho}} \tag{38}$$

$$g_2^* \;\; = \;\; \tilde{d}^* \;\; \frac{p_{g_2}^{-\rho}}{p_{g_1}^{1-\rho} \;\; + \;\; p_{g_2}^{1-\rho}} \tag{39}$$

and expenditures in the two time periods:

$$d_1^* \;\; = \;\; \tilde{d}^* \;\; \frac{p_{g_1}^{1-\rho}}{p_{g_1}^{1-\rho} \;\; + \;\; p_{g_2}^{1-\rho}} \;\; = \;\; \tilde{d}^* \; \tau \tag{40}$$

$$d_2^* \;\; = \;\; \tilde{d}^* \;\; \frac{p_{g_2}^{1-\rho}}{p_{g_1}^{1-\rho} \;\; + \;\; p_{g_2}^{1-\rho}} \;\; = \;\; \tilde{d}^* \; (1-\tau) \tag{41}$$

where the share of donations $\tilde{d}^*$ spent in time period 1 is $p_{g_1}^{1-\rho}/(p_{g_1}^{1-\rho} \;\; + \;\; p_{g_2}^{1-\rho}) \triangleq \tau$, where τ is a function of α_{s_1}, α_{r_1}, α_{s_2}, α_{r_2}, σ and ρ.

At the second level down, use (35) to write the cost function within each time period as a function of p_{s_t} and p_{r_t} and the Marshallian demands for the within-time period aggregate characteristic g_t^*:

$$d_1^* \;\; \equiv \;\; p_{g_1} \;\; g_1^* \;\; = \;\; g_1^* \;\; ({p_{s_1}}^{1-\sigma} + {p_{r_1}}^{1-\sigma})^{1/(1-\sigma)} \tag{42}$$

and

$$(43) \qquad d_2^* \;\equiv\; p_{g_2}\;g_2^* \;=\; g_2^*\;({p_{s_2}}^{1-\sigma} + {p_{r_2}}^{1-\sigma})^{1/(1-\sigma)}.$$

Invert the cost function (42) to get the within-time period 1 sub-utility function (i.e., g_1^*) and apply Roy's identity:

$$(44) \qquad \omega_{s_1}^* \;=\; d_1^* \;\frac{p_{s_1}^{-\sigma}}{p_{s_1}^{1-\sigma} \;+\; p_{r_1}^{1-\sigma}}$$

$$(45) \qquad \omega_{r_1}^* \;=\; d_1^* \;\frac{p_{r_1}^{-\sigma}}{p_{s_1}^{1-\sigma} \;+\; p_{r_1}^{1-\sigma}}.$$

Equations (44) and (45) are the Marshallian demands for the warm glow characteristics ω_{s_1} and ω_{r_1} in time period 1. The corresponding expenditures on (donations to) the respective charities are:

$$(46) \qquad s_1^* \;=\; d_1^* \;\frac{p_{s_1}^{1-\sigma}}{p_{s_1}^{1-\sigma} \;+\; p_{r_1}^{1-\sigma}} \;=\; d_1^*\;\phi_1 \;=\; \tilde{d}^*\;\tau\;\phi_1$$

$$(47) \qquad r_1^* \;=\; d_1^* \;\frac{p_{r_1}^{1-\sigma}}{p_{s_1}^{1-\sigma} \;+\; p_{r_1}^{1-\sigma}} \;=\; d_1^*\;(1-\phi_1) \;=\; \tilde{d}^*\;\tau\;(1-\phi_1).$$

where as in Section 3 the share of donations d_1^* spent on charity S is $p_{s_1}^{1-\sigma}/(p_{s_1}^{1-\sigma} \;+\; p_{r_1}^{1-\sigma}) \triangleq \phi_1$ (however now the share and prices are all indexed by $t = 1$).

Expressions analogous to (44) and (45) can be derived for the warm glow characteristics in time period $t = 2$. Donations to the two charities in time period 2 are then:

$$(48) \qquad s_2^* \;=\; d_2^*\;\phi_2 \;=\; \tilde{d}^*\;(1-\tau)\;\phi_2$$

$$(49) \qquad r_2^* \;=\; d_2^*\;(1-\phi_2) \;=\; \tilde{d}^*\;(1-\tau)\;(1-\phi_2)$$

where $p_{s_2}^{1-\sigma}/(p_{s_2}^{1-\sigma} \;+\; p_{r_2}^{1-\sigma}) \triangleq \phi_2$ the $t = 2$ share of donations d_2^* spent on charity S. Equations

(46) - (49) are equations (5) - (8) in the main text.

Equations (46) - (49) can be described in terms of lift and shift. Overall lift is captured by $\tilde{d}^*$—total two-period expenditures on two-period aggregate warm glow $\tilde{g}^*$. Shift in the two-period model can now occur in two separate dimensions: τ captures time-shifting from d_2^* (the donation aggregate in time period 2) to d_1^* (the donation aggregate in time period 1) and ϕ_1 captures the shift in the charity-space dimension from r_1^* (donations to charity R) to s_1^* (donations to charity S) within time period 1. The analogous shift in charity-space within time period 2 is captured by ϕ_2. For example, r_2^* can decrease in response to a fundraising appeal from charity S if the lift in two-period aggregate donations $\tilde{d}^*$ is more than offset by a reduction in the share $(1-\tau)$ of those donations spent in time period 2 plus a reduction in the share $(1-\phi_2)$ of the time period 2 donations spent on charity R.

To make the combination of lift, time-shift, and charity-space shift concrete we consider a two-period fundraising appeal as an increase in the salience with which donations produce warm glow that begins in the first time period, and remains increased in the second time period. In other words, the fundraising appeal is a two-tuple $(\text{dlog } \alpha_{s_1},\ \text{dlog } \alpha_{s_2})$. We model an appeal launched in the first period but continuing into the second as remaining salient in the second period but less so than it was in the first; specifically, we model the second period increase in salience as an exponential decline relative to the increased salience in the first period: $(\text{dlog } \alpha_{s_1},\ \xi \text{ dlog } \alpha_{s_1})$ and $0 < \xi < 1$. In this way, a fundraising appeal is a single exogenous intervention that plays out over two time periods. The two-tuple is $(\text{dlog } \lambda,\ \xi \text{ dlog } \lambda)$.

The two-period fundraising appeal causes a drop in the hedonic price of ω_{s_t} at both $t = 1$ and 2: $\text{dlog } p_{s_1}/\text{dlog } \lambda = -1$ and $\text{dlog } p_{s_2}/\text{dlog } \lambda = -\xi$. Accordingly, the drop in the price of the aggregate warm glow characteristic in time period 1 (g_1) is

$$\frac{\partial \log p_{g_1}}{\partial \log \lambda} = -\phi_1 \tag{50}$$

(just as it was in (15) for the one-period model) and the drop in time period 2 (for g_2) is

$$\frac{\partial \log\, p_{g_2}}{\partial \log\, \lambda} \quad = \quad -\ \xi\ \phi_2. \tag{51}$$

The drop in the two-period aggregate warm glow characteristic ($\tilde{g}$) is a weighted combination of these:

$$\begin{aligned} \frac{\partial \log\, p_{\tilde{g}}}{\partial \log\, \lambda} \quad &= \quad \tau\ \frac{\partial \log\, p_{g_1}}{\partial \log\, \lambda} \quad + \quad (1-\tau)\ \frac{\partial \log\, p_{g_2}}{\partial \log\, \lambda} \\ &= \quad -\ \ [\tau\ \phi_1 \quad + \quad (1-\tau)\ \xi\ \phi_2] \end{aligned} \tag{52}$$

where the weights τ and $(1-\tau)$ are the shares of the two-period aggregate donations $\tilde{d}^*$ spent in the first and second time periods.

The lift in overall (i.e., two-period aggregate) donations is driven by the drop in the price (52):

$$\frac{\partial \log\, \tilde{d}^*}{\partial \log\, \lambda} \quad = \quad -\ \ [\tau\ \phi_1 \quad + \quad (1-\tau)\ \xi\ \phi_2]\ \ (\gamma+1) \tag{53}$$

where the term in square brackets is positive. That lift occurs if (and only if) the two-period aggregate warm glow characteristic $\tilde{g}$ is price elastic is no different than seen in Sections 2 and 3.

The pattern of donations we observe in the time period after the weeks immediately following the launch of a disaster appeal ($t = 2$ in the two-period model) is: $\partial \log\, s_2/\partial\lambda > 0$ and $\partial \log\, r_2/\partial\lambda < 0$ (at $t = 1$ the observed pattern is as analyzed in Section 3: $\partial \log\, s_1/\partial\lambda > 0$ and $\partial \log\, r_1/\partial\lambda > 0$). We begin with $r_2 \downarrow$:

$$\begin{aligned} \frac{\partial \log\, r_2^*}{\partial \log\, \lambda} \quad &= \quad \frac{\partial \log\, \tilde{d}^*}{\partial \log\, \lambda} \quad - \quad \frac{\tau}{(1-\tau)}\ \frac{\partial \log\, \tau}{\partial \log\, \lambda} \quad - \quad \frac{\phi_2}{(1-\phi_2)}\ \frac{\partial \log\, \phi_2}{\partial \log\, \lambda} \\ &= \quad -\,(\gamma+1)\ \ [\tau\ \phi_1\ +\ (1-\tau)\ \phi_2\ \xi] \quad + \quad (1-\rho)\ \tau\ \ [\phi_1\ -\ \phi_2\ \xi] \quad + \quad (1-\sigma)\ \phi_2\ \xi. \end{aligned} \tag{54}$$

The first term on the right-hand side describes the lift. In a two-period model, however, there are two dimensions to shift: the middle term on the right-hand side describes the shift of donations in the time dimension $t = 2 \to t = 1$ and the last term describes the shift in the charity-space dimension $r_2 \to s_2$.[3] The empirical pattern we observe requires the right-hand side to be negative, that is, that the lift be less than the two dimensions of shift. This implies a lower bound on ρ in terms of σ, γ and the shares:

$$(55) \qquad \rho \; > \; 1 \; - \; (\gamma+1)\,\frac{[\tau\;\phi_1 \;+\; (1-\tau)\;\phi_2\;\xi]}{\tau\;[\phi_1 \;-\; \phi_2\;\xi]} \;+\; (1-\sigma)\,\frac{\phi_2\;\xi}{\tau\;[\phi_1 \;-\; \phi_2\;\xi]} \Leftrightarrow \frac{\partial \log\, r_2^*}{\partial \log\, \lambda} < 0.$$

We will refer to the lower bound on the right-hand side of the inequality in (55) as "$\mathrm{L}_{r_2}(\sigma,\ \rho)$". L_{r_2} is a written as a function of σ and ρ because ϕ_1 and ϕ_2 are functions of σ, and τ is a function of ρ.

The empirical pattern also includes $\partial \log\, s_1/\partial\lambda > 0$ and $\partial \log\, r_1/\partial\lambda > 0$. These inequalities also imply lower bounds on ρ to ensure that enough donations are time-shifted to $t = 1$ so that even if there is substantial charity-space shift ($r_1 \to s_1$), the time-shift plus the overall lift will nevertheless produce both $s_1 \uparrow$ and $r_1 \uparrow$:

$$(56)\; \rho \; > \; 1 \; + \; (\gamma+1)\,\frac{[\tau\;\phi_1 + (1-\tau)\phi_2\;\xi]}{(1-\tau)\;\;[\phi_1 \;-\; \phi_2\;\xi]} \;+\; (1-\sigma)\frac{(1-\phi_1)}{(1-\tau)\,[\phi_1 \;-\; \phi_2\;\xi]} \Leftrightarrow \frac{\partial \log\, s_1^*}{\partial \log\, \lambda} > 0$$

$$(57)\; \rho \; > \; 1 \; + \; (\gamma+1)\,\frac{[\tau\;\phi_1 + (1-\tau)\phi_2\;\xi]}{(1-\tau)\;\;[\phi_1 \;-\; \phi_2\;\xi]} \;-\; (1-\sigma)\frac{\phi_1}{(1-\tau)\,[\phi_1 \;-\; \phi_2\;\xi]} \Leftrightarrow \frac{\partial \log\, r_1^*}{\partial \log\, \lambda} > 0$$

with the lower bounds on the right-hand sides of the inequalities in (56) and (57), respectively, denoted "$\mathrm{L}_{s_1}(\sigma,\ \rho)$" and "$\mathrm{L}_{r_1}(\sigma,\ \rho)$". The boundary (57) corresponds to equation (9) in the main text.

The lower bounds L_{r_2}, L_{s_1} and L_{r_1} indicate how much time-shifting is necessary to produce $r_2 \downarrow$, $s_1 \uparrow$ and $r_1 \uparrow$. However, too much time-shifting would contradict the last part of the observed empirical pattern: that $s_2 \uparrow$. To maintain $s_2 \uparrow$ the comparative statics of s_2 imply an upper bound

[3]The middle time-shift term is $\frac{\partial \log\, \tau}{\partial \log\, \alpha_{s_1}} \;\equiv\; \frac{\partial \log\, \tau}{\partial \log\, \lambda} \;=\; -\,(1-\rho)\;\frac{\tau}{1-\tau}\;\;[\phi_1 \;-\; \phi_2\;\xi]$.

on ρ:

$$(58) \qquad \rho \;<\; 1 \;-\; (\gamma+1)\,\frac{[\tau\;\phi_1 \;+\; (1-\tau)\;\phi_2\;\xi]}{\tau\;[\phi_1 \;-\; \phi_2\;\xi]} \;-\; (1-\sigma)\,\frac{(1-\phi_2)\;\xi}{\tau\;[\phi_1 \;-\; \phi_2\;\xi]} \Leftrightarrow \frac{\partial \log\, s_2^*}{\partial \log\, \lambda} > 0.$$

where the upper bound on the right-hand side referred to as "$\mathrm{U}_{s_2}(\sigma,\;\rho)$".

Replacing the inequality signs in (55) - (58) with "=" and solving for ρ as a function of σ leads to boundaries that partition the $(\sigma,\;\rho)$ parameter space into sets. Each set corresponds to a different qualitative response pattern in donations to charities S and R in time periods 1 and 2 that can follow a fundraising appeal by Charity S. General closed-form solutions of the boundaries are not possible, because ϕ_1 and ϕ_2 are non-linear functions of σ, and τ is a non-linear function of ρ. However, (55) - (58) simplify when the salience with which donations to s produce warm glow is time-invariant in the baseline—$\alpha_{s_1} = \alpha_{s_2}$ and likewise for donations to charity R: $\alpha_{r_1} = \alpha_{r_2}$. Time-invariant baseline salience is a reasonable assumption in many applications, including our's. Time-invariant salience implies $p_{s_1} = p_{s_2}$ and $p_{r_1} = p_{r_2}$ which in turn implies (a) two-period aggregate donations $\tilde{d}$ are split equally across the two time periods: $p_{g_1} = p_{g_2} \Leftrightarrow \tau = (1-\tau) = \frac{1}{2}$; and (b) within each time period the share of donations spent on charity S is equal: $\phi_1 = \phi_2 \equiv \phi$. Hence under time-invariant salience, (55) - (58) simplify to:

$$(55') \qquad \rho \;>\; 1 \;-\; (\gamma+1)\left(\frac{1\;+\;\xi}{1\;-\;\xi}\right) \;+\; (1-\sigma)\,\frac{\xi}{\frac{1}{2}\;(1\;-\;\xi)} \Leftrightarrow \frac{\partial \log\, r_2^*}{\partial \log\, \lambda} < 0$$

$$(56') \qquad \rho \;>\; 1 \;+\; (\gamma+1)\left(\frac{1\;+\;\xi}{1\;-\;\xi}\right) \;+\; (1-\sigma)\,\frac{1\;-\;\phi}{\frac{1}{2}\;\phi\;(1\;-\;\xi)} \Leftrightarrow \frac{\partial \log\, s_1^*}{\partial \log\, \lambda} > 0$$

$$(57') \qquad \rho \;>\; 1 \;+\; (\gamma+1)\left(\frac{1\;+\;\xi}{1\;-\;\xi}\right) \;-\; (1-\sigma)\,\frac{1}{\frac{1}{2}\,(1\;-\;\xi)} \Leftrightarrow \frac{\partial \log\, r_1^*}{\partial \log\, \lambda} > 0$$

$$(58') \qquad \rho \;<\; 1 \;-\; (\gamma+1)\left(\frac{1\;+\;\xi}{1\;-\;\xi}\right) \;-\; (1-\sigma)\,\frac{(1\;-\;\phi)\,\xi}{\frac{1}{2}\,\phi\,(1\;-\;\xi)} \Leftrightarrow \frac{\partial \log\, s_2^*}{\partial \log\, \lambda} > 0$$

Note that the boundaries in (55′) and (57′) can now be solved in closed-form; they are straight lines.

Below we will prove our main result—that under time-invariant salience the model implies that the qualitative response pattern in donations that we observed ($s_1 \uparrow$, $r_1 \uparrow$, $s_2 \uparrow$, $r_2 \downarrow$) implies $\sigma > 1$, that is: ω_s and ω_r are substitutes. But first it helps establish intuition by considering a special case in which donations to s and r are equally effective in producing warm glow: $\phi = \frac{1}{2}$. In this case (56′) and (58′) also are straight lines:

$$(56'') \qquad \rho \;>\; 1 \;+\; (\gamma+1)\left(\frac{1\;+\;\xi}{1\;-\;\xi}\right) \;+\; (1-\sigma)\,\frac{1}{\frac{1}{2}\,(1\;-\;\xi)}$$

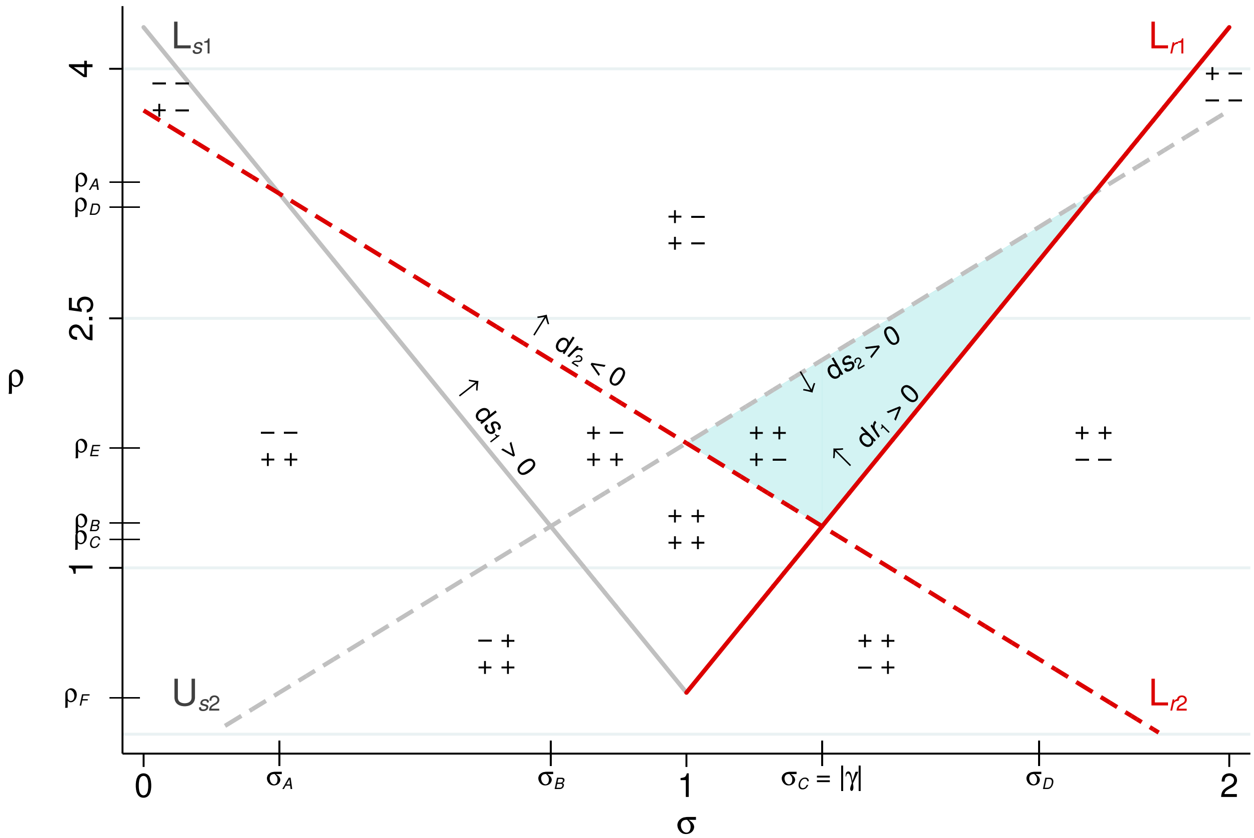
$$(58'') \qquad \rho \;<\; 1 \;-\; (\gamma+1)\left(\frac{1\;+\;\xi}{1\;-\;\xi}\right) \;-\; (1-\sigma)\,\frac{\xi}{\frac{1}{2}\,(1\;-\;\xi)}$$

The boundaries (55′), (57′), (56″) and (58″) are graphed in Figure C1 for a price elasticity of two-period aggregate donations $\gamma = -1.25$ and decay rate $\xi = .50$. The boundaries partition the $(\sigma,\ \rho)$ parameter space into ten sets. The qualitative donation response pattern in each set is identified by the $+$ and $-$ signs according to the following scheme:

		$t=1$	$t=2$
Donations to	s	$+$	$+$
	r	$+$	$-$

The example illustrated in the scheme is the donation pattern we observed: $s_1\uparrow$, $r_1\uparrow$, $s_2\uparrow$, $r_2\downarrow$. The set of $(\sigma,\ \rho)$ pairs consistent with this pattern is indicated by the shaded triangle in the figure. Note that in this set, all the values of the elasticity of substitution are $\sigma > 1$.

Figure C1. Substitution between warm glow characteristics, $\alpha_s = \alpha_r = 1$.



The vertices in Figure C1 are:

$$\sigma_A = 1 + (\gamma+1)\left(\frac{1+\xi}{1-\xi}\right)$$

(59)

$$\rho_A = 1 - (\gamma+1)\left(\frac{1+\xi}{1-\xi}\right)^2$$

$$\sigma_B = 1 + (\gamma + 1), \qquad \rho_B = |\gamma| \tag{60}$$

$$\sigma_C = |\gamma|, \qquad \rho_C = |\gamma| \tag{61}$$

$$\begin{aligned} \sigma_D &= 1 - (\gamma + 1) \left(\frac{1 + \xi}{1 - \xi} \right) \\ \rho_D &= 1 - (\gamma + 1) \left(\frac{1 + \xi}{1 - \xi} \right)^2 \end{aligned} \tag{62}$$

$$\sigma_E = 1, \qquad \rho_E = 1 - (\gamma + 1) \left(\frac{1 + \xi}{1 - \xi} \right) \tag{63}$$

$$\sigma_F = 1, \qquad \rho_F = 1 + (\gamma + 1) \left(\frac{1 + \xi}{1 - \xi} \right). \tag{64}$$

Because in a two-period model there are two dimensions in which shift can occur, the parameter space needed to describe possible donation response patterns is two-dimensional. Recall that in the static model there was a "middle set" (23) such that for σs in this middle set whether the underlying warm glow characteristics were substitutes or complements could not be identified from that set's qualitative donation response pattern $s \uparrow r \uparrow$, because overall lift obscured the underlying preference relationship. That static middle set has, in the two-period model, been transformed into four sets that share a common vertice at ($\sigma_E = 1$, ρ_E): the diamond in the middle of Figure C1, the V-shaped wedge directly north of that diamond, and the two triangles (one of them shaded) in between the diamond and the the V-shaped wedge. Whether the static middle set identification problem from the first period can be resolved by the donation pattern that emerges in the second period depends on which of these four sets obtains. If the diamond or the V-shaped wedge obtains, the identification problem remains. If either of the two triangles obtains, the identification problem is resolved.

For $(\sigma,\ \rho)$ pairs in the V-shaped wedge the $r_1 \downarrow r_2 \downarrow$ qualitative response pattern is sufficient evidence to conclude that the time period 1 and 2 warm glow aggregate characteristics g_1^* and g_2^* are intertemporal substitutes ($\rho > 1$), but because the intertemporal elasticity of substitution is strong enough ($\rho >> 1$) to move both r_1 and r_2 in the same $\downarrow$ direction the underlying substitution/complementarity preference relationship between ω_{s_t} and ω_{r_t} is obscured. And the stronger the ρ (the farther north in the V-shaped wedge), the more time-shifting can obscure stronger and stronger underlying substitution or complementarity (movement to the east or west) between ω_{s_t} and ω_{r_t}.

For the $(\sigma,\ \rho)$ parameters in the diamond-shaped set, the intertemporal elasticity of substitution is weaker, and the qualitative pattern is that all four donations s_1, s_2, r_1 and $r_2 \uparrow$. This pattern obscures whether g_1^* and g_2^* are intertemporal substitutes or complements, and well as obscuring substitution/complementarity between ω_{s_t} and ω_{r_t}. In other words, the lift in overall donations is enough to obscure any underlying substitution/complementarity preference relationships in both the time and charity-space dimensions.

For the two triangles, time-shifting from $t = 2$ to $t = 1$ caused by intertemporal substitution works in the same direction as overall lift during $t = 1$ to make it more difficult to adjudicate whether the warm glow characteristics are substitutes or complements. There is, however, another side to the time-shifting coin: it necessarily implies that during $t = 2$, time-shifting is working in the opposite direction as overall lift and that makes successful identification of substitution/complementarity more likely. Consider the shaded triangle set of (σ, ρ) pairs consistent with the response pattern we observed. All of these pairs are such that r decreases in the second period. And all of these pairs have $\sigma > 1$, implying that the donation response pattern in this set would be sufficient evidence to conclude that ω_s and ω_r are substitutes. Pick a σ inside this set, hold it fixed and then move north; once you cross the U_{s_2} boundary $t = 2 \to t = 1$ time-shifting becomes "too strong": $s_2 \downarrow$ and the substitution relationship is again obscured.[4,5]

Although the intuition gleaned from Figure C1 holds more generally, the $\phi = \frac{1}{2}$ special case is not well-matched to our application in which $\phi \approx .06$, and, more importantly, does not constitute a proof. Our main result is:

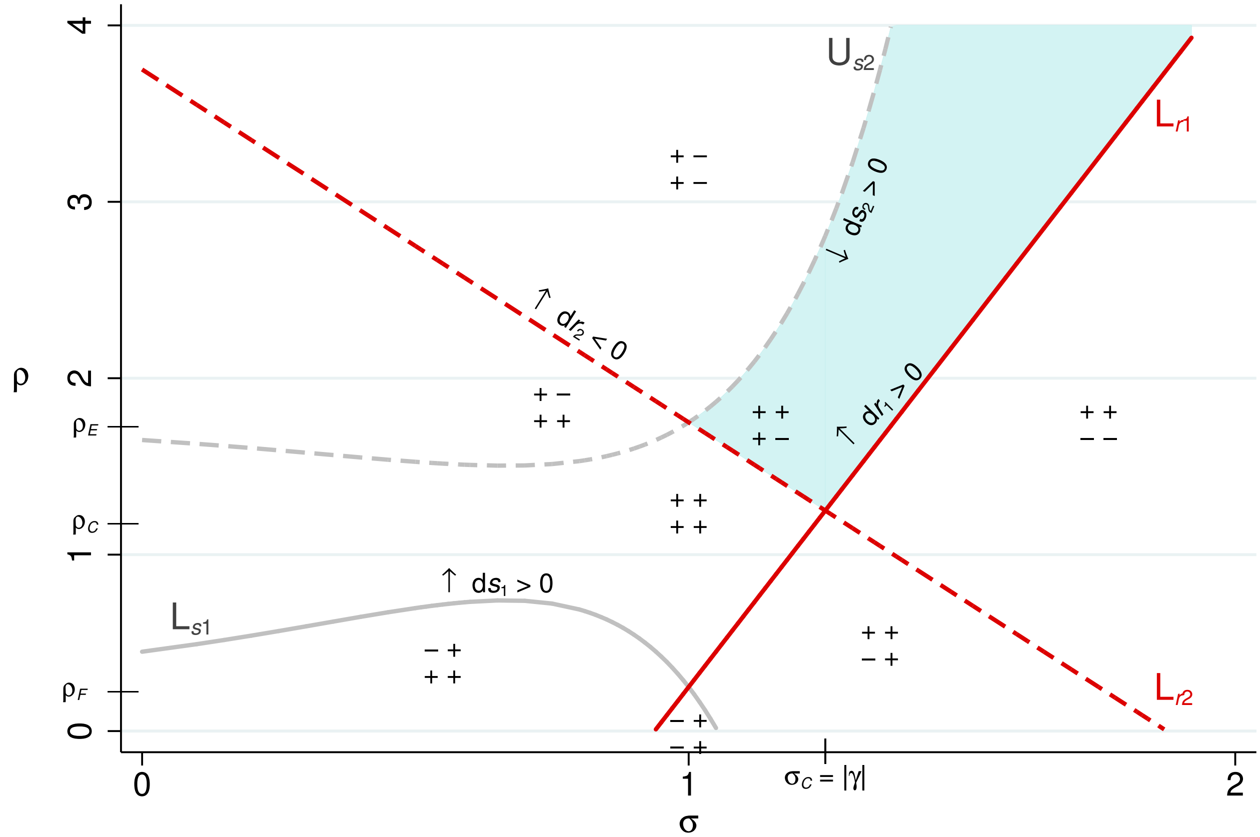
Proposition 2. In a two-period model with quasilinear utility and two warm glow characteristics in which the price elasticity of the two-period aggregate of the warm glow characteristics is $\gamma < -1$, the decay rate in the fundraising appeal's salience is $\xi < 1$ and the baseline salience with which donations produce warm glow is time-invariant, then the qualitative donation pattern we observed ($s_1 \uparrow$, $r_1 \uparrow$, $s_2 \uparrow$, $r_2 \downarrow$) implies the two warm glow characteristics are substitutes.

Proof. The proof is based the boundaries (55′) - (58′). Figure C2 (Figure 4 in the main text) plots these boundaries for $\alpha_s = .05$ and $\alpha_r = 1$, values of the baseline salience parameters that can produce $\phi \approx .06$, as in our application. We use Figure C2/Figure 4 as a visual guide to organizing the proof, but the proof is general. The proof focuses on the case where it is assumed that $\alpha_s < \alpha_r$, but the proof follows for the opposite $\alpha_s > \alpha_r$ as well. The opposite case is discussed in remarks that follow the proof.

[4]There are $(\sigma,\ \rho)$ pairs in the shaded set such that $\sigma > |\gamma|$ and still we can get $s_1 \uparrow$ and $r_1 \uparrow$. This could not happen in the static model (see equation (23)), but can happen in a two-period model if time-shifting is strong enough to, when in combination with the overall lift, obscure the $r_1 \to s_1$ shift in charity-space at $t = 1$. Similarly, the V-shaped wedge set in which $s_1 \uparrow$ and $r_1 \uparrow$ also has $(\sigma,\ \rho)$ pairs such that $\sigma > |\gamma|$.

[5]It may seem at first that with two organizations, two time periods, and donations that can go in two directions (up or down) there are 16 possible donation response patterns, and that because Figure C1 contains only ten, six patterns are being overlooked. However, the six patterns not shown in Figure C1 would require unusual preferences. For instance, $\begin{smallmatrix} - & - \\ - & - \end{smallmatrix}$ would require a fundraising appeal to lower donations to all charities (Giffen good-like behavior in expenditures that would not be possible if there is overall lift, $\gamma < -1$). The pattern $\begin{smallmatrix} - & + \\ - & + \end{smallmatrix}$ would require time preferences in which the future is preferred to the present; $\begin{smallmatrix} - & + \\ - & - \end{smallmatrix}$ and $\begin{smallmatrix} - & - \\ - & + \end{smallmatrix}$ would also require a preference for the future but with charity-space substitution or complementarity, respectively, that combines with the future-preference to cause either s_2 or r_2 to $\uparrow$. The pattern $\begin{smallmatrix} + & - \\ - & + \end{smallmatrix}$ would require ω_s and ω_r switch from being substitutes at $t = 1$ to being complements at $t = 2$; the pattern $\begin{smallmatrix} - & + \\ + & - \end{smallmatrix}$ would require an opposite order complements-then-substitutes switch.

Figure C2. Substitution between warm glow characteristics, α_s = .05, α_r = 1



The most important part of the proof is straightforward: equating the right-hand sides of (55′) and (58′) leads to:

$$-\ (1-\sigma)\ \frac{(1-\phi)}{\phi} = (1-\sigma) \tag{65}$$

which implies that U_{s_2} and L_{r_2} intersect at $\sigma = 1$ (for ϕ strictly less than 1). Also $\mathrm{U}_{s_2} < \mathrm{L}_{r_2}\ \forall\, \sigma < 1$ (replace the "=" sign in (65) with "$<$ " and it becomes the condition for $\mathrm{U}_{s_2} < \mathrm{L}_{r_2}$). Likewise $\mathrm{U}_{s_2} > \mathrm{L}_{r_2}\ \ \forall\, \sigma > 1$. The implication is that $s_2 \uparrow$ and $r_2 \downarrow$ requires $\sigma > 1$.

It remains to be shown that there are some $(\sigma > 1, \rho)$ combinations that satisfy $s_2 \uparrow$, $r_2 \downarrow$ while also satisfying $(s_1 \uparrow r_1 \uparrow)$. First we show that $\mathrm{L}_{s_1} < \mathrm{L}_{r_2}\ \ \forall\, \sigma > 1$ which implies that any $(\sigma > 1, \rho)$

pair that has $r_2 \downarrow$ also has $s_1 \uparrow$. To see this begin with (55′) and (56′):

$$\begin{array}{lccc} & \mathrm{L}_{s_1} & <^? & \mathrm{L}_{r_2} \\ (66) & (\gamma + 1)(1 + \xi) & <^? & (1 - \sigma)\left[\xi - \dfrac{(1 - \phi)}{\phi}\right]. \end{array}$$

The inequality holds because the left-hand side is negative, and $\forall\ \sigma > 1$ the right-hand side is positive. To see the latter note that $\phi(\sigma)$ is monotonic in σ, decreasing in σ if $\alpha_s < \alpha_r$ (and increasing if $\alpha_s > \alpha_r$). If follows that if $\alpha_s < \alpha_r$ then $\frac{(1 - \phi)}{\phi}$ is monotonically increasing in σ. Because at $\sigma = 1$, $\phi = \frac{1}{2} \Rightarrow \frac{(1 - \phi)}{\phi} = 1$, monotonicity implies that $\frac{(1 - \phi)}{\phi} > 1 \ \ \forall\ \sigma > 1$. Then $\xi < 1$ implies that the square-bracket term in (66) is negative and the right-hand-side is positive.[6]

To see that there are some $(\sigma > 1, \rho)$ pairs that satisfy $r_1 \uparrow$, note that from (55′) and (57′), L_{r_2} and L_{r_1} intersect at $\sigma = |\gamma|$. This is denoted as "σ_C" in Figure C2. Because $|\gamma| > 1$, σ_C is necessarily > 1. Hence, the $(\sigma > 1, \rho)$ pairs just to the northeast of the segment of L_{r_1} between $\sigma = 1$ and σ_C have $(s_1 \uparrow r_1 \uparrow s_2 \uparrow r_2 \downarrow)$.

The proof is completed by showing that there are such "northeast" $(\sigma > 1, \rho)$ pairs because either (a) U_{s_2} does not intersect L_{r_1} or (b) U_{s_2} intersects L_{r_1} at a value of σ larger than σ_C. Continuing with (55′) and (57′), note that $\mathrm{L}_{r_2} > \mathrm{L}_{r_1} \Leftrightarrow \sigma < \sigma_C$. Because $\mathrm{U}_{s_2} > \mathrm{L}_{r_2} \ \ \forall\ \sigma > 1$, it follows that $\mathrm{U}_{s_2} > \mathrm{L}_{r_2}$ for $1 < \sigma < \sigma_C$. Therefore if U_{s_2} intersects L_{r_1}, it must be to the right of σ_C.

If U_{s_2} intersects L_{r_1}, the value of σ at which this occurs solves (from (57′) and (58′)):

(67) $$(1 - \sigma)\left[1 - \frac{1 - \phi(\sigma)}{\phi(\sigma)}\,\xi\right] = (\gamma + 1)(1 + \xi)$$

where the functional dependence of ϕ on σ now has been made explicit. If (67) has a solution—call it "σ_D"—in general it cannot be written in closed-form.[7] We can, however, describe necessary

[6]Details: $\mathrm{d}\phi(\sigma)/\mathrm{d}\sigma = \phi\,(1 - \phi)\,\ln(\alpha_s/\alpha_r)$ which is negative if $\alpha_s < \alpha_r$, and positive if $\alpha_s > \alpha_r$. Therefore $\mathrm{d}(\frac{1-\phi}{\phi})/\mathrm{d}\sigma = -\ \frac{1-\phi}{\phi}\ \ln(\alpha_s/\alpha_r)$ is positive if $\alpha_s < \alpha_r$ (and negative if $\alpha_s > \alpha_r$). The boundaries L_{s_1} and U_{s_2} in (56″) and (58″), and graphed in Figure C1, are straight lines because when $\alpha_s = \alpha_r$, both $\mathrm{d}\phi/\mathrm{d}\sigma$ and $\mathrm{d}(\frac{1-\phi}{\phi})/\mathrm{d}\sigma$ are zero $\forall\ \sigma$.

[7]However in the special case $\alpha_s = \alpha_r$ it is easy to show that U_{s_2} intersects L_{r_1} (see Figure C1), and a closed-form

conditions that a solution σ_D would have to satisfy. First, if there is a solution σ_D that lies in the domain $\sigma > 1$ it must be the case that the square-bracket term in (67) is positive (when evaluated at σ_D). This is because for $\sigma > 1$, a negative square-bracket term would imply the left-hand side of (67) is positive, but the right-hand is negative and consequently no solution would be possible. Hence, if there is a solution σ_D it must satisfy:

$$(68) \qquad \xi \;<\; \frac{\phi(\sigma_D)}{1 \;-\; \phi(\sigma_D)}.$$

Second, to investigate the square-bracket term let "σ_0" be the value of σ at which:

$$(69) \qquad \xi \;=\; \frac{\phi(\sigma_0)}{1 \;-\; \phi(\sigma_0)}.$$

Because $\frac{\phi}{1-\phi}$ equals 1 at $\sigma = 1$ and is monotonically decreasing in σ (if $\alpha_s < \alpha_r$), and because $\xi < 1$, we know that the solution to (69) σ_0 must be greater than 1. Hence, for $\sigma \in (1, \sigma_0)$ we have $\xi < \frac{\phi(\sigma)}{1-\phi(\sigma)}$, the square-bracket term in (67) is positive, and it is possible that a solution σ_D to (67) exists.[8]

Now, if $\sigma_0 < \sigma_C$ then we know that U_{s_2} cannot intersect L_{r_1} because if it did the intersection would have to occur in the σ-domain $1 < \sigma < \sigma_0 < \sigma_C$, but we have already proved that $\mathrm{U}_{s_2} > \mathrm{L}_{r_2}$ for $1 < \sigma < \sigma_C$.[9,10]

Therefore, either U_{s_2} does not intersect L_{r_1} or, if it does it intersects L_{r_1} at a value of σ larger than σ_C. Note that the condition $\sigma_C < \sigma_0$ is necessary, though not sufficient, for U_{s_2} to intersect L_{r_1}. Whether or not U_{s_2} intersects L_{r_1} depends on how rapidly the slope of U_{s_2} is increasing as σ increases, and this depends on the relative baseline salience levels α_s and α_r. Recall, if $\alpha_s = 1$ and

for σ_D can be found (see equation (62)).

[8]Conversely, for the domain of σ such that $1 < \sigma_0 < \sigma$, the square-bracket term is negative and no solution to (67) is possible.

[9]In Figure C2, $\sigma_0 \;=\; 1.2314$, is less than σ_C (1.25), and therefore no solution to (67) is possible.

[10]It is straightforward to show that U_{s_2} cannot intersect L_{r_1} over the domain $\sigma < 1$. Because $\frac{\phi}{1-\phi} = 1$ at $\sigma = 1$ and monotonically increases as σ gets smaller, we know $\frac{\phi}{1-\phi} > 1 \;\; \forall \;\; \sigma < 1$. This implies (because $\xi < 1$) $\xi < \frac{\phi(\sigma)}{1-\phi(\sigma)} \; \forall \; \sigma < 1$, the square-bracket term in (67) is positive, but because $(1 - \sigma)$ is positive, the left-hand side of (67) is positive while the right-hand side is negative, implying no solution is possible.

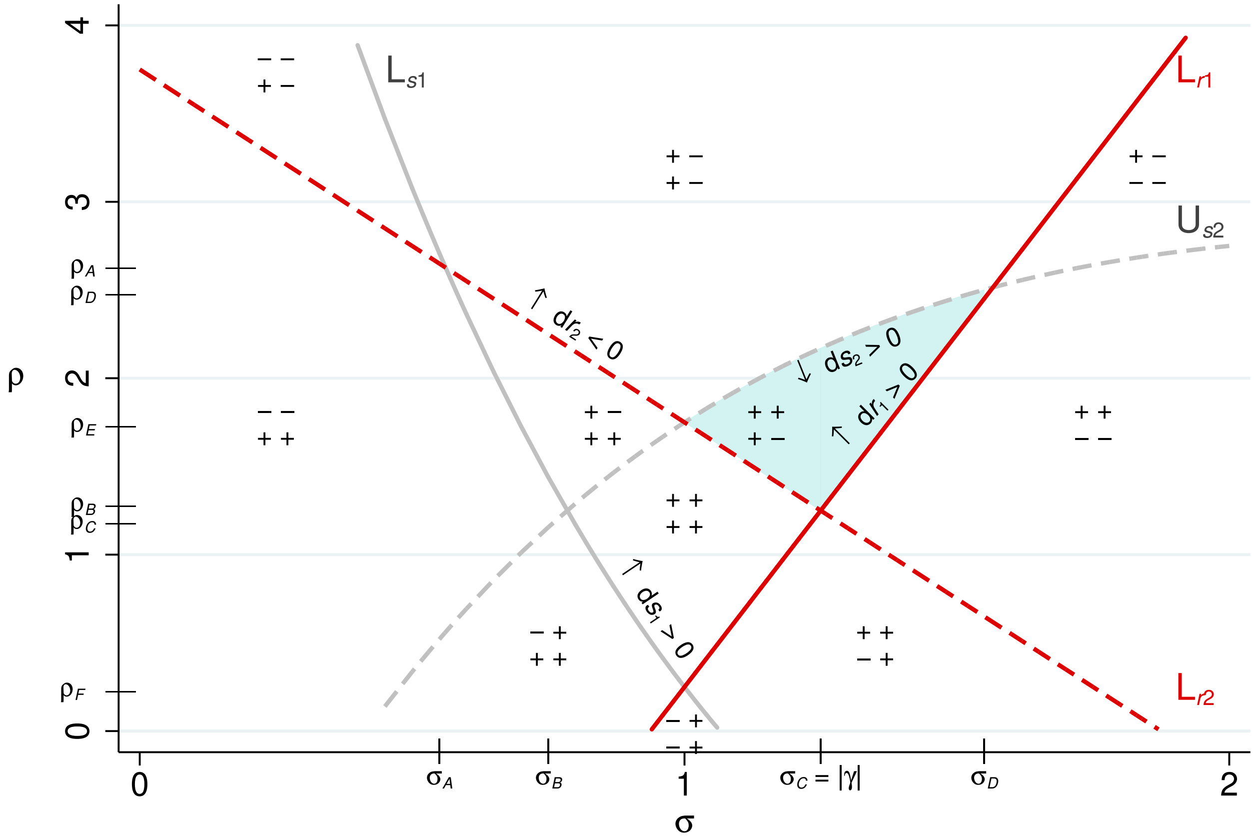
$\alpha_r = 1$, then U_{s_2} and L_{r_1} intersect as seen in Figure C1. ■

Corollary 1. The donation response pattern we observed implies $\rho > 1$: that the time period 1 and 2 warm glow aggregate characteristics g_1 and g_2 are intertemporal substitutes. This follows immediately from L_{r_2} and L_{r_1}, from (55′) and (57′), intersecting at $\rho_C = |\gamma| > 1$ as indicated by (61). Hence, a donation response pattern that has $r_1 \uparrow$ and $r_2 \downarrow$ must have $\rho > 1$. ■

Corollary 2. In the case where the underlying warm glow characteristics ω_s are ω_r are substitutes, but the overall lift obscures this in a static analysis ($\sigma < |\gamma|$; see eqn. (23)), for a two-period analysis to resolve the identification problem, it is necessary for $\rho > |\gamma|$. However, if ρ is too large, the identification problem re-emerges. The $\rho > |\gamma|$ necessary condition follows immediately from the intersection of L_{r_2} and L_{r_1} at $\sigma_C = |\gamma|$ (eqns. (55′) and (57′)). The identification problem re-emerges when ρ becomes large enough to cross the U_{s_2} boundary, eqn. (58′). ■

Remarks. The qualitative difference that arises for the case where $\alpha_s > \alpha_r$ is that U_{s_2} and L_{r_1} will intersect. The proof follows along lines similar to what we have already shown. Figure C3 provides an example for $\alpha_s = 2$ and $\alpha_r = 1$.

Figure C3. Substitution between warm glow characteristics, $\alpha_s = 2$, $\alpha_r = 1$.



The most important substantive implication of *Proposition 2* is that depending on the (unknown) preference parameters σ and ρ, donation evidence from a single time period—such as $s_1 \uparrow r_1 \uparrow$—can have weak power to detect underlying substitution/complementarity in warm glow characteristics, weak compared to evidence from multiple time periods. This is because, as discussed earlier, there are two sides to the time-shifting coin: time-shifting makes adjudicating substitution/complementarity more difficult at $t = 1$ because it adds to overall lift, but time-shifting aids identification at $t = 2$ because it subtracts from overall lift. In practice whether evidence from multiple time periods serves to actually resolve otherwise ambiguous identification depends on the application: specifically, how strong substitutes (or complements) the warm glow characteristics are in the first place. In our application, if we had stopped collecting data at the end of the weeks immediately after the launch of the fundraising appeal, we would have observed $s_1 \uparrow r_1 \uparrow$ and would have been unable to adjudicate whether ω_s and ω_r were substitutes or complements. However because we collected data until the donation patterns had returned to baseline, we were able to observe $s_2 \uparrow r_2 \downarrow$, and that is sufficient evidence to conclude that ω_s and ω_r are substitutes.[11]

Compared to our application, an application in which substitution in charity-space is stronger—or in which substitution in time is (paradoxically) weaker—may be able to detect the substitution relationship with a single time period of data. Pick a point a point in the shaded set in Figure C2 and along the vertical $\sigma = \sigma_C = |\gamma|$ line. To consider stronger substitution in charity-space, move to the east. Once you cross the L_{r_1} substitution is strong enough to be revealed in the donation pattern at $t = 1$ despite the time-shifting from $t = 2 \rightarrow t = 1$ (and despite the overall lift from γ). Furthermore, the substitution relationship would also be revealed in the $t = 2$ donation pattern. To consider weaker substitution in time, go back to the previous starting point in the shaded set and move south. Once you cross the $\mathrm{L}_{r_1}, \mathrm{L}_{r_2}$ intersection point the time-shifting from $t = 2 \rightarrow t = 1$ is no longer strong enough (in combination with the overall lift from γ) to prevent the underlying substitution relationship from being revealed in the donation pattern at $t = 1$).

The two qualitative donation response patterns sufficient to conclude that ω_s and ω_r are complements lie, in their entirety, to the left of the vertical line at $\sigma = 1$. These sets have in common that $\underline{s}$ decreases in either the first or second time periods. The intuition about how charity-space shift and shifting in the time dimension interact so that these donation patterns can be considered sufficient evidence of complementarity in preferences is based on the magnitude of ρ. If ρ is large but not too large—so that we are in the set of $(\sigma,\ \rho)$ pairs to the north of U_{s_2} and south of L_{r_2}—time-shifting from $t = 2$ to $t = 1$ is strong enough to reveal complementarity via $s_2 \downarrow$ and $r_2 \uparrow$.

[11]Interestingly, $r_2 \downarrow$ on its own is <u>not</u> sufficient evidence to conclude that ω_s and ω_r are substitutes. If ρ is very large, it is possible that time-shifting to $t = 1$ be so large so as to cause $r_2 \downarrow$ even though ω_s and ω_r are complements, as in the large set in the north-center area of Figure C2 in which $s_1 \uparrow$, $r_1 \uparrow$, $s_2 \downarrow$, $r_2 \downarrow$. In contrast, $r_1 \downarrow$ on its own would be sufficient evidence of substitution (under the maintained assumption that we can set aside the unlikely case where ρ is close to zero—i.e., the case where time period 1 and 2 warm glow aggregate characteristics g_1 and g_2 are close to being perfect intertemporal complements—which would imply that charity S's fundraising appeal has a perverse immediate negative effect on its own donations).

It is this possibility that holds out some hope for identifying an ω_s-ω_r complement relationship using qualitative donation response patterns. Recall that in the static model successful identification of complementarity required a perverse outcome that is unlikely to occur: in the immediate aftermath of a fundraising appeal own donations fall. And recall that this is because overall lift in the immediate aftermath of the fundraising appeal works to obscure the underlying substitution-complementarity relationship. It may be more likely, however, for such complementarity to be revealed at $t = 2$, because time-shifting to $t = 1$ from $t = 2$ offsets, rather than reinforces, the overall lift at $t = 2$.

However, if time-shifting to $t = 1$ from $t = 2$ is too strong, then it no longer serves to assist with identification. Specifically, when ρ is to the north of L_{r_2}, time-shifting is too strong and both s_2 and $r_2 \downarrow$. This qualitative pattern is not only consistent with complementarity, but is also consistent with some substitutability between ω_s and ω_r.

Conversely, if time-shifting is weaker then donations increase to both organizations in both time periods s_1, r_1, s_2 and $r_2 \uparrow$; the middle set around the horizontal line at $\rho = 1$) because of the overall lift from γ. If time-shifting is yet weaker ($\rho << 1$; strong complementarity across time), for any ω_s- ω_r complementarity to be revealed, $s_1 \downarrow$ is required. As discussed in the static model and just above, a situation like this is likely not to be practically relevant because it implies that charity S's fundraising appeal has a perverse immediate negative effect on its own donations.

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